Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Exploring a Supersymmetric Four-Higgs Doublet Model

Lucas Willanzheimer, Franziska Lohner, Gudrid Moortgat-Pick

30.05.2024 Master Colloquium


Foto: Midjourney

Outline

1 Why considering four Higgs doublets?

2 Superpotential and Higgs sector

3 Yukawa Structure of the 4HDSSM

4 Spectrum

1

WHY CONSIDERING FOUR HIGGS DOUBLETS?

WHY FOUR HIGGS DOUBLETS?

- Indications for new Higgs bosons at the LHC (local significance $\sim 3\sigma$) (see [Bieкötter et al, 2021])
  - Resonances at around 400 GeV:
    - $A \to t\bar{t}$ [CMS, 2019]
    - $\phi \to \tau^+\tau^-$ [ATLAS, 2020]
  - Resonances at around 96 GeV:
    - $pp \to \phi \to \gamma\gamma$ [CMS, 2018]
    - $e^+e^- \to Z\phi \to b\bar{b}$ [LEP, 2003]

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Why four Higgs doublets?

- (N)MSSM does not offer enough flexibility in the couplings to fit all excesses simultaneously due to the type II like Yukawa structure **[Biekötter et al, 2021]**
- Our goal: building a SUSY model with “private“ Yukawa couplings
  - Each type of matter couples to its “own“ doublet
  - Minimal doublet extension adds two doublets (with opposite hypercharge) to preserve anomaly-cancellation
  - Can this model fit all excesses?
  - Today: only first steps towards the SUSY model

2

SUPERPOTENTIAL AND HIGGS SECTOR

HIGGS SECTOR OF THE SUPERSYMMETRIC 4HDM

- Adding two Higgs superfields $(\widehat{H}_{u2}, \widehat{H}_{d2})$ with the same charges under $SU(2)_L \times U(1)_Y$ as the MSSM superfields.
- Superpotential:

$$\mathcal{W} = \mathcal{W}_{Yuk} + \mu_{11}\hat{H}_{u1}\hat{H}_{d1} + \mu_{12}\hat{H}_{u1}\hat{H}_{d2} + \mu_{21}\hat{H}_{u2}\hat{H}_{d1} + \mu_{22}\hat{H}_{u2}\hat{H}_{d2}$$

- Scalar part of superfields:

$$\Phi_{ui} = \begin{pmatrix} H_{ui}^{+} \\ H_{ui}^{0} \end{pmatrix} \qquad \Phi_{di} = \begin{pmatrix} H_{di}^{0} \\ H_{di}^{-} \end{pmatrix} \qquad (i = 1, 2).$$

$$H_k^0 = \frac{1}{\sqrt{2}}(v_k + h_k + i a_k) \qquad k \in \{u1, u2, d1, d2\}.$$

$$v^2 = v_{u1}^2 + v_{u2}^2 + v_{d1}^2 + v_{d2}^2 = (246\,\mathrm{GeV})^2$$

Higgs potential

[Gupta and Wells, 2009]

$$V=\sum_{i=1}^{2}\left(|\mu_{i1}|^2+|\mu_{i2}|^2+m_{ui}^2\right)\left(\left|H_{ui}^0\right|^2+\left|H_{ui}^+\right|^2\right)+\sum_{i=1}^{2}\left(|\mu_{1i}|^2+|\mu_{2i}|^2+m_{di}^2\right)\left(\left|H_{di}^0\right|^2+\left|H_{di}^-\right|^2\right)$$
$$+\left(\left(\mu_{11}^*\mu_{21}+\mu_{12}^*\mu_{22}\right)\left(H_{u1}^{0*}H_{u2}^0+H_{u1}^{+*}H_{u2}^+\right)+\text{c.c.}\right)+\left(\left(\mu_{11}^*\mu_{12}+\mu_{21}^*\mu_{22}\right)\left(H_{d1}^{0*}H_{d2}^0+H_{d1}^{-*}H_{d2}^-\right)+\text{c.c.}\right)$$
$$+\left(\sum_{i=1}^{2}\sum_{j=1}^{2}b_{ij}\left(H_{ui}^+H_{dj}^--H_{ui}^0H_{dj}^0\right)+\text{c.c.}\right)+\frac{g^2+g'^2}{8}\left(\sum_{i=1}^{2}\left(\left|H_{ui}^0\right|^2+\left|H_{ui}^+\right|^2-\left|H_{di}^0\right|^2-\left|H_{di}^-\right|^2\right)\right)^2$$
$$+\frac{g^2}{2}\left(\sum_{i=1}^{2}\left(\left|H_{ui}^{+*}H_{ui}^0+H_{di}^{0*}H_{di}^-\right|^2\right)-\sum_{i=1}^{2}\sum_{j=1}^{2}\left(\left|H_{ui}^0\right|^2-\left|H_{di}^0\right|^2\right)\left(\left|H_{uj}^+\right|^2-\left|H_{dj}^-\right|^2\right)\right).$$

- 12 (real) parameters in the potential + 3 angles from the vev parametrisation

$$v_{u1}=\frac{v\sin\beta}{\sqrt{1+\tan^2\omega}} \qquad v_{d1}=\frac{v\cos\beta}{\sqrt{1+\tan^2\omega}}$$
$$v_{u2}=\frac{v\sin\alpha\tan\omega}{\sqrt{1+\tan^2\omega}} \qquad v_{d2}=\frac{v\cos\alpha\tan\omega}{\sqrt{1+\tan^2\omega}}$$

Higgs potential

$$V = \sum_{i=1}^{2} \left(|\mu_{i1}|^2 + |\mu_{i2}|^2 + m_{ui}^2\right)\left(|H_{ui}^0|^2 + |H_{ui}^+|^2\right) + \sum_{i=1}^{2} \left(|\mu_{1i}|^2 + |\mu_{2i}|^2 + m_{di}^2\right)\left(|H_{di}^0|^2 + |H_{di}^-|^2\right)$$
$$+ \left(\left(\mu_{11}^*\mu_{21} + \mu_{12}^*\mu_{22}\right)\left(H_{u1}^{0*}H_{u2}^0 + H_{u1}^{+*}H_{u2}^+\right) + \text{c.c.}\right) + \left(\left(\mu_{11}^*\mu_{12} + \mu_{21}^*\mu_{22}\right)\left(H_{d1}^{0*}H_{d2}^0 + H_{d1}^{-*}H_{d2}^-\right) + \text{c.c.}\right)$$
$$+ \left(\sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij}\left(H_{ui}^+H_{dj}^- - H_{ui}^0H_{dj}^0\right) + \text{c.c.}\right) + \frac{g^2+g'^2}{8}\left(\sum_{i=1}^{2}\left(|H_{ui}^0|^2 + |H_{ui}^+|^2 - |H_{di}^0|^2 - |H_{di}^-|^2\right)\right)^2$$
$$+ \frac{g^2}{2}\left(\sum_{i=1}^{2}\left(|H_{ui}^{+*}H_{ui}^0 + H_{di}^{0*}H_{di}^-|^2\right) - \sum_{i=1}^{2}\sum_{j=1}^{2}\left(|H_{ui}^0|^2 - |H_{di}^0|^2\right)\left(|H_{uj}^+|^2 - |H_{dj}^-|^2\right)\right).$$

- 12 (real) parameters in the potential + 3 angles from the vev parametrisation

$$v_{u1} = \frac{v\sin\beta}{\sqrt{1+\tan^2\omega}} \qquad v_{d1} = \frac{v\cos\beta}{\sqrt{1+\tan^2\omega}}$$
$$v_{u2} = \frac{v\sin\alpha\tan\omega}{\sqrt{1+\tan^2\omega}} \qquad v_{d2} = \frac{v\cos\alpha\tan\omega}{\sqrt{1+\tan^2\omega}}$$

