Precision predictions for the Higgs boson mass in the NMSSM

matching the NMSSM to the SM

Christoph Borschensky, Thi Nhung Dao,
Martin Gabelmann, Margarete Mühlleitner,
Heidi Rzehak

THDM-Seminar, 2024-04-11

Outline

1) SUSY: motivation
   SUSY formalism
   The lightest Higgs Boson Mass in the (N)MSSM

2) Higher-Order Corrections to m_h^2

3) Intermediate Summary

4) Implementation in NMSSMCALC
   Matching procedures
   Calculation of $m_h \pm \sigma_{m_h}$
   Numerical results

What's special about the Higgs boson (mass)?

What's special about the Higgs boson (mass)?

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

What's special about the Higgs boson (mass)?

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

> Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

protected by gauge symmetries, $m_V \to 0$

What's special about the Higgs boson (mass)?

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by chiral symmetry, $m_f \to 0$

What's special about the Higgs boson (mass)?

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

What's special about the Higgs boson (mass)?

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  which symmetry protects m_h?

  $\to$ SUSY

What's special about the Higgs boson (mass)?

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  which symmetry protects m_h?

  $\to$ SUSY

Imposing SUSY: Higgs boson mass corrections are controlled by the SUSY-breaking scale: $\delta m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0$.

What's special about the Higgs boson (mass)?

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  which symmetry protects m_h?

  $\to$ SUSY

Imposing SUSY: Higgs boson mass corrections are controlled by the SUSY-breaking scale: $\delta m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0$.

Other motivational buzzwords: Gauge coupling unification, dark matter, non-renormalisation theorems, radiative EWSB, Coleman-Mandula loop-hole, strings, ...

1) SUSY: motivation
SUSY formalism
The lightest Higgs Boson Mass in the (N)MSSM

2) Higher-Order Corrections to m_h^2

3) Intermediate Summary

4) Implementation in NMSSMCALC
Matching procedures
Calculation of $m_h \pm \sigma_{m_h}$
Numerical results

Example: SUSY Toy-Model

see SUSY Primer by S.P. Martin

Consider the field content:

- gauge field G_μ^a
- scalar Φ

Needed SUSY partners:

- must be fermions: $n_F = n_B$
- Weyl fermions: $\tilde{G}^a$, $\tilde{\Phi}$
- $n_F^{Weyl} = 2$
- $\rightarrow \Phi$ must be complex

Gauge Superfield $\hat{G} = (G_\mu^a, \tilde{G}^a)$ and chiral superfield $\hat{\Phi} = (\Phi, \tilde{\Phi})$ share the same gauge/quantum numbers (respectively).

SUSY goes off-shell

- Higher-order corrections will involve off-shell states.
- $\overline{\sigma}^\mu \partial_\mu \tilde{\Phi} = 0 \rightarrow$ the SUSY algebra does not close off-shell!
- Idea: introduce auxiliary fields $\mathcal{L}_{aux} \supset FF^* + D^a D^a$ to match d.o.f.

	Φ	$\tilde{\Phi}$	F	G^a_μ	$\tilde{G}^a$	D^a
on-shell	2	2	0	2	2	0
off-shell	2	4	2	3	4	1

D-Terms

$$\mathcal{L}_{\text{gauge}} \supset g(\Phi^* T^a \Phi) D_a + \frac{1}{2} D^a D_a$$

$$\text{EOM} \rightarrow D^a = -g(\Phi^* T^a \Phi)$$

SUSY connects scalar with gauge sector

F-Terms

$$\mathcal{L}_{\text{chiral}} \supset -\frac{1}{2}\frac{\delta^2 W}{\delta \Phi^2}\tilde{\Phi}\tilde{\Phi} + \frac{\delta W}{\delta \Phi} F + F^* F$$

$$\text{EOM} \rightarrow F = -\frac{\delta W^*}{\delta \Phi}$$

SUSY connects Yukawa with scalar sector

The N/MSSM Scalar Potential

Encoded in the superpotential $W \equiv W(\Phi)$, $\dim W = 3$:

$$W_{\mathrm{MSSM}} = \mu \hat{H}_u \hat{H}_d + Y_L \hat{L} \hat{H}_u \hat{e} + Y_u \hat{Q} \hat{H}_u \hat{u} + Y_d \hat{U} \hat{H}_d \hat{d}$$
$$W_{\mathrm{NMSSM}} = W_{\mathrm{MSSM}}\big|_{\mu = \lambda \hat{S}} + \kappa \hat{S}^3$$

and constructed by:

$$V = \sum_{\Phi = H_{u,d}, S, L, Q, u, d} \frac{\delta W}{\delta \Phi} \left(\frac{\delta W}{\delta \Phi} \right)^* + \frac{1}{2}(D_1 D_1 + D_2^i D_{2,i} + D_3^a D_{3,a}) + \underbrace{V_{\mathrm{soft}}}_{\text{soft-SUSY-breaking terms preserving all other symmetries}}$$

with:

$$D_1 = \frac{g_1}{2}(|H_u|^2 - |H_d|^2) + \ldots$$
$$D_2^i = g_2(H_d^* \sigma^i H_d + H_u^* \sigma^i H_u) + \ldots$$
$$D_3^a = -g_3(|\tilde{t}_L|^2 - |\tilde{t}_R|^2) T^a + \ldots$$

$$\Rightarrow \text{E.g.}: V(H_u^0, H_d^0) \sim \frac{1}{8}(g_1^2 + g_2^2)(|H_u^0|^2 - |H_d^0|^2)^2 + \mu^2(|H_u^0|^2 - |H_d^0|^2) + m_{H_u}^2 |H_u^0|^2 + m_{H_d}^2 |H_d^0|^2 + \ldots$$

SPLIT SUPERSYMMETRY (2003)

- no SUSY at the weak scale
  - LEP: $m_h > 115$ GeV
  - increasing constraints on colored particles
- maybe no naturalness in the Higgs sector?

- Idea:
  - low-energy Lagrangian contains fermions only
  - scalars live at (large) SUSY breaking scale

SPLIT SUPERSYMMETRY

- Advantages:
  - almost all motivations are safe
  - dark matter candidate
  - gauge couplings unification
  - rich phenomenology

- Disadvantages (matter of taste):
  - fine tuned Higgs sector h_{125}

SUPERSPLIT SUPERSYMMETRY (2005)

Supersplit Supersymmetry

Patrick J. Fox,[1] David E. Kaplan,[2] Emanuel Katz,[3,4] Erich Poppitz,[5]
Veronica Sanz,[6] Martin Schmaltz,[4] Matthew D. Schwartz,[7] and Neal Weiner[8]

[1]Santa Cruz Institute for Particle Physics, Santa Cruz, CA, 95064
[2]Dept. of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218
[3]Stanford Linear Accelerator Center, 2575 Sand Hill Rd. Menlo Park, CA 94309
[4]Dept. of Physics, Boston University, Boston, MA 02215
[5]Department of Physics, University of Toronto, 60 St George St, Toronto, ON M5S 1A7, Canada
[6]Universitat de Granada, Campus de Fuentenueva, Granada, Spain
[7]University of California, Dept. of Physics, Berkeley, CA 94720-7300
[8]Center for Cosmology and Particle Physics, Dept. of Physics, New York University, New York , NY 10003

(Dated: April 1, 2005)

The possible existence of an exponentially large number of vacua in string theory behooves one to consider possibilities beyond our traditional notions of naturalness. Such an approach to electroweak physics was recently used in "Split Supersymmetry", a model which shares some successes and cures some ills of traditional weak-scale supersymmetry by raising the masses of scalar superpartners significantly above a TeV. Here we suggest an extension - we raise, in addition to the scalars, the

