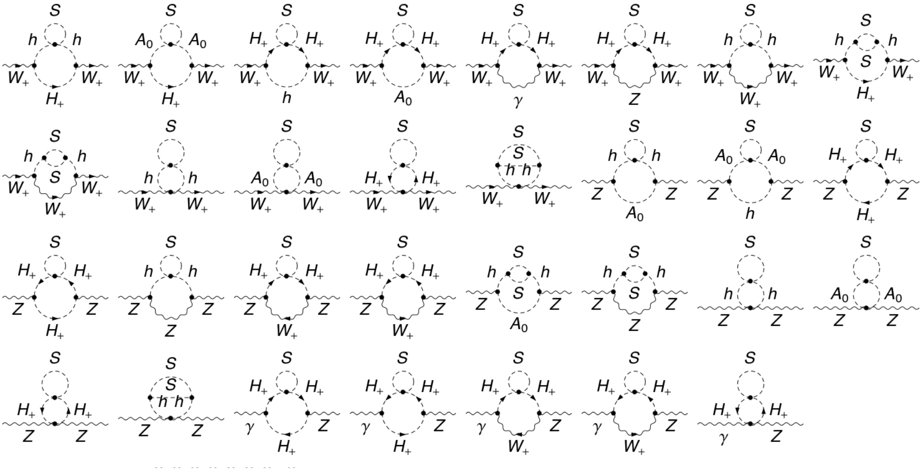
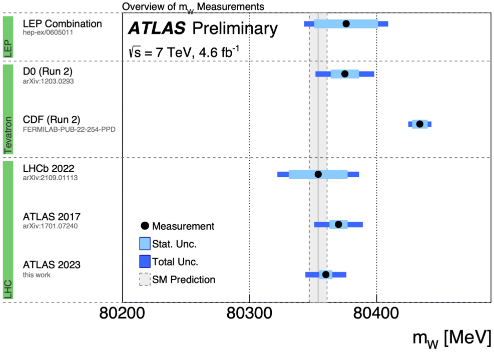
Precise W-boson mass predictions in the $O(N)$-singlet extension



The W boson mass...

- two most-recent most-precise individual measurements
- huge discrepancy of about $\propto 7\sigma$
- also w/o CDF: small upwards tendency



[ATLAS-CONF-2023-004]

BSM perspective:
despite what the solution for the $M_W^{\text{exp.}}$-miracle might be, it will most-likely result in a very precise number $\rightarrow$ precision predictions for concrete BSM scenarios also required!

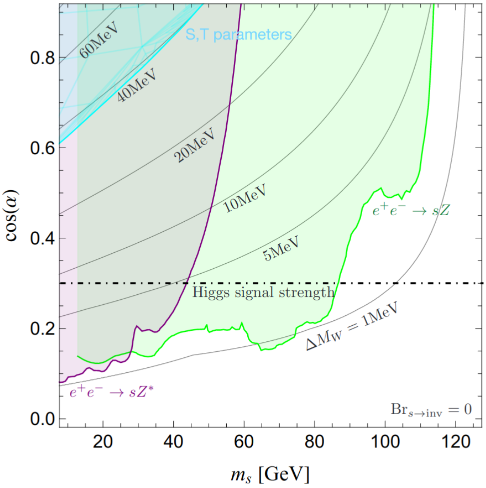
M_W in singlet extensions

Previously:

- no direct singlet coupling to W/Z
- mixing with SM doublet: $\cos\alpha$
- alters/generates WWh/WWs and ZZh/ZZs couplings
  $\rightarrow$ contributes to ΔM_W
- However: fast decoupling and strong constraints on α

Idea:

- Higgs doublet serves as "portal" to BSM
- maybe Higgs teleports BSM effects onto M_W beyond one-loop?
  $\rightarrow$ use simple singlet extension to test this



[Sakurai, Takahashi, Yin '22]

Outline

- introducing M_W calculation (in the SM)
- the $O(N)$ singlet extension (SSM_N)
- M_W results in the SSM_N

Getting started: how to "predict" M_W?

