Domain walls and Gravitational Waves

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Outline

Dynamics of domain walls

- Formation
- Cosmological evolution
- Solution to domain wall problem

Gravitional waves from domain walls

- Estimation of spectra
- Physical interpretation of spectra
- Model-independent predictions
- Example: NMSSM

Summary and Outlook

Formation of Domain walls

- Mainly follow paper: „A review of gravitational waves from cosmic domain walls", K. Saikawa, 2017, ArXiv: 1703.02576
- Domain walls are thought to be created when a discrete symmetry is spontaneously broken.

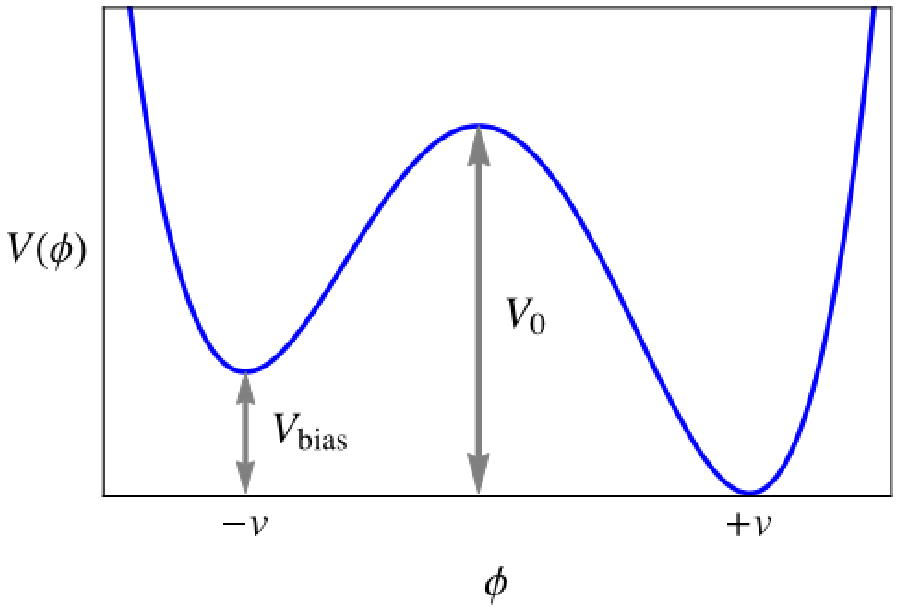
  Consider a Toy-model with $V(\phi) = \frac{\lambda}{4}(\phi^2 + v^2)^2$
  Discrete $\mathbb{Z}_2$-symmetry $\phi \rightarrow -\phi$, is spontaneously broken when ϕ acquires vev $< \phi > = \pm v$
  ⇒ Regions of space in the evolution of the universe, which live in different vacua
  ⇒ Transition between this regions, where ϕ=0, which is called „**Domain Wall**"
- Problem: Stable domain walls cosmological unfavourable

Cosmological evolution of Domain walls

- Dynamics driven by two forces:
  - Tension $p_T \sim \frac{\sigma}{R_{wall}}$, σ: surface energy density, R_{wall}:curvature radius
  - Friction $p_F \sim v T^4 \rightarrow$ exponentially damped at temperatures $T < m_{int}$
- Domain walls enter so-called „scaling regime“, when p_T dominates: $R_{wall} \sim L \sim H^{-1} \sim t \quad \Rightarrow \boxed{\rho_{wall}(t) = A\frac{\sigma}{t}}$
  A: dimensionless „area parameter“, proportional to number of degenerate vacua
- $\rho_{wall} \sim t^{-1}$ *while* $\rho_m \sim \rho_r \sim t^{-2} \Rightarrow$ Domain walls would dominate the energy density of the universe
- Additionally: large scale density fluctuations must be measurable in CMB $\Rightarrow \frac{\delta\rho}{\rho} \sim \frac{\rho_{wall}}{\rho_c} \sim 10^{12}\frac{\sigma}{TeV^3} \Rightarrow \text{upper bound } \sigma^{1/3} \lesssim O(MeV)$ (ZKO-Bound)

