Domain walls in the 2HDMS

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Outline

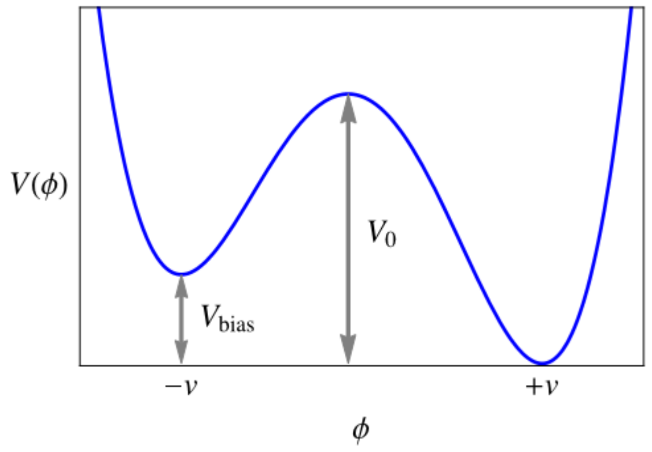
Domain walls

Domain wall problem

Domain walls are thought to be created when a discrete symmetry is spontaneously broken.
⇒ Regions of space in the evolution of the universe, which live in different vacua
⇒ Transition between these regions is called „**Domain Wall**“

- Problem: Stable domain walls are cosmologically unfavourable. If still existing domain walls would dominate the energy density of the universe.
  $\rho_{DW} \sim t^{-1}$ while $\rho_m, \rho_r \sim t^{-2}$

- Solution: Introduction of a bias term ΔV which breaks the discrete symmetry explicitly
  ⇒ Unstable, collapsing domain walls



Domain walls

Constraints to particle physics models

- Demanding that domain walls collapse before they dominate the universe
  ⇒ constraint on ΔV

$$\Delta V > \frac{4C_{Ann}A^2\sigma^2}{3M_{Pl}^2}$$

- Condition for domain walls to form: $\frac{\Delta V}{V_0} < 0.795$

- Discrete symmetries appear in many BSM models
  - $\mathbb{Z}_2$, $\mathbb{Z}_3$, CP…

  ⇒ Domain walls a tool to constrain parameter space of your model

- Additionally: Unstable Domain walls are a possible source of Gravitational waves!

- For further introduction to the topic→arXiv:1703.02576, Saikawa, K.

2HDMS

Lagrangian and symmetries

2HDM + complex singlet

- discrete $\mathbb{Z}_2$ and $\mathbb{Z}_3$ symmetries
- Soft symmetry breaking parameters:
  - m_{12} and μ_{12} break $\mathbb{Z}_2$
  - m_{12} breaks $\mathbb{Z}_3$
- Combination of m_{12} and μ_{12} can serve as a bias-term

$$
\begin{aligned}
V = {} & m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\
& + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + m_S^2(\Phi_S^\dagger\Phi_S) + \lambda_1'(\Phi_S^\dagger\Phi_S)(\Phi_1^\dagger\Phi_1) + \lambda_2'(\Phi_S^\dagger\Phi_S)(\Phi_2^\dagger\Phi_2) \\
& + \frac{\lambda_3''}{4}(\Phi_S^\dagger\Phi_S)^2 + (-m_{12}^2(\Phi_1^\dagger\Phi_2) + \frac{\mu_{s1}}{6}\Phi_S^3 + \mu_{12}\Phi_S\Phi_1^\dagger\Phi_2 + h.c)
\end{aligned}
$$

$\mathbb{Z}_2$

$$
\Phi_1 \to \Phi_1 \qquad \Phi_2 \to -\Phi_2 \qquad \Phi_S \to \Phi_S
$$

$\mathbb{Z}_3$

$$
\begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_S\end{pmatrix} = \begin{pmatrix}1 & & \\ & e^{\frac{2\pi i}{3}} & \\ & & e^{-\frac{2\pi i}{3}}\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_S\end{pmatrix}
$$