Minimizing the potential: The tadpole equations

$$\left.\frac{\partial V}{\partial H_k^0}\right|_{v_k} \stackrel{!}{=} 0 \longrightarrow$$

$$k \in \{u1, u2, d1, d2\}$$

$$0 = b_{11}\frac{v_{d1}}{v_{u1}} + b_{12}\frac{v_{d2}}{v_{u1}} - \mu_u\frac{v_{u2}}{v_{u1}} + \lambda\sum_{i=1}^{2}\left(v_{di}^2 - v_{ui}^2\right) - m_{u1}^2 - \mu_{11}^2 - \mu_{22}^2$$

$$0 = b_{22}\frac{v_{d2}}{v_{u2}} + b_{21}\frac{v_{d1}}{v_{u2}} - \mu_u\frac{v_{u1}}{v_{u2}} + \lambda\sum_{i=1}^{2}\left(v_{di}^2 - v_{ui}^2\right) - m_{u2}^2 - \mu_{11}^2 - \mu_{22}^2$$

$$0 = b_{11}\frac{v_{u1}}{v_{d1}} + b_{21}\frac{v_{u2}}{v_{d1}} - \mu_d\frac{v_{d2}}{v_{d1}} - \lambda\sum_{i=1}^{2}\left(v_{di}^2 - v_{ui}^2\right) - m_{d1}^2 - \mu_{11}^2 - \mu_{22}^2$$

$$0 = b_{22}\frac{v_{u2}}{v_{d2}} + b_{12}\frac{v_{u1}}{v_{d2}} - \mu_d\frac{v_{d1}}{v_{d2}} - \lambda\sum_{i=1}^{2}\left(v_{di}^2 - v_{ui}^2\right) - m_{d2}^2 - \mu_{11}^2 - \mu_{22}^2$$

$$\mu_u = \mu_{11}\mu_{21} + \mu_{12}\mu_{22}$$

$$\mu_d = \mu_{11}\mu_{12} + \mu_{21}\mu_{22}$$

$$\lambda = \frac{g^2 + g'^2}{8}$$

- Tadpole equations are easily solvable for soft-breaking mass terms
  - But: we lose control over leading contributions to the mass matrix diagonal elements
  - No viable analytical solution for other parameters
- Eliminate 4 parameters with 4 equations:
  - $15 - 4 = 11$ free parameters in the Higgs sector

Minimizing the potential: The tadpole equations

$$\left.\frac{\partial V}{\partial H_k^0}\right|_{v_k} \stackrel{!}{=} 0 \longrightarrow$$

$k \in \{u1, u2, d1, d2\}$

$$0 = b_{11}\frac{v_{d1}}{v_{u1}} + b_{12}\frac{v_{d2}}{v_{u1}} - \mu_u\frac{v_{u2}}{v_{u1}} + \lambda\sum_{i=1}^{2}\left(v_{di}^2 - v_{ui}^2\right) - m_{u1}^2 - \mu_{11}^2 - \mu_{22}^2$$

$$0 = b_{22}\frac{v_{d2}}{v_{u2}} + b_{21}\frac{v_{d1}}{v_{u2}} - \mu_u\frac{v_{u1}}{v_{u2}} + \lambda\sum_{i=1}^{2}\left(v_{di}^2 - v_{ui}^2\right) - m_{u2}^2 - \mu_{11}^2 - \mu_{22}^2$$

$$0 = b_{11}\frac{v_{u1}}{v_{d1}} + b_{21}\frac{v_{u2}}{v_{d1}} - \mu_d\frac{v_{d2}}{v_{d1}} - \lambda\sum_{i=1}^{2}\left(v_{di}^2 - v_{ui}^2\right) - m_{d1}^2 - \mu_{11}^2 - \mu_{22}^2$$

$$0 = b_{22}\frac{v_{u2}}{v_{d2}} + b_{12}\frac{v_{u1}}{v_{d2}} - \mu_d\frac{v_{d1}}{v_{d2}} - \lambda\sum_{i=1}^{2}\left(v_{di}^2 - v_{ui}^2\right) - m_{d2}^2 - \mu_{11}^2 - \mu_{22}^2$$

$$\mu_u = \mu_{11}\mu_{21} + \mu_{12}\mu_{22}$$

$$\mu_d = \mu_{11}\mu_{12} + \mu_{21}\mu_{22}$$

$$\lambda = \frac{g^2 + g'^2}{8}$$

- Tadpole equations are easily solvable for soft-breaking mass terms
  - But: we lose control over leading contributions to the mass matrix diagonal elements
  - No viable analytical solution for other parameters
- Eliminate 4 parameters with 4 equations:
  - $15 - 4 = 11$ free parameters in the Higgs sector

Mass matrix for CP-even Higgs

$$\left(\mathcal{M}_H^2\right)_{kl} = \frac{\partial^2 V}{\partial h_k \partial h_l} \longrightarrow \mathcal{M}_H^2 = \begin{pmatrix} \mathcal{M}^2_{H,11} & -b_{11} - 2\lambda v_{d1} v_{u1} & \mu_d + 2\lambda v_{d1} v_{d2} & -b_{21}^2 - 2\lambda v_{d1} v_{u2} \\ -b_{11} - 2\lambda v_{d1} v_{u1} & \mathcal{M}^2_{H,22} & -b_{12} - 2\lambda v_{d2} v_{u1} & \mu_u + 2\lambda v_{u1} v_{u2} \\ \mu_d + 2\lambda v_{d1} v_{d2} & -b_{12} - 2\lambda v_{u1} v_{d2} & \mathcal{M}^2_{H,33} & -b_{22} - 2\lambda v_{d2} v_{u2} \\ -b_{21} - 2\lambda v_{d1} v_{u2} & \mu_u + 2\lambda v_{u1} v_{u2} & -b_{22} - 2\lambda v_{d2} v_{u2} & \mathcal{M}^2_{H,44} \end{pmatrix}$$

with

$$\mathcal{M}^2_{H,11} = b_{11}\frac{v_{u1}}{v_{d1}} + b_{21}\frac{v_{u2}}{v_{d1}} - \mu_d \frac{v_{d2}}{v_{d1}} + 2\lambda v_{d1}^2$$

$$\mathcal{M}^2_{H,22} = b_{11}\frac{v_{d1}}{v_{u1}} + b_{12}\frac{v_{d2}}{v_{u1}} - \mu_u \frac{v_{u2}}{v_{u1}} + 2\lambda v_{u1}^2$$

$$\mathcal{M}^2_{H,33} = b_{22}\frac{v_{u2}}{v_{d2}} + b_{12}\frac{v_{u1}}{v_{d2}} - \mu_d \frac{v_{d1}}{v_{d2}} + 2\lambda v_{d2}^2$$

$$\mathcal{M}^2_{H,44} = b_{22}\frac{v_{d2}}{v_{u2}} + b_{21}\frac{v_{d1}}{v_{u2}} - \mu_u \frac{v_{u1}}{v_{u2}} + 2\lambda v_{u2}^2$$

$$\mu_u = \mu_{11}\mu_{21} + \mu_{12}\mu_{22}$$
$$\mu_d = \mu_{11}\mu_{12} + \mu_{21}\mu_{22}$$

$$\lambda = \frac{g^2 + g'^2}{8}$$

Diagonalizing the mass matrices

- Rotate into mass basis as usual
- Mass matrix: real and symmetric

  $\longrightarrow \mathcal{R}^{H^0,A^0,H^\pm} \in SO(4)$
- 6 mixing angles each
- Obtain masses via orthogonal transformation

$$\left(\mathcal{R}^{H^0}\right)^T \mathcal{M}_H^2 \mathcal{R}^{H^0} = \mathrm{diag}\left(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2, m_{H_4}^2\right)$$

$$\left(\mathcal{R}^{A^0}\right)^T \mathcal{M}_A^2 \mathcal{R}^{A^0} = \mathrm{diag}\left(0, m_{A_1}^2, m_{A_2}^2, m_{A_3}^2\right)$$

$$\left(\mathcal{R}^{H^\pm}\right)^T \mathcal{M}_{H^\pm}^2 \mathcal{R}^{H^\pm} = \mathrm{diag}\left(0, m_{H_1^\pm}^2, m_{H_2^\pm}^2, m_{H_3^\pm}^2\right)$$

- 13 physical Higgs bosons

$$\mathfrak{Re}\left(\begin{pmatrix} \sqrt{2}H_{d1}^0 - v_{d1} \\ \sqrt{2}H_{u1}^0 - v_{u1} \\ \sqrt{2}H_{d2}^0 - v_{d2} \\ \sqrt{2}H_{u2}^0 - v_{u2} \end{pmatrix}\right) = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{pmatrix} = \mathcal{R}^{H^0}\begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{pmatrix}$$

$$\mathfrak{Im}\left(\begin{pmatrix} \sqrt{2}H_{d1}^0 - v_{d1} \\ \sqrt{2}H_{u1}^0 - v_{u1} \\ \sqrt{2}H_{d2}^0 - v_{d2} \\ \sqrt{2}H_{u2}^0 - v_{u2} \end{pmatrix}\right) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \mathcal{R}^{A^0}\begin{pmatrix} G^0 \\ A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

$$\begin{pmatrix} H_{d_1}^{-\dagger} \\ H_{u_1}^+ \\ H_{d_2}^{-\dagger} \\ H_{u_2}^+ \end{pmatrix} = \mathcal{R}^{H^\pm}\begin{pmatrix} G^+ \\ H_1^+ \\ H_2^+ \\ H_3^+ \end{pmatrix}$$

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Upper Bound for the Lightest Higgs Mass

At tree level:

$$m_{H_1}^{\text{tree}} \leq M_Z \left|\cos^2\omega \cos 2\beta + \sin^2\omega \cos 2\alpha\right|$$