- in most models (incl. SM): M_W cannot be "predicted"
- actual prediction: relation between M_W and other theory parameters/observables
- e.g. in the SM the tree-level relation [Ross, Veltman '75]

$$\rho = \frac{G_{CC}}{G_{NC}} = \frac{M_W^2}{c_w^2 M_Z^2} = 1\,, \quad (c_w = \cos\theta_w,\ s_w = \sin\theta_w)$$

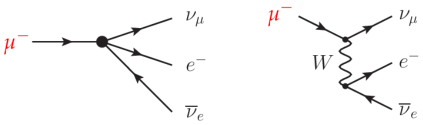
is perturbed by higher-order terms.

Strategy: use/predict relations that involve theoretically and experimentally well-defined observables (e.g. OS pole-masses) that have smallest uncertainties. (physical meaning of $\cos\theta_w$? $\rightarrow$ use $\rho^{(0)} = 1$ (if possible) to eliminate presence of $\cos\theta_w$ in the relation of *some other* set of observables.)

Relation between $M_W \leftrightarrow M_Z$, G_F and α_{QED}

consider muon decay: $G_F/\sqrt{2}\,(\bar{e}\gamma_\rho\nu_e)\,(\bar{\mu}\gamma^\rho\nu_\mu)$

$$G_F = \frac{\pi\alpha_{\text{QED}}}{\sqrt{2}M_W^2\left(1 - M_W^2/M_Z^2\right)}(1+\Delta r)$$



[fig.: Illana, Cano]

- G_F: from muon life time
- $\alpha_{\text{QED}}(0)$: fine-structure constant (including $\Delta\alpha$ resummation)
- M_W and M_Z: pole-masses
- Δr: contains higher-order corrections $\Delta r \equiv \Delta r(M_W, M_Z, \alpha, \ldots)$

Solve (iteratively) for M_W:

$$M_W = M_Z\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha_{\text{QED}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r)}\right)$$

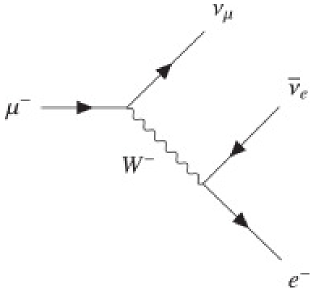
Instead of *predicting* M_W we are asking "which (OS) value of M_W do we need to get the muon decay right?"

Higher-order corrections to the muon decay (sketchy)

$$\Delta r^{(1)} = 2\frac{\delta^{(1)} Z_e}{e} + \frac{\Sigma_W^{(1),T}(0) - \delta^{(1)} M_W^2}{M_W^2} - \frac{\delta^{(1)} s_w^2}{s_w^2} + \delta_{\text{vertex+box}}$$
$$\approx \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\text{remainder}}$$

ingredients:

- $\delta^{(1)} Z_e$: photon and photon-Z selfenergies
- $\delta^{(1)} M_{W/Z}^2$, $\Sigma_{W/Z}^{(1),T}(0)$, $\delta^{(1)} s_W^2$: (transverse) W/Z selfenergies
- $\delta_{\text{vertex+box}}$: vertex/box diagrams



dominant:

- $\Delta\alpha$: light fermion contributions (SM-like)
- $\Delta\rho = \frac{\Sigma_Z^{(1),T}(0)}{M_Z^2} - \frac{\Sigma_W^{(1),T}(0)}{M_W^2}$ ← sensitivity to BSM physics

The $O(N)$ singlet extension (SSM$_N$)

(see also [Drozd, Grzadkowski, Wudka '11])