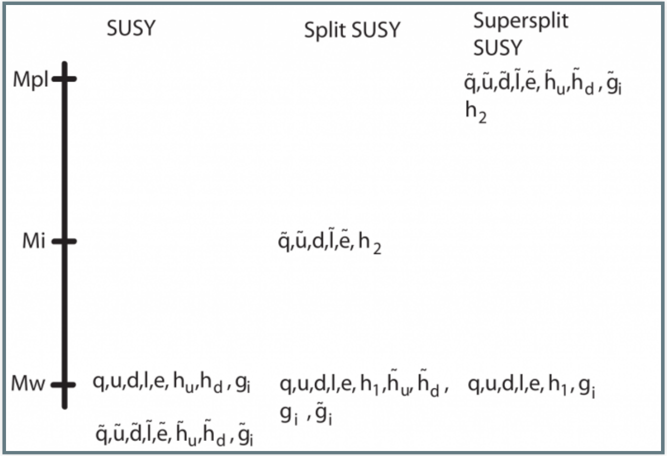
SUPERSPLIT LAGRANGIAN

The Lagrangian for this model is simply

$$\mathcal{L} = -\frac{1}{4g_1^2}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4g_2^2}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4g_s^2}G_{\mu\nu}G^{\mu\nu} + \bar{\psi}_f(i\gamma^\mu D_\mu - m_f)\psi_f + D_\mu h D^\mu h^* - V(h) \quad (1)$$

where f indexes the various fermions, and D is the appropriate covariant derivative.

SUPERSPLIT SPECTRUM



[Fox et al.]

SUPERSPLIT SUPERSERIOUS (2008-2018)

- **high scale SUSY** is still predictive
- SUSY relations still useful
  - match SM to SUSY model
  - D-Terms: $\lambda^{SM}(M_{SUSY}) \propto m_Z^2$
- EFT perspective:
  - resummation of large logs
  - precise Higgs mass predictions

1) SUSY: motivation

SUSY formalism

The lightest Higgs Boson Mass in the (N)MSSM

2) Higher-Order Corrections to m_h^2

3) Intermediate Summary

4) Implementation in NMSSMCALC

Matching procedures

Calculation of $m_h \pm \sigma_{m_h}$

Numerical results

The SM-like Neutral Higgs Boson Mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

> MSSM: $m_h^{\text{tree}} \leq m_Z \ll 125\,\text{GeV}$ ↯

The SM-like Neutral Higgs Boson Mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{NMSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z \ll 125\,\text{GeV}$ ↯
- NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

The SM-like Neutral Higgs Boson Mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{NMSSM}}$$

→ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z \ll 125\,\text{GeV}$ ↯
- NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

→ In either case: Higher-order corrections must shift m_h to the measured Higgs mass:

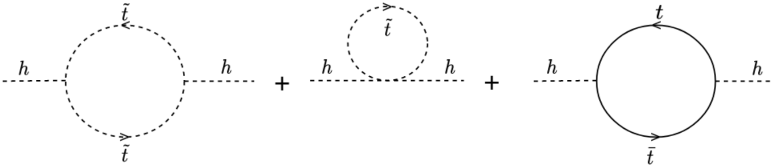
The SM-like Neutral Higgs Boson Mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{NMSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z \ll 125\,\text{GeV}$ ϟ
- NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass:



$$\approx \frac{1}{(4\pi)^2}\frac{m_t^4}{v^2}\log\frac{m_t^2}{M_{\tilde{t}}^2}$$

The SM-like Neutral Higgs Boson Mass in the NMSSM

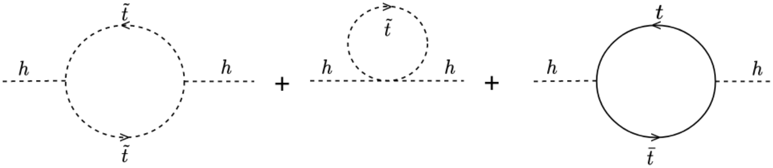
$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2\cos^2 2\beta}_{\text{MSSM}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{NMSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

> MSSM: $m_h^{\text{tree}} \leq m_Z \ll 125\,\text{GeV}$ ↯

> NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass:



$$\approx \frac{1}{(4\pi)^2}\frac{m_t^4}{v^2}\log\frac{m_t^2}{M_{\tilde{t}}^2}$$

> $M_{\tilde{t}} = m_t + m_{\text{SUSY}} \Rightarrow$ in the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{M_{\text{SUSY}}\to 0} 0$

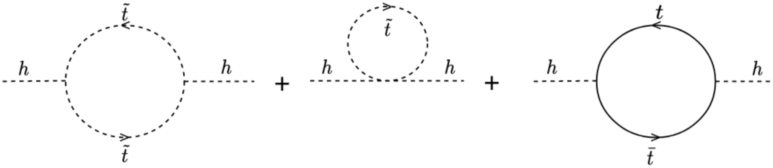
The SM-like Neutral Higgs Boson Mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{NMSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z \ll 125\,\text{GeV}$ ↯
- NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass:



$$\approx \frac{1}{(4\pi)^2}\frac{m_t^4}{v^2}\log\frac{m_t^2}{M_{\tilde{t}}^2}$$

- $M_{\tilde{t}} = m_t + m_{\text{SUSY}} \Rightarrow$ in the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{M_{\text{SUSY}}\rightarrow 0} 0$
- but we need $\delta m_h^2 \approx \mathcal{O}(20 - 40\,\text{GeV})$! $\rightarrow$ higher-orders required

Determination of m_h^{SUSY}

$$\det\left[p^2\mathbb{1} - \boldsymbol{m}_h^{2,\,(tree)} + \underbrace{\hat{\Sigma}_h(p^2)}_{\substack{\text{no OS-renormalisation} \\ \text{possible (bc SUSY)!}}}\right] = 0$$

Perturbative series of $\hat{\Sigma}_h$ can be organized in:

> number of loops: $(4\pi)^{-2}$
> powers of coefficients: $\mathcal{O}(\alpha_s\alpha_t)$ etc.
> powers of logs: $\log\frac{m^2_{\text{SUSY}}}{m^2_{\text{SM}}}$
> suppression by heavy scales: $\frac{m^2_{\text{SM}}}{m^2_{\text{SUSY}}}$
> (+ combinations)

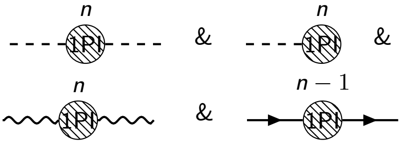
If $M_{\text{SUSY}} \gg m_{\text{SM}}$, the $\log$-expansion at fixed-order might not converge well.

$\rightarrow$ Large-log resummation required for many SUSY scenarios.

m_h^{SUSY}: Fixed-Order Ingredients

A fixed-order n-loop result will incorporate the full logarithmic dependence $(4\pi)^{-2n}\sum_{k=0}^{n}\log^k$ (good and bad ones!) and constant $\frac{m^2_{\text{SM}}}{M^2_{\text{SUSY}}}$-terms:

0	α^0				$\alpha\propto(4\pi)^{-1}$
1	$\alpha\log$	α			
2	$\alpha^2\log^2$	$\alpha^2\log$	α^2		
	$\vdots$	$\vdots$	$\vdots$		
n	$\alpha^n\log^n$	$\alpha^n\log^{n-1}$	$\alpha^n\log^{n-2}$	...	α^n
	LL	NLL	NNLL	...	N^{n}LL