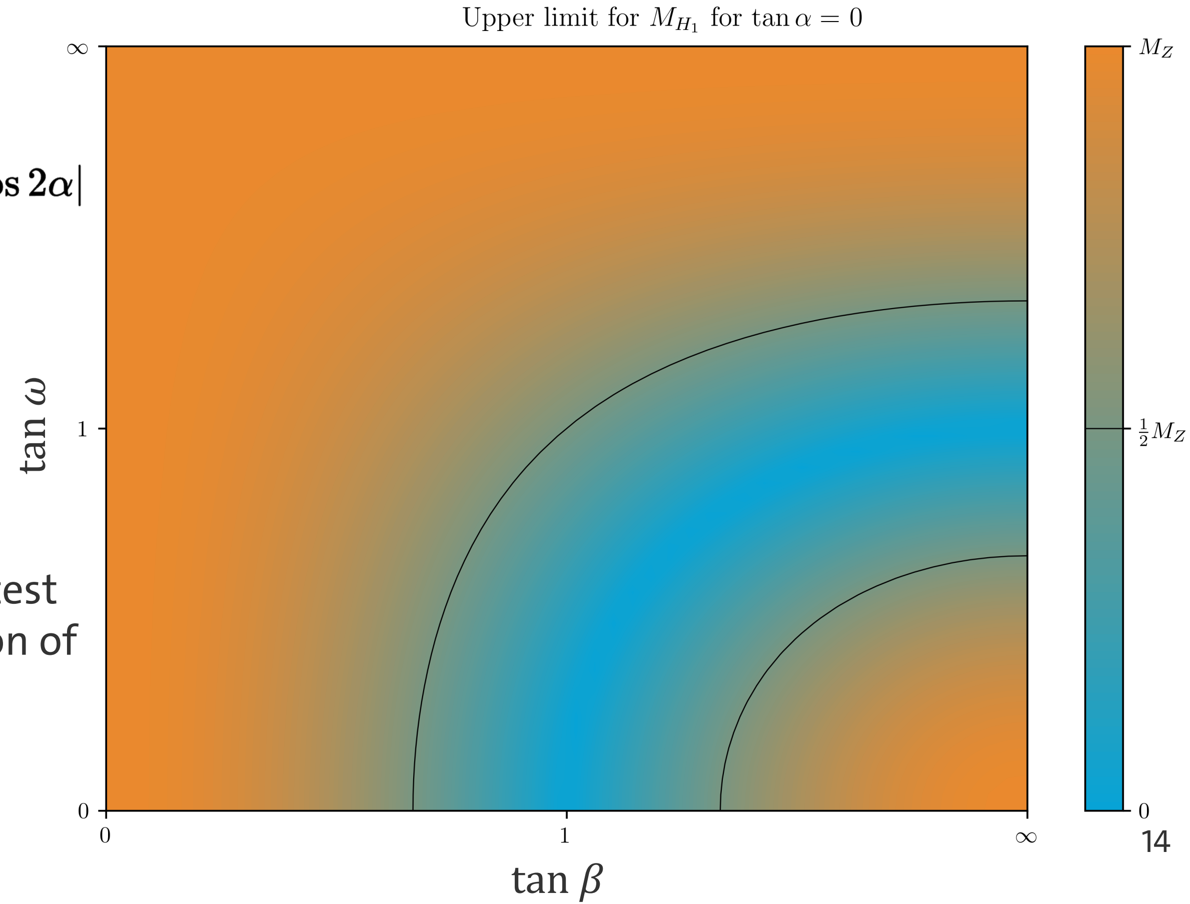


Figure 1:
Upper bound for lightest Higgs mass as function of $\tan\beta$ and $\tan\omega$

3

Yukawa Structure of the 4HDSSM

HIGGS-FERMION COUPLINGS IN THE PRIVATE TYPE 4HDSSM

- Let $\mathcal{W}_{Yuk} = -\hat{u}\mathbf{y_u}\hat{Q}\hat{H}_{u1} - \hat{d}\,\mathbf{y_d}\hat{Q}\hat{H}_{d1} - \hat{e}\mathbf{y_e}\hat{L}\hat{H}_{d2}$ [Arroyo-Urena et al, 2019]
- Calculate Higgs-fermion interactions (e. g. with SARAH [Staub et al, 2019])
- Define the effective coupling: $c_{\phi_i \bar{f} f} = \left| \frac{g_{\phi_i \bar{f} f}}{g_{H_{SM} \bar{f} f}} \right|$

$$c_{\phi_i \bar{b} b} = \left| \frac{R^{\phi}_{i1}}{\cos\beta \cos\omega} \right| \qquad c_{\phi_i \bar{t} t} = \left| \frac{R^{\phi}_{i2}}{\sin\beta \cos\omega} \right| \qquad c_{\phi_i \tau^+ \tau^-} = \left| \frac{R^{\phi}_{i3}}{\cos\alpha \sin\omega} \right|$$

- R^{ϕ} : respective Higgs mixing matrix
  - can be calculated numerically

HIGGS-FERMION COUPLINGS IN A PRIVATE TYPE 4HDSSM VS MSSM

- Let $\mathcal{W}_{Yuk} = -\hat{u}\mathbf{y_u}\hat{Q}\hat{H}_{u1} - \hat{d}\,\mathbf{y_d}\hat{Q}\hat{H}_{d1} - \hat{e}\mathbf{y_e}\hat{L}\hat{H}_{d2}$ [Arroyo-Urena et al, 2019]
- Calculate Higgs-fermion interactions (e. g. with SARAH [Staub et al, 2019])
- Define the effective coupling: $c_{\phi_i \bar{f} f} = \left| \frac{g_{\phi_i \bar{f} f}}{g_{H_{SM} \bar{f} f}} \right|$

$$c_{\phi_i \bar{b}b} = \left| \frac{R^{\phi}_{i1}}{\cos\beta\cos\omega} \right| \qquad c_{\phi_i \bar{t}t} = \left| \frac{R^{\phi}_{i2}}{\sin\beta\cos\omega} \right| \qquad c_{\phi_i \tau^+\tau^-} = \left| \frac{R^{\phi}_{i3}}{\cos\alpha\sin\omega} \right|$$

$$c^{\text{MSSM}}_{\phi_i \bar{b}b} = \left| \frac{\left(R^{\phi}_{\text{MSSM}}\right)_{i1}}{\cos\beta} \right| \qquad c^{\text{MSSM}}_{\phi_i \bar{t}t} = \left| \frac{\left(R^{\phi}_{\text{MSSM}}\right)_{i2}}{\sin\beta} \right| \qquad c^{\text{MSSM}}_{\phi_i \tau^+\tau^-} = \left| \frac{\left(R^{\phi}_{\text{MSSM}}\right)_{i1}}{\cos\beta} \right|$$

HIGGS-FERMION COUPLINGS IN A PRIVATE TYPE 4HDSSM

- For future work: full analysis of the production processes $b\bar{b} \to \phi_i \to f\bar{f}$, $gg \to \phi_i \to f\bar{f}$
- Requirements for $m_\phi \approx 400$ GeV [Biekötter et al, 2021]

$$\sigma(b\bar{b} \to \phi) > \sigma(gg \to \phi) \longrightarrow |c_{\phi b\bar{b}}| \gg |c_{\phi t\bar{t}}|$$

Sizable $BR(\phi \to \tau^+\tau^-) \longrightarrow |c_{\phi\tau^+\tau^-}| \gg |c_{\phi t\bar{t}}|$

$$c_{\phi_i \bar{b}b} = \left|\frac{R^\phi_{i1}}{\cos\beta\cos\omega}\right| \quad c_{\phi_i\tau^+\tau^-} = \left|\frac{R^\phi_{i3}}{\cos\alpha\sin\omega}\right|$$

$$c_{\phi_i \bar{t}t} = \left|\frac{R^\phi_{i2}}{\sin\beta\cos\omega}\right|$$

- Detailed analysis requires e. g. Monte Carlo Scan
- Effective Yukawa couplings in the 4HDSSM offer more flexibility due to dependence on additional angles

4

Spectrum

How to find a parameter point (in four easy steps) (1)

1. Implement 4HDSSM in SARAH **[Staub et al, 2019]**
2. Implement 4HDSSM in FlexibleSUSY **[Athron et al, 2017]**
3. Use CMSSM-like boundary conditions in the SUSY-sector but the full parameter space in the Higgs sector
4. Play around with parameters to find useful ranges & define exclusion limits