Higgs doublet H as in the SM extended with $\vec{S} = (S_1, S_2, \ldots, S_N)^T$ singlets:

$$V_{\mathrm{SSM}_N} = \mu^2|H|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{1}{2}\vec{S}\cdot\vec{S}\left(\mu_S^2 + \lambda_{SH}|H|^2 + \lambda_S\vec{S}\cdot\vec{S}\right)$$

- obeys global $O(N)$ symmetry ($\mathbb{Z}_2$ for $N = 1$)
- $\mu_S^2 > 0$ and $\mu^2 < 0$: EWSB as in the SM, $O(N)$ stays *exact*
  $\rightarrow \langle H\rangle = v$ and $\langle\vec{S}\rangle = \vec{0}$
- S_i are degenerate in mass: $m_{S_i} = m_S$ (can be broken by soft terms $\mu_{ij}^2 S_i S_j$)
- S_i *do not* mix with h_{SM}
- viable dark matter candidate ($\Omega_{\mathrm{DM}}^N = N\Omega_{\mathrm{DM}}^{N=1}$) $\rightarrow \vec{S}$ acts like one singlet with N d.o.f.

Input parameters: $\lambda_H = \frac{M_h^2}{v^2}$, $\lambda_{SH} = \frac{2\left(m_S^2 - \mu_S^2\right)}{v^2}$ and λ_S

The $O(N)$ singlet extension (SSM$_N$): pre-M_W constraints

$$V_{\mathrm{SSM}_N} = \mu^2|H|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{1}{2}\vec{S}\cdot\vec{S}\left(\mu_S^2 + \lambda_{SH}|H|^2 + \lambda_S\vec{S}\cdot\vec{S}\right), \qquad \lambda_H = \frac{M_h^2}{v^2}, \quad \lambda_{SH} = \frac{2\left(m_S^2-\mu_S^2\right)}{v^2}, \quad \lambda_S$$

- bounded from below (BFB): $V_{\mathrm{SSM}_N}(H, S \to \infty) \geq 0$
  - $\lambda_H > 0$ ($\checkmark$)
  - $\lambda_{SH} > 0 \to \lambda_S > 0$
  - $\lambda_{SH} < 0 \to \lambda_S > \frac{\lambda_{SH}^2}{4\lambda_H^2} = \frac{\left(m_S^2-\mu_S^2\right)^2}{M_h^2 v^2}$
- perturbative unitarity: $\lambda_i < 8\pi$ ($a_0^{\max} < 0.5$ computed with `SPheno`)
- for $m_S \leq M_h/2$: enhancement of $h \to$ inv. possible
  - compute spectrum+decays with `SARAH`/`SPheno` for $N = 1$
  - use `HiggsBounds`/`HiggsSignals` with $\Gamma^N_{\text{inv.}} = N\cdot\Gamma^{N=1}_{\text{inv.}}$
- no singlet VEVs $\mu_S^2 > 0$ ($\checkmark$)

M_W prediction in the SSM_N

- At tree-level:
  - $\rho_{\mathrm{SSM}_N} = 1 \leftrightarrow M_W^{(1),\,\mathrm{SSM}_N} = M_W^{(1),\,\mathrm{SM}}$
  - all tree-level h-to-SM couplings are *exactly* as in the SM
  - $\vec{S}$ does only couples to H (h), not to $W/Z/\gamma$
- At one-loop:
  - H contributes to M_W at one-loop but not $\vec{S}$ (due to tree-level relations) $\rightarrow M_W^{(1),\,\mathrm{SSM}_N} = M_W^{(1),\,\mathrm{SM}}$
- At two-loop:
  - vertex+box diagrams: same as in the SM (known)
  - self-energies: $\vec{S}$ appears for the time via coupling to H $\rightarrow \Sigma_V^{(2)} = \Sigma_V^{(2),\,\mathrm{SM}} + N \cdot \Sigma_V^{(2),\,\mathrm{SSM}_{N=1}}$

Strategy: only calculate two-loop diagrams involving at least one $\vec{S}$. Take SM corrections from literature (incl. higher-orders).

