Domain walls in the 2HDM +CS

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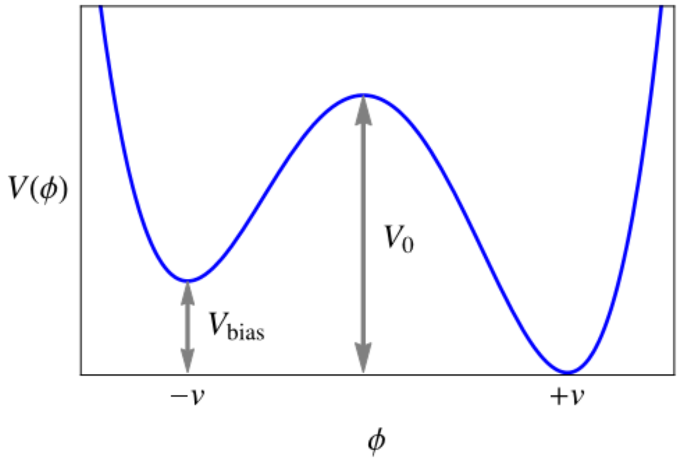
Domain walls – quick recap

Domain walls are thought to be created when a discrete symmetry is spontaneously broken.
⇒ Regions of space in the evolution of the universe, which live in different vacua
⇒ Transition between this regions, where ϕ=0, which is called „**Domain Wall“**

- Problem: Stable domain walls cosmological unfavourable. If still existing domain walls would dominate the energy density of the universe.

- Solution: Introduction of a bias term ΔV which breaks the discrete symmetry explicitly
  ⇒ Unstable, collapsing domain walls

- Most important quantity: surface energy density σ
  How to calculate this?



Domain walls – quick recap

- Solve equations of motion of the domain wall problem; z: space coordinate
  ⇒ Field configurations $\phi_i(z)$

$$\frac{d^2 S}{dz^2} = \frac{\partial V}{\partial S^*}, \quad \frac{d^2 H_u}{dz^2} = \frac{\partial V}{\partial H_u^*}, \quad \text{and} \quad \frac{d^2 H_d}{dz^2} = \frac{\partial V}{\partial H_d^*}$$

  ⇒ Integrating over z gives surface density

$$\rho_{\text{wall}}(z) = \left|\frac{dS}{dz}(z)\right|^2 + \left|\frac{dH_u}{dz}(z)\right|^2 + \left|\frac{dH_d}{dz}(z)\right|^2 + V(S(z), H_u(z), H_d(z)) - V(v_s/\sqrt{2}, v_u/\sqrt{2}, v_d/\sqrt{2})$$

- Very similar to equations of motion for tunneling

$$-\frac{d^2\phi_i}{dr^2} - \frac{d-1}{r}\frac{d\phi_i}{dr} + \frac{\partial V}{\partial \phi_i} = 0.$$

- Main difference: boundary conditions and friction term

$$\left.\frac{d\phi_b}{dr}\right|_{r=0} = 0\,, \qquad \phi_b|_{r=\infty} = 0\,,$$

2HDM + CS

- 2HDM + complex singlet model with discrete Z2- and Z3 symmetries

⇒ two sources of DW formation possible

$$V = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + m_S^2(\Phi_S^\dagger\Phi_S) + \lambda_1'(\Phi_S^\dagger\Phi_S)(\Phi_1^\dagger\Phi_1) + \lambda_2'(\Phi_S^\dagger\Phi_S)(\Phi_2^\dagger\Phi_2)$$
$$+ \frac{\lambda_3''}{4}(\Phi_S^\dagger\Phi_S)^2 + (-m_{12}^2(\Phi_1^\dagger\Phi_2) + \frac{\mu_{s1}}{6}\Phi_S^3 + \mu_{12}\Phi_S\Phi_1^\dagger\Phi_2 + h.c)$$

$$\Phi_1 \to -\Phi_1$$

$$\begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_S\end{pmatrix} = \begin{pmatrix}1 & & \\ & e^{\frac{2\pi i}{3}} & \\ & & e^{-\frac{2\pi i}{3}}\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_S\end{pmatrix}$$

Solving methods: Path deformation (CosmoTransitions)

- General approach:
  1. Take some initial guess for the field configuration
  2. Split EoMs into one part parallel and one perpendicular to the path
  3. Solve 1D problem (upper equation)
  4. Let perpendicular force act on path (lower equation)

$$\frac{d^2x}{d\rho^2} + \frac{\alpha}{\rho}\frac{dx}{d\rho} = \frac{\partial}{\partial x}V[\vec{\phi}(x)]$$
$$\frac{d^2\vec{\phi}}{dx^2}\left(\frac{dx}{d\rho}\right)^2 = \nabla_\perp V(\vec{\phi}),$$

- In „The gravitational waves from the collapsing domain walls in the complex singlet model", Chen, N. et al., arXiv:2004.10148, they use this procedure to solve the DW equations

- Problem: For me it is very much unstable…

Solving methods: Path deformation (CosmoTransitions)

- Solving the 1D problem with the overshooting/undershooting method
- Try and error of initial value until field hits exactly the other extremum
- Problem for DW-eqs: no friction term
  ⇒how to solve this for biased case?
  Always case (b) ?

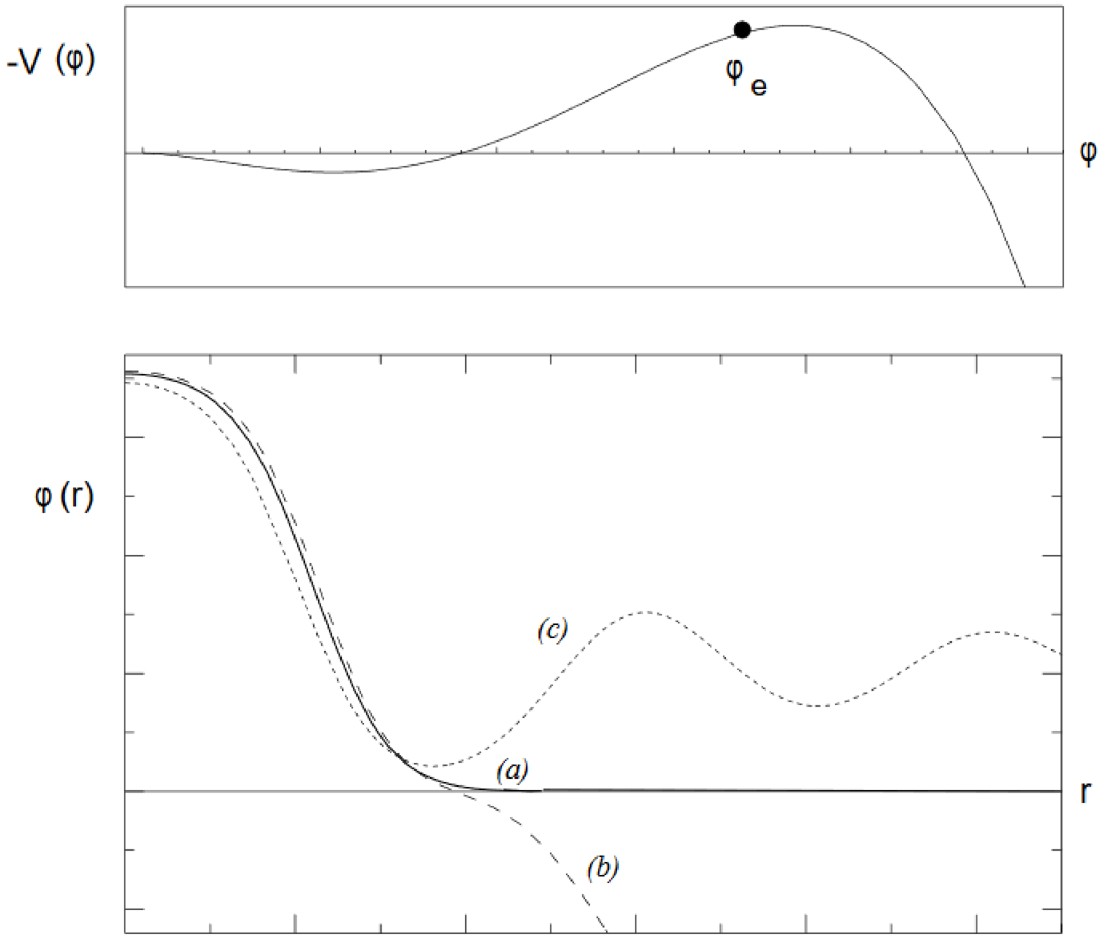
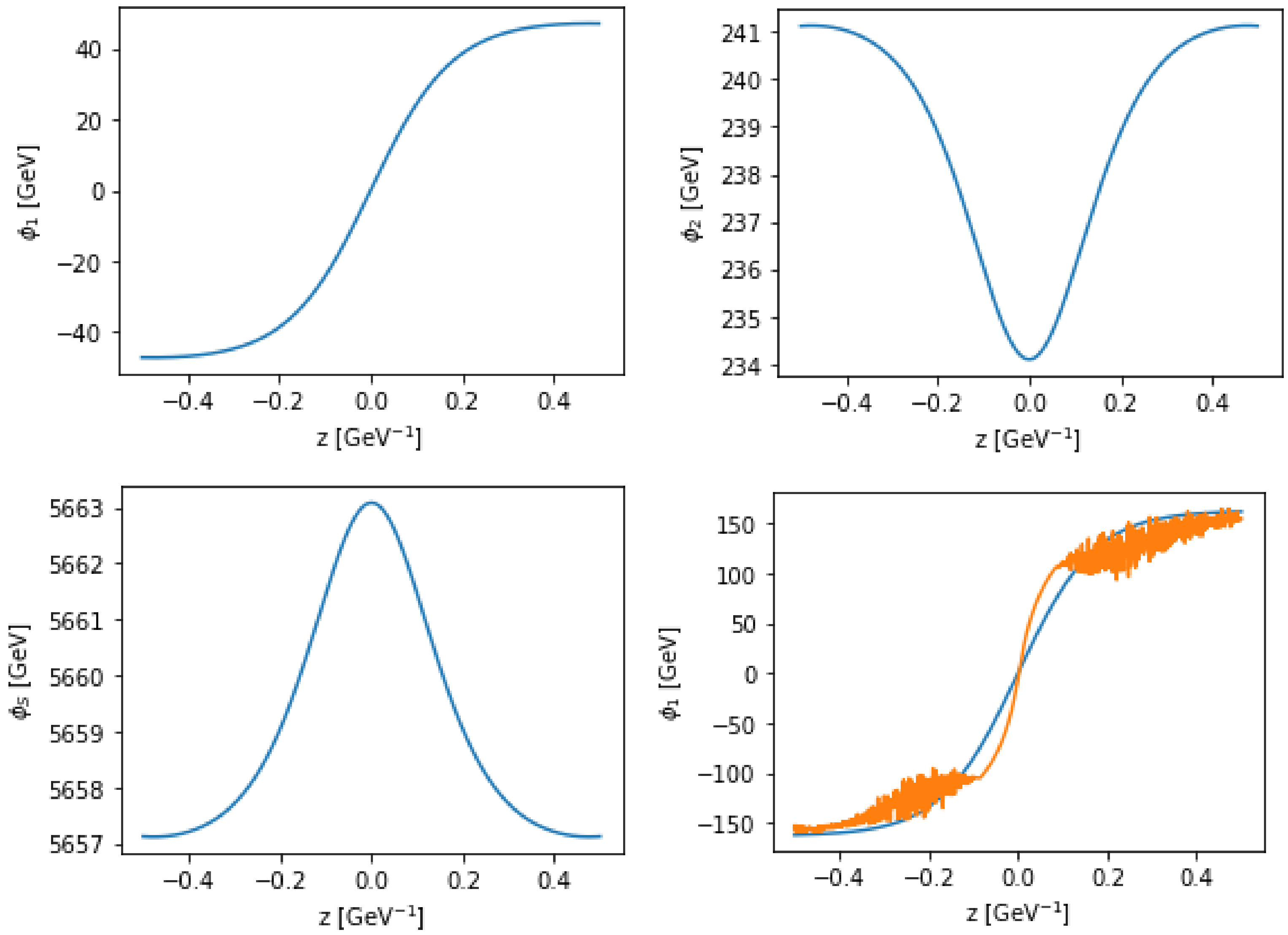


Figure 3: Overshooting-undershooting method.

Solving methods: Path deformation (CosmoTransitions)

- My solution of the 1D problem: keeping 2 of the three fields constant (at vevs) and solving the 1D equation
- Problem: Solution highly dependent on initial value..
  ⇒ just construct some initial guess parametrized by ΔV=V_barr-V_min
  ⇒ let perpendicular forces act
- One example for a point from arXiv:1503.06998
  and another one for when it doesnt work(right bottom)



Solving methods: Globally convergent Newton method

- Proposed in „Gravitational waves from domain walls in the next-to-minimal supersymmetric standard model", Saikawa, K., arXiv:1503.06998
- Root-finding algorythm for equation or system of equations F(x); x is our field configuration we look for
- At each step, check if f is reduced
  ⇒ backtracking if step is too big/solution diverges
- Approximating the second derivative at each point and each iteration via central differences

$$\mathbf{F}(\mathbf{x}) = 0$$

$$\mathbf{x}_{\text{new}} = \mathbf{x}_{\text{old}} + \delta\mathbf{x}$$

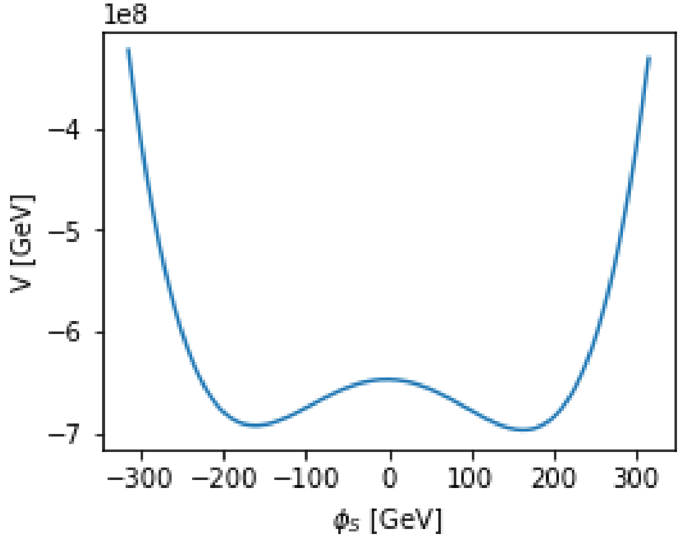
$$\delta\mathbf{x} = -\mathbf{J}^{-1} \cdot \mathbf{F}$$

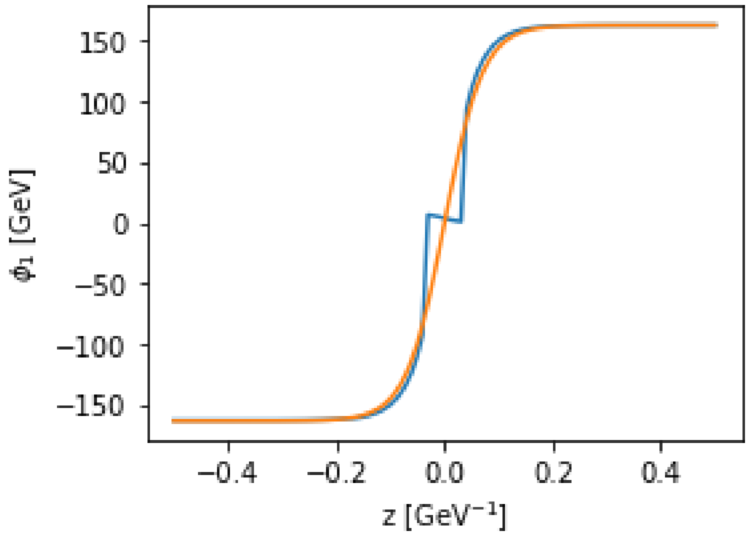
$$\nabla f \cdot \delta\mathbf{x} = (\mathbf{F} \cdot \mathbf{J}) \cdot (-\mathbf{J}^{-1} \cdot \mathbf{F}) = -\mathbf{F} \cdot \mathbf{F} < 0$$

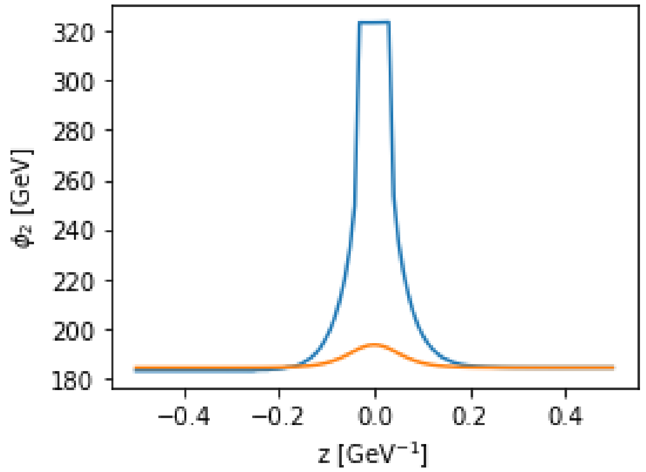
$$f = \frac{1}{2}\mathbf{F} \cdot \mathbf{F}$$

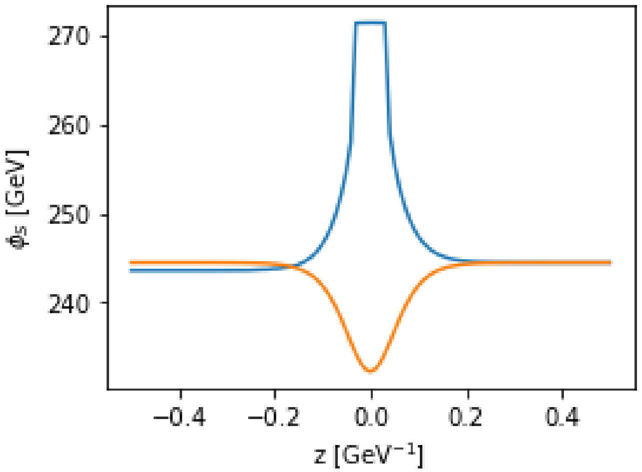
Solving methods: Globally convergent Newton method

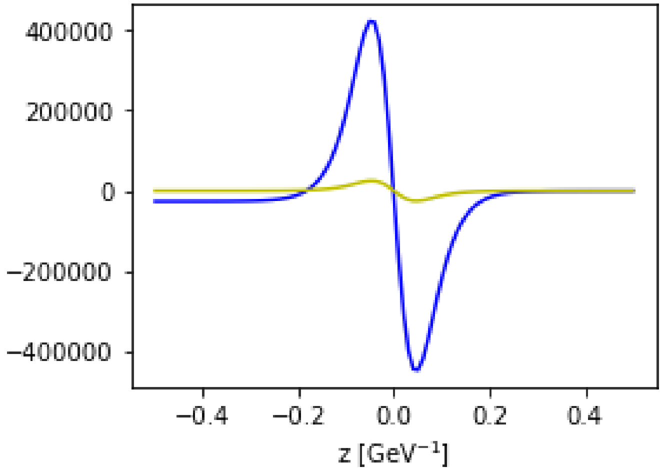
```
vS=172.863004
l1=0.55155398
l2=0.16093653
l3=0.440419985
l4=-0.0691352032
mus1=-9.6993968
mu12=-1.392339
ls33=2.24901256
ls1=0.27872891
ls2=-0.117033124
m11sq=353886384.
m22sq=79516.1336
mSsq=1355.26956
m12sq=(mu12*vS/np.sqrt(2))
```











Solving methods: Gradient flow method

- Proposed in „Simulations of Domain Walls in Two Higgs DoubletModels”, Battye et al., 2006.13273
- Introduction of simulation „time“ variable t to find minimum energy configurations
- Work in progress, no working code yet..

$$\frac{\partial \phi_n}{\partial t} = -\frac{\delta E}{\delta \phi_n} = \frac{\partial}{\partial x}\left(\frac{\partial \mathcal{E}}{\partial\left(\partial \phi_n / \partial x\right)}\right) - \frac{\partial \mathcal{E}}{\partial \phi_n}.$$

$$\lim_{t \to \infty} \frac{\partial \phi_n}{\partial t} = 0,$$