Parameter	Point B	Parameter range
m_0	321.21 GeV	100 GeV to 5000 GeV
$m_{1/2}$	2461.05 GeV	100 GeV to 5000 GeV
A_0	-3361.24 GeV	-5000 GeV to 5000 GeV
μ_{11}	211.19 GeV	50 GeV to 500 GeV
μ_{12}	0.017 GeV	-100 GeV to 100 GeV
μ_{21}	-0.955 GeV	-100 GeV to 100 GeV
μ_{22}	247.28 GeV	50 GeV to 500 GeV
b_{11}	$9.71 \times 10^5\,\mathrm{GeV}^2$	$1 \times 10^4\,\mathrm{GeV}^2$ to $1.5 \times 10^6\,\mathrm{GeV}^2$
b_{12}	$2.10 \times 10^4\,\mathrm{GeV}^2$	$1\,\mathrm{GeV}^2$ to $1 \times 10^6\,\mathrm{GeV}^2$
b_{21}	$9.04 \times 10^3\,\mathrm{GeV}^2$	$1\,\mathrm{GeV}^2$ to $1 \times 10^6\,\mathrm{GeV}^2$
b_{22}	$9.91 \times 10^5\,\mathrm{GeV}^2$	$1 \times 10^4\,\mathrm{GeV}^2$ to $1 \times 10^6\,\mathrm{GeV}^2$
$\tan\beta$	40.702	10 to 50
$\tan\alpha$	0.399	0.05 to 2
$\tan\omega$	0.124	0.05 to 5

How to find a parameter point (in four easy steps) (2)

- Exclusion limits:
  - Potential bounded from below
  - Perturbativity
  - Sparticle searches
  - 125-GeV Higgs
  - 400 GeV pseudoscalar

sparticle	lower limit	
$m_{\tilde{\chi}_1^0}$	220 GeV	$> \sqrt{s}_{\mathrm{LEP}}$
$m_{\tilde{\chi}_1^\pm}$	220 GeV	$> \sqrt{s}_{\mathrm{LEP}}$
$m_{\tilde{\nu}_1}$	220 GeV	$> \sqrt{s}_{\mathrm{LEP}}$
$m_{\tilde{e}_1}$	220 GeV	$> \sqrt{s}_{\mathrm{LEP}}$
$m_{\tilde{u}_1}$	1200 GeV	LHC limits
$m_{\tilde{d}_1}$	1200 GeV	LHC limits
$m_{\tilde{g}}$	2000 GeV	LHC limits

[Particle Data Group]

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Spectrum of the 4HDSSM

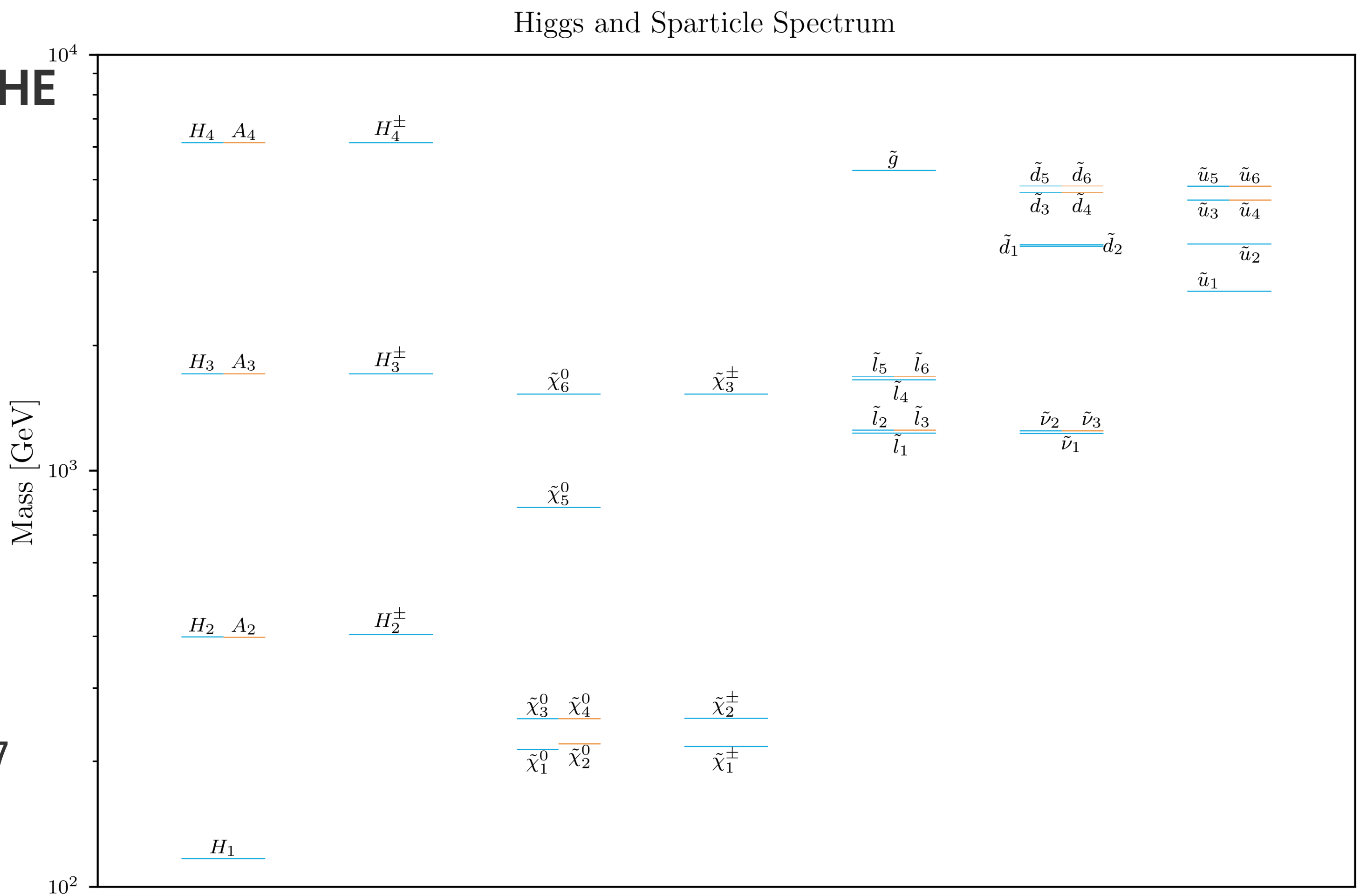


Figure 2:
Spectrum of the 4HDSSM, calculated with FlexibleSUSY 2.7
[Athron et al, 2017]

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Running of the Gauge Couplings

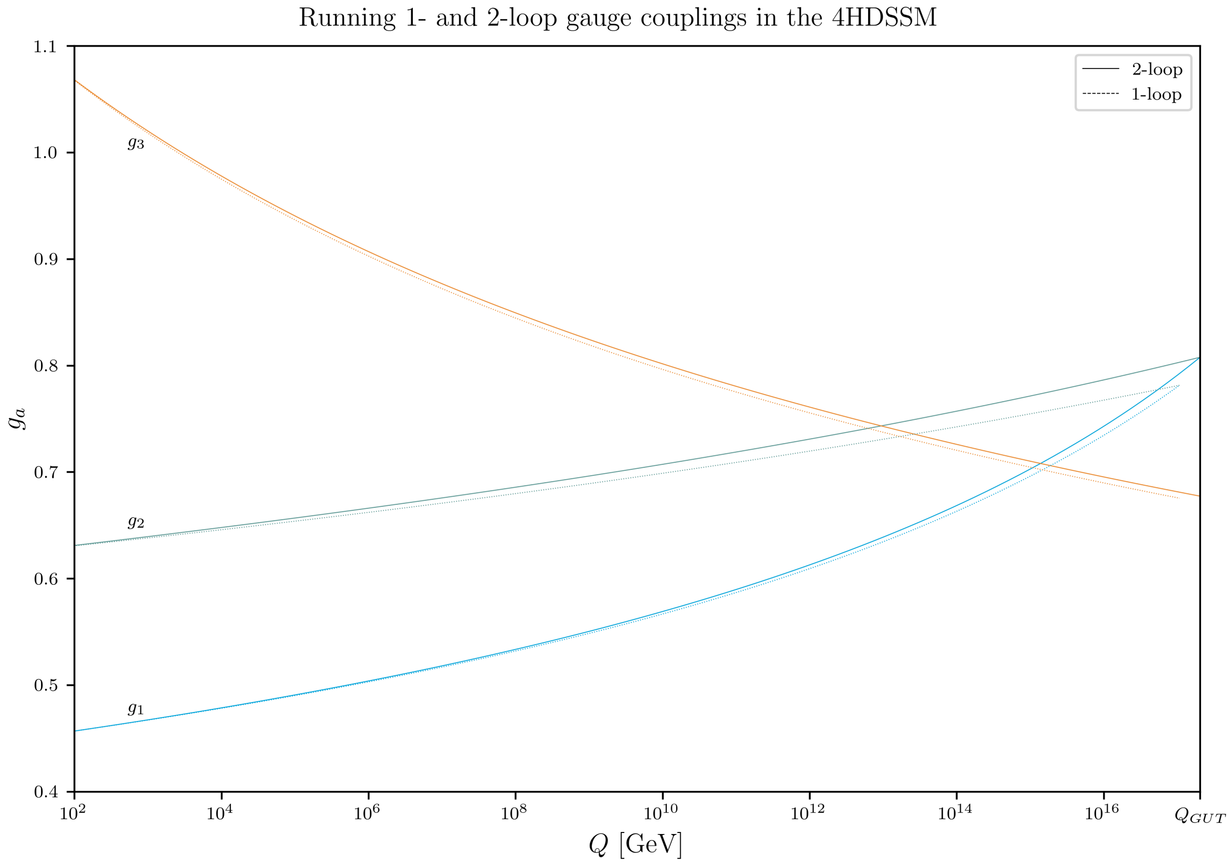


Fig 3: Running of the gauge couplings in the 4HDSSM at 1 and 2 loop-level calculated with FlexibleSUSY 2.7 **[Athron et al, 2017]**