Renormalisation of $\Delta\rho$ in the SSM$_N$

Sub-loop renormalisation

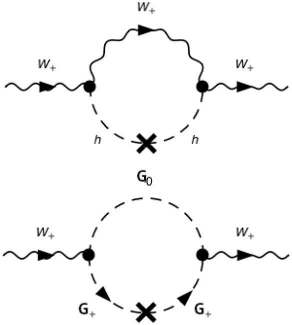
Remember: one-loop singlet contribution vanishes $\to$ only take singlet effects $\mathcal{O}(\lambda_{SH})$ into account.

Adopting an on-shell (OS) scheme the singlet-only contributions to the one-loop counterterms read:

- "OS" tadpoles: $T_h = \delta^{(1)} t_h + t_h^{(1)} = 0$
  $\to \delta^{(1)} t_h = \lambda_{SH} A_0(m_S^2)/32\pi^2$
  $\to$ leads to "OS" Goldstones: $\delta^{(1)} m^2_{G^{\pm/0}} = \delta^{(1)} t_h / v$

- OS Higgs mass: $\delta^{(1)} M_h^2 = \Sigma_{hh}(p^2 = M_h^2)$
  $= \lambda_{SH} \left(\lambda_{SH} v^2 B_0(M_h^2; m_S^2, m_S^2) - A_0(m_S^2)\right)/32\pi^2$

- OS W/Z masses: $\delta^{(1)} M^2_{W/Z} = 0$ (also $\delta^{(1)} e = 0$) since no one-loop diagrams with singlets exist



Results for $\Delta\rho|_{\mathrm{SSM}_N}$

$$\Delta\rho|_{\mathrm{SSM}_N} = \frac{\Sigma_Z^{(2),\,T}(0)}{M_Z^2} - \frac{\Sigma_W^{(2),\,T}(0)}{M_W^2}$$

$= $ [illegible]

$$\approx N \frac{\lambda_{SH}^2}{(4\pi)^4} \times \mathcal{O}\left(\frac{M_{W/Z/h}^2}{m_S^2}\right)$$

Consistency checks:

- UV-finite
- no $\log\mu^2$-dependence
- $\Delta\rho|_{\mathrm{SSM}_N} \xrightarrow{M_Z \to M_W} 0$
- $\Delta\rho|_{\mathrm{SSM}_N} \xrightarrow{m_S \to \infty} 0$

Combination with known higher-order SM-results

Reminder: $$M_W = M_Z\left(\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha_{\text{QED}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r(M_W, M_Z, M_t, M_h, m_S, \mu_S))}\right)$$

- $\Delta r = \Delta r|_{\text{SM}} + \Delta r|_{\text{SSM}_N} \approx \Delta r|_{\text{SM}} - \frac{c_w^2}{s_w^2}\,\Delta\rho|_{\text{SSM}_N}$
- $\Delta r|_{\text{SM}}$ full one&two-loop and partially three&four loop available [Awramik, Chen, Chetyrkin, Czakon, Degrassi, Djouad, Fleischer, Freitas, Giardino, Gambino, Hollik, Jegerlehner, Kühn, Kniehl, Sirlin, Steinhauser, Tarasov, Weiglein, ...]
- recently improved/collected/implemented in [Bagnaschi, Chakraborti, Heinemeyer, Saha, Weiglein '22]
- also fit formulas available [Awramik, Czakon, Freitas, Weiglein '21]