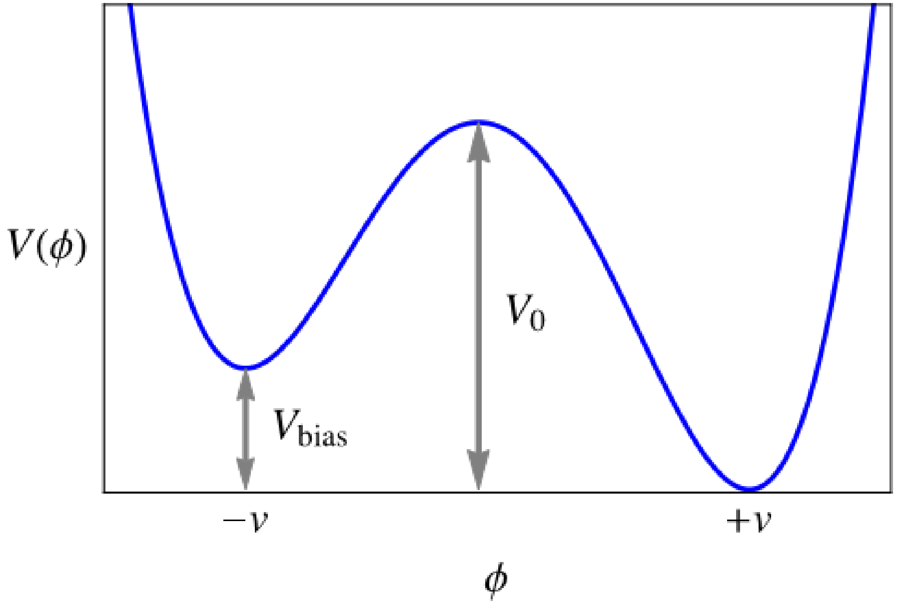
Solution to domain wall problem

- Idea: Introduction of additional term ΔV, which breaks the discrete symmetry explicitly, so-called *bias*
- Symmetry must still hold approximately, so that domain walls can form
  - $\frac{p_-}{p_+} \cong \exp\left(-\frac{V_{bias}}{V_0}\right)$
  - $p_- > p_c$ to guarantee that regions of both vacua will develop
    $\Rightarrow \frac{V_{bias}}{V_0} < \ln\left(\frac{1-p_c}{p_c}\right) = 0.795$



Solution to domain wall problem

- The energy difference of the vacua drives the dynamics of the domain wall network
  ⇒ regions of false vacuum shrink, domain walls become unstable
- Described by volume pressure $p_V \sim V_{bias}$
  When $p_V \sim p_T$ walls annihilate
  $\Rightarrow t_{ann} = C_{ann} \frac{A\sigma}{V_{bias}}$
  $\Rightarrow T_{ann} \propto g_*(T_{ann})^{-1/4} * t_{ann}{}^{-1/2}$
- Require $t_{ann} < t_{dom}$
  ⇒ lower bound for bias and temperature
  $V_{bias} > \frac{4C_{ann}A^2\sigma^2}{3M_{pl}{}^2} \quad ; \quad T_{ann} > g_*(T_{ann})^{-1/4}(A\sigma)^{1/2}$
- These bounds can be further constrained by BBN, since decay products of domain walls may interact with particles in the universe



Gravitational waves from domain walls

- Energy density of GW can be estimated as $\rho_{GW} \sim G A^2 \sigma^2$
- Numerical calculation of the GW spectrum:
  - Simulation of scalar field dynamics
  - Calculate the stress-energy tensor
  - Integrate over conformal time $\Rightarrow \rho_{\mathrm{GW}}$

    $\Rightarrow \Omega_{GW}(t,f) = \frac{1}{\rho_c(t)} \frac{d\rho_{GW}(t)}{d\ln(f)}$; $f = \frac{k}{2\pi a(t)}$

    $\Rightarrow$ simulations confirm the theoretical estimation
- Extrapolating the spectrum to present cosmic time leads to:

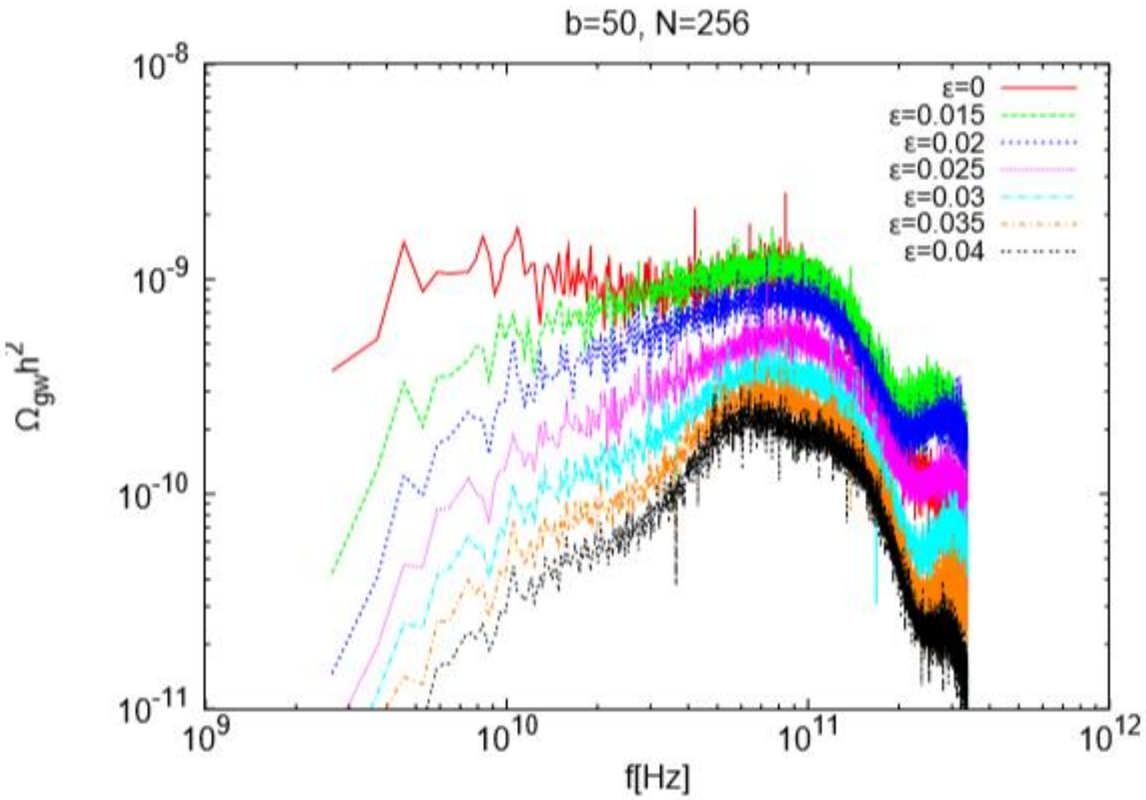
$$\Omega_{peak}(t_0) \sim (A\sigma)^2 {T_{ann}}^{-4} \; ; \quad f_{peak} \sim T_{ann}$$

Gravitational waves from domain walls – Toy model

- Example: Toy model with bias term, with discrete $\mathbb{Z}_2$-symmetry $\phi \to -\phi$

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2 + \epsilon\eta\phi\left(\frac{1}{3}\phi^2 - \eta^2\right) + \frac{\lambda}{8}T^2\phi^2.$$