Conclusion & Outlook

- SUSY 4HDM offers new flexibility in fermion couplings compared to MSSM
- Future work:
    - Implement Monte Carlo Markov Chain to investigate the parameter space in detail
    - Calculate production cross sections
    - Analyse the Higgs sector in HiggsTools [Bahl et al, 2022]
    - Add a singlet and investigate the 96 GeV excesses
    - Build future linear collider and find more Higgs
- Could be a promising candidate to explain excesses

$$c_{\phi_i \bar{b}b} = \left| \frac{R^{\phi}_{i1}}{\cos\beta\cos\omega} \right| \qquad c_{\phi_i \tau^+\tau^-} = \left| \frac{R^{\phi}_{i3}}{\cos\alpha\sin\omega} \right|$$

$$c_{\phi_i \bar{t}t} = \left| \frac{R^{\phi}_{i2}}{\sin\beta\cos\omega} \right|$$

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

The End

Thank You for Your attention

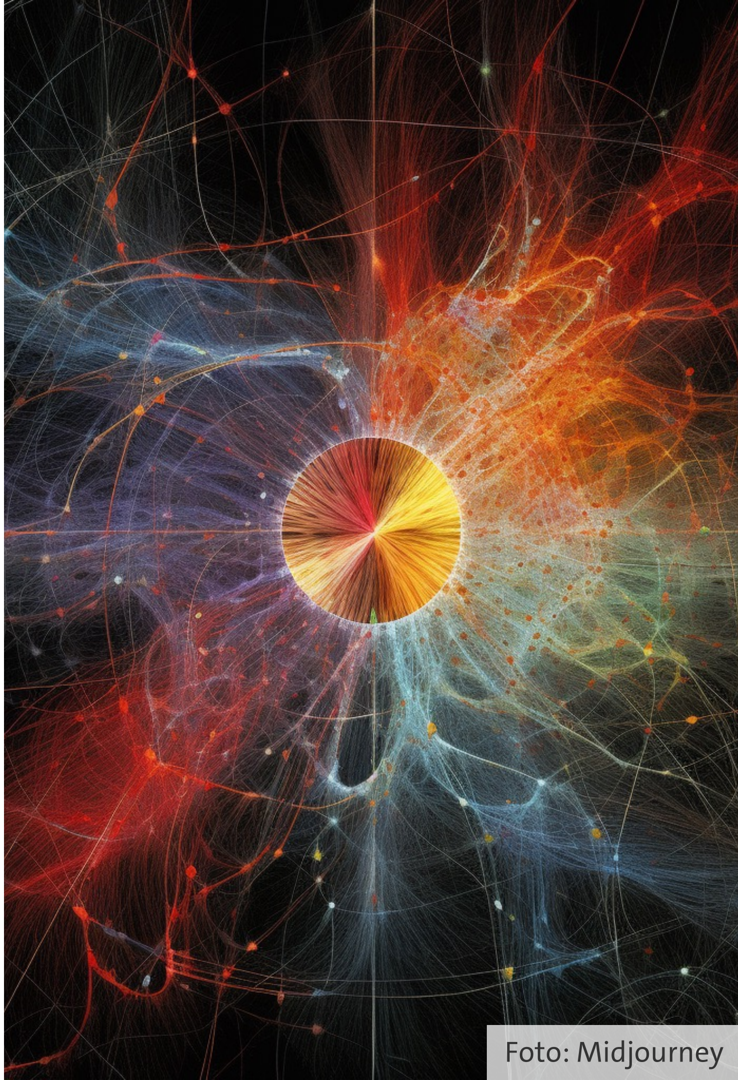


Foto: Midjourney

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Literature (Excesses)

CMS Collaboration. „Search for heavy Higgs bosons decaying to a top quark pair in proton-proton collisions at $\sqrt{s} = 13$ TeV“. In: *JHEP 04 (2020) 171, Erratum: JHEP 03 (2022) 187* 2020.4 (Aug. 3, 2019). ISSN: 1029-8479. DOI: 10.1007/jhep04(2020)171. arXiv: 1908.01115 [hep-ex].

ATLAS Collaboration. „Search for heavy Higgs bosons decaying into two tau leptons with the ATLAS detector using pp collisions at $\sqrt{s} = 13$ TeV“. In: *Phys. Rev. Lett. 125 (2020) 051801* 125.5 (Feb. 27, 2020), p. 051801. ISSN: 1079-7114. DOI: 10.1103/physrevlett.125.051801. arXiv: 2002.12223 [hep-ex].

ATLAS Collaboration. „Search for heavy resonances decaying into a W or Z boson and a Higgs boson in final states with leptons and b-jets in 36 fb^{-1} of $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector“. In: *JHEP 03 (2018) 174* 2018.3 (Dec. 18, 2017). ISSN: 1029-8479. DOI: 10.1007/jhep03(2018)174. arXiv: 1712.06518 [hep-ex].

LITERATURE (EXCESSES)

G. Abbiendi. „Search for the Standard Model Higgs Boson at LEP“. In: *Phys. Lett. B565, 2003* 565 (June 16, 2003), pp. 61–75. ISSN: 0370-2693. DOI: 10.1016/s0370-2693(03)00614-2. arXiv: hep-ex/0306033 [hep-ex].

CMS Collaboration. „Search for a standard model-like Higgs boson in the mass range between 70 and 110 GeV in the diphoton final state in proton-proton collisions at $\sqrt{s} = 8$ and 13 TeV“. In: *Phys. Lett. B 793 (2019) 320* 793 (Nov. 20, 2018), pp. 320–347. ISSN: 0370-2693. DOI: 10.1016/j.physletb.2019.03.064. arXiv: 1811.08459 [hep-ex].

Literature (SARAH)

F. Staub. „Sarah“. In: (June 2008). DOI: 10.48550/ARXIV.0806.0538. arXiv: 0806.0538 [hep-ph].

Florian Staub. „From Superpotential to Model Files for FeynArts and CalcHep/-CompHep“. In: *Comput.Phys.Commun.181:1077-1086,2010* 181.6 (Sept. 15, 2009), pp. 1077–1086. DOI: 10.1016/j.cpc.2010.01.011. arXiv: 0909.2863 [hep-ph].

Florian Staub. „Automatic Calculation of supersymmetric Renormalization Group Equations and Self Energies“. In: *Comput.Phys.Commun.182:808-833,2011* 182.3 (Feb. 3, 2010), pp. 808–833. DOI: 10.1016/j.cpc.2010.11.030. arXiv: 1002.0840 [hep-ph].

Florian Staub et al. „A tool box for implementing supersymmetric models“. In: *Computer Physics Communications 183 (2012) pp. 2165-2206* 183.10 (Sept. 23, 2011), pp. 2165–2206. DOI: 10.1016/j.cpc.2012.04.013. arXiv: 1109.5147 [hep-ph].

LITERATURE (FLEXIBLESUSY)

Peter Athron et al. „FlexibleSUSY – A spectrum generator generator for supersymmetric models“. In: *Computer Physics Communications 190 (2015) 139-172* 190 (June 9, 2014), pp. 139–172. ISSN: 0010-4655. DOI: 10.1016/j.cpc.2014.12.020. arXiv: 1406.2319 [hep-ph].

Peter Athron et al. „FlexibleSUSY 2.0: Extensions to investigate the phenomenology of SUSY and non-SUSY models“. In: *Computer Physics Communications* 230 (Oct. 10, 2017), pp. 145–217. ISSN: 0010-4655. DOI: 10.1016/j.cpc.2018.04.016. arXiv: 1710.03760 [hep-ph].

Peter Athron et al. „FlexibleDecay: An automated calculator of scalar decay widths“. In: *Comput. Phys. Commun. 283 (2023) 108584* 283 (June 9, 2021), p. 108584. ISSN: 0010-4655. DOI: 10.1016/j.cpc.2022.108584. arXiv: 2106.05038 [hep-ph].

Peter Athron et al. „Precise calculation of the W boson pole mass beyond the Standard Model with FlexibleSUSY“. In: *Physical Review D* 106.9 (Apr. 11, 2022), p. 095023. ISSN: 2470-0029. DOI: 10.1103/physrevd.106.095023. arXiv: 2204.05285 [hep-ph].