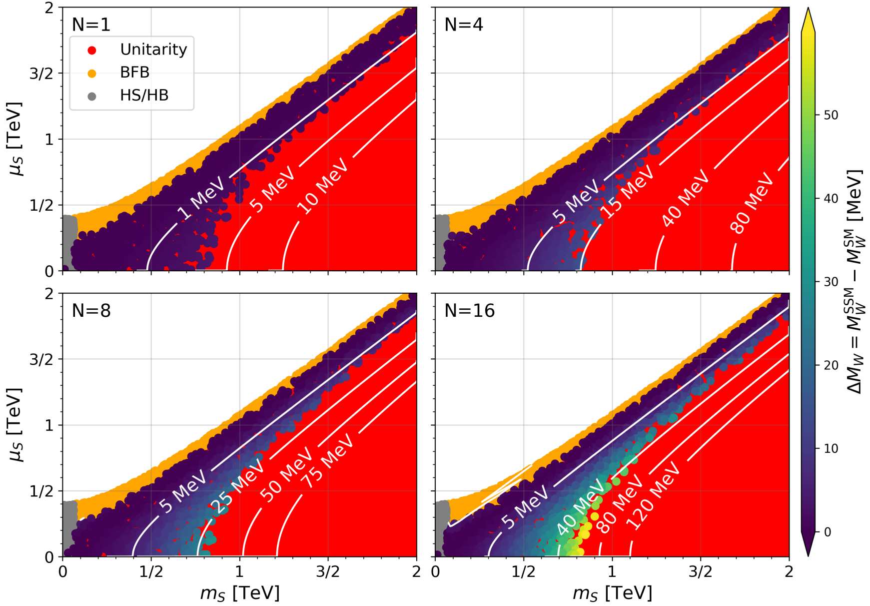
$$M_W^{\text{SM}} = 80.3526\,\text{GeV}$$

$$\Delta M_W \equiv M_W^{\text{SSM}_N} - M_W^{\text{SM}}$$

Phenomenology

ΔM_W contours

- $\Delta M_W \overset{N=1}{\lesssim} 4\,\mathrm{MeV}$
- $\Delta M_W \overset{N\leq4}{\lesssim} 15\,\mathrm{MeV}$
- $\Delta M_W \overset{N\leq8}{\lesssim} 25\,\mathrm{MeV}$
- $\Delta M_W \overset{N\leq16}{\lesssim} 75\,\mathrm{MeV}$



$M_W^{\text{exp.}}$ Limits

$$M_W^{\text{CDF}} = 80433 \pm 9.4\,\text{MeV}$$

$$M_W^{\text{ATLAS}} = 80360 \pm 16\,\text{MeV}$$

$$M_W^{\text{WA}} = 80377 \pm 12\,\text{MeV}$$

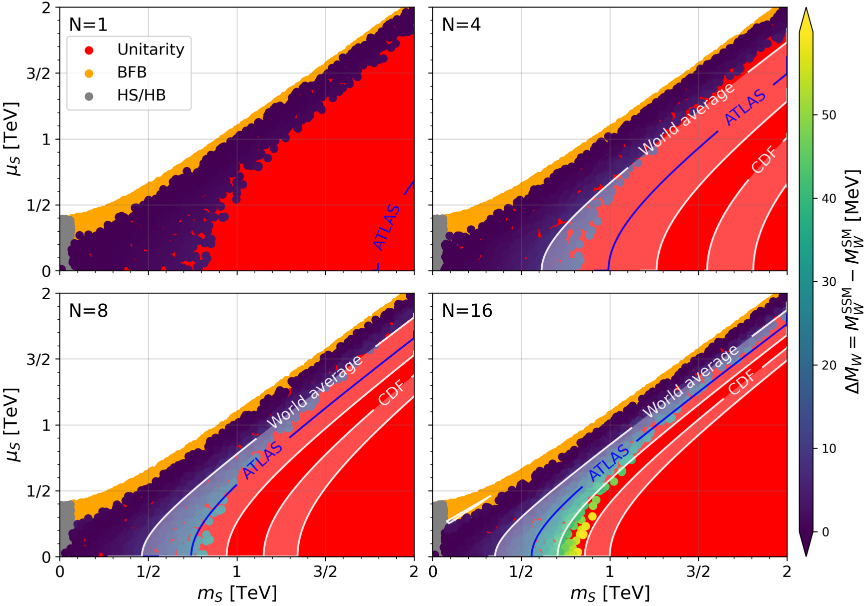
$$M_W^{\text{SM}} = 80353 \pm 5\,\text{MeV}$$

Conservative approach:

$$\Delta M_W \overset{!}{\subset} M_W^{\text{exp.}} - M_W^{\text{SM}} \pm \delta^{\text{exp.}} \pm \delta^{\text{SM}}$$

Interpretation strongly depends on experiment!

(but also theory unc. contributes here)



Summary

- W boson mass M_W as precision observable sensitive to new physics
- 7σ discrepancy between CDF experiment and SM (but also between CDF and every other experiment...)
- studied two-loop corrections to $\Delta\rho$ in the $O(N)$ singlet extension
- corrections $\lesssim 5\,\mathrm{MeV}$ for small N (i.e. smaller than SM-theory uncertainty)
- large N:
  - larger corrections possible
  - can "release" tension between SM and experiment
  - not possible to "overshoot" CDF but can be close

Backup

RGEs of the SSM$_N$

One loop results from [Drozd, Grzadkowski, Wudka '11]:

$$\begin{aligned}
\beta^{(1)}_{\lambda_H} &= 24\lambda_H + 2N\lambda_{SH}^2 - 6Y_t^2(1-2\lambda_H)^2 + \mathcal{O}(g_1^2, g_2^2)\\
\beta^{(1)}_{\lambda_S} &= \lambda_{SH}^2 + 4(8+N)\lambda_S^2\\
\beta^{(1)}_{\lambda_{SH}} &= \lambda_{SH}\left(6\lambda_H + 12\lambda_S + 4\lambda_{SH} + 6Y_t^2 + \mathcal{O}(g_1^2, g_2^2)\right)
\end{aligned}$$

$\rightarrow$ large λ_{SH},λ_S and/or large N may induce too large λ_H and lead to a relatively low UV cut-off (landau pole near the electroweak scale)

Beyond $\Delta\rho$

$$\Delta r^{(2)} = 2\frac{\delta^{(2)} Z_e}{e} + \frac{\Sigma_W^{(2),T}(0) - \delta^{(2)} M_W^2}{M_W^2} - \frac{\delta^{(2)} s_w^2}{s_w^2} + \delta^{(2)}_{\text{vertex+box}}$$

$$\approx \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\text{remainder}}$$

Estimate two-loop contributions from $\Delta r_{\text{remainder}}$:

- all one-loop-squared pieces vanish ✓
- $\delta^{(2)} Z_e = 0$ (due to "OS" tadpoles/Goldstones), already checked ✓
- vertex+boxes: independent of M_h at one-loop
  $\rightarrow \vec{S}$ contributes only via Goldstone-propagator corrections which should also cancel ✓
- need $\Sigma^{(2),\mathrm{T}}_{W/Z}(p^2 \neq 0)$ for proper OS definition
  $\rightarrow$ **WIP** (requires e.g. `TSIL` [Martin, Robertson '05], but could not yet obtain UV-finite result)

Two-Loop Diagrammatic n-Point Functions

Idea: calculate "generic" diagrams
Assume most general Lorentz-invariant couplings and arbitrary masses.
Calculate to "robust form" and perform specific field-insertions later-on.

Strategy:

- FeynArts: generate generic diagrams *("InsertionLevel→{Generic}")* [Hahn, '01]
- FeynCalc: basic simplifications, Dirac traces [Shtabovenko, '16]
- TARCER: reduction to scalar master integrals [Tarasov, '97] [Mertig, Scharf, '98]
- handle special cases such as vanishing Gram determinants etc.

Then:

- SSM FeynArts model file with SARAH [Staub, '08]:
  calculates LO-vertices and NLO-CT-vertices
- generate arbitrary set of diagrams with FeynArts *("InsertionLevel→{Classes or Particles}")*
- iterate over generic amplitudes while applying insertion rules