- diagrammatic: calculate $\Sigma_{ij}^{(n)}(p^2)$ and $T_i^{(n)}$
  - computational expensive
  - mass hierarchies: expand loop-integrals
  - numerical evaluation: no access to logs
- effective potential: $\partial^2_{h_ih_j}V_{\mathit{eff}}^{(n)} \leftrightarrow \Sigma^{(n)}_{h_ih_j}(p^2=0)$
  - V_{eff} known up to 2- and 3-loops for general QFT [Martin, Patel, '18] [Martin, '17]
  - $\partial V_{\mathit{eff}}$ numerically difficult
  - $p^2=0\to$ massless particles are troublesome
- other ingredients:
  - (OS) v_{SM}: n-loop vector boson masses
  - (OS) s/top sector: $(n-1)$-loop selfenergies
  - n-loop SUSY RGEs from m_Z to M_{SUSY}

m_h^{SUSY}: EFT ingredients

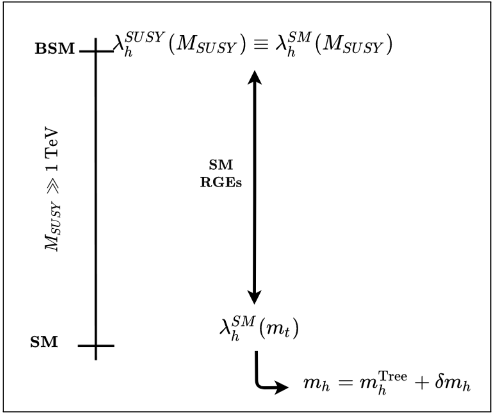
fixed-order ↓

0	α^0				$\alpha \propto (4\pi)^{-1}$
1	$\alpha \log$	α			
2	$\alpha^2 \log^2$	$\alpha^2 \log$	α^2		
	$\vdots$	$\vdots$	$\vdots$		
n	$\alpha^n \log^n$	$\alpha^n \log^{n-1}$	$\alpha^n \log^{n-2}$	...	α^n
	LL	NLL	NNLL	...	N^nLL

EFT →

m_h^{SUSY}: EFT ingredients

Idea: avoid large logs by separation of SUSY/SM contributions.

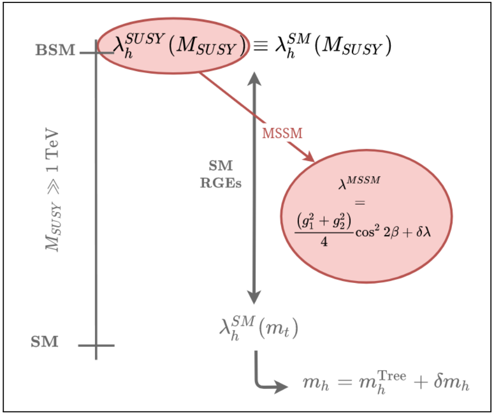


Schematically:

- SUSY: n loop threshold corrections to matching conditions
  - match 4-pt. function in unbroken phase
  - all SM particles massless, scaleless SM-loops $\rightarrow$ no large logs, only $\log \frac{M_{\text{SUSY}}}{Q_{\text{match}}}$
- SM: $n+1$ loop RGE running down to m_t $\rightarrow$ resums large logs $\log \frac{M_{\text{SUSY}}}{m_t}$
- SM: n loop m_h pole-mass calculation $\rightarrow$ no large logs, only $\log \frac{m_{\text{SM}}}{m_t}$

m_h^{SUSY}: EFT ingredients

Idea: avoid large logs by separation of SUSY/SM contributions.

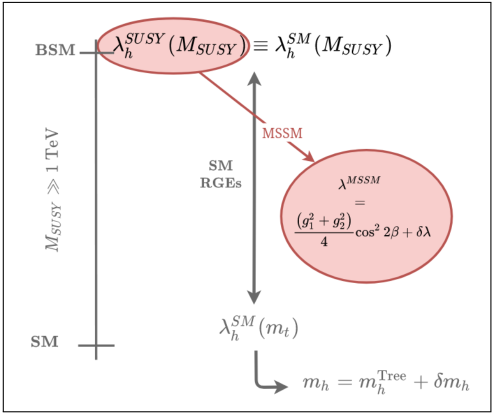


Schematically:

- SUSY: n loop threshold corrections to matching conditions
  - match 4-pt. function in unbroken phase
  - all SM particles massless, scaleless SM-loops $\rightarrow$ no large logs, only $\log \frac{M_{\text{SUSY}}}{Q_{\text{match}}}$
- SM: $n+1$ loop RGE running down to m_t $\rightarrow$ resums large logs $\log \frac{M_{\text{SUSY}}}{m_t}$
- SM: n loop m_h pole-mass calculation $\rightarrow$ no large logs, only $\log \frac{m_{\text{SM}}}{m_t}$

m_h^{SUSY}: EFT ingredients

Idea: avoid large logs by separation of SUSY/SM contributions.



Schematically:

- SUSY: n loop threshold corrections to matching conditions
  - match 4-pt. function in unbroken phase
  - all SM particles massless, scaleless SM-loops $\to$ no large logs, only $\log \frac{M_{\text{SUSY}}}{Q_{\text{match}}}$
- SM: $n+1$ loop RGE running down to m_t $\to$ resums large logs $\log \frac{M_{\text{SUSY}}}{m_t}$
- SM: n loop m_h pole-mass calculation $\to$ no large logs, only $\log \frac{m_{\text{SM}}}{m_t}$

Attention: no p^2- and no $\frac{v^2}{M_{\text{SUSY}}^2}$-dependence, no mixing contributions

Alternative EFT: pole-mass matching

Idea: "recycling" fixed-order results and incorporate v^2/M^2_{SUSY}-terms

- previously: get λ_h via matching of the four-point function: λ_h^{IV} [Gabelmann et al. '18] [Bagnaschi et al. '22] (prev. only assumed CP-conserving NMSSM)
- can also use the two-point function with non-vanishing v^2/M^2_{SUSY}
  - tree-level:
    - $(m_h^{\text{SM}})^2 = 2(v^{\text{SM}})^2\,\lambda_h^{\text{SM},\text{II}} \overset{!}{=} (m_h^{\text{NMSSM}})^2,\ v^{\text{SM}} = v^{\text{NMSSM}}$
    - $\Rightarrow \lambda_h^{\text{SM},\text{II}} = \frac{(m_h^{\text{NMSSM}})^2}{2(v^{\text{NMSSM}})^2}$

Alternative EFT: pole-mass matching

Idea: "recycling" fixed-order results and incorporate v^2/M^2_{SUSY}-terms

- previously: get λ_h via matching of the four-point function: λ_h^{IV} [Gabelmann et al. '18] [Bagnaschi et al. '22] (prev. only assumed CP-conserving NMSSM)
- can also use the two-point function with non-vanishing v^2/M^2_{SUSY}
  - tree-level:
    - $(m_h^{\text{SM}})^2 = 2(v^{\text{SM}})^2\,\lambda_h^{\text{SM,II}} \overset{!}{=} (m_h^{\text{NMSSM}})^2, \;\; v^{\text{SM}} = v^{\text{NMSSM}}$
    - $\Rightarrow \lambda_h^{\text{SM,II}} = \frac{(m_h^{\text{NMSSM}})^2}{2(v^{\text{NMSSM}})^2}$
  - one-loop:
    - $2(v^{\text{SM}})^2\,\lambda_h^{\text{SM,II}} - \hat{\Sigma}_h^{\text{SM}}\big((m_h^{\text{NMSSM}})^2\big) \overset{!}{=} (m_h^{\text{NMSSM}})^2 - \hat{\Sigma}_h^{\text{NMSSM}}\big((m_h^{\text{NMSSM}})^2\big)$
    - $(v^{\text{SM}})^2 = (v^{\text{NMSSM}})^2 + \delta v^2, \;\; \frac{\delta v^2}{v^2} = \Big[\hat{\Sigma}_h^{\text{NMSSM}\prime}(0) - \hat{\Sigma}_h^{\text{SM}\prime}(0)\Big] + \mathcal{O}(v^2/M^2_{\text{SUSY}})$
    - **expand self-energies**: $\hat{\Sigma}_h^{X}\big((m_h^{\text{NMSSM}})^2\big) = \hat{\Sigma}_h^{X}(0) + (m_h^{\text{NMSSM}})^2\,\hat{\Sigma}_h^{X\prime}(0) + \mathcal{O}\big((m_h^{\text{NMSSM}})^4\big)$
    - $\Rightarrow \boldsymbol{\Delta\lambda_h^{\text{SM,1l,II}} = -\frac{1}{2(v^{\text{NMSSM}})^2}\Big[\Delta\hat{\Sigma}_h + 2(m_h^{\text{NMSSM}})^2\,\Delta\hat{\Sigma}_h'\Big]}, \;\; \Delta X \equiv X^{\text{NMSSM}} - X^{\text{SM}}$