Literature

Thomas Biekötter et al. „Possible indications for new Higgs bosons in the reach of the LHC: N2HDM and NMSSM interpretations“. In: *Eur.Phys.J.C 82 (2022) 2, 178* 82.2 (Sept. 2, 2021). DOI: 10.1140/epjc/s10052-022-10099-1. arXiv: 2109.01128 [hep-ph].

Rick S. Gupta and James D. Wells. „Next generation Higgs bosons: Theory, constraints, and discovery prospects at the Large Hadron Collider“. In: *Phys. Rev. D* 81 (5 Dec. 1, 2009), p. 055012. DOI: 10.1103/PhysRevD.81.055012. arXiv: 0912.0267 [hep-ph].

M. A. Arroyo-Urena et al. „A Private SUSY 4HDM with FCNC in the Up-sector“. In: (Jan. 2019). arXiv: 1901.01304 [hep-ph].

Henning Bahl et al. „HiggsTools: BSM scalar phenomenology with new versions of HiggsBounds and HiggsSignals“. In: *Computer Physics Communications* 291 (Oct. 17, 2022), p. 108803. ISSN: 0010-4655. DOI: 10.1016/j.cpc.2023.108803. arXiv: 2210.09332 [hep-ph].

Conclusion & Outlook

- SUSY 4HDM offers new flexibility in fermion couplings compared to MSSM
- Future work:
  - Implement Monte Carlo Markov Chain to investigate the parameter space in detail
  - Calculate production cross sections
  - Analyse the Higgs sector in HiggsTools [Bahl et al, 2022]
  - Add a singlet and investigate the 96 GeV excesses
  - Build future linear collider and find more Higgs
- Could be a promising candidate to explain excesses

$$c_{\phi_i \bar{b} b} = \left| \frac{R^{\phi}_{i1}}{\cos\beta \cos\omega} \right| \qquad c_{\phi_i \tau^+ \tau^-} = \left| \frac{R^{\phi}_{i3}}{\cos\alpha \sin\omega} \right|$$

$$c_{\phi_i \bar{t} t} = \left| \frac{R^{\phi}_{i2}}{\sin\beta \cos\omega} \right|$$

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

APPENDIX A: UPPER BOUND FOR THE LIGHTEST HIGGS MASS

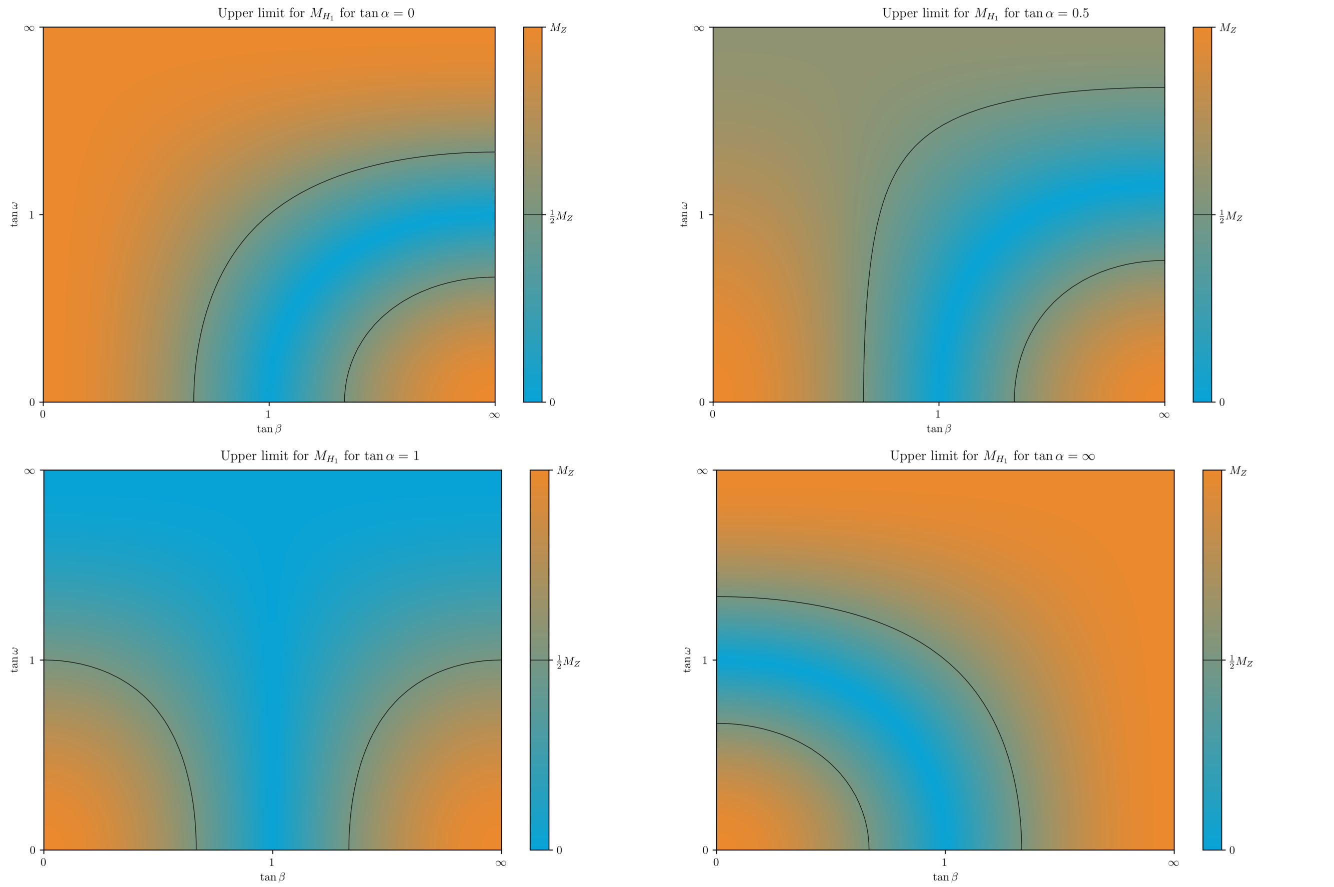


Fig B1: Upper limit for the lightest Higgs mass for different $\tan\alpha$

Appendix B: Mass matrix for CP-odd Higgs

$$\left(\mathcal{M}_A^2\right)_{kl} = \frac{\partial^2 V}{\partial a_k \partial a_l} \longrightarrow \mathcal{M}_A^2 = \begin{pmatrix} \mathcal{M}_{A,11}^2 & b_{11} & \mu_d & b_{21} \\ b_{11} & \mathcal{M}_{A,22}^2 & b_{12} & \mu_u \\ \mu_d & b_{12} & \mathcal{M}_{A,33}^2 & b_{22} \\ b_{21} & \mu_u & b_{22} & \mathcal{M}_{A,44}^2 \end{pmatrix}$$

with

$$\mathcal{M}_{A,11}^2 = b_{11}\frac{v_{u1}}{v_{d1}} + b_{21}\frac{v_{u2}}{v_{d1}} - \mu_d\frac{v_{d2}}{v_{d1}}$$

$$\mathcal{M}_{A,22}^2 = b_{11}\frac{v_{d1}}{v_{u1}} + b_{12}\frac{v_{d2}}{v_{u1}} - \mu_u\frac{v_{u2}}{v_{u1}}$$

$$\mathcal{M}_{A,33}^2 = b_{22}\frac{v_{u2}}{v_{d2}} + b_{12}\frac{v_{u1}}{v_{d2}} - \mu_d\frac{v_{d1}}{v_{d2}}$$

$$\mathcal{M}_{A,44}^2 = b_{22}\frac{v_{d2}}{v_{u2}} + b_{21}\frac{v_{d1}}{v_{u2}} - \mu_u\frac{v_{u1}}{v_{u2}}$$

$$\mu_u = \mu_{11}\mu_{21} + \mu_{12}\mu_{22}$$

$$\mu_d = \mu_{11}\mu_{12} + \mu_{21}\mu_{22}$$

$$\lambda = \frac{g + g'^2}{8}$$

Appendix C: Mass matrix for charged Higgs

$$\mathcal{M}^2_{H^\pm} = \mathcal{M}^2_A + \frac{g^2}{4}\begin{pmatrix} v_{u1}^2 + v_{u2}^2 - v_{d2}^2 & v_{d1}v_{u1} & v_{d1}v_{d2} & v_{d1}v_{u2} \\ v_{d1}v_{u1} & v_{d1}^2 + v_{d2}^2 - v_{u2}^2 & v_{d2}v_{u1} & v_{u1}v_{u2} \\ v_{d1}v_{d2} & v_{d2}v_{u1} & v_{u1}^2 + v_{u2}^2 - v_{d1}^2 & v_{d2}v_{u2} \\ v_{d1}v_{u2} & v_{u1}v_{u2} & v_{d2}v_{u2} & v_{d1}^2 + v_{d2}^2 - v_{u1}^2 \end{pmatrix}$$