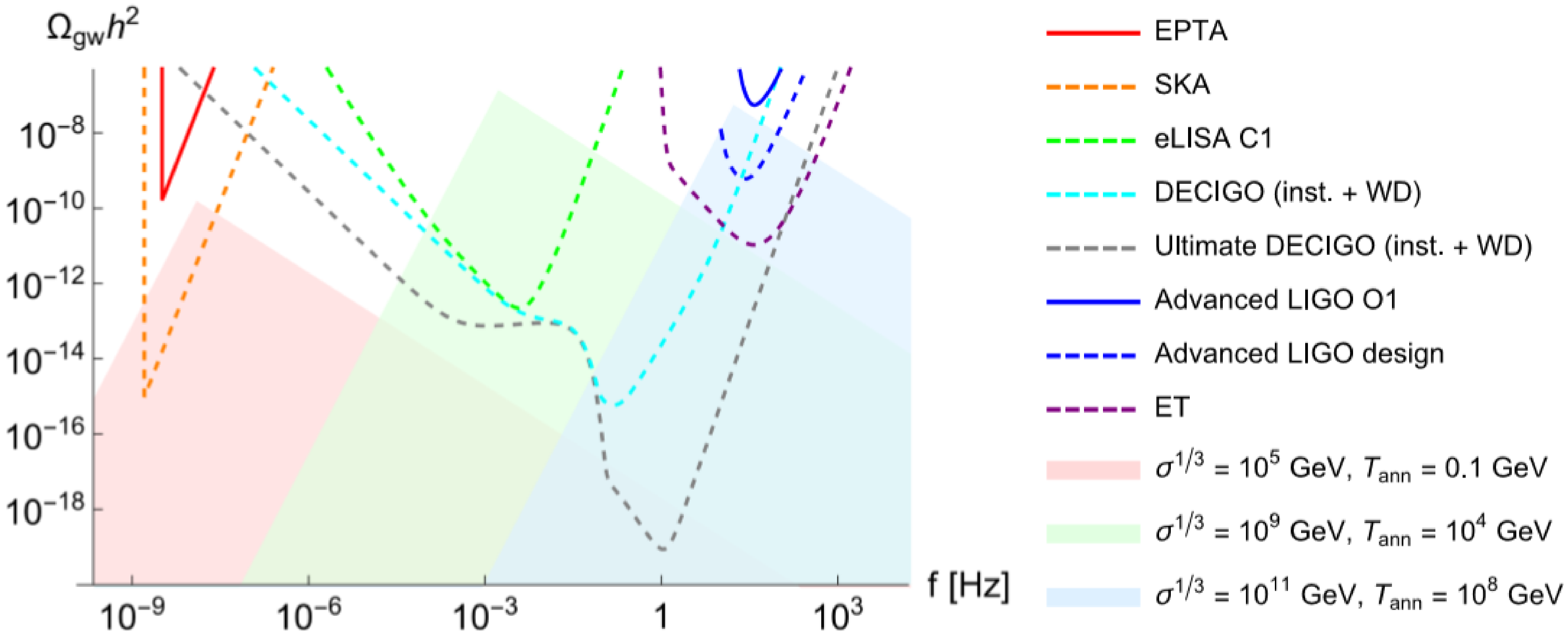
- Peak frequency corresponds to horizon size, cutoff at higher frequencies and scaling towards is determined by wall width δ
- ε parametrizes V_{bias} $\Rightarrow$ for larger ε, shorter lifetime of domain walls and therefore smaller amplitude



[Gravitational Waves from Collapsing Domain Walls, T. Hiramatsu et al., ArXiv:1002.1555]

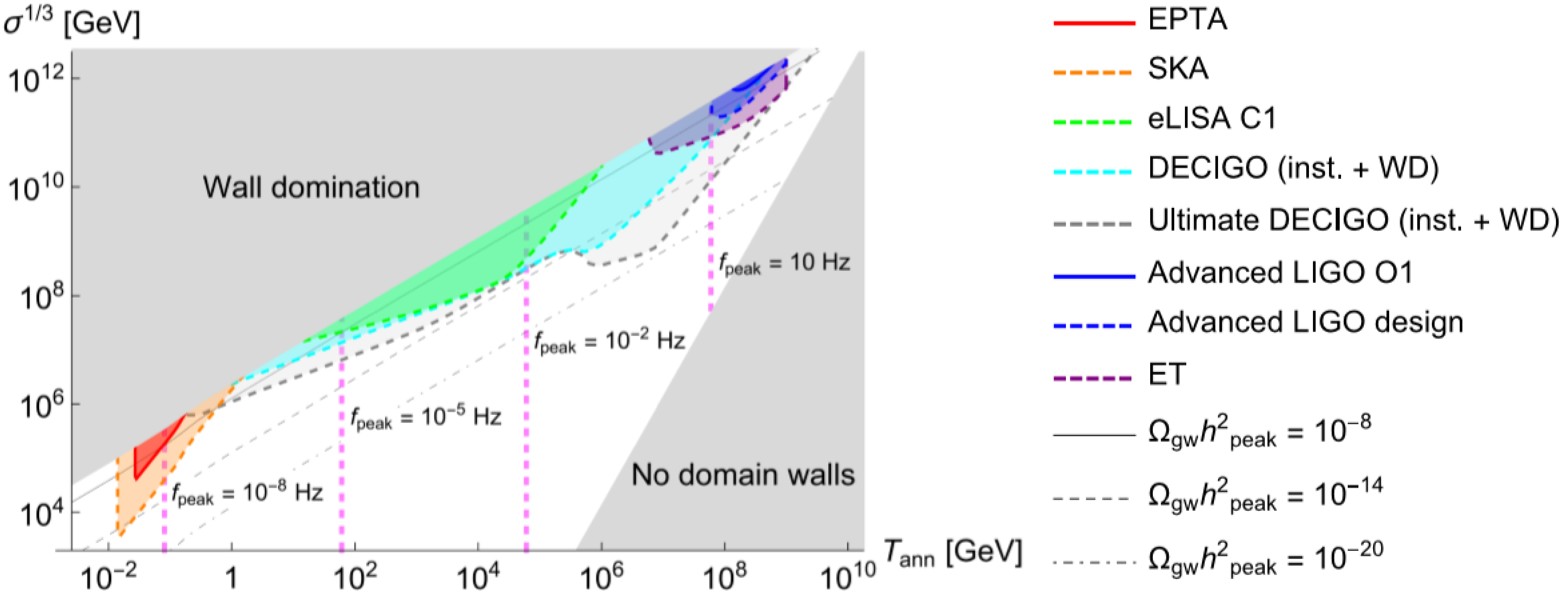
Gravitational waves from domain walls

Model-independent predictions



Gravitational waves from domain walls

Model-independent predictions



Example model with unstable domain walls - NMSSM

$$V = \left|\kappa S^2 - \lambda H_u H_d\right|^2 + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 + |\lambda|^2|S|^2(|H_u|^2 + |H_d|^2)$$
$$+ \frac{g^2}{4}\left(|H_u|^2 - |H_d|^2\right)^2 + \left[\frac{1}{3}\kappa A_\kappa S^3 - \lambda A_\lambda H_u H_d S + \text{h.c.}\right],$$

- Discrete Z3-symmetry imposed: $\Phi \rightarrow e^{\frac{2\pi i}{3}}\Phi$

  $\Rightarrow$ three degenerate vacua $(\langle S\rangle, \langle H_u\rangle, \langle H_d\rangle) = (v_s/\sqrt{2}, v_u/\sqrt{2}, v_d/\sqrt{2}), \left(v_s e^{\frac{2\pi i}{3}}/\sqrt{2}, v_u e^{\frac{2\pi i}{3}}/\sqrt{2}, v_d e^{\frac{2\pi i}{3}}/\sqrt{2}\right), \left(v_s e^{\frac{4\pi i}{3}}/\sqrt{2}, v_u e^{\frac{4\pi i}{3}}/\sqrt{2}, v_d e^{\frac{4\pi i}{3}}/\sqrt{2}\right)$

- $\mu \equiv \frac{\lambda v_S}{\sqrt{2}} \rightarrow O(100)GeV;\ in\ the\ decopling\ limit\ \lambda \ll 1 \Rightarrow v_S$ large, $\lambda \sim \kappa$

- $\Rightarrow \quad V \simeq \kappa^2|S|^4 + m_S^2|S|^2 + \left[\frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}\right]$

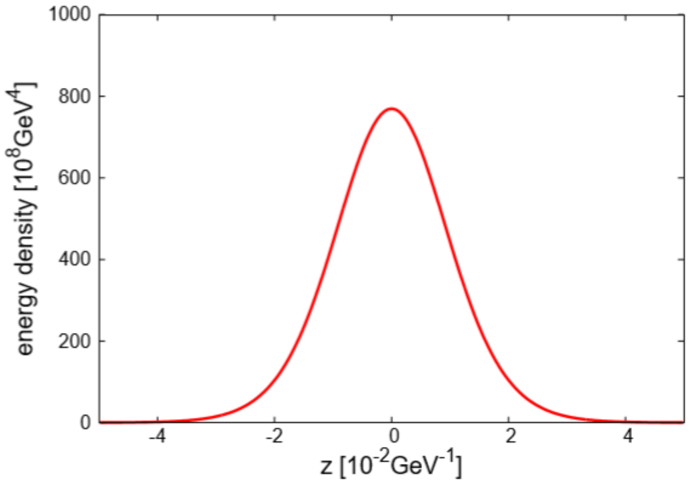
  [„*Gravitational waves from domain walls in the next-to-minimal supersymmetric standard model*”, K. Kadota et al., 2015, ArXiv: 1503.06998]

Example model with unstable domain walls - NMSSM

- Find domain wall solution with planar wall approximation:

$$\frac{d^2S}{dz^2} = \frac{\partial V}{\partial S^*}, \quad \frac{d^2H_u}{dz^2} = \frac{\partial V}{\partial H_u^*}, \quad \text{and} \quad \frac{d^2H_d}{dz^2} = \frac{\partial V}{\partial H_d^*}$$

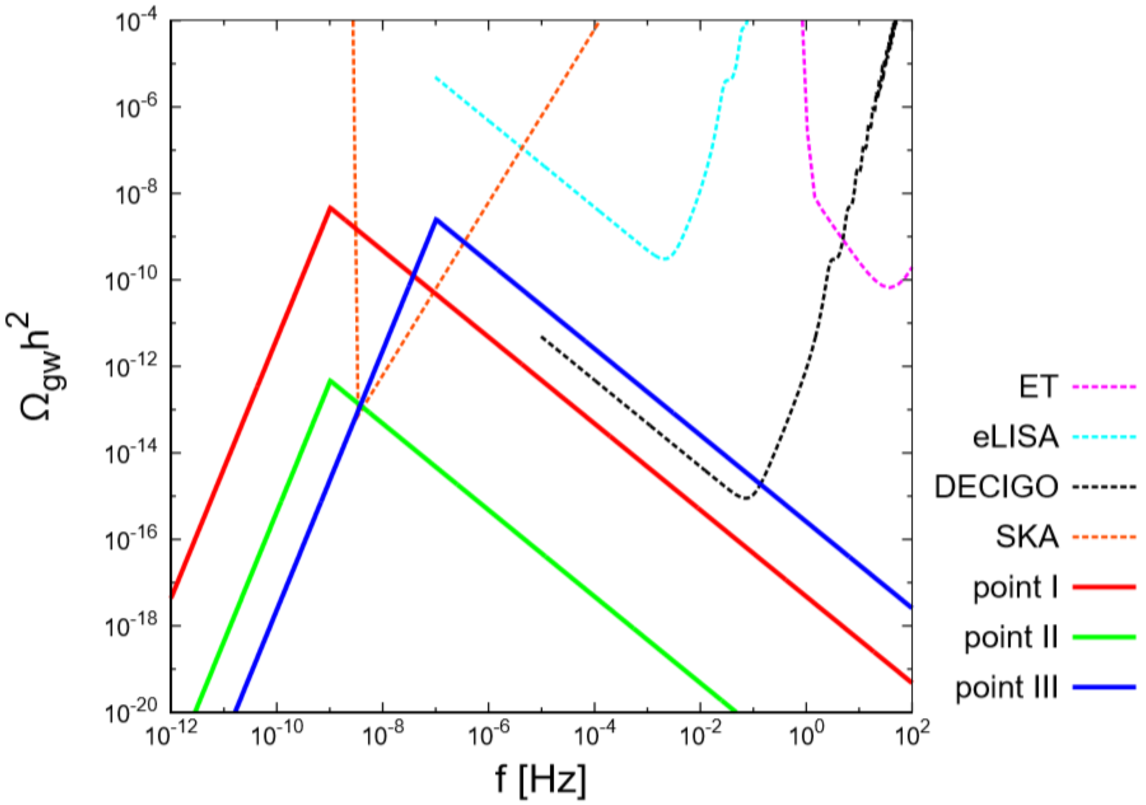
$$\rho_{\text{wall}}(z) = \left|\frac{dS}{dz}(z)\right|^2 + \left|\frac{dH_u}{dz}(z)\right|^2 + \left|\frac{dH_d}{dz}(z)\right|^2 + V(S(z), H_u(z), H_d(z)) - V(v_s/\sqrt{2}, v_u/\sqrt{2}, v_d/\sqrt{2})$$



- From this one can finally get the surface wall density: $\sigma_{wall} = \int dz\, \rho_{wall}(z)$
- In the decoupling limit, one can estimate $\sigma_{wall} \sim \kappa {v_S}^3$
- Bias term is not specified, but the range it can take is theoretically constrained

Example with unstable domain walls - NMSSM

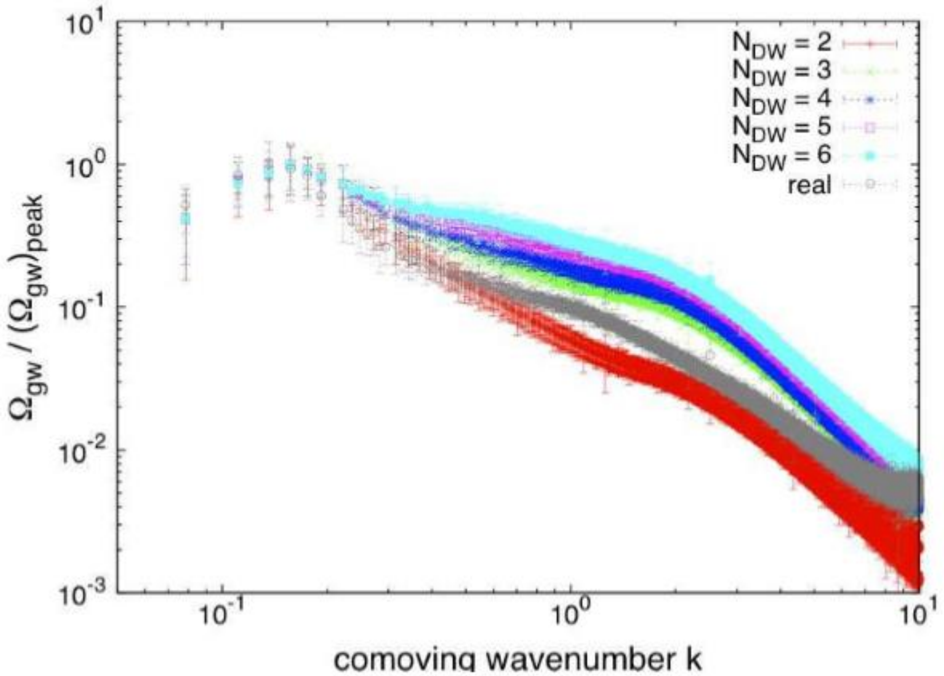
	I	II	III
λ	5×10^{-4}	5×10^{-3}	5×10^{-6}
κ	2×10^{-4}	2×10^{-3}	2×10^{-6}
A_λ	150GeV	150GeV	150GeV
A_κ	-150GeV	-150GeV	-150GeV
$\tan\beta$	5	5	5
μ	200GeV	200GeV	200GeV
$t_{\rm dec}$	10^{-2}sec	10^{-2}sec	10^{-6}sec
$\sigma_{\rm wall}$	1.96×10^4 TeV^3	1.96×10^2 TeV^3	1.96×10^8 TeV^3
$\Omega_{\rm gw} h^2(t_0)_{\rm peak}$	4.66×10^{-9}	4.66×10^{-13}	2.51×10^{-9}
$f(t_0)_{\rm peak}$	1.02×10^{-9}Hz	1.02×10^{-9}Hz	1.02×10^{-7}Hz



Summary and Outlook

- Gravitational waves from unstable domain walls could be a tool to test high energy physics
- The key model-dependent quantities are σ and T_{ann}
- Simulation techniques are limited $\Rightarrow$ analytic approach to calculate GW spectrum needed

Backup



$N_{DW} = 2$
$N_{DW} = 3$
$N_{DW} = 4$
$N_{DW} = 5$
$N_{DW} = 6$
real
$\Omega_{gw} / (\Omega_{gw})_{peak}$
comoving wavenumber k
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}