Solving the DW EoMs

Equations and problems

- Most important quantity: surface energy density σ
  Solve EoMs of DW

→ spatial field configurations

- Problem:
  - Vacuum structure of the 2HDMS is very rich
  - ⇒ Numerical instability
  - ⇒ Rescaling
- Use Minimization conditions to obtain VeVs for given parameter-configuration
- Check against bounded-from-below conditions

- „Benchmark" point, vary m_{12} and μ_{12} :

$$\frac{\partial^2 \phi_n}{\partial z^2} = \frac{dV}{d\phi_n}$$

$$(\phi_1, \phi_2, \phi_S) \xrightarrow{z\to\infty} (v_1, v_2, v_S)$$

$$(\phi_1, \phi_2, \phi_S) \xrightarrow{z\to-\infty} (v_1, -v_2, v_S)$$

$$\rho_{DW}(z) = \left(\frac{d\phi_1(z)}{dz}\right)^2 + \left(\frac{d\phi_2(z)}{dz}\right)^2 + \left(\frac{d\phi_S(z)}{dz}\right)^2 + V((\phi_1(z), \phi_2(z), \phi_S(z)) - V(v_1, v_2, v_S)$$

$$\widehat{z} = m_{22} z, \quad \widehat{E} = \frac{\lambda_2 E}{m_{22}^3}, \quad \mu^2 = -\frac{m_{11}^2}{m_{22}^2},$$

$$\widehat{m}_{12}^2 = \frac{m_{12}^2}{m_{22}^2}, \quad \mu_S^2 = -\frac{m_S^2}{m_{22}^2}, \quad \lambda = \frac{\lambda_1}{\lambda_2},$$

$$g = \frac{\lambda_3 + \lambda_4}{2\lambda_2}, \quad \widehat{\mu}_{s1} = \frac{\mu_{s1}}{\sqrt{\lambda_2} m_{22}}, \quad \widehat{\mu}_{12} = \frac{\mu_{12}}{\sqrt{\lambda_2} m_{22}}$$

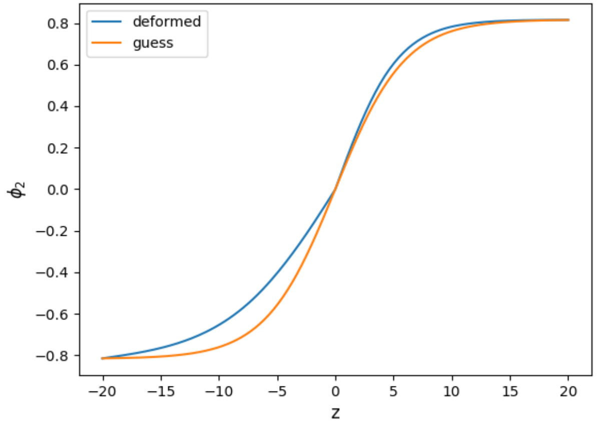
$$g_i = \frac{\lambda_i'}{\lambda_2} \quad \text{for} \quad i = 1, 2, 3$$

$$\mu^2 = -1.0, \quad \mu_S^2 = -0.5, \quad \lambda = 1.0, \quad g = 0.25, \quad \widehat{\mu}_{s1} = -0.2, \quad g_1 = 0.1, \quad g_2 = -0.1, \quad g_3 = 0.5$$

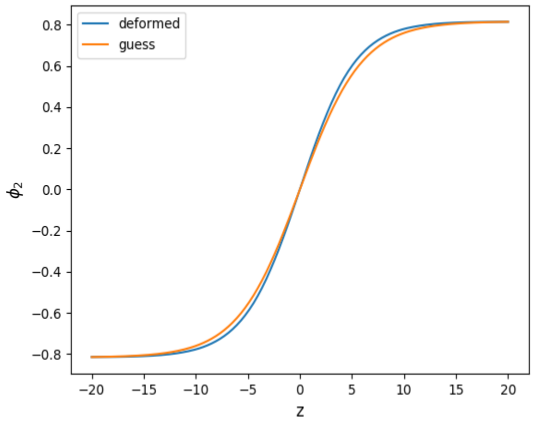
Solving the DW EoMs

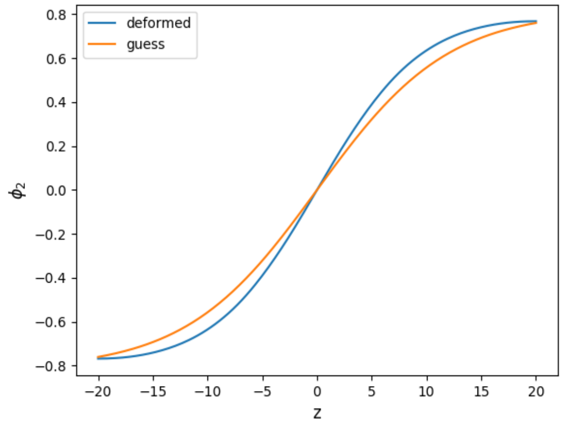
Path deformation (CosmoTransitions)

CosmoTransitions → routine to calculate phase transitions and tunneling solutions in multiple field dimensions (arXiv:1109.4189, Wainwright, C.L.)
Method used in: arXiv:2006.06913, Chen et al; arXiv:2004.10148, Chen et al



- General approach: split the EoMs into a part parallel to the path and a part perpendicular
  → Solve the 1D problem with overshooting/undershooting method
- Deform solution path, to fulfill perpendicular EoMs
- Problems:
  - Need for a good initial guess
  - Solution has a kink as soon as bias-term is present, due to numerical procedure
  - Code is quite complex and therefore very fragile





Solving the DW EoMs

Gradient flow method

Method introduced in:
arXiv:2006.13273, Battye et al.

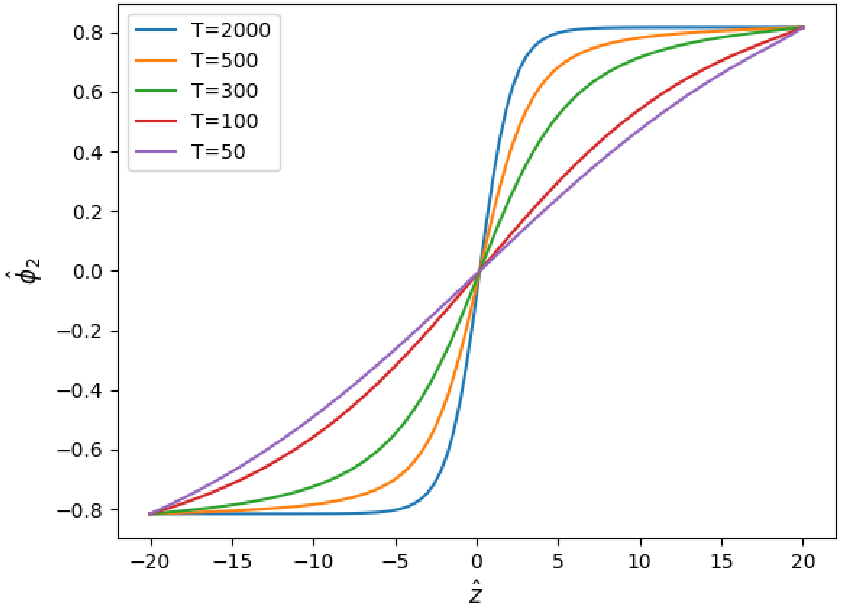
Looking for minimum energy configurations of the fields

- Introduce an artificial time parameter t → simulation time
- Gradient flow is limit of gradient descent method with step-size close to zero
- Very stable solver!
- Disadvantage: Calculation takes a lot of time

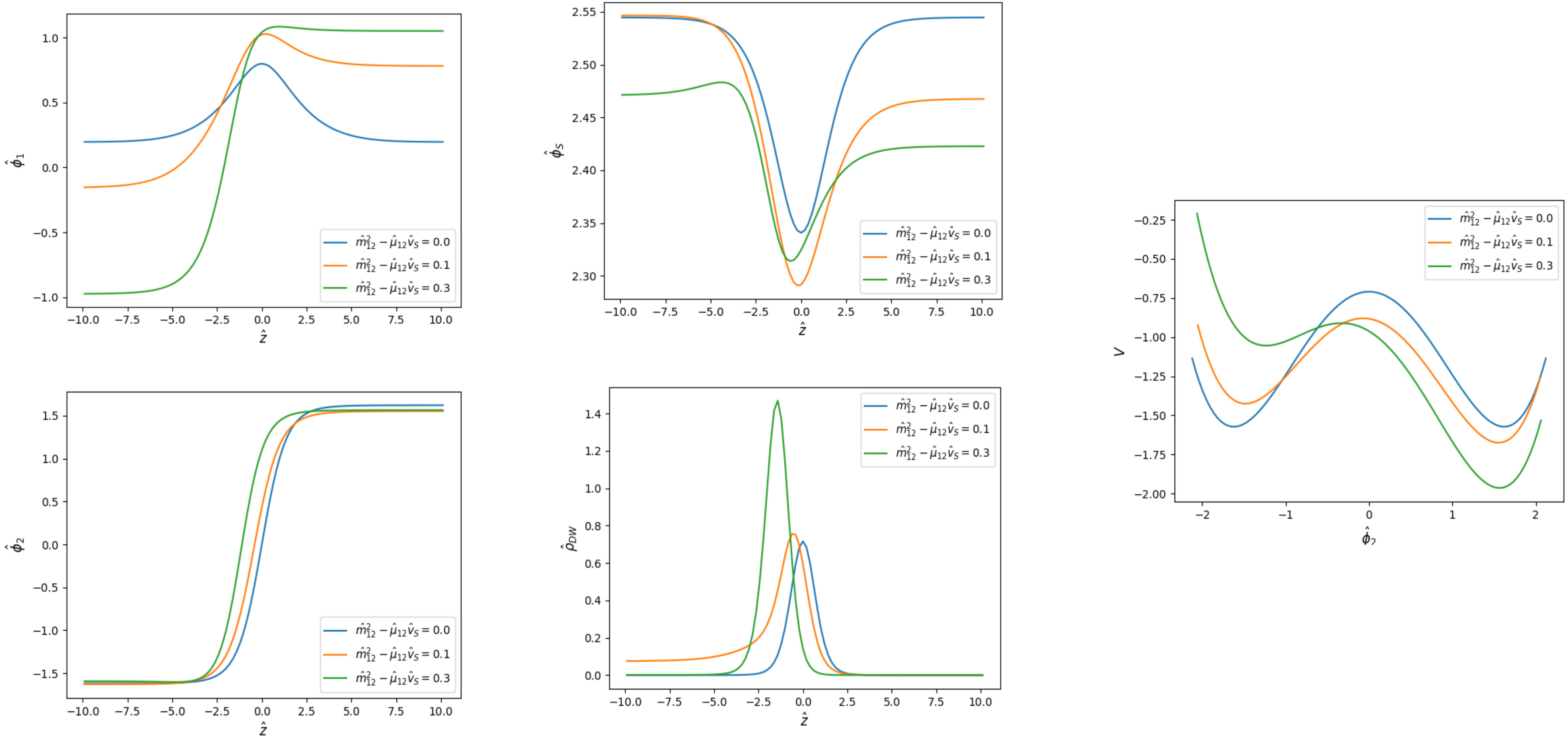
$$\frac{\partial \phi_n}{\partial t} = -\frac{\delta E}{\delta \phi_n} = \frac{\partial}{\partial x}\left(\frac{\partial \mathcal{E}}{\partial\left(\partial \phi_n / \partial x\right)}\right) - \frac{\partial \mathcal{E}}{\partial \phi_n}.$$

$$\lim_{t\to\infty} \frac{\partial \phi_n}{\partial t} = 0,$$

$$\frac{\partial \phi_n}{\partial t} = \frac{\partial^2 \phi_n}{\partial z^2} - \frac{dV}{d\phi_n}$$



DW solution for the 2HDMS



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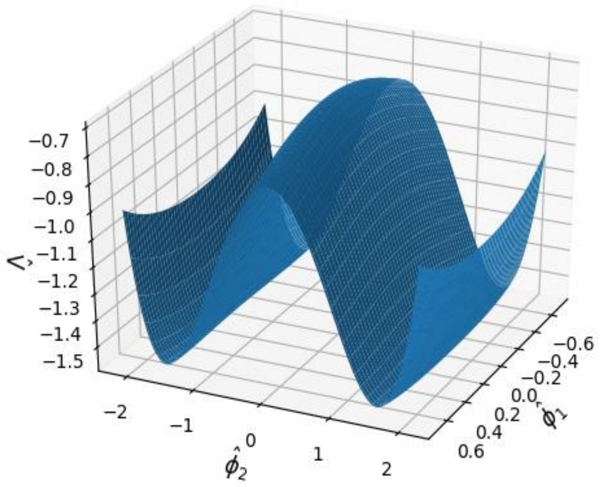
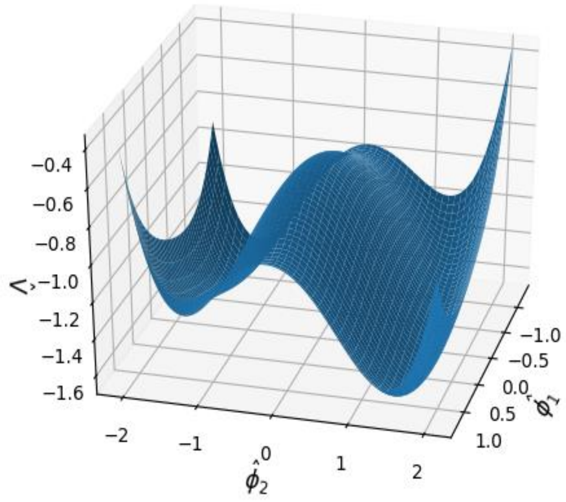
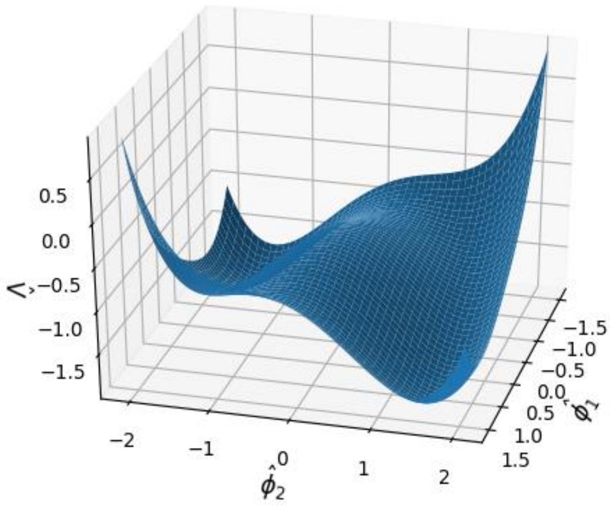
DW solution for the 2HDMS

Potential shapes for different bias values







Summary

- Domain walls can put cosmological constraints on BSM-models
- Domain wall solutions for $\mathbb{Z}_2$-symmetry of the 2HDMS can be found via Gradient Flow techniques
- Domain walls might form between vacua of accidental symmetry of the 2HDMS when bias term is large

Outlook:

- Start with physical scans
- Finite temperature effects
- Calculate GW spectra

Thank you