Appendix D: Brute force ansatz for diagonalization

- $\mathbf{M}^2$ in general hermitian, here: real and symmetric
- Can be diagonalized with an 4×4 orthogonal matrix R:

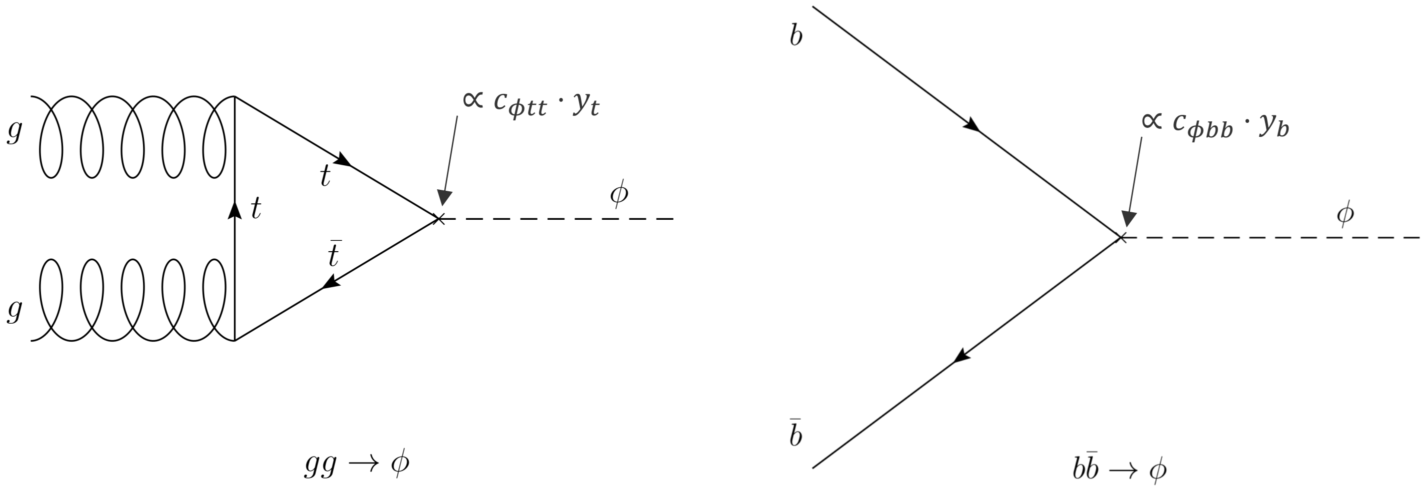
$$\mathrm{diag}\left(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2, m_{H_4}^2\right) = R^T \mathbf{M}^2 R$$

- Parametrize R as [Dita 2001]:

$$R = J_{3,4}(\vartheta_1)\, J_{2,3}(\varphi_1) J_{1,2}(\phi_1) J_{3,4}(\vartheta_2) J_{2,3}(\varphi_2) J_{3,4}(\vartheta_3) \quad \text{with} \quad J_{3,4}(\vartheta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\vartheta & -\sin\vartheta \\ 0 & 0 & \sin\vartheta & \cos\vartheta \end{pmatrix}$$

- 6 different mixing angles

Appendix E



To compensate $y_t \gg y_b$ $\rightarrow$ $c_{\phi\bar{b}b} \gg c_{\phi\bar{t}t}$ to obtain $\sigma(b\bar{b} \to \phi) > \sigma(gg \to \phi)$

APPENDIX F: PARAMETER EVOLUTION IN THE cMSSM

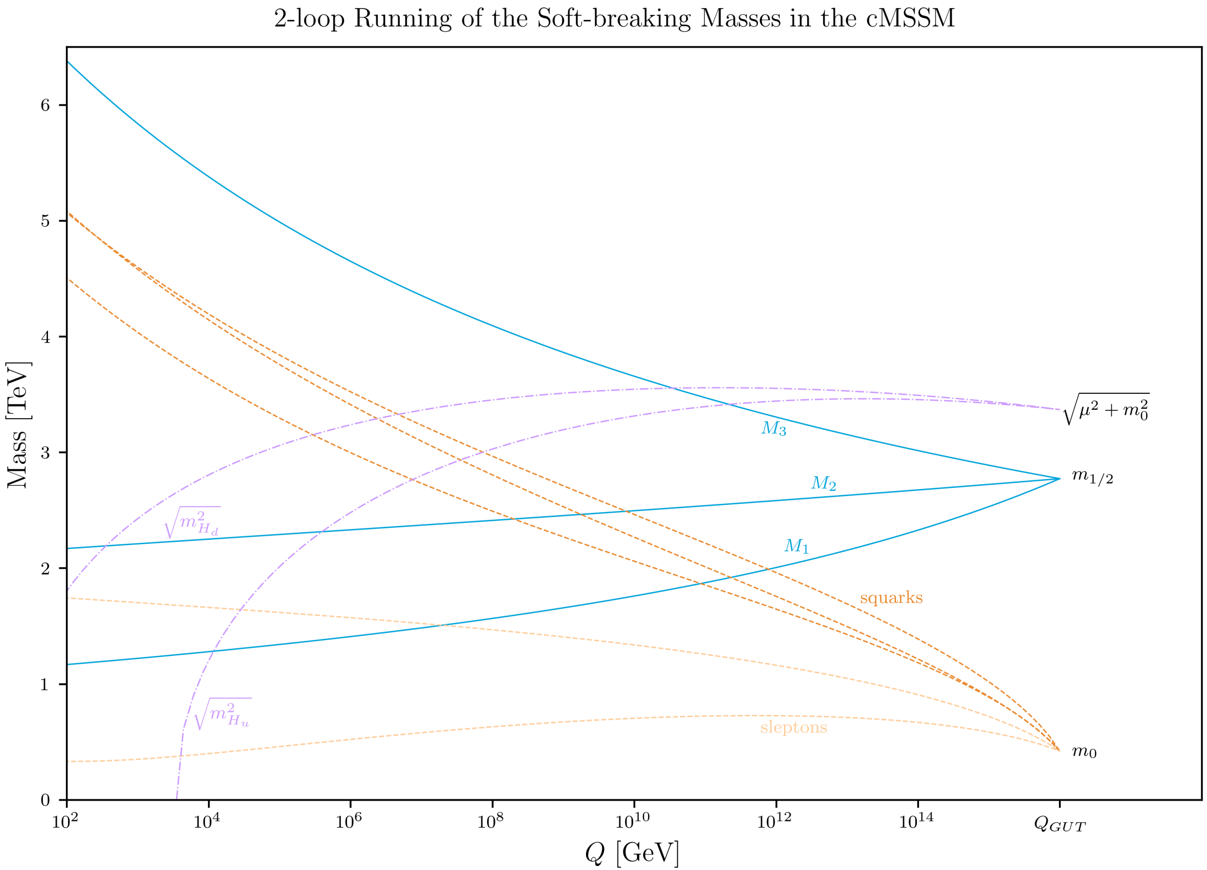


Figure F1:
Running soft-breaking parameters in the cMSSM

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Appendix G: Boundness from below in detail

- Pure SUSY: Bounded from below by design, but we include soft-breaking!
- Potential is bfb if the leading (quartic) terms are positive for arbitrary field values

$$V = \sum_{i=1}^{2} \left(|\mu_{i1}|^2 + |\mu_{i2}|^2 + m_{ui}^2\right) |H_{ui}^0|^2 + \sum_{i=1}^{2} \left(|\mu_{1i}|^2 + |\mu_{2i}|^2 + m_{di}^2\right) |H_{di}^0|^2$$
$$+ \left((\mu_{11}\mu_{21} + \mu_{12}\mu_{22}) H_{u1}^{0*} H_{u2}^0 + (\mu_{11}\mu_{12} + \mu_{21}\mu_{22}) H_{d1}^{0*} H_{d2}^0 + \text{c.c.}\right)$$
$$- \left(\sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij} \left(H_{ui}^0 H_{dj}^0\right) + \text{c.c.}\right) + \frac{g^2 + g'^2}{8} \left(\sum_{i=1}^{2} \left(|H_{ui}^0|^2 - |H_{di}^0|^2\right)\right)^2$$

- Consider D-flat direction of potential: $|H_{u1}| = |H_{u2}| = |H_{d1}| = |H_{d2}|$
- The quartic terms vanish in this direction, behaviour is determined by the quadratic terms
- The quadratic terms are positive if $\sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij} < \sum_{i=1}^{2}\sum_{j=1}^{2} \mu_{ij}^2 + (\mu_{11} + \mu_{22})(\mu_{12} + \mu_{21}) + \frac{1}{2}\sum_{i=1}^{2} (m_{ui}^2 + m_{di}^2)$