Alternative EFT: pole-mass matching

Idea: "recycling" fixed-order results and incorporate v^2/M^2_{SUSY}-terms

- previously: get λ_h via matching of the four-point function: λ_h^{IV} [Gabelmann et al. '18] [Bagnaschi et al. '22] (prev. only assumed CP-conserving NMSSM)
- can also use the two-point function with non-vanishing v^2/M^2_{SUSY}
  - tree-level:
    - $(m_h^{\text{SM}})^2 = 2(v^{\text{SM}})^2\,\lambda_h^{\text{SM,II}} \overset{!}{=} (m_h^{\text{NMSSM}})^2,\ v^{\text{SM}} = v^{\text{NMSSM}}$
    - $\Rightarrow \lambda_h^{\text{SM,II}} = \frac{(m_h^{\text{NMSSM}})^2}{2(v^{\text{NMSSM}})^2}$
  - one-loop:
    - $2(v^{\text{SM}})^2\,\lambda_h^{\text{SM,II}} - \hat{\Sigma}_h^{\text{SM}}\big((m_h^{\text{NMSSM}})^2\big) \overset{!}{=} (m_h^{\text{NMSSM}})^2 - \hat{\Sigma}_h^{\text{NMSSM}}\big((m_h^{\text{NMSSM}})^2\big)$
    - $(v^{\text{SM}})^2 = (v^{\text{NMSSM}})^2 + \delta v^2,\ \frac{\delta v^2}{v^2} = \left[\hat{\Sigma}_h^{\text{NMSSM}\prime}(0) - \hat{\Sigma}_h^{\text{SM}\prime}(0)\right] + \mathcal{O}(v^2/M^2_{\text{SUSY}})$
    - **expand self-energies**: $\hat{\Sigma}_h^{X}\big((m_h^{\text{NMSSM}})^2\big) = \hat{\Sigma}_h^{X}(0) + (m_h^{\text{NMSSM}})^2\,\hat{\Sigma}_h^{X\prime}(0) + \mathcal{O}\big((m_h^{\text{NMSSM}})^4\big)$
    - $\Rightarrow \boldsymbol{\Delta\lambda_h^{\text{SM,1l,II}} = -\frac{1}{2(v^{\text{NMSSM}})^2}\left[\Delta\hat{\Sigma}_h + 2(m_h^{\text{NMSSM}})^2\Delta\hat{\Sigma}_h^{\prime}\right]}$, $\Delta X \equiv X^{\text{NMSSM}} - X^{\text{SM}}$
  - **Attention:**
    - there are large logs in both self-energies, Σ^{NMSSM} and Σ^{SM}!
    - proper cancellation of large logs in ΔX requires expansion of self-energies around $p^2 \approx v^2$ and a parametrisation in terms of input-parameters that **does not** induce higher-orders
    - checks: numerically perform $v \to 0$ limit and compare with $\lambda_h^{\text{SM,1l,IV}}$

Summary: The Three Frontiers in Higgs Mass Calculations

(Apart from increasing loops)

Fixed Order (FO)

- Weak/TeV-scale SUSY
- $m_{SUSY} \lesssim 1 - 2\,\mathrm{TeV}$
- calculate full $\delta m_h^{\mathrm{SUSY}}(m_{\mathrm{SUSY}})$
- full $\frac{m_{\mathrm{SM}}}{M_{\mathrm{SUSY}}}$ dependence

RGE Improved (EFT)

- High/Split-scale SUSY
- $m_{\mathrm{SUSY}} \gtrsim 1 - 2\,\mathrm{TeV}$
- matching, calculate $\delta\lambda_h^{\mathrm{SUSY}}$
- neglects $\frac{m_{\mathrm{SM}}}{M_{\mathrm{SUSY}}}$-terms

Ren. Conditions*

- $\overline{\mathrm{DR}}$: minimal subtraction
- OS: Express result through physical quantities

$$m_h(m_t^{\overline{DR}}) \leftrightarrow m_h(m_t^{OS})$$
$$m_h(m_{H^\pm}^{\overline{DR}}) \leftrightarrow m_h(m_{H^\pm}^{OS})$$

Summary: The Three Frontiers in Higgs Mass Calculations

(Apart from increasing loops)

Fixed Order (FO)

- Weak/TeV-scale SUSY
- $m_{SUSY} \lesssim 1 - 2\,\mathrm{TeV}$
- calculate full $\delta m_h^{\mathrm{SUSY}}(m_{\mathrm{SUSY}})$
- full $\frac{m_{\mathrm{SM}}}{M_{\mathrm{SUSY}}}$ dependence

RGE Improved (EFT)

- High/Split-scale SUSY
- $m_{\mathrm{SUSY}} \gtrsim 1 - 2\,\mathrm{TeV}$
- matching, calculate $\delta\lambda_h^{\mathrm{SUSY}}$
- neglects $\frac{m_{\mathrm{SM}}}{M_{\mathrm{SUSY}}}$-terms

Ren. Conditions*

- $\overline{\mathrm{DR}}$: minimal subtraction
- OS: Express result through physical quantities

$$m_h(m_t^{\overline{DR}}) \leftrightarrow m_h(m_t^{OS})$$
$$m_h(m_{H^\pm}^{\overline{DR}}) \leftrightarrow m_h(m_{H^\pm}^{OS})$$

Also "hybrid approaches" exist: combine FO and EFT results

- using pole mass matching [Athron, Park, Steudtner, Stöckinger, Voigt, '16] [Porod, Staub, '17]:

$$\lambda^{\mathrm{SM,\,1l,II}} = \frac{1}{v^2}\left(\delta^{(1)} m_h^{\mathrm{SUSY}\,2} - \delta^{(1)} m_h^{\mathrm{SM}\,2}\right)$$

- combine ${}^{m^2_{\mathrm{SM}}}/{}_{m^2_{\mathrm{SUSY}}}$ from FO with resummed EFT results [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein, '13] [Bahl, Hollik, '16] [Harlander, Klappert, Voigt, '19]

$$m_h^2 \equiv \left(m_h^{\mathrm{FO}}\right)^2 - \left(m_h^{\mathrm{FO,\ large\text{-}logs}}\right)^2 + \left(m_h^{\mathrm{EFT\text{-}resummed}}\right)^2$$

Summary: The Three Frontiers i

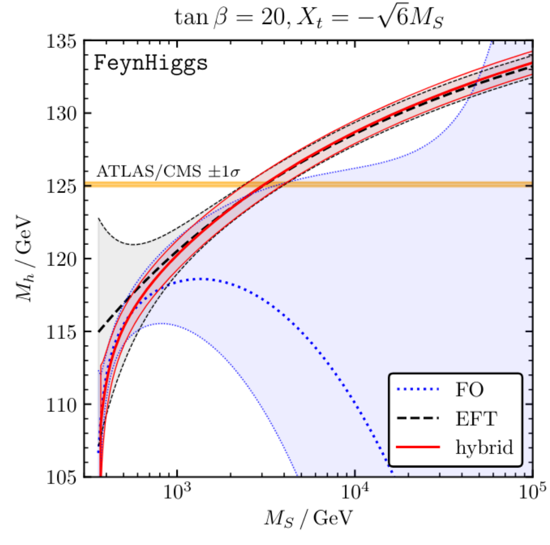
(Apart from increasing loops)

Fixed Order (FO)

> Weak/TeV-scale SUSY
> $m_{SUSY} \lesssim 1 - 2\,\mathrm{TeV}$
> calculate full $\delta m_h^{\mathrm{SUSY}}(m_{\mathrm{SUSY}})$
> full $\frac{m_{\mathrm{SM}}}{M_{\mathrm{SUSY}}}$ dependence

RGE Improved (E

> High/Split-scale S
> $m_{\mathrm{SUSY}} \gtrsim 1 - 2\,\mathrm{Te}$
> matching, calculat
> neglects $\frac{m_{\mathrm{SM}}}{M_{\mathrm{SUSY}}}$-ter



[Slavich et al. '21]

Also "hybrid approaches" exist: combine FO and EFT re

> using pole mass matching [Athron, Park, Steudtner, Stöckinger, Voigt, '

$$\lambda^{\mathrm{SM,\,1l,II}} = \frac{1}{v^2}\left(\delta^{(1)}\right.$$

> combine $m^2_{\mathrm{SM}}/m^2_{\mathrm{SUSY}}$ from FO with resummed EFT results [Hahn, Klappert, Voigt, '19]

$$m_h^2 \equiv \left(m_h^{\mathrm{FO}}\right)^2 - \left(m_h^{\mathrm{FO,\,large}}\right.$$

NMSSMCALC...

...is more than a *spectrum generator* for the CP-violating NMSSM:

- takes parameter point using SLHA and calculates:
- Higgs boson masses ($m_{H_i^0}$ and $m_{H^\pm}$) up to two-loop $\mathcal{O}(\alpha_s\alpha_t + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$
- Higgs boson self-couplings $\lambda^{\text{eff}}_{hhh}$ up to two-loop $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$
- $H_i^{0,\pm} \to X_j X_k$ decays at NLO including SUSY QCD+EW corrections (or using $\lambda^{\text{eff}}_{hhh}$)
- W-boson mass (M_W) and $\Delta\rho$ at two-loops
- electric dipole moments (EDMs) of e and various bound-states
- $(g-2)_e$ and $(g-2)_\mu$
- also for inverse seesaw scenario (NMSSMCALC-nuSS)

example

```
wget https://www.itp.kit.edu/\~maggie/NMSSMCALC/nmssmcalc.tar
tar xf nmssmcalc.tar
cd nmssmcalc-C
make
./run inp.dat
```

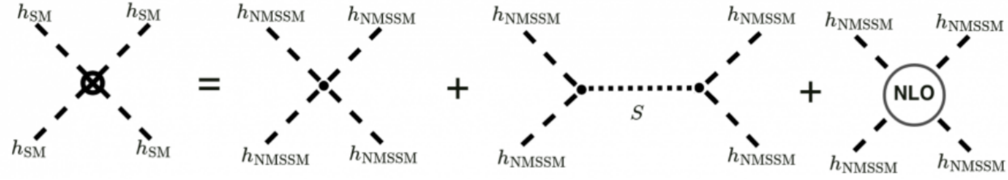
1) SUSY: motivation
SUSY formalism
The lightest Higgs Boson Mass in the (N)MSSM

2) Higher-Order Corrections to m_h^2

3) Intermediate Summary

4) Implementation in NMSSMCALC
Matching procedures
Calculation of $m_h \pm \sigma_{m_h}$
Numerical results

Quartic coupling matching



$$\lambda_h^{\mathrm{II,tree}} = \underbrace{\frac{1}{8}(g_1^2 + g_2^2)\cos^2 2\beta}_{\text{MSSM } D\text{-terms}} + \underbrace{\frac{1}{4}|\lambda|\sin^2 2\beta}_{\text{NMSSM } F\text{-terms}}$$

$$\underbrace{- \frac{1}{48|\kappa|^2 M_s^2 (3M_s^2 + M_{a_s}^2)} \Big(3|\kappa|^2 M_{H^\pm}^2 - 3|\kappa|^2 M_{H^\pm}^2 \cos 4\beta + (3M_s^2 + M_{a_s}^2)\big(|\kappa||\lambda| \cos\varphi_y \sin 2\beta - 2|\lambda|^2\big)\Big)^2}_{s/t/u\text{-channel } s}$$

$$\underbrace{- \frac{3}{16 M_{a_s}^2}|\lambda|^2 (3M_s^2 + M_{a_s}^2)\sin^2 2\beta \sin^2\varphi_y}_{s/t/u\text{-channel } a_s} + \delta^{(1)}\lambda_h^{\mathrm{IV}}$$

- M_s, M_{a_s}: singlet masses
- $M_{H^\pm}$: mass of the heavy doublet
- φ_y: CP-violating phase
- $\delta^{(1)}\lambda_h^{\mathrm{IV}}$: with `SARAH` *
- cross-checks with
  - `FeynArts/FeynCalc`
  - $\lambda_h^{\mathrm{IV}} \overset{!}{=} \lambda_h^{\mathrm{II}}|_{v\to 0}$

Pole-mass matching

- reminder: $\Delta\lambda_h^{\text{SM},1l,\text{II}} = -\frac{1}{2(v^{\text{NMSSM}})^2}\left[\Delta\hat{\Sigma}_h + 2(m_h^{\text{NMSSM}})^2\Delta\hat{\Sigma}_h'\right]$, $\Delta X \equiv X^{\text{NMSSM}} - X^{\text{SM}}$
- all self-energies already contained in `NMSSMCALC`!
- $\rightarrow$ need only SM h self-energy (trivial...)
- (+ minor trivialities regarding $\overline{\text{DR}} \rightarrow \overline{\text{MS}}$ shifts)

extremely convenient and even get v^2/M_{SUSY}^2-corrections for free!

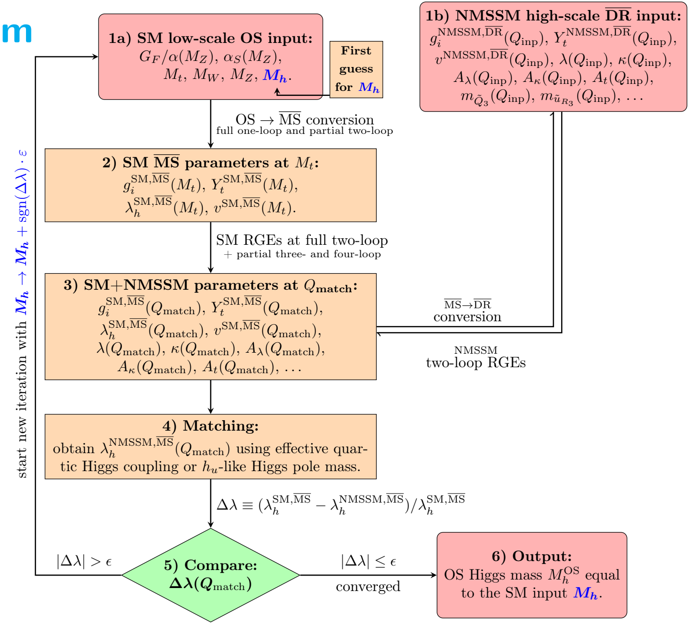
1) SUSY: motivation
SUSY formalism
The lightest Higgs Boson Mass in the (N)MSSM

2) Higher-Order Corrections to m_h^2

3) Intermediate Summary

4) Implementation in NMSSMCALC
Matching procedures
Calculation of $m_h \pm \sigma_{m_h}$
Numerical results

The algorithm



Uncertainty estimate

> SM uncertainty:

- scale uncertainty: $0 \overset{!}{=} p^2 - 2\lambda^2(Q_{\text{EW}})v^2(Q_{\text{EW}}) - \text{Re}\Sigma_h^{\text{SM},\overline{\text{MS}}}(p^2, Q_{\text{EW}})|_{\text{UV-fin}}$
  $\rightarrow \Delta^{\text{SM}}_{Q_{\text{EW}}} = \max\{|M_h^{\text{OS}} - M_h^{\overline{\text{MS}},\text{pole}}(2M_t)|, |M_h^{\text{OS}} - M_h^{\overline{\text{MS}},\text{pole}}(M_t/2)|\}$
- missing gauge corr. $\Delta^{\text{SM}}_{G_F/\alpha_{M_Z}} = |M_h^{G_F} - M_h^{\alpha_{M_Z}}|$
- missing top corr. $\Delta^{\text{SM}}_{Y_t} = M_h(Y_t^{\mathcal{O}(\alpha_s^2)}) - M_h(Y_t^{\mathcal{O}(\alpha_s^3)})$

> SUSY uncertainty:

- scale-uncertainty: $\Delta^{\text{SUSY}}_{Q_{\text{match}}} = \max\{|M_h^{M_{\text{SUSY}}/2} - M_h^{M_{\text{SUSY}}}|, |M_h^{2M_{\text{SUSY}}} - M_h^{M_{\text{SUSY}}}|\}$
- for the quartic-coupling matching: $\Delta M_h^{\text{IV}} = \left[(\Delta M_h^{\text{II}})^2 + (\underbrace{M_h^{\text{II}} - M_h^{\text{IV}}}_{\Delta^{\text{SUSY}}_{v/M_{\text{SUSY}}}})^2\right]^{\frac{1}{2}}$

> total uncertainty: $\Delta M_h^{\text{II}} = \left[\left(\Delta^{\text{SM}}_{G_F/\alpha_{M_Z}}\right)^2 + \left(\Delta^{\text{SM}}_{Q_{\text{EW}}}\right)^2 + \left(\Delta^{\text{SM}}_{Y_t}\right)^2 + \left(\Delta^{\text{SUSY}}_{Q_{\text{match}}}\right)^2\right]^{\frac{1}{2}}$

1) SUSY: motivation

SUSY formalism

The lightest Higgs Boson Mass in the (N)MSSM

2) Higher-Order Corrections to m_h^2

3) Intermediate Summary

4) Implementation in NMSSMCALC

Matching procedures

Calculation of $m_h \pm \sigma_{m_h}$

Numerical results

(Preliminary) Results

Consider 3 scenarios:

	$\tan\beta$	λ	κ	M_1	M_2	M_3	A_t	A_λ	A_κ	$\mu_{eff.}$	$m_{\tilde{Q}_{L3}}$	$m_{\tilde{t}_{R3}}$	Ref.
BP1	3.0	0.6	0.6	1.0	2.0	2.5	12.75	0.3	-2.0	1.5	5.0	5.0	[Bagnaschi et al.]
BP2	20.0	0.05	0.05	3.0	3.0	3.0	-7.20	-2.85	-1.0	3.0	3.0	3.0	[Slavich et al.]
BP3	1.27	0.73	0.62	0.24	1.18	2.3	-0.39	0.06	-1.44	0.49	1.79	1.51	this work

(all TeV)

	$M_{h_1}^{\text{II}}$	m_{h_2}	m_{h3}	m_{A_1}	m_{A_2}	m_{H^+}	$m_{\tilde{t}_1}$	$m_{\chi_1^0}$	$m_{\chi_1^+}$
BP1	124.3 (h_u)	2407.6 (h_s)	2971.8 (h_d)	2905.7 (a)	3000.2 (a_s)	2967.1	4829.6	997.2	1490.2
BP2	125.3 (h_u)	2996.4 (h_d)	5744.4 (h_s)	2985.3 (a_s)	3010.5 (a)	2997.8	2831.6	2932.7	2940.9
BP3	127.2 (h_u)	305.5 (h_s)	659.5 (h_d)	663.8 (a)	1308.7 (a_s)	658.4	1514.2	232.8	477.3

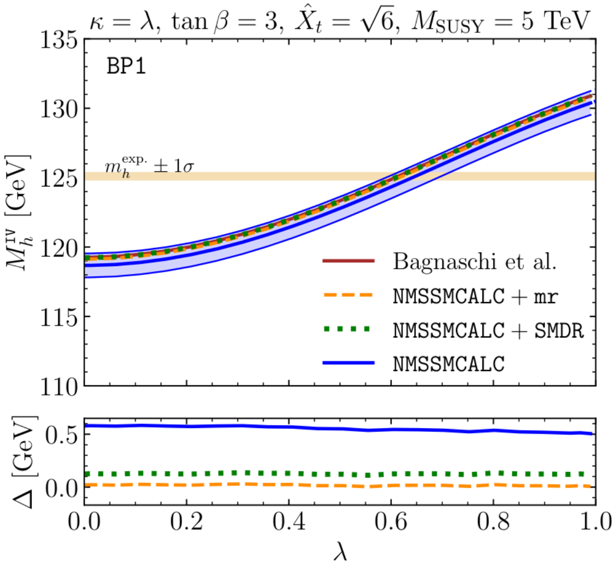
(all GeV)

- BP1: high-scale NMSSM (small $\tan\beta$, large $\lambda, \kappa, \hat{X}_t$)
- BP2: high-scale MSSM (large $\tan\beta$, small $\lambda, \kappa, \hat{X}_t$)
- BP3: light-singlet

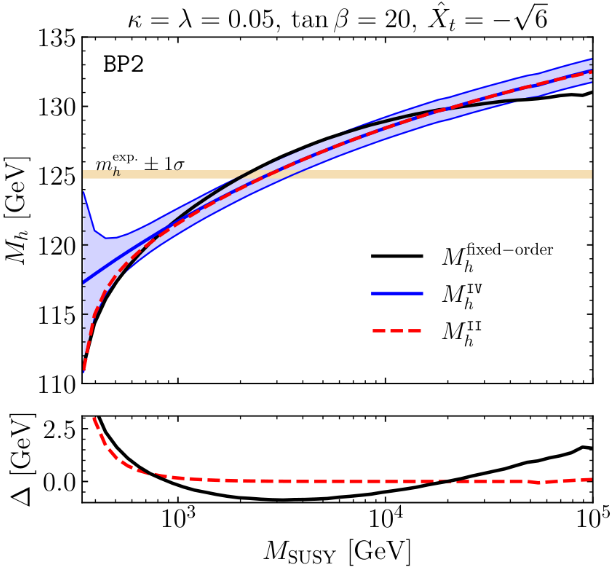
	$\Delta_{Y_t}^{\text{SM}}$	$\Delta_{Q_{\text{EW}}}^{\text{SM}}$	$\Delta_{G_F/\alpha_{M_Z}}^{\text{SM}}$	$\Delta_{Q_{\text{match}}}^{\text{SUSY}}$	$\Delta_{v/M_{\text{SUSY}}}^{\text{SUSY}}$	ΔM_h^{II}	ΔM_h^{IV}
BP1	739	190	19	371	21	847	848
BP2	680	212	69	398	12	818	819
BP3	401	204	21	839	2010	952	2453

(all MeV)

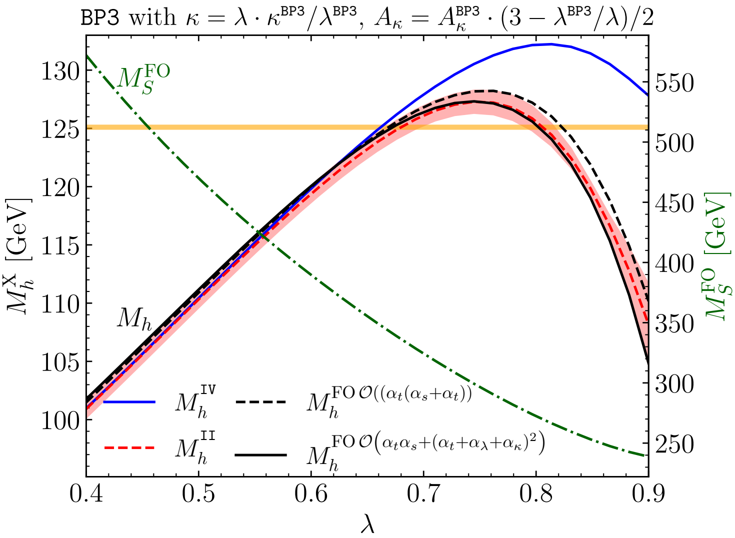
Cross-check I: high-scale NMSSM



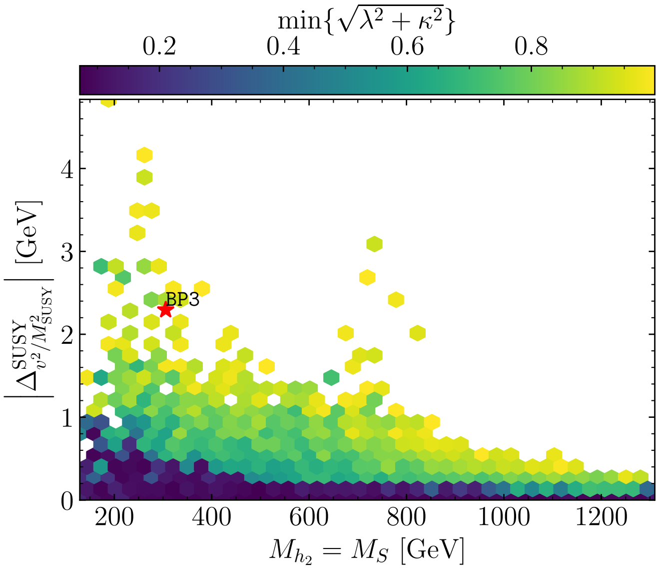
Cross-check II: decoupling of $v/M_{\rm SUSY}$ (high-scale MSSM)



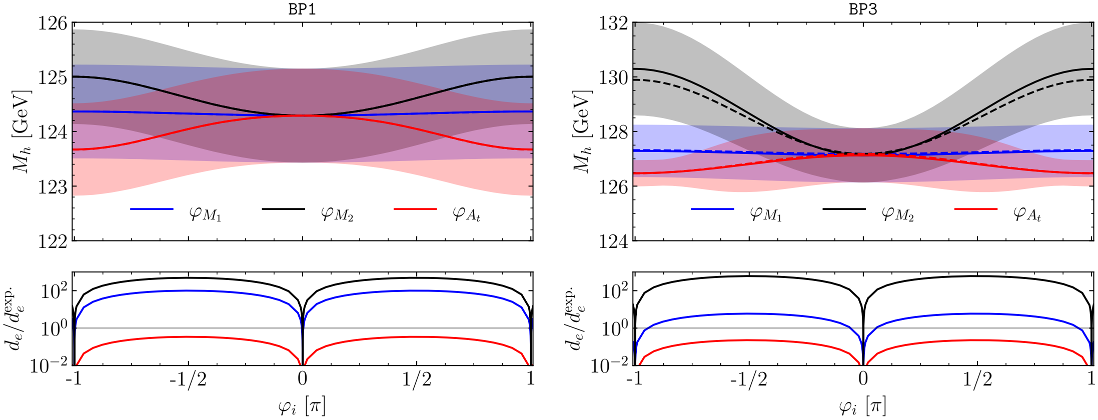
Light singlet I: comparison with fixed-order



Light singlet II: global picture

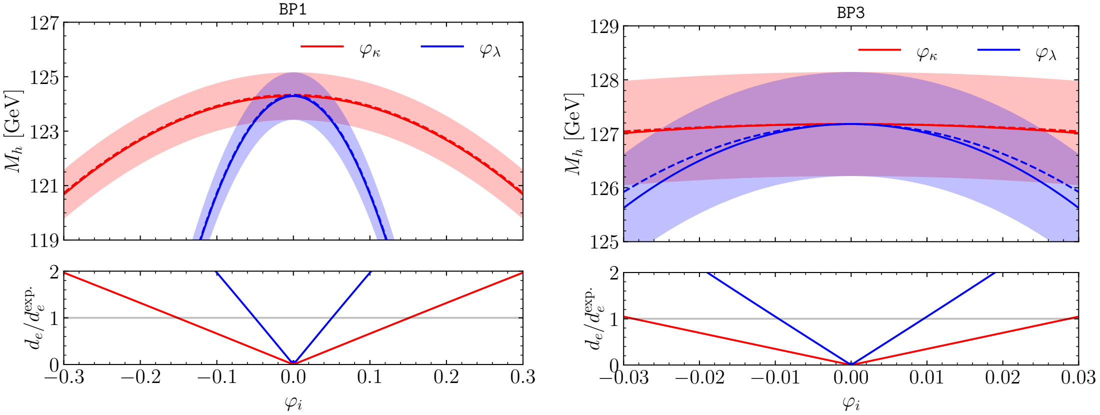


CP-violation: loop-induced



- solid: pole-mass matching
- dashed: quartic coupling matching shifted by $\Delta^{\text{SUSY}}_{v/M_{\text{SUSY}}}\Big|_{\varphi_i=0}$
- lower panels: eEDM constraint (>1 means excluded)

CP-violation: tree-induced



- $\varphi_y = 2\varphi_s + \varphi_\kappa - \varphi_\lambda - \varphi_u \neq 0 \rightarrow$ CPV at tree-level
- check: for $v \rightarrow 0$ the pole-mass and quartic matching agree
- **Attention:** tree-level phases mix h_{SM} and e.g. a_s
  $\rightarrow$ SM not the right EFT. Breakdown for too-large values of $\varphi_y, \lambda \sim \mathcal{O}(1)$!

Summary

Higgs boson mass in SUSY

Summary

Higgs boson mass in SUSY

- not a free input parameter; determined perturbatively in terms of SUSY inputs

Summary

Higgs boson mass in SUSY

- not a free input parameter; determined perturbatively in terms of SUSY inputs
- very sensitive to higher-order corrections
  $\rightarrow$ indirect constraints on SUSY parameters/spectra

Summary

Higgs boson mass in SUSY

- not a free input parameter; determined perturbatively in terms of SUSY inputs
- very sensitive to higher-order corrections
  $\rightarrow$ indirect constraints on SUSY parameters/spectra

Higher-order corrections

- either within fixed-order $(M_{\text{SUSY}} \approx \mathcal{O}(1\,\mathrm{TeV}))$ or EFT $(M_{\text{SUSY}} \gg \mathcal{O}(1\,\mathrm{TeV}))$ framework

Summary

Higgs boson mass in SUSY

- not a free input parameter; determined perturbatively in terms of SUSY inputs
- very sensitive to higher-order corrections
  $\rightarrow$ indirect constraints on SUSY parameters/spectra

Higher-order corrections

- either within fixed-order $(M_{\text{SUSY}} \approx \mathcal{O}(1\,\mathrm{TeV}))$ or EFT $(M_{\text{SUSY}} \gg \mathcal{O}(1\,\mathrm{TeV}))$ framework
- combinations/recycling possible: pole-mass matchings and hybrid-calculations

Summary

Higgs boson mass in SUSY

- not a free input parameter; determined perturbatively in terms of SUSY inputs
- very sensitive to higher-order corrections
  $\rightarrow$ indirect constraints on SUSY parameters/spectra

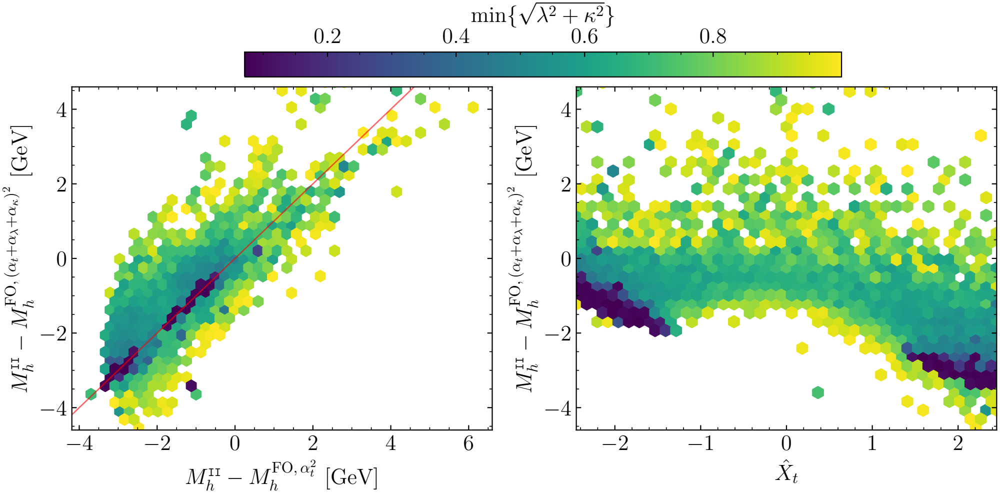
Higher-order corrections

- either within fixed-order $(M_{\text{SUSY}} \approx \mathcal{O}(1\,\text{TeV}))$ or EFT $(M_{\text{SUSY}} \gg \mathcal{O}(1\,\text{TeV}))$ framework
- combinations/recycling possible: pole-mass matchings and hybrid-calculations
- NMSSM:
  - v/M_{SUSY}-terms can be much more important than in the MSSM!
  - CPV-effects can be sizeable

Thank You!

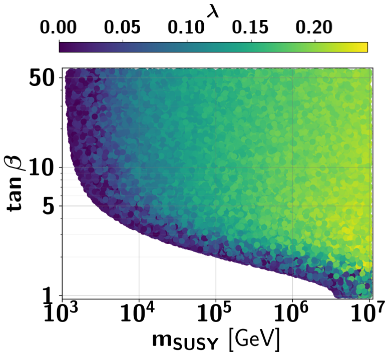
Backup

Comparison with fixed-order: global picture



Status in the NMSSM: EFT

> **One-loop:** full matching in $\overline{\text{DR}}$ (CP-conserving):
- studied: NMSSM $\to$ SM [Zarate, '16] [MSc Thesis, '18] [Bagnaschi, '22]



($\hat{X}_t = \sqrt{6}, \kappa = \frac{\lambda}{2}$ all other SUSY masses degenerate)

- studied: NMSSM $\to$ SM+Singlet+EW-inos [Gabelmann, Muehlleitner, Staub, '19]
- also possible: 2HDM, 2HDM+S (+EW-inos)

> **Two-loop:** $\overline{\text{DR}}$-hybrid (pole-mass matching) only, [Staub, Porod, '17]

Two-Loop Corrections to m_h: Ingredients

- numerically solve:
  $$det\left[p^2\mathbb{1}_{5x5} - m_h^2 + \hat{\Sigma}_h^{(1)}(p^2) + \hat{\Sigma}_h^{(2)}(0)|_{g_{1,2}=0}\right] = 0$$
- renormalized self-energy:
  $$\hat{\Sigma}_h^{(2)}(p^2) = \Sigma_h^{(2)}(0) - \delta^{(2)} m_h^2$$
- two-loop mass matrix counter-term:
  $$\delta^{(2)} m_h^2 = \mathcal{O}\left((\delta^{(1)} Z_h)^2 + \delta^{(1)} m_h \delta^{(1)} Z_h\right) + \left(\delta^{(2)} Z_h^\dagger m_h^{(0)} + m_h^{(0)} \delta^{(2)} Z_h\right)$$
  $$+ m_h^{(0)}(\alpha \to \alpha + \delta^{(1)}\alpha + \delta^{(2)}\alpha)|_{\alpha=0}, \ \alpha = \{\lambda, \kappa, v, v_S, t_i, ...\}$$
- pure $\overline{\text{DR}}$: $\delta\alpha$ are purely divergent, canceled by wave-function ren. (SUSY-non ren.)
- OS quantities (e.g. $\delta v \leftrightarrow \delta m_{W,Z}$, or δt_i) : can yield extra finite contributions to $\delta^{(2)} m_h^2$

Most manpower required for (decreasing complexity):

- two-loop wave-function ren. constants $\delta^{(2)} Z_h = \left.\frac{\partial \Sigma_h(p^2)}{\partial p^2}\right|_{p^2=0}$
- two-loop vector self-energy diagrams $\delta^{(2)}\Sigma_V(0)$ (required for $\delta^{(2)} v$)
- two-loop scalar self-energy diagrams $\delta^{(2)}\Sigma_h(0)$
- two-loop tadpole diagrams $\delta^{(2)} t_i$

The same applied for the charged Higgs bosons, $h \to H^\pm$.

Status in the MSSM (fixed-order)

Most precise results are based on **3-loop** self-energies&tadpoles: [Kant, Harlander, Mihaila, Steinhauser, '10]

- gaugeless limit $g_1, g_2 \to 0$ (no graphs involving EW vector bosons)
- vanishing external momenta: $p^2 \approx m_h^2 \approx v^2 g_{1,2}^2 \to 0$
- consider strong sector only, t, $\tilde{t}$, g, $\tilde{g}$ (up to m_t^4-terms)
- assume hierarchies, e.g. (1) $m_{\tilde{g}} \gg m_{\tilde{t}}$, (2) $m_{\tilde{g}} \gg m_{\tilde{t}}$, etc.
- $\overline{\text{DR}}$ and $\overline{\text{MDR}}$
- new 3-loop results (semi-numerical) for general mass hierarchies [Reyes, Fazio, '19]

Two-loop self-energies (diagrammatic or effective potential): [Slavich, '01], [Martin, '01] , [Degrassi, Di Vita, Slavich, '14], [Borowka, Hahn, Heinemeyer, Heinrich, Hollik, '14], [...]

- $g_1, g_2, p^2 \to 0$
- $p^2 \neq 0$: $m_h^{p^2=0} - m_h^{p^2\neq 0} \approx 100 - 500\,\text{MeV}$
- full mass hierarchies
- with CPV and RPV
- $\overline{\text{DR}}$ and OS conditions for $m_{\tilde{t}}$, m_t and $m_{H^\pm}$

Status in the MSSM: EFT

Benefit from FO calculations:

> Pro: Hybrid approaches allow to "recycle" FO results
> $\rightarrow$ 3-loop pole-mass matching + 4-loop RGEs (N^3LL)
> [Harlander, Klappert, Ochoa, Voigt, '19]

> Con: applicable for high-scale SUSY
> i.e. one light Higgs but nothing else

Dedicated matching of scalar couplings:

> one-loop:
- real 2HDM [Haber, Hempfling, '93]
- complex 2HDM [Gorbahn, Jager, Nierste, Trine, '09] [Murphy, Rzehak, '1
- generic [Gabelmann, Muehlleitner, Staub, '18]
- matching extended 2HDM Higgs-masses, Split-SUSY with light fermions, ...

> two-loop λ_{SM}:
- leading QCD [Bagnaschi, Slavich, '17]
- mixed QCD-EW, combined with 3-loop hybrid [Bagnaschi, Degrassi, Slavich, '19]

> three-loop: hopeless, but
- $m_h^{\text{2L FO},p^2=0} - m_h^{\text{2L FO},p^2\neq 0} \approx 100 - 500\,\text{MeV}$
- $m_h^{\text{3L, hybrid}} - m_h^{\text{2L, mixed QCD-EW}} \approx 10 - 100\,\text{MeV}$
- $m_h^{\text{3L, hybrid}} - m_h^{\text{2L}} \approx 50 - 500\,\text{MeV}$
- gauge/momentum-less approximation fully exploited(?)