Appendix H: A_0 and the Higgs mass

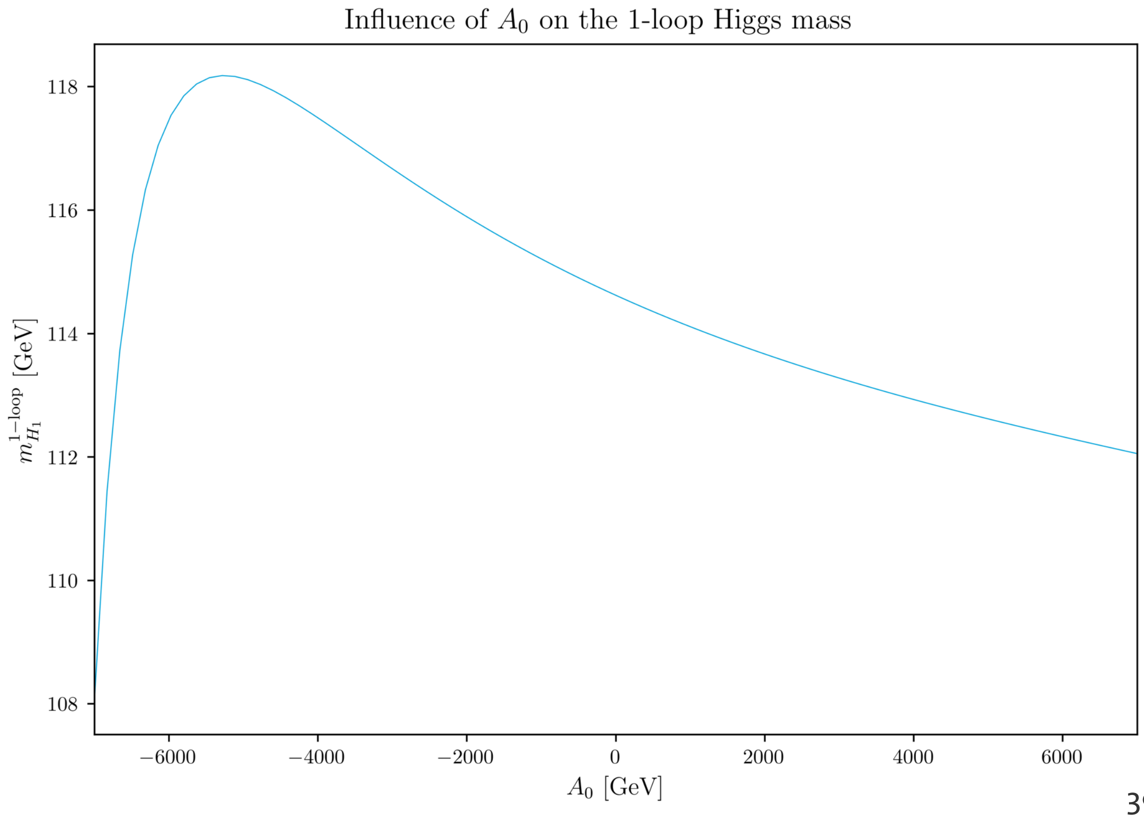


Fig. H1:
Influence of A_0 on the 1-loop Higgs mass

Appendix I: Boundary conditions

- Low scale: $Q_{Low} = M_Z$
  - Input: $\{\tan\beta, \tan\alpha, \tan\omega\}$

$$v_{u1} = \frac{v\sin\beta}{\sqrt{1+\tan^2\omega}} \qquad v_{d1} = \frac{v\cos\beta}{\sqrt{1+\tan^2\omega}}$$

$$v_{u2} = \frac{v\sin\alpha\tan\omega}{\sqrt{1+\tan^2\omega}} \qquad v_{d2} = \frac{v\cos\alpha\tan\omega}{\sqrt{1+\tan^2\omega}}$$

- SUSY scale: $Q_{SUSY} = \sqrt{m_{\widetilde{q_1}} m_{\widetilde{q_6}}}$
  - Input: $\{\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, b_{11}, b_{12}, b_{21}, b_{22}\}$

- High scale: $Q_{GUT} = Q(g_1 = g_2)$
  - Input: $\{m_0, m_{1/2}, A_0\}$

$$\mathcal{M}_Q^2 = \mathcal{M}_L^2 = \mathcal{M}_u^2 = \mathcal{M}_d^2 = \mathcal{M}_e^2 = m_0^2 \mathbb{I}_3,$$

$$\mathcal{T}_u = A_0 \mathcal{Y}_u, \qquad \mathcal{T}_d = A_0 \mathcal{Y}_d, \qquad \mathcal{T}_e = A_0 \mathcal{Y}_e,$$

$$M_1 = M_2 = M_3 = m_{1/2}.$$

Appendix J: Spectrum Insights

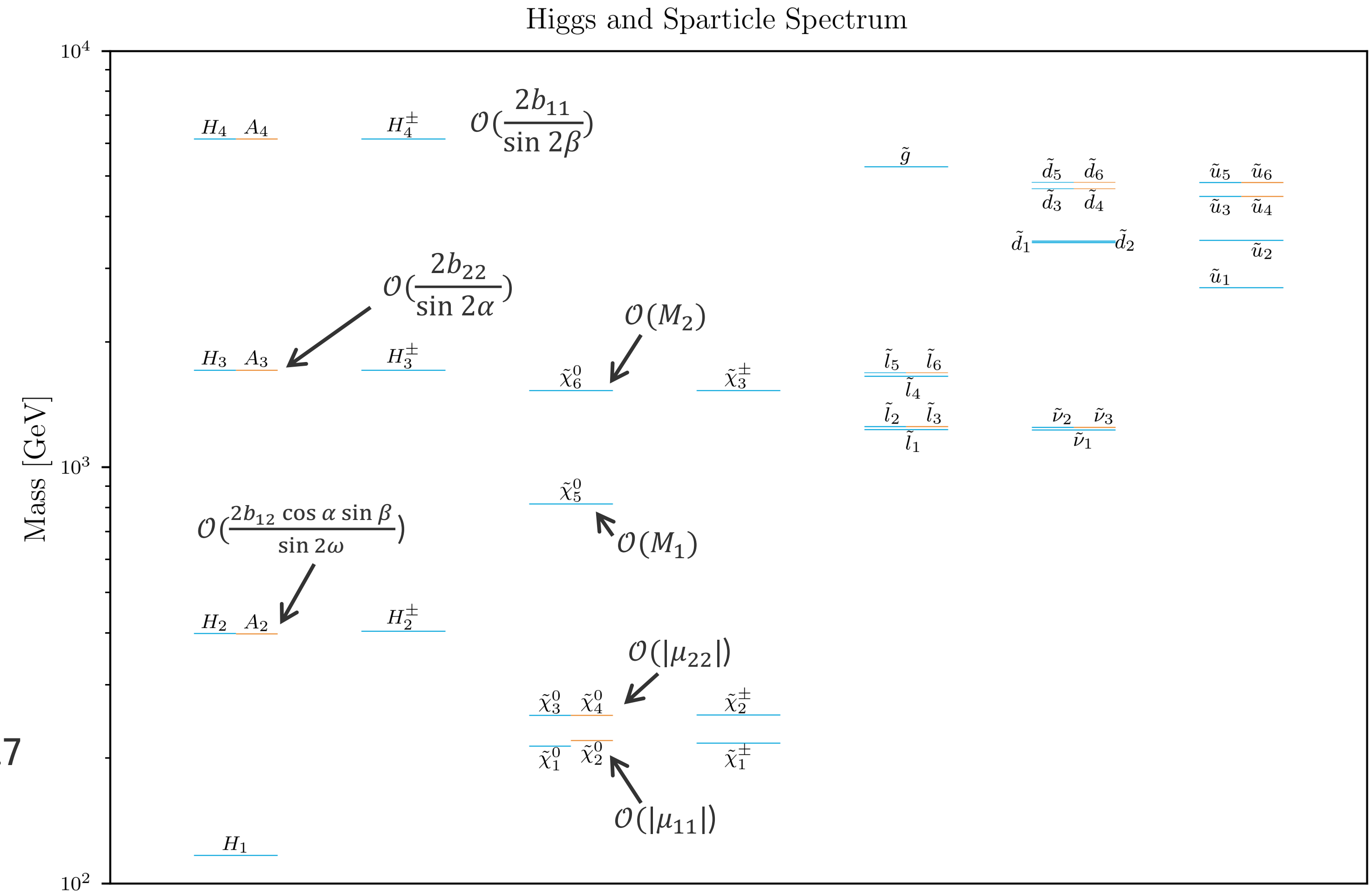


Figure J1:
Spectrum of the 4HDSSM, calculated with FlexibleSUSY 2.7
[Athron et al, 2017]

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG