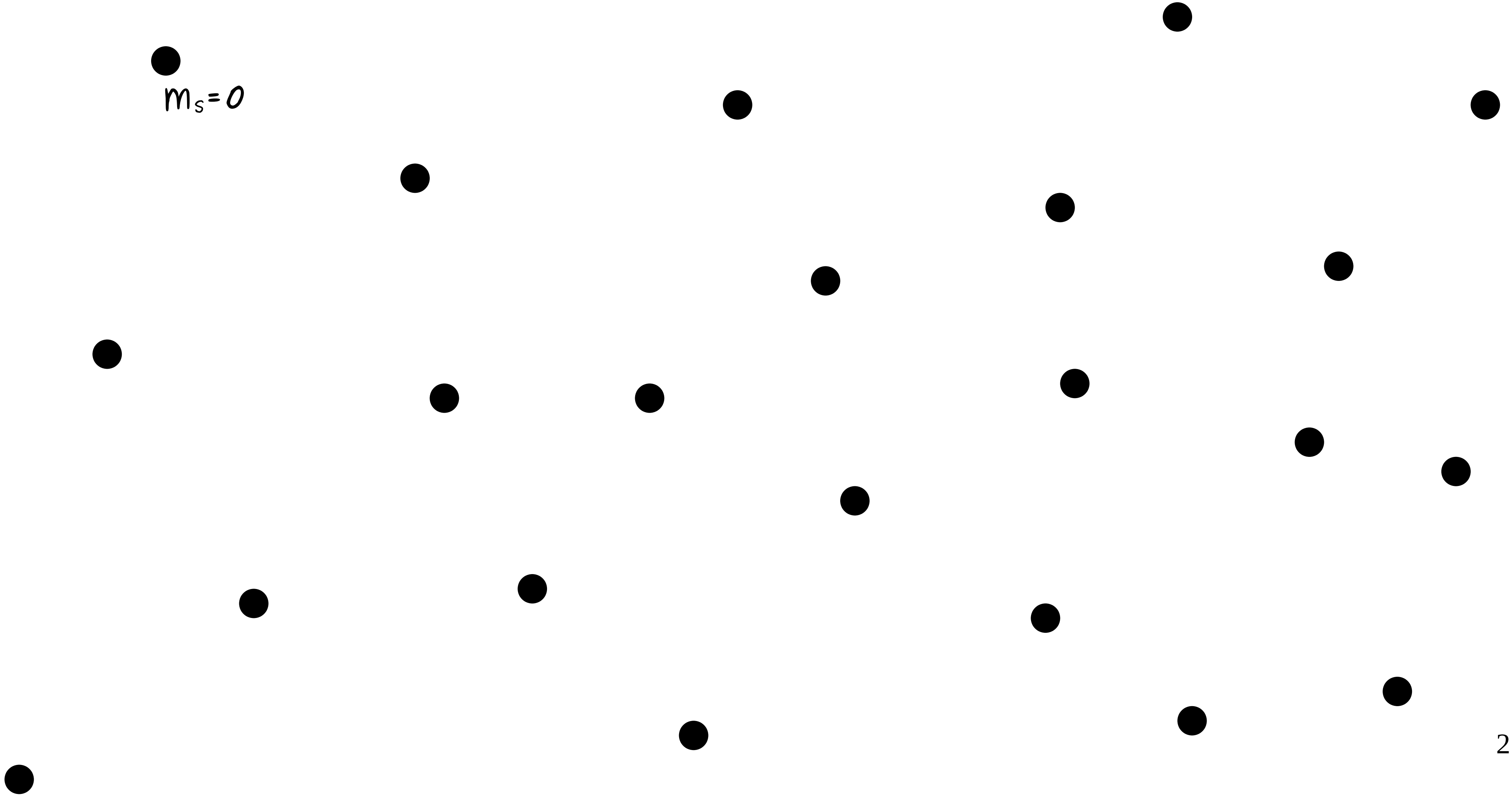
Phase transitions with symmetry restoration – when does the bubble stop running?

Speaker: Julia Ziegler
In collaboration with: Andrew Long, Bibhushan Shakya

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

symmetric phase

Boedeker & Moore, arXiv: 1703.08215



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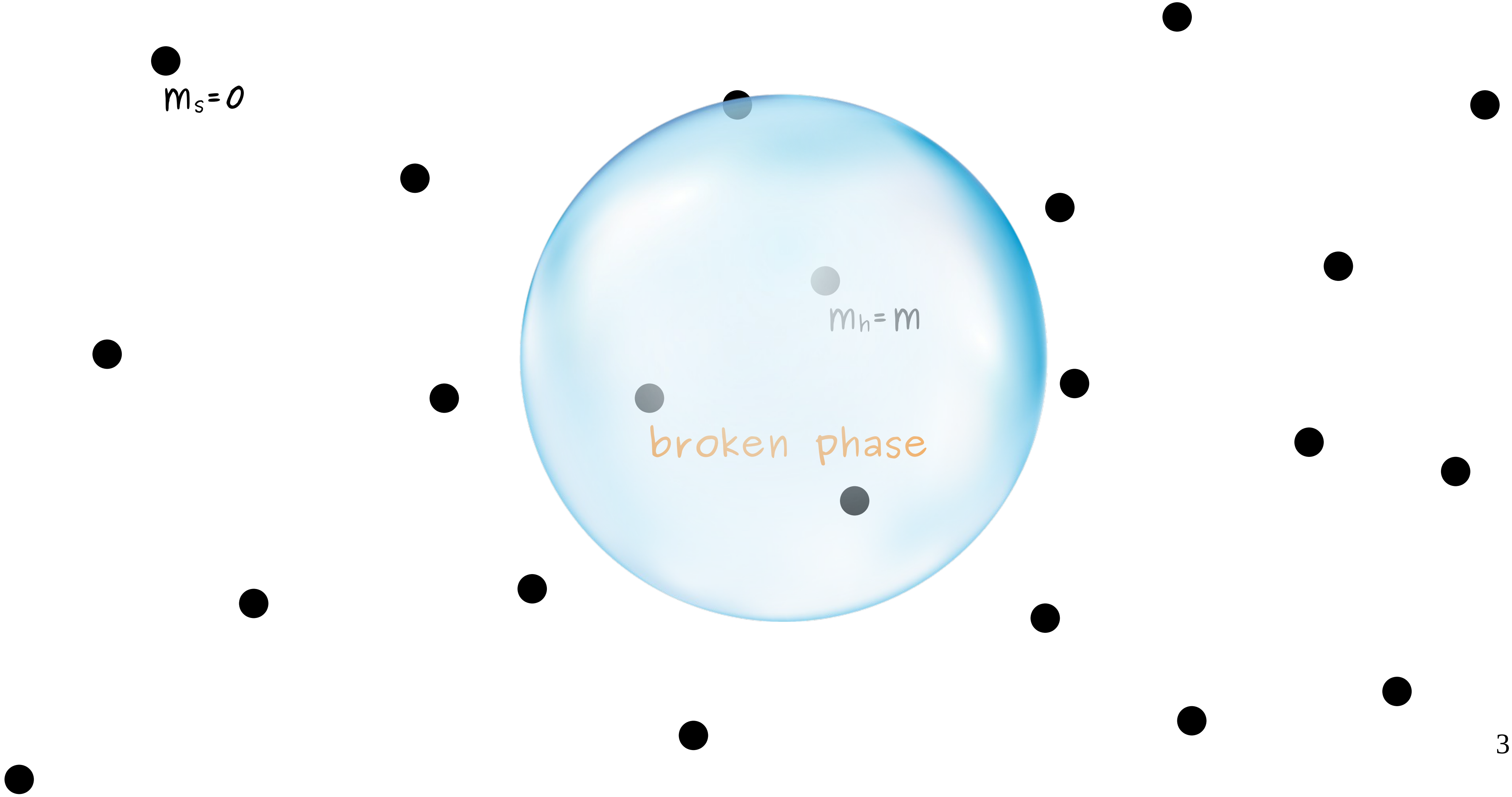

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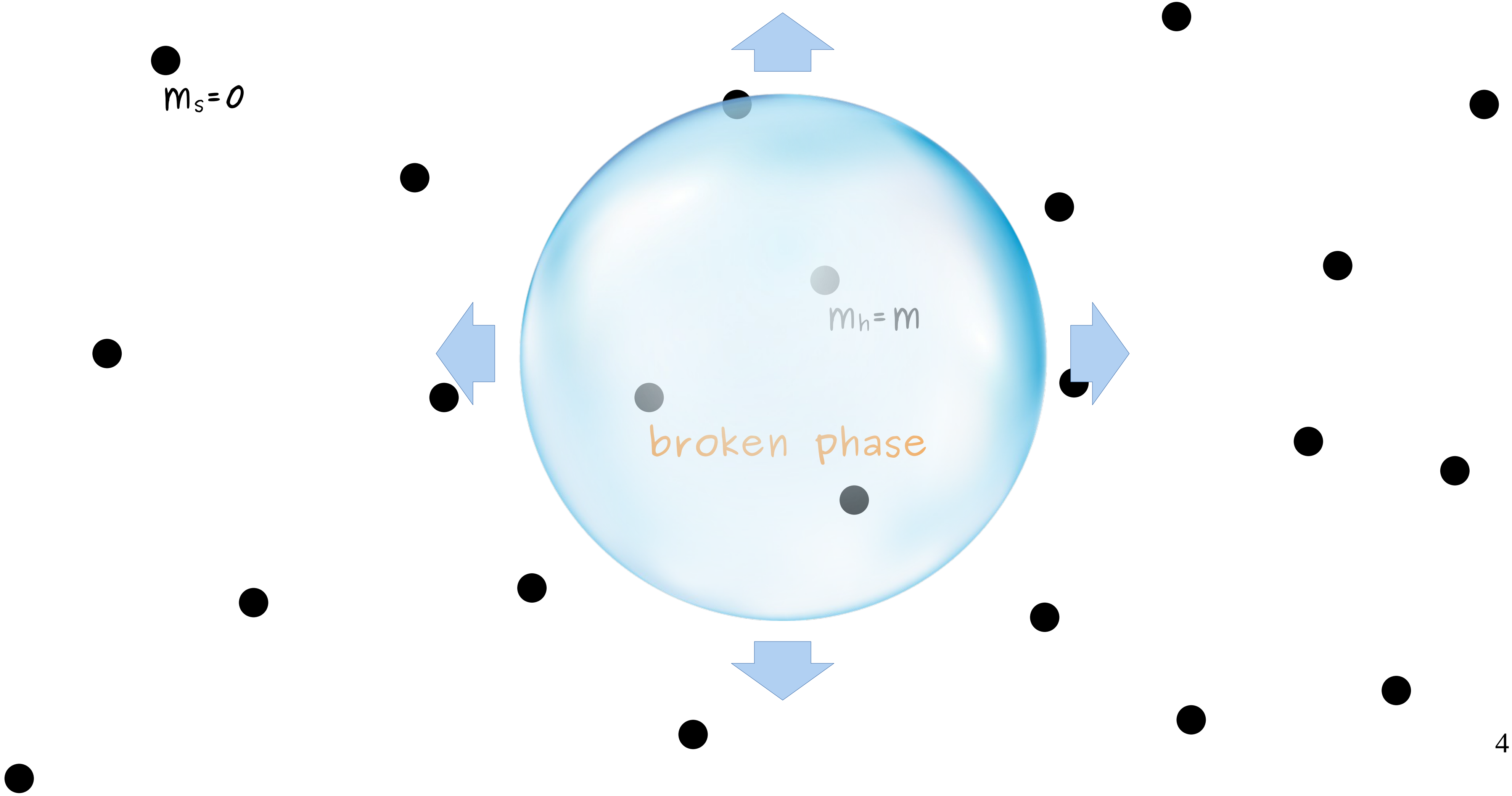

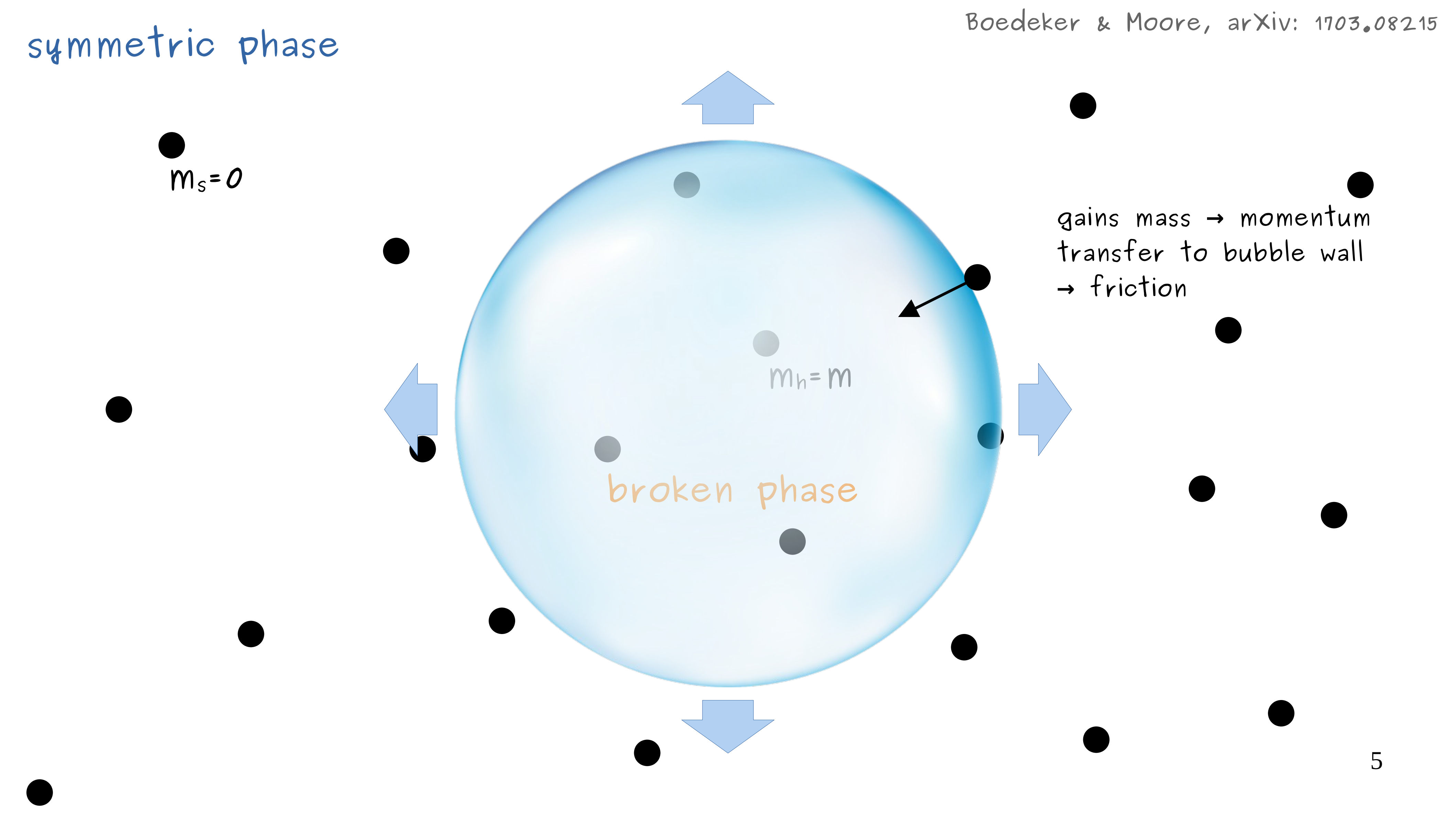
symmetric phase
Boedeker & Moore, arXiv: 1703.08215
$m_s = 0$
gains mass → momentum transfer to bubble wall → friction
$m_h = m$
broken phase

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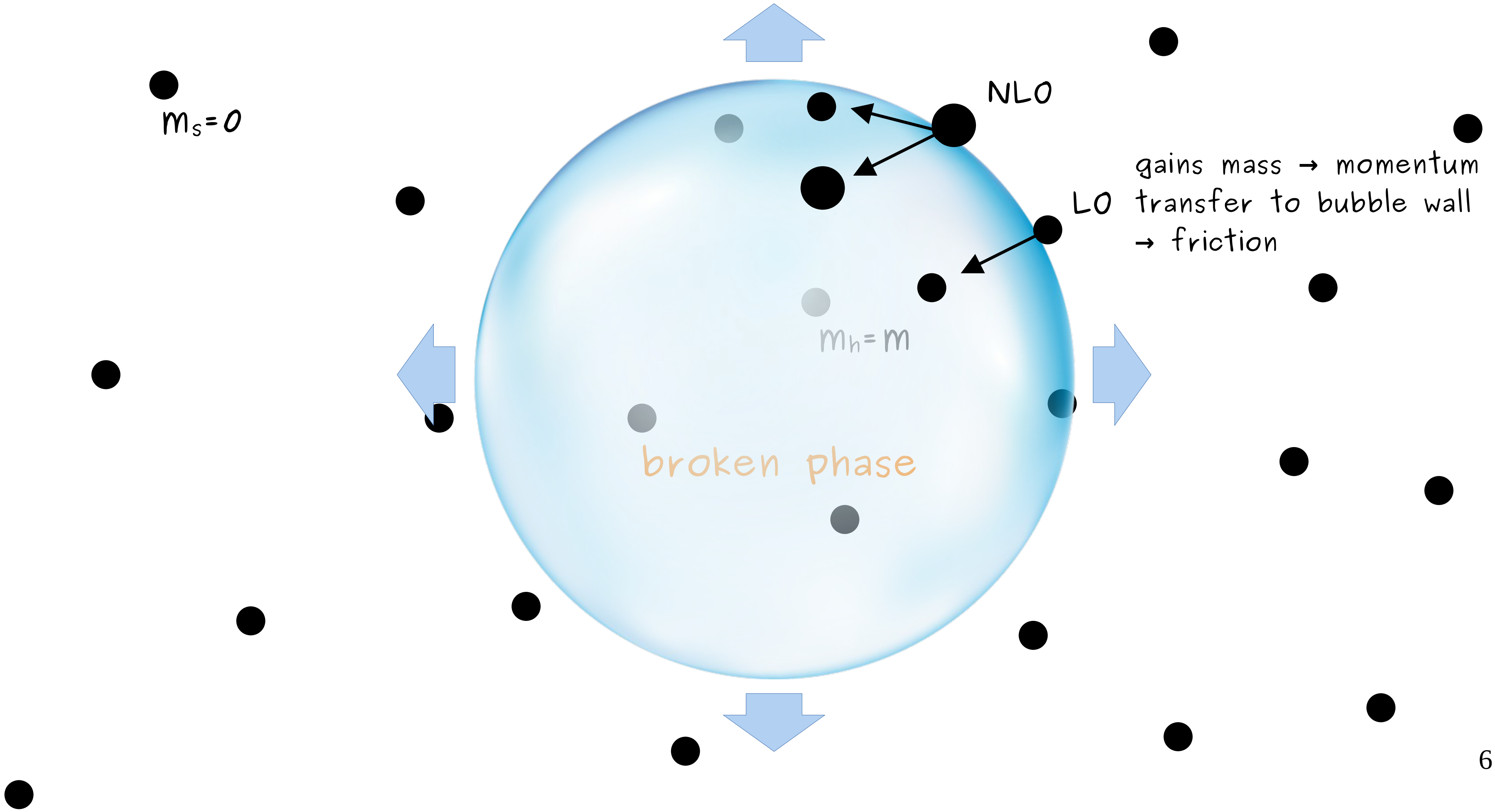

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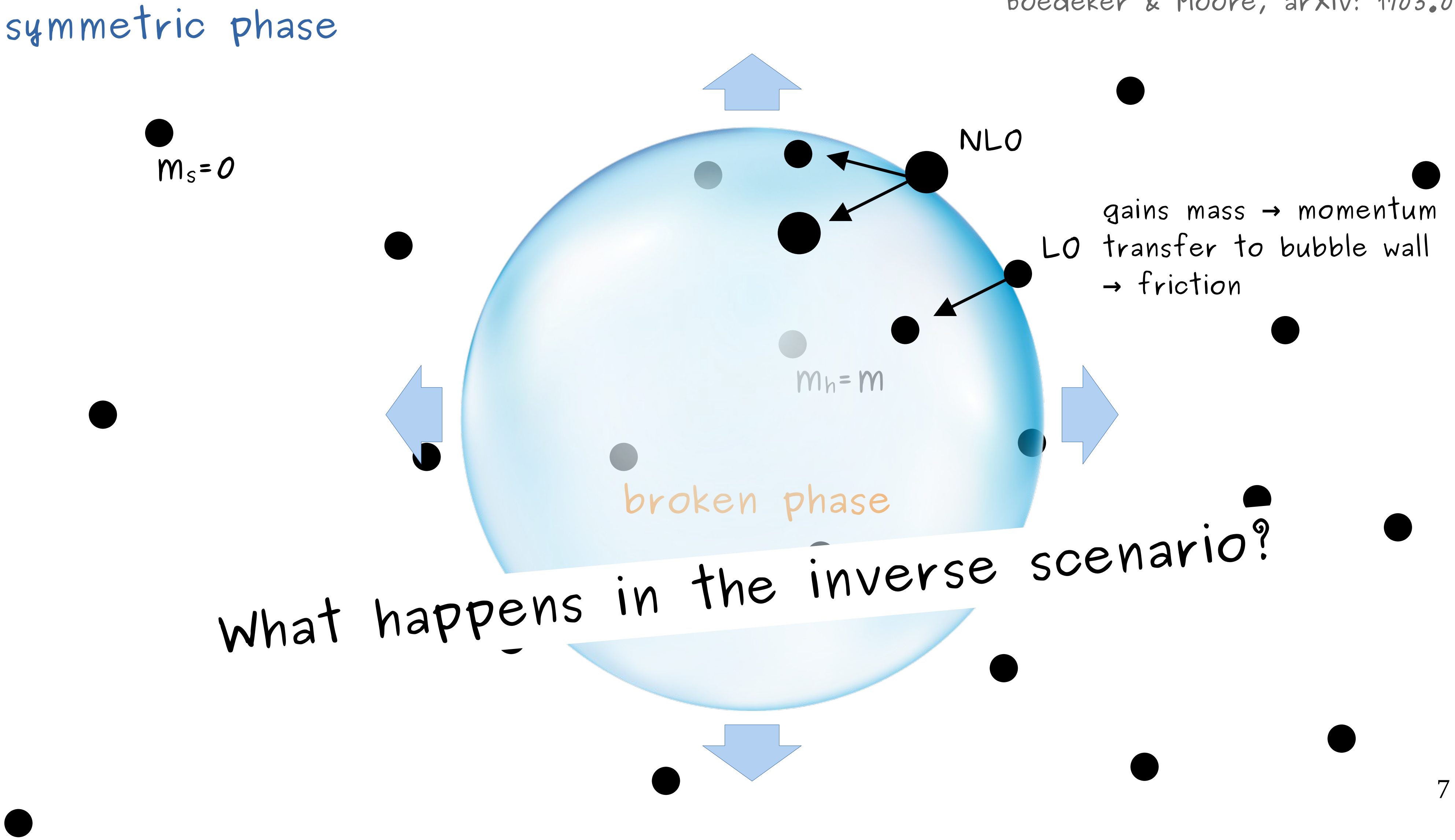
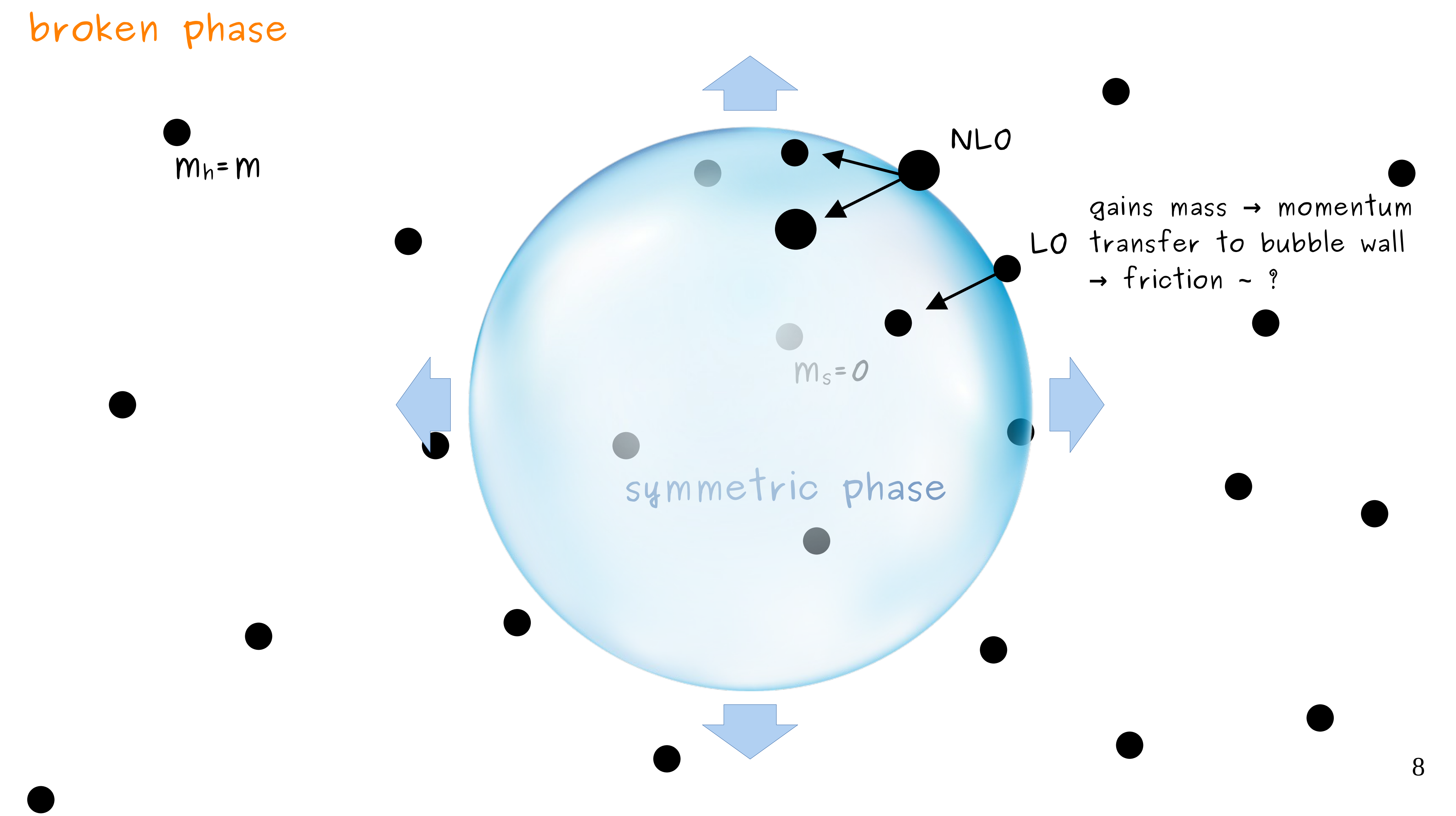


What happens in the inverse scenario?


broken phase
$m_h = m$
NLO
gains mass → momentum
LO transfer to bubble wall
→ friction ~ ?
$m_s = 0$
symmetric phase

What is a first order phase transition (FOPT)?

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Transition from one state of a medium (or vacuum state) to another
e.g. boiling of water, ferromagnetic transition

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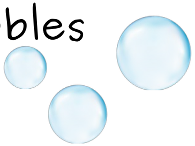
Discontinuous change in macroscopic quantity (e.g. density)
during transition both phases can coexist and separate into little droplets or bubbles
e.g. boiling of water



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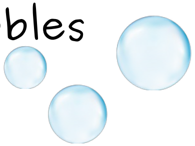

Transition from one state of a medium (or vacuum state) to another
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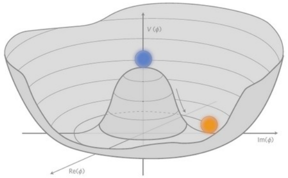
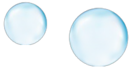
Electroweak phase transition (symmetric → broken):



- SM: second order or crossover
- BSM: can be first order



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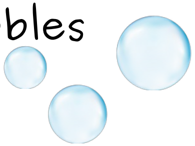

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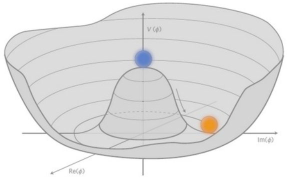

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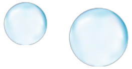
- BSM: can be first order  → implications for: baryogenesis, GW, topological defects

Bubbles from a FOPT

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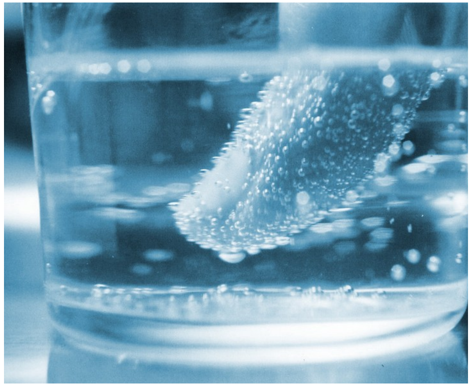
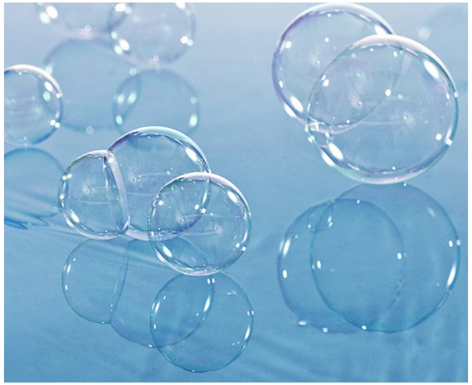


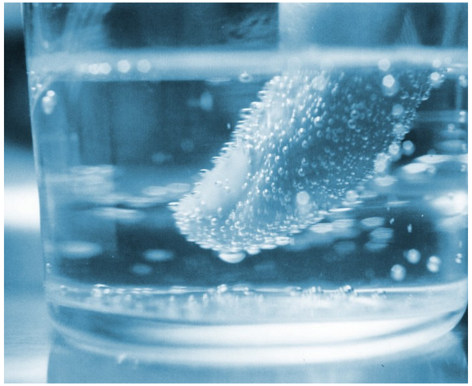
Nucleation

Expansion



Collision

Bubbles from a FOPT



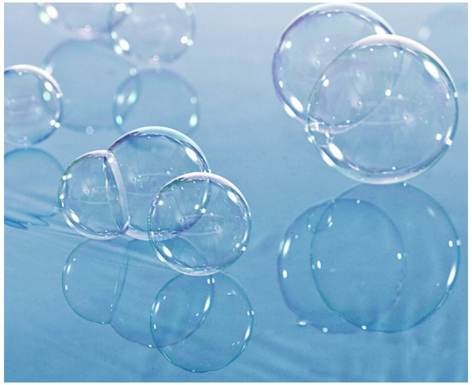
Nucleation

Expansion

→ driven by difference of potential in symmetric and broken phase

→ damped by friction of surrounding plasma

→ impact on GW signal, baryogenesis, plasma dynamics, discriminate between BSM models



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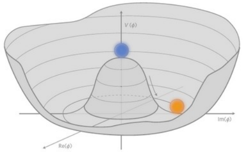
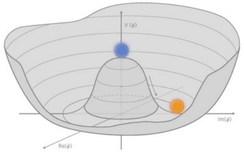
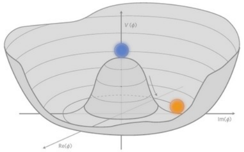


Electroweak phase transition



Electroweak phase transition

Can be FOPT in BSM



Electroweak phase transition

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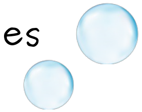
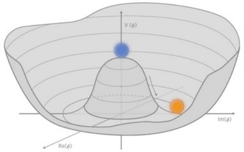
formation of bubbles





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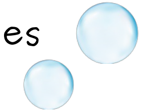
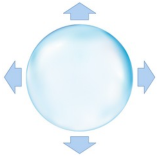
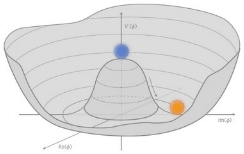




Bubbles expand (wall velocity v_w)



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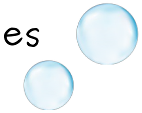
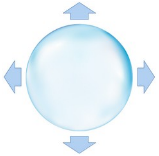
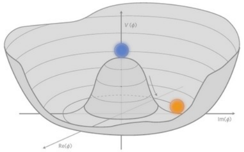




Bubbles expand (wall velocity v_w)

consequences for
→ GW, baryogenesis



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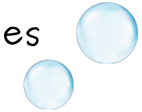

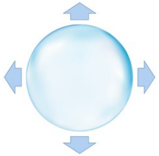

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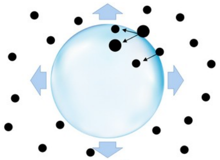
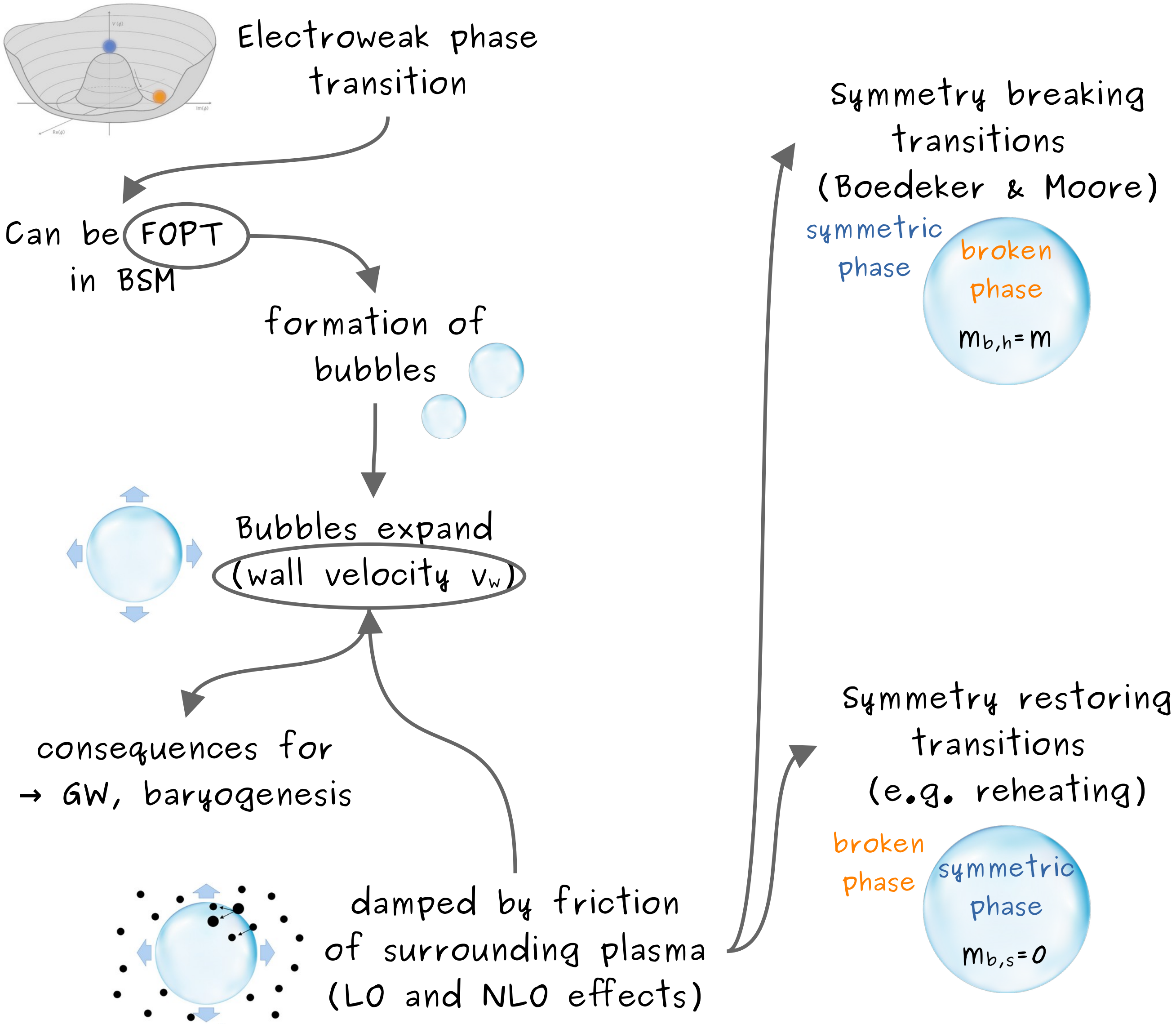


damped by friction of surrounding plasma (LO and NLO effects)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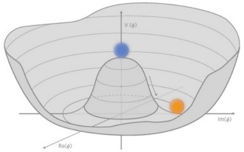
Symmetry breaking transitions (Boedeker & Moore)
symmetric phase
broken phase
$m_{b,h}=m$
Symmetry restoring transitions (e.g. reheating)
broken phase
symmetric phase
$m_{b,s}=0$



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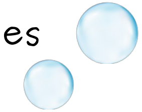

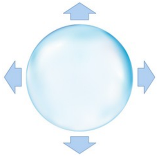

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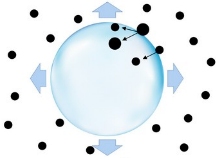

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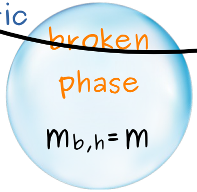
Symmetry breaking transitions (Boedeker & Moore)

symmetric phase

broken phase

$m_{b,h} = m$



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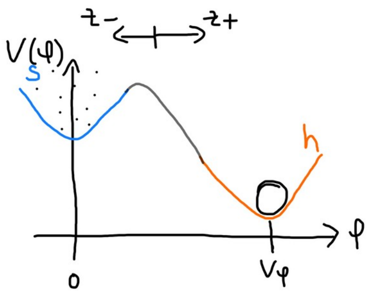
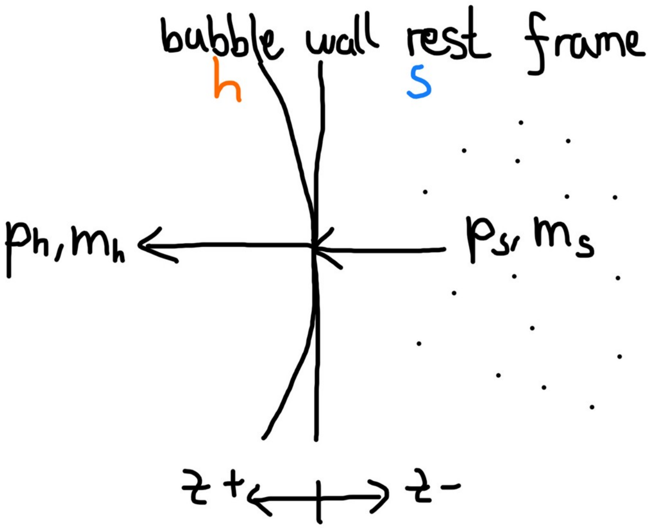
Bubble wall expansion, symmetry breaking





Symmetric (s) → broken/Higgs (h)

- particles obtain mass in bubble
- particles exert friction on bubble wall (decelerate expansion)

LO:

Friction ~ m^2T^2

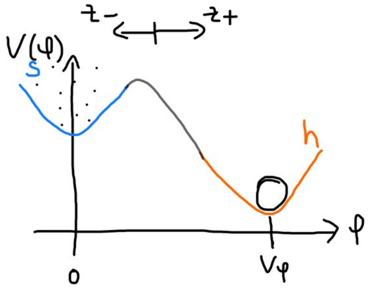
- force of expansion can be greater than friction → run away of bubble wall

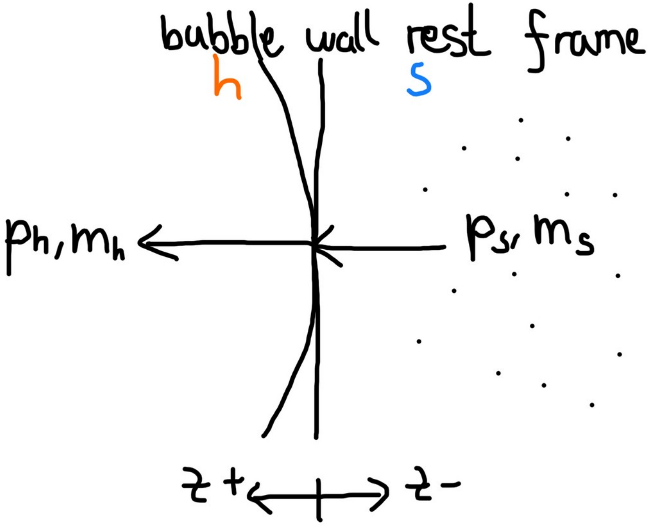
$$\mathcal{P}_{1\to 1} \sim \int \frac{d^3p}{(2\pi)^3} \underbrace{\Delta p_{1\to 1}}_{\approx \frac{m_h^2 - m_s^2}{2E}}$$

$$\approx \int \frac{d^3p}{(2\pi)^3 2E}(m_h^2 - \underbrace{m_s^2}_{\approx 0})$$

$$\sim m_h^2 T^2$$

Bubble wall expansion, symmetry breaking





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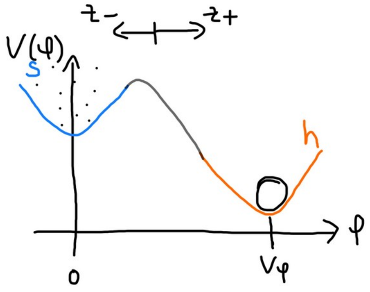

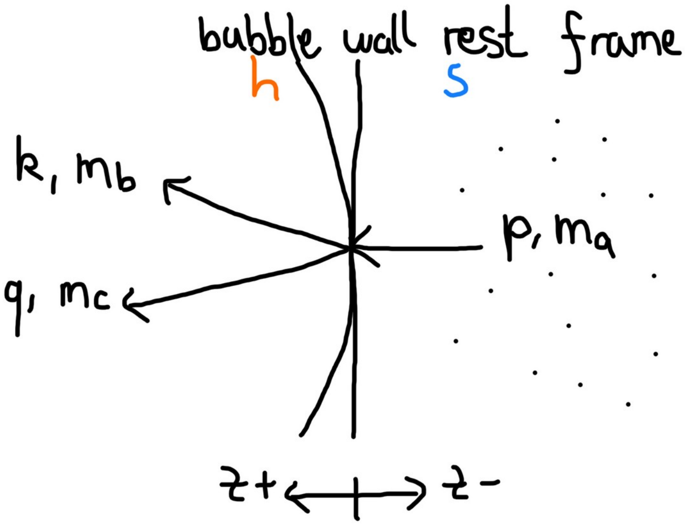

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NLO:

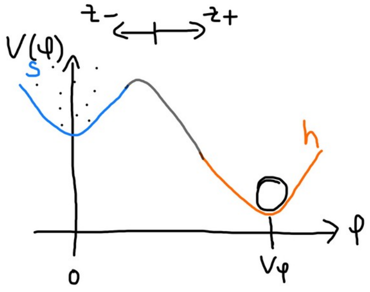
Friction ~ $\gamma m T^3$

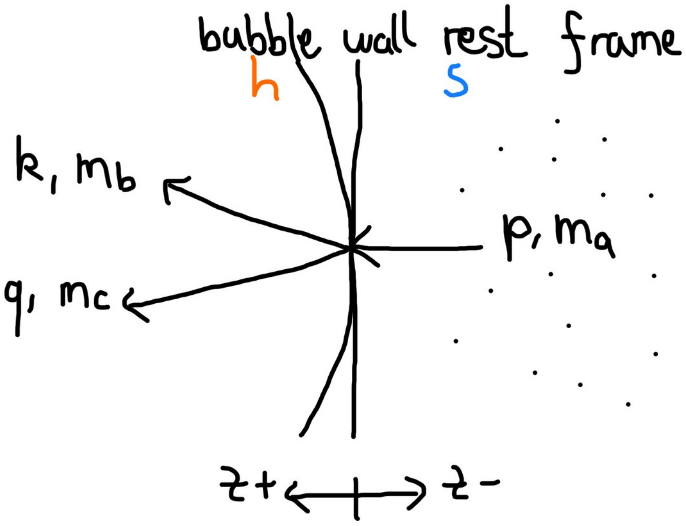
(γ = γ–factor of the wall)

- friction grows with γ → no run away of bubble wall

[Boedeker & Moore, arXiv: 1703.08215]

Bubble wall expansion, symmetry breaking





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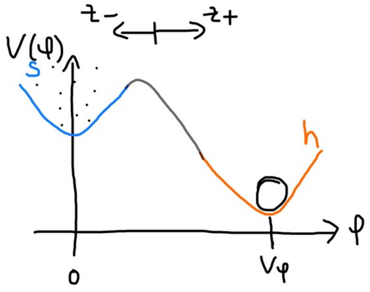
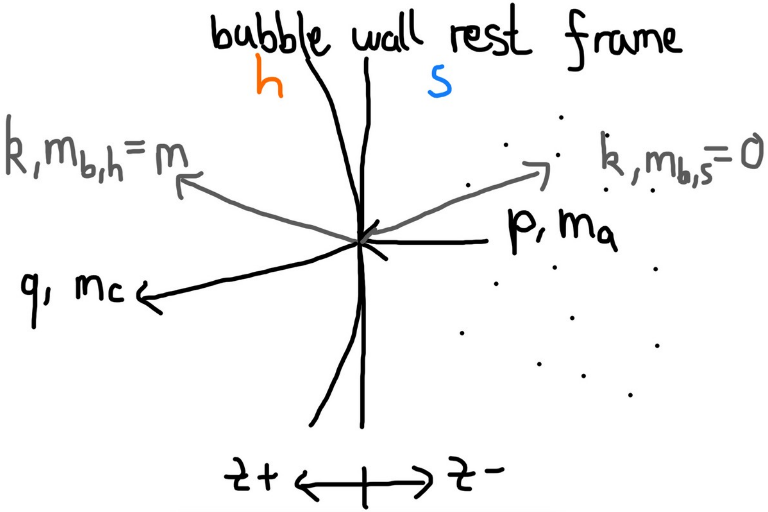




NLO:

Pressure on wall:

$$\mathcal{P}_{1\to 2} = \sum_{a,b,c} \nu_a \underbrace{\int \frac{d^3p}{(2\pi)^3}}_{\propto \gamma T^3} \frac{1}{(2p^0)^2} \int \frac{d^2k_\perp}{(2\pi)^2} \int \frac{dk^0}{(2\pi)2k^0}$$

$$\times \underbrace{[f_p][1 \pm f_k][1 \pm f_{p-k}]}_{\sim 1} \underbrace{(p_z - k_z - q_z)}_{\Delta p_z} |\mathcal{M}|^2$$

$$\mathcal{M} = \int dz \chi_k^*(z) \chi_q^*(z) V(z) \chi_p(z)$$

$$|\mathcal{M}|^2 = 4(p^0)^2 |V|^2 \left(\frac{A_{\text{inside}} - A_{\text{outside}}}{A_{\text{inside}} A_{\text{outside}}}\right)^2$$

$$A_{\text{in/out}} = p_{z,\text{in/out}} - k_{z,\text{in/out}} - q_{z,\text{in/out}}$$

Assumptions:

$$p^0 \approx q^0 \gg k^0$$

$$p^0, k^0, q^0 \gg p_\perp, k_\perp, q_\perp \sim m_a, m_b, m_c$$

$$m_{b,\text{inside}} \neq 0$$

$$m_{b,\text{outside}} = 0$$

[Boedeker & Moore, arXiv: 1703.08215]

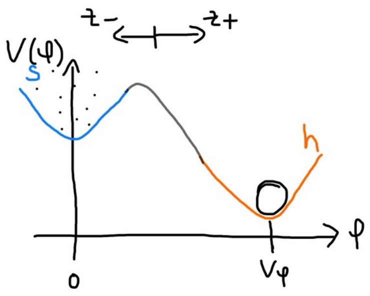
Squared vertex function $|V|^2$:

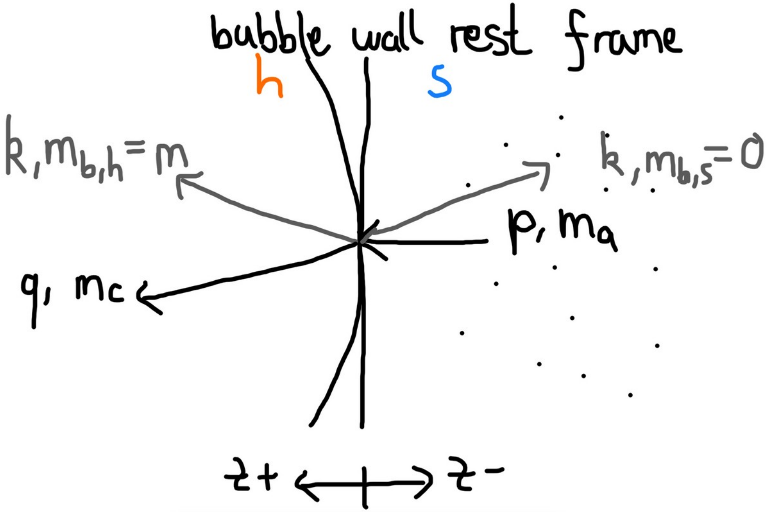
$S \to V_T S$ $F \to V_T F$ $V \to V_T V$	$4g^2 C_2[R] \frac{1}{x^2} k_\perp^2$
$S \to V_L S$ $F \to V_L F$ $V \to V_L V$	$4g^2 C_2[R] \frac{1}{x^2} m^2$

$$x = \frac{k^0}{p^0} \ll 1$$

$$\gamma = \frac{1}{\sqrt{1 - v^2}}$$

Bubble wall expansion, symmetry breaking





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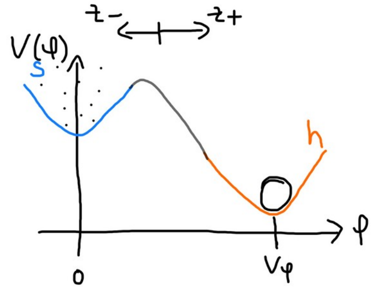
$V \to V_T V$	
$S \to V_L S$	
$F \to V_L F$	$4g^2 C_2[R] \frac{1}{x^2} m^2$ ($= m_b$)
$V \to V_L V$	

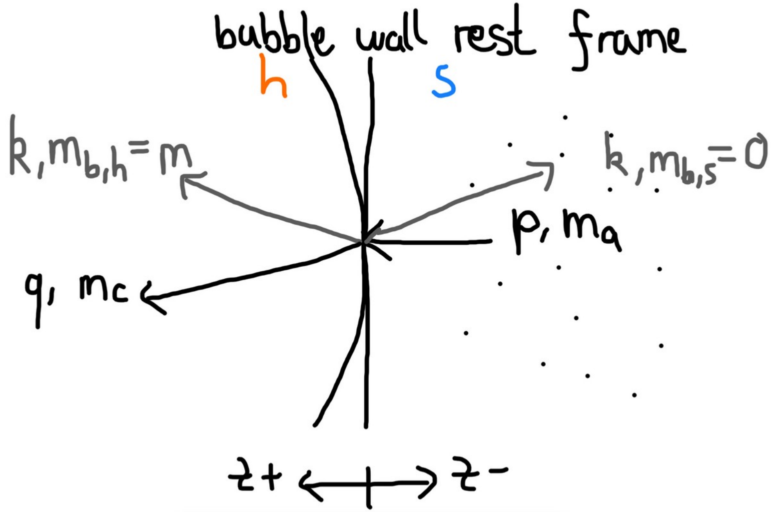
$$x = \frac{k^0}{p^0} \ll 1$$

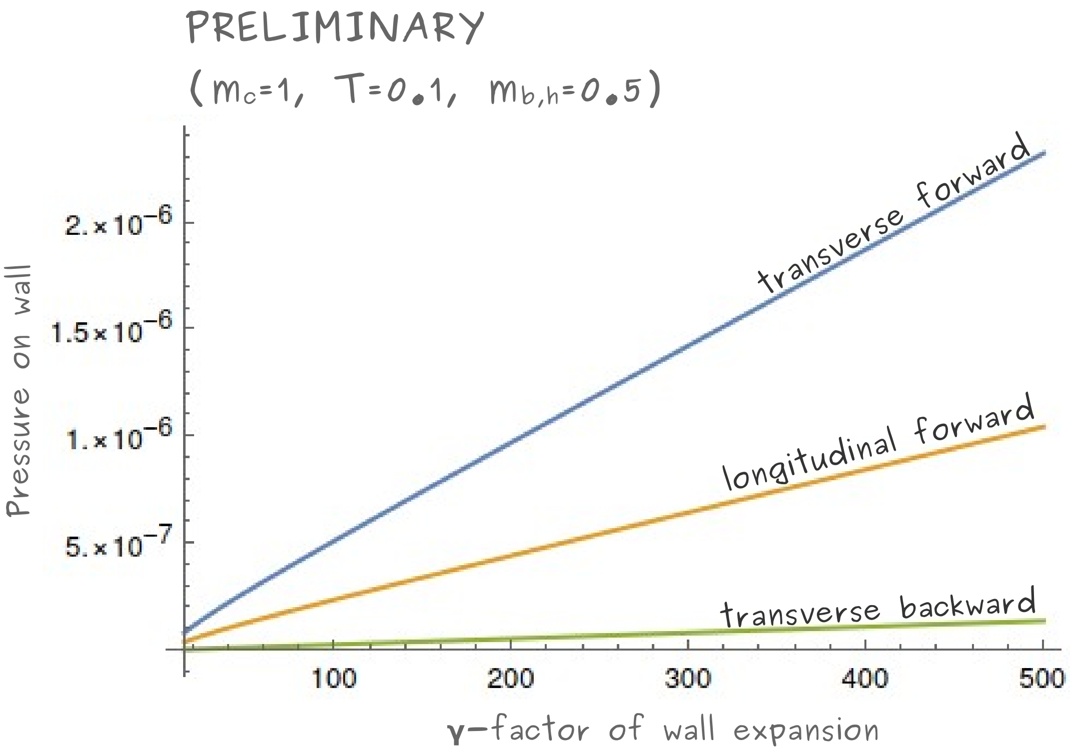
$$\gamma = \frac{1}{\sqrt{1 - v^2}}$$

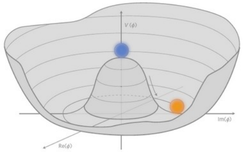
[Boedeker & Moore, arXiv: 1703.08215]

Bubble wall expansion, symmetry breaking









Electroweak phase transition

Can be FOPT in BSM

formation of bubbles

Bubbles expand (wall velocity v_w)

consequences for → GW, baryogenesis

damped by friction of surrounding plasma (LO and NLO effects)

Symmetry breaking transitions (Boedeker & Moore)

symmetric phase

broken phase

$m_{b,h} = m$

LO ~ m^2T^2

NLO ~ $\gamma m T^3$

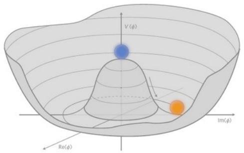
STOP

Symmetry restoring transitions (e.g. reheating)

broken phase

symmetric phase

$m_{b,s} = 0$



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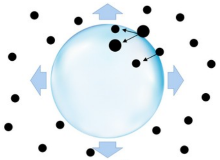

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Bubble wall expansion, symmetry restoring

Broken/Higgs (h) → symmetric (s)

- particles become massless in bubble

LO:

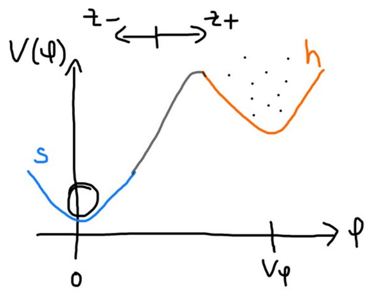
Anti–Friction ~ $-m^2T^2$

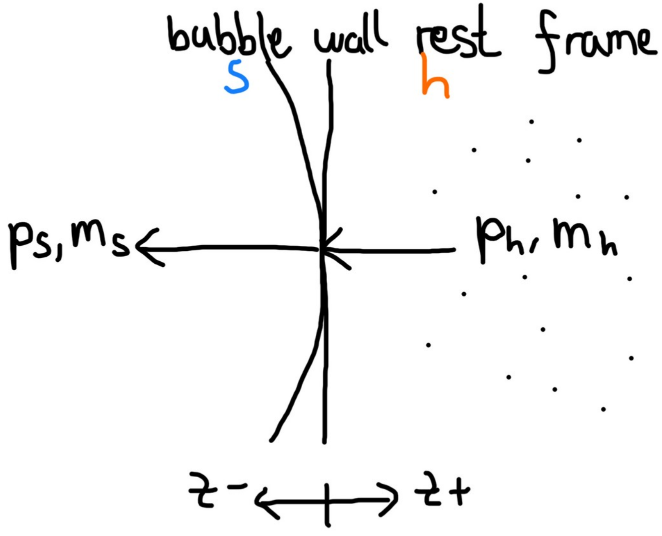
- negative friction → acceleration and run away of bubble wall

$$\mathcal{P}_{1\to1} \sim \int \frac{d^3p}{(2\pi)^3} \underbrace{\Delta p_{1\to1}}_{\approx \frac{m_s^2 - m_h^2}{2E}}$$

$$\approx \int \frac{d^3p}{(2\pi)^3 2E} (\underbrace{m_s^2}_{\approx 0} - m_h^2)$$

$$\sim -m_h^2 T^2$$





Bubble wall expansion, symmetry restoring

Broken/Higgs (h) → symmetric (s)

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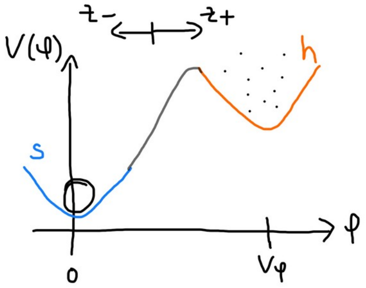

LO:

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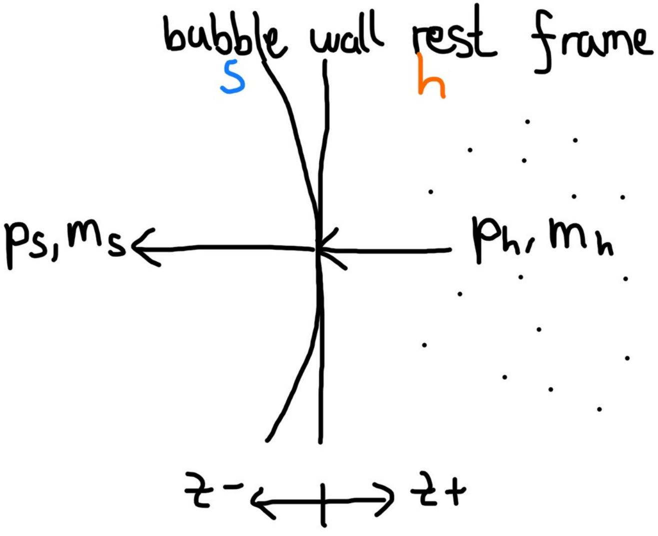
$$\approx \int \frac{d^3p}{(2\pi)^3 2E}\big(\underbrace{m_s^2}_{\approx 0} - m_h^2\big)$$

$$\sim -m_h^2 T^2$$



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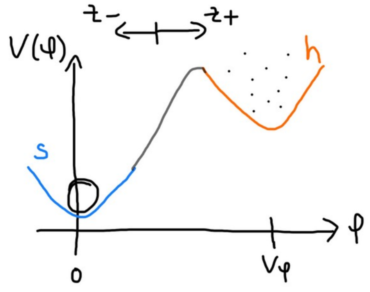
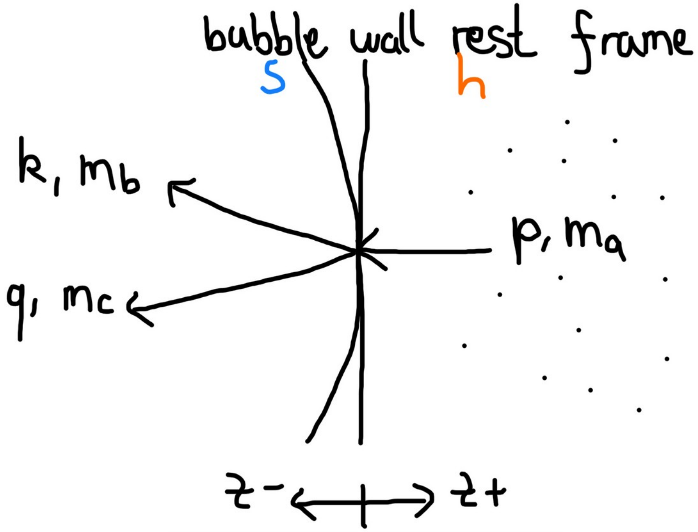
NLO:

Friction ~ γ

(γ = γ-factor of the wall)

- friction grows with γ → no run away of bubble wall





[Azatov et al, arXiv: 2405.19447]

Bubble wall expansion, symmetry restoring

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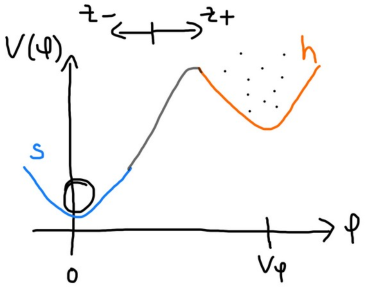
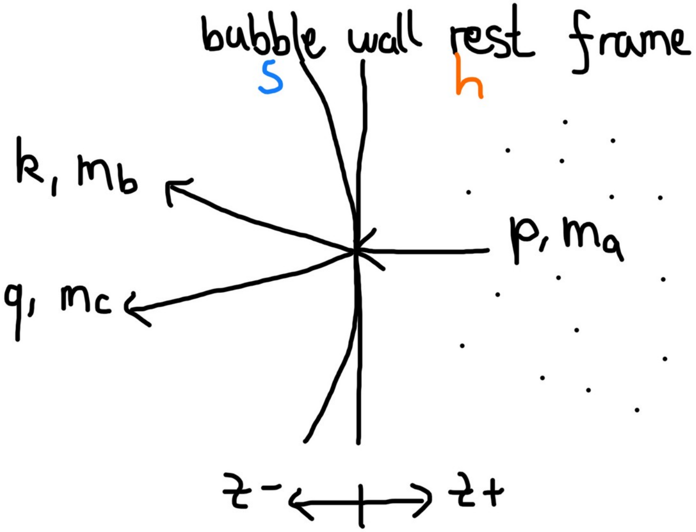




[Azatov et al, arXiv: 2405.19447]

Bubble wall expansion, symmetry restoring

NLO:

Pressure on wall:

$$\mathcal{P}_{1\to 2} = \sum_{a,b,c} \nu_a \int \underbrace{\frac{d^3p}{(2\pi)^3}}_{\propto \gamma T^3} \frac{1}{(2p^0)^2} \int \frac{d^2k_\perp}{(2\pi)^2} \int \frac{dk^0}{(2\pi)2k^0}$$

$$\times \underbrace{[f_p][1\pm f_k][1\pm f_{p-k}]}_{\sim 1} \underbrace{(p_z - k_z - q_z)}_{\Delta p_z} |\mathcal{M}|^2$$

$$\mathcal{M} = \int dz \chi_k^*(z)\chi_q^*(z)V(z)\chi_p(z)$$

$$|\mathcal{M}|^2 = 4(p^0)^2|V|^2\left(\frac{A_{\text{inside}} - A_{\text{outside}}}{A_{\text{inside}}A_{\text{outside}}}\right)^2$$

$$A_{\text{in/out}} = p_{z,\text{in/out}} - k_{z,\text{in/out}} - q_{z,\text{in/out}}$$

Assumptions:

$$p^0 \approx q^0 \gg k^0$$

$$p^0, k^0, q^0 \gg p_\perp, k_\perp, q_\perp \sim m_a, m_b, m_c$$

$$m_{b,\text{inside}} = 0$$

$$m_{b,\text{outside}} \neq 0$$

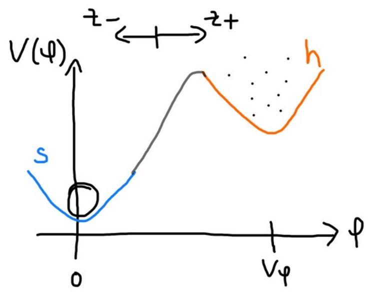
[Boedeker & Moore, arXiv: 1703.08215]

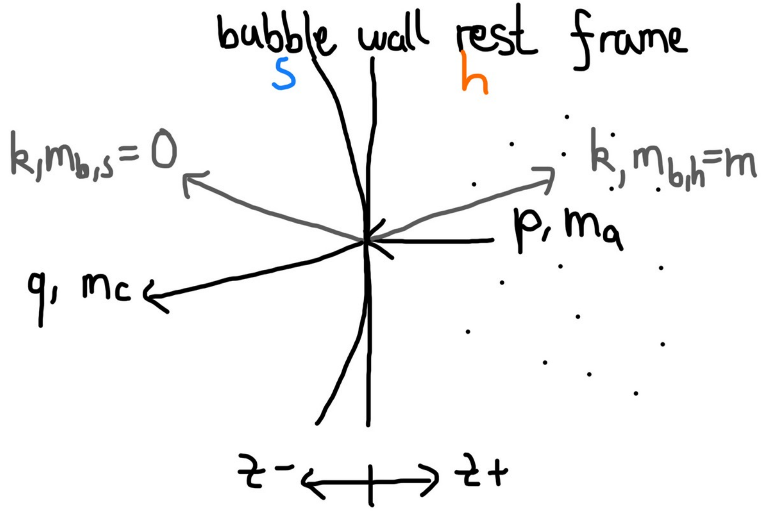
Squared vertex function $|V|^2$:

$S \to V_T S$	
$F \to V_T F$	$4g^2C_2[R]\frac{1}{x^2}k_\perp^2$
$V \to V_T V$	
$S \to V_L S$	
$F \to V_L F$	$4g^2C_2[R]\frac{1}{x^2}m^2$
$V \to V_L V$	

$$x = \frac{k^0}{p^0} \ll 1$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

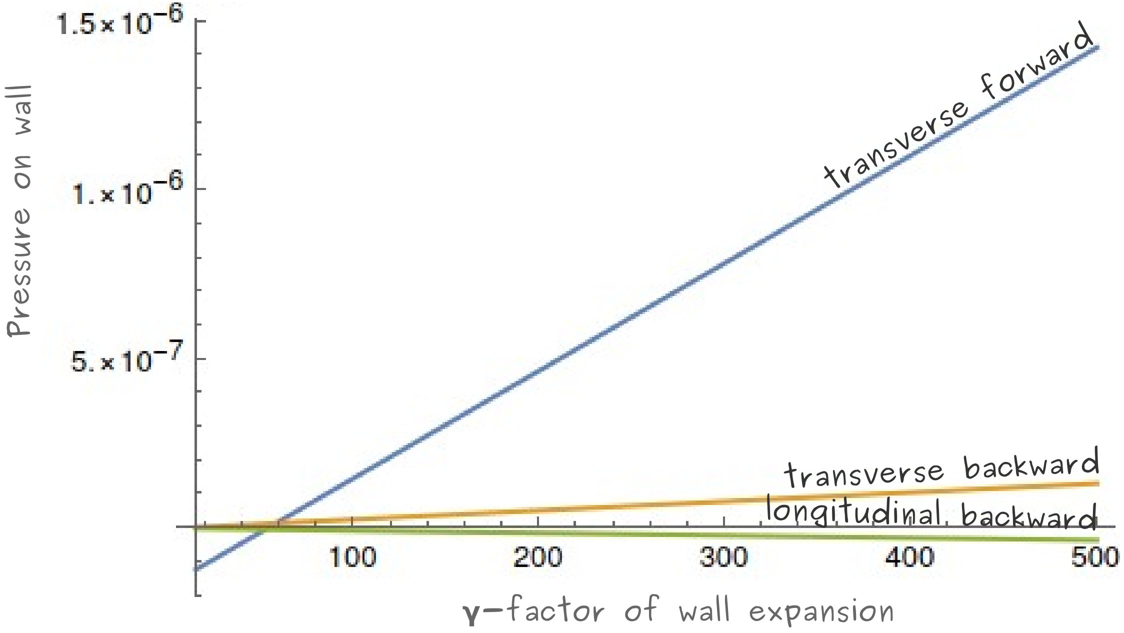


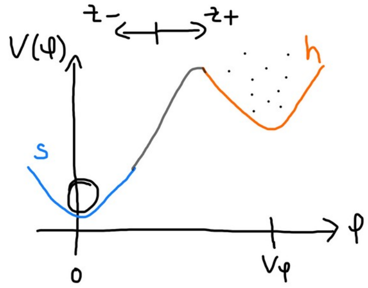


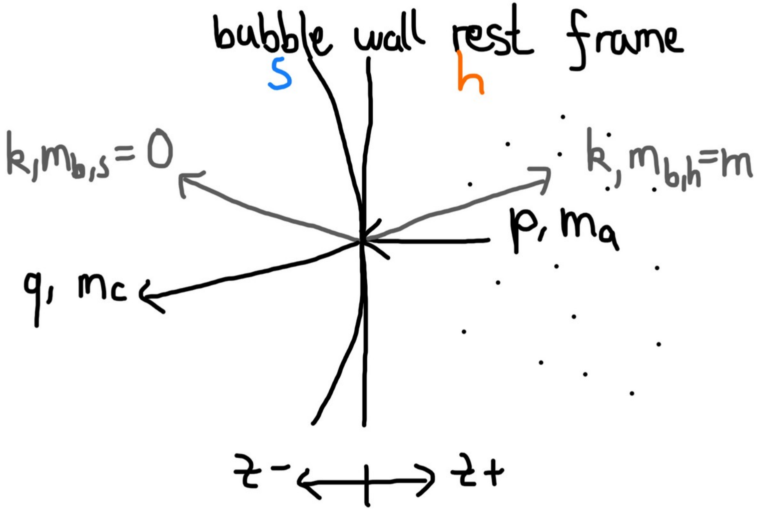
Bubble wall expansion, symmetry restoring

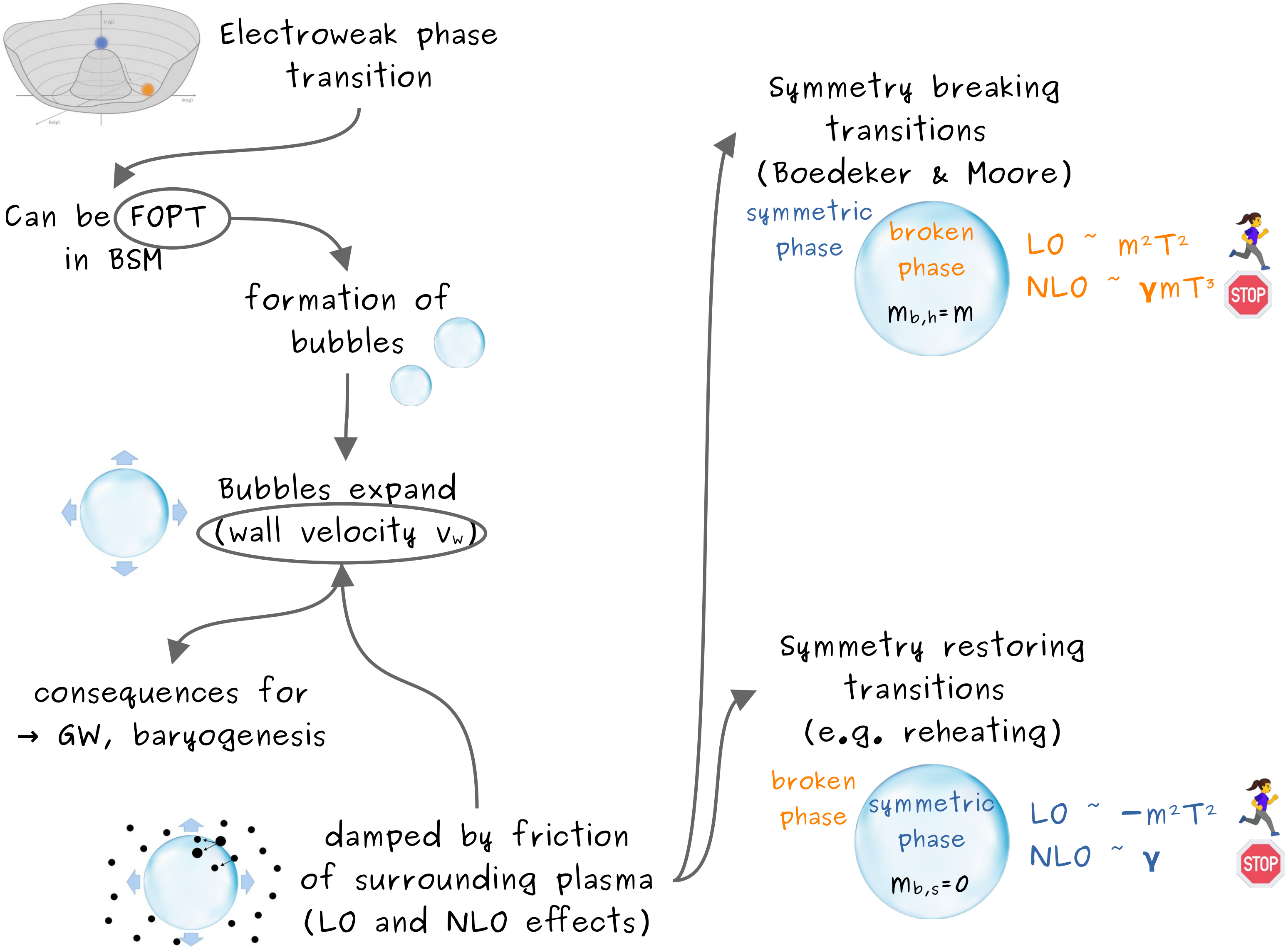
PRELIMINARY

(m_a=1, T=0.1, $m_{b,h}$=0.5)








Electroweak phase transition
Can be FOPT in BSM
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Symmetry breaking transitions (Boedeker & Moore)
symmetric phase
broken phase
$m_{b,h} = m$
LO ~ m^2T^2
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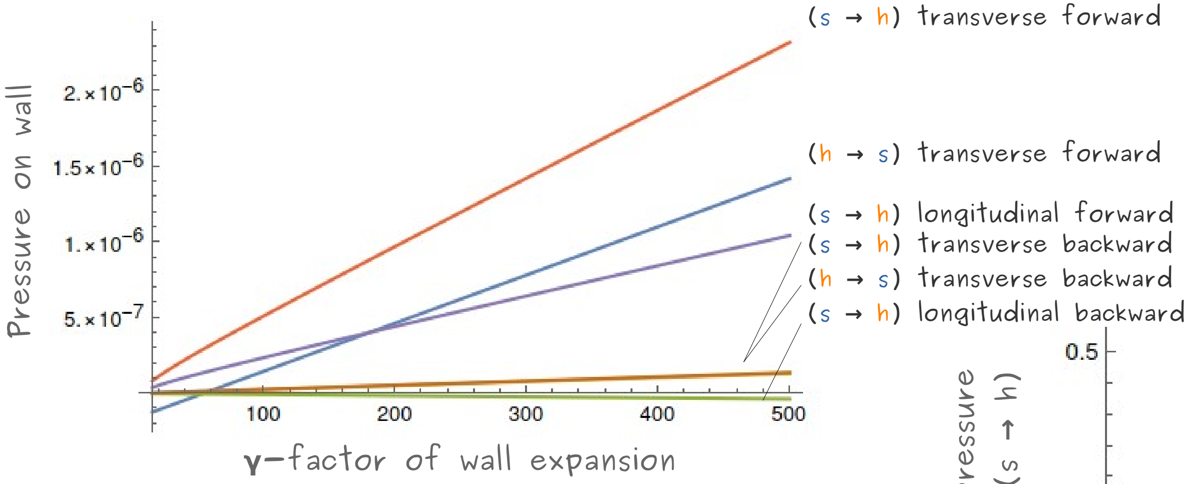
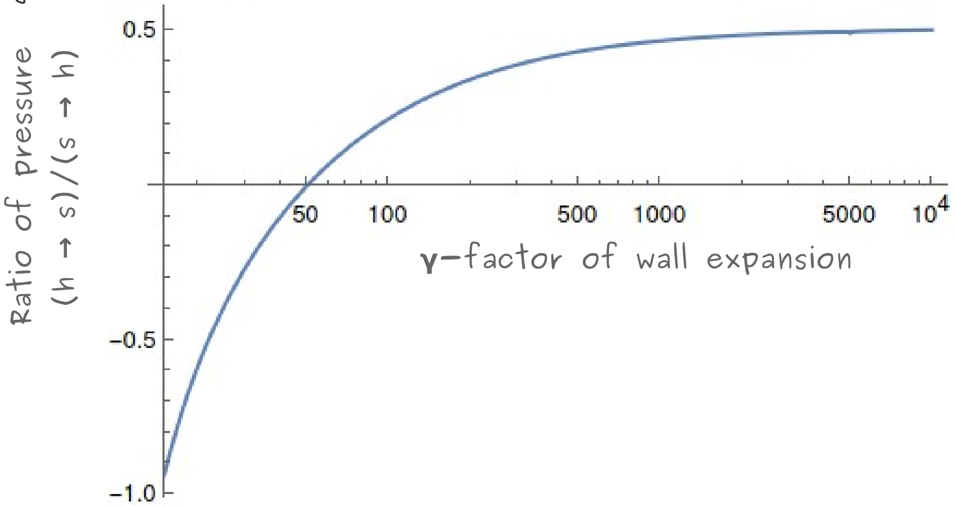
STOP
Symmetry restoring transitions (e.g. reheating)
broken phase
symmetric phase
$m_{b,s} = 0$
LO ~ $-m^2T^2$
NLO ~ γ
STOP

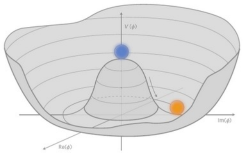
Bubble wall expansion, comparison

PRELIMINARY

($m_{a,h}=m_{c,h}=1$, $T=0.1$, $m_{b,h}=0.5$)







Electroweak phase transition

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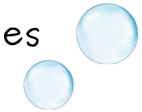
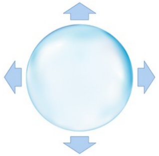
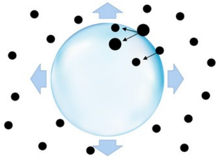




Bubbles expand (wall velocity v_w)

consequences for
→ GW, baryogenesis



damped by friction of surrounding plasma (LO and NLO effects)

Symmetry breaking transitions (Boedeker & Moore)

symmetric phase

broken phase

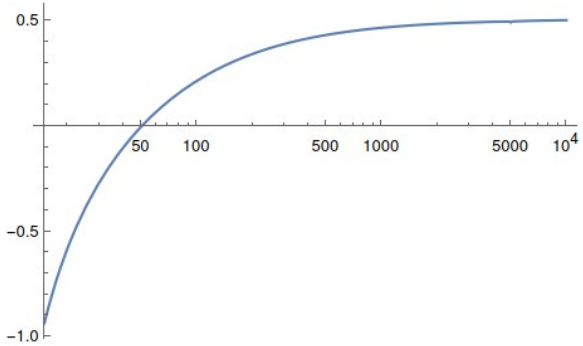
$m_{b,h} = m$

LO ~ $m^2 T^2$

NLO ~ $\gamma m T^3$

STOP

Ratio of pressure (h → s)/(s → h)



Symmetry restoring transitions (e.g. reheating)

broken phase

symmetric phase

$m_{b,s} = 0$

LO ~ $-m^2 T^2$

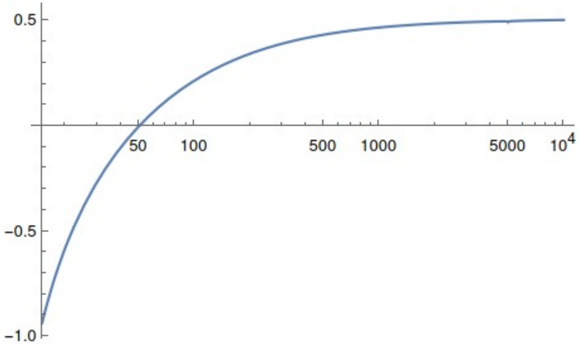
NLO ~ γ

STOP

Conclusion

- Investigated bubble wall expansion from FOPT at LO and NLO
- Symmetry breaking transitions (s → h) are well studied, e.g. [Boedeker & Moore, arXiv: 1703.08215]
- Symmetry restoring transitions (h → s) investigated in this work, also studied by e.g. [Azatov et al, arXiv: 2405.19447], here: repeated calculation as by Boedeker & Moore
- In both cases: bubbles run away at LO
- In both cases: NLO contributions stop bubble run away
- For $\gamma < 50$: get negative contributions (at NLO for (h → s))
- For $\gamma > 1000$: get ratio of 0.5 here (at NLO for (h → s)/(s → h))

Ratio of pressure
(h → s)/(s → h)



Conclusion

- Investigated bubble wall expansion from FOPT at LO and NLO
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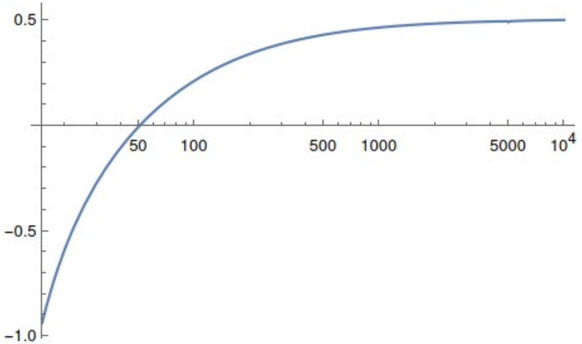
Thank you!

Ratio of pressure $(h \to s)/(s \to h)$



Backup, symmetry restoring, LO

For one particle moving through the wall without radiating (see left image of Figure 2), the momentum transfer $\Delta p_{1\to 1}$ on the wall in z-direction is obtained by simple energy conservation:

$$
\begin{aligned}
E_{\text{outside}} &= p^2_{z,\text{outside}} + p^2_{\perp,\text{outside}} + m^2_{\text{outside}} \\
&= p^2_{z,\text{inside}} + p^2_{\perp,\text{inside}} + m^2_{\text{inside}} = E_{\text{inside}} \qquad (3.1) \\
\Rightarrow m^2_{\text{inside}} - m^2_{\text{outside}} &= p^2_{z,\text{outside}} - p^2_{z,\text{inside}} + \underbrace{p^2_{\perp,\text{outside}} - p^2_{\perp,\text{inside}}}_{=0} \\
&= \underbrace{\left(p_{z,\text{outside}} - p_{z,\text{inside}}\right)}_{\Delta p_{1\to 1}} \underbrace{\left(p_{z,\text{outside}} + p_{z,\text{inside}}\right)}_{\approx 2E} \qquad (3.2) \\
\Rightarrow \Delta p_{1\to 1} &\approx \frac{m^2_{\text{inside}} - m^2_{\text{outside}}}{2E}. \qquad (3.3)
\end{aligned}
$$

In the symmetry restoring scenario the mass inside the bubble is approximately 0 and the **pressure** $\mathcal{P}_{1\to 1}$ **on the wall** is then negative and leads to anti-friction, which accelerates the wall further:

$$
\begin{aligned}
\mathcal{P}_{1\to 1} &= \sum_a \nu_a \int \frac{d^3p}{(2\pi)^3} f_a(p) \quad \underbrace{\Delta p_{1\to 1}}_{\approx \frac{m^2_{\text{inside}} - m^2_{\text{outside}}}{2E}} \\
&\approx \sum_a \nu_a \int \frac{d^3p}{(2\pi)^3 2E} f_a(p) (\underbrace{m^2_{\text{inside}}}_{\approx 0} - m^2_{\text{outside}}) \\
&\sim -m^2_{\text{outside}} T^2 \qquad (3.4)
\end{aligned}
$$

Backup, symmetry restoring, NLO

$\Delta p_{1\to 2,\text{forward}}$ on the wall is:

$$\begin{aligned}\Delta p_{1\to2,\text{forward}} = \Delta p_{z,\text{forward}} &= \underbrace{p_z}_{\approx p^0-\frac{m_a^2+p_\perp^2}{2p^0}} - \underbrace{k_{z,\text{forward}}}_{\approx k^0-\frac{m_{b,\text{inside}}^2+k_\perp^2}{2k^0}} - \underbrace{q_z}_{\approx q^0-\frac{m_c^2+q_\perp^2}{2q^0}} \\ &\approx \underbrace{p^0-k^0-q^0}_{=0} - \frac{1}{2p^0}(m_a^2+\underbrace{p_\perp^2}_{=0}) + \frac{1}{2k^0}(\underbrace{m_{b,\text{inside}}^2}_{=0}+k_\perp^2) + \underbrace{\frac{1}{2q^0}}_{q^0=p^0-k^0\approx p^0}(m_c^2+\underbrace{q_\perp^2}_{=k_\perp^2}) \\ &\approx -\frac{m_a^2}{2p^0}+\frac{k_\perp^2}{2k^0}+\underbrace{\frac{m_c^2+k_\perp^2}{2p^0}}_{\approx 0} \approx -\frac{m_a^2}{2p^0}+\frac{k_\perp^2}{2k^0} \qquad (3.5)\end{aligned}$$

- particle b moves backwards outside the bubble (see right image of Figure 2), hence k_z has a negative sign and m_b is not zero. The momentum transfer $\Delta p_{1\to2,\text{backward}}$ on the wall is:

$$\begin{aligned}\Delta p_{1\to2,\text{backward}} = \Delta p_{z,\text{backward}} &= \underbrace{p_z}_{\approx p^0-\frac{m_a^2+p_\perp^2}{2p^0}} - \underbrace{k_{z,\text{backward}}}_{\approx -k^0+\frac{m_{b,\text{outside}}^2+k_\perp^2}{2k^0}} - \underbrace{q_z}_{\approx q^0-\frac{m_c^2+q_\perp^2}{2q^0}} \\ &\approx \underbrace{p^0-q^0}_{=k^0}+k^0 - \frac{1}{2p^0}(m_a^2+\underbrace{p_\perp^2}_{=0}) - \frac{1}{2k^0}(m_{b,\text{outside}}^2+k_\perp^2) + \underbrace{\frac{1}{2q^0}}_{q^0\approx p^0}(m_c^2+\underbrace{q_\perp^2}_{=k_\perp^2}) \\ &\approx 2k^0-\frac{m_a^2}{2p^0}-\frac{m_{b,\text{outside}}^2+k_\perp^2}{2k^0}+\underbrace{\frac{m_c^2+k_\perp^2}{2p^0}}_{\approx 0} \approx 2k^0-\frac{m_a^2}{2p^0}-\frac{m_{b,\text{outside}}^2+k_\perp^2}{2k^0} \qquad (3.6)\end{aligned}$$

The **pressure** $\mathcal{P}_{1\to2}$ **on the wall** can then be computed via the following equation from [4, eq. (12), (16)]:

$$\begin{aligned}\mathcal{P}_{1\to2} &= \sum_{a,b,c}\nu_a\int\frac{d^3p}{(2\pi)^3 2p^0}\int\frac{d^3k\,d^3q}{(2\pi)^6 2k^0 2\underbrace{q^0}_{\approx p^0}} f_p[1\pm f_k][1\pm f_q]\underbrace{(p_z-k_z-q_z)}_{\Delta p_z} \\ &\quad\times(2\pi)^3\delta^2(\mathbf{p}_\perp-\mathbf{k}_\perp-\mathbf{q}_\perp)\delta(p^0-k^0-q^0)|\mathcal{M}|^2 \\ &= \sum_{a,b,c}\nu_a\int\frac{d^3p}{(2\pi)^3(2p^0)^2}f_p\int\frac{d^2k_\perp}{(2\pi)^2}\int\frac{dk^0}{(2\pi)2k^0}[1\pm f_k][1\pm f_{p-k}]\Delta p_z|\mathcal{M}|^2, \qquad (3.7)\end{aligned}$$

Backup, symmetry restoring, NLO

$$
\begin{aligned}
\mathcal{M} &= \int dz \chi_k^*(z)\chi_q^*(z)V(z)\chi_p(z) \\
&\approx \exp(-i\int_0^z k_z(z')dz')\exp(-i\int_0^z q_z(z')dz')V(z)\exp(i\int_0^z p_z(z')dz') \\
&= V_{\text{inside}}\int_{-\infty}^{0} dz \exp(iz\underbrace{(p_{z,\text{inside}} - k_{z,\text{inside}} - q_{z,\text{inside}})}_{=A_{\text{inside}}/2p^0}) \\
&\quad + V_{\text{outside}}\int_{0}^{\infty} dz \exp(iz\underbrace{(p_{z,\text{outside}} - k_{z,\text{outside}} - q_{z,\text{outside}})}_{=A_{\text{outside}}/2p^0}) \\
&= 2ip^0\left(\frac{V_{\text{outside}}}{A_{\text{outside}}} - \frac{V_{\text{inside}}}{A_{\text{inside}}}\right) && (3.8) \\
|\mathcal{M}|^2 &= 4(p^0)^2|V|^2\left(\frac{A_{\text{inside}} - A_{\text{outside}}}{A_{\text{inside}}A_{\text{outside}}}\right)^2 && (3.9)
\end{aligned}
$$

Backup, symmetry restoring, NLO

- for the case of particle b moving forward, with $\frac{k^0}{p^0} = x \ll 1$, we get:

$$\begin{aligned}
\frac{A_{\text{inside,forward}}}{2p^0} &= p_{z,\text{inside}} - k_{z,\text{inside,forward}} - q_{z,\text{inside}} \\
&\approx \underbrace{p^0 - k^0 - q^0}_{=0} - \frac{m_c^2 + 0}{2p^0} + \frac{0 + k_\perp^2}{2k^0} + \frac{m_c^2 + q_\perp^2}{2q^0} \\
&= \frac{1}{2p^0}\Big(-m_c^2 + \frac{k_\perp^2}{\underbrace{k^0/p^0}_{=x}} + \frac{m_c^2 + q_\perp^2}{\underbrace{q^0/p^0}_{=1-x}}\Big) \\
&= \frac{1}{2p^0}\Big(\frac{k_\perp^2}{x} + \underbrace{\frac{q_\perp^2}{1-x}}_{q_\perp^2 = k_\perp^2} + \underbrace{\frac{m_c^2}{1-x} - m_c^2}_{\approx 0}\Big) \\
&\approx \frac{1}{2p^0} \frac{k_\perp^2}{\underbrace{x(1-x)}_{\approx \frac{k_\perp^2}{x}}} && (3.10)
\end{aligned}$$

$$\begin{aligned}
\frac{A_{\text{outside,forward}}}{2p^0} &= p_{z,\text{outside}} - k_{z,\text{outside,forward}} - q_{z,\text{outside}} \\
&\approx \underbrace{p^0 - k^0 - q^0}_{=0} - \frac{m_a^2 + 0}{2p^0} + \frac{m_{b,\text{outside}}^2 + k_\perp^2}{2k^0} + \frac{m_a^2 + q_\perp^2}{2q^0} && (3.11) \\
&= \frac{1}{2p^0}\Big(\frac{m_{b,\text{outside}}^2 + k_\perp^2}{x} + \underbrace{\frac{q_\perp^2}{1-x}}_{q_\perp^2 = k_\perp^2} + \underbrace{\frac{m_a^2}{1-x} - m_a^2}_{\approx 0}\Big) \\
&\approx \frac{A_{\text{inside,forward}} + m_{b,\text{outside}}^2/x}{2p^0} && (3.12)
\end{aligned}$$

we then get for the term $(\frac{A_{\text{inside}} - A_{\text{outside}}}{A_{\text{inside}} A_{\text{outside}}})^2$:

$$\left(\frac{A_{\text{inside}} - A_{\text{outside}}}{A_{\text{inside}} A_{\text{outside}}}\right)^2\Big|_{\text{forward}} = x^2 \frac{m_{b,\text{outside}}^4}{k_\perp^4 (k_\perp^2 + m_{b,\text{outside}}^2)^2} \qquad (3.13)$$

Backup, symmetry restoring, NLO

$$
\begin{aligned}
\frac{A_{\text{inside,backward}}}{2p^0} &= p_{z,\text{inside}} - k_{z,\text{inside,backward}} - q_{z,\text{inside}} \\
&\approx \underbrace{p^0 + k^0 - q^0}_{=2k^0} - \frac{m_c^2+0}{2p^0} - \frac{0+k_\perp^2}{2k^0} + \frac{m_c^2+q_\perp^2}{2q^0} \\
&= \frac{1}{2p^0}(4k^0p^0 - \frac{k_\perp^2}{x} + \underbrace{\frac{q_\perp^2}{1-x}}_{q_\perp^2=k_\perp^2} + \underbrace{\frac{m_c^2}{1-x} - m_c^2)}_{\approx 0} \\
&\approx \frac{1}{2p^0}(4k^0p^0 - \underbrace{\frac{k_\perp^2}{x(1-x)}}_{\approx \frac{k_\perp^2}{x}})
\end{aligned}
$$

$$
\begin{aligned}
\frac{A_{\text{outside,backward}}}{2p^0} &= p_{z,\text{outside}} - k_{z,\text{outside,backward}} - q_{z,\text{outside}} \\
&\approx \underbrace{p^0 + k^0 - q^0}_{=2k^0} - \frac{m_a^2+0}{2p^0} - \frac{m_{b,\text{outside}}^2+k_\perp^2}{2k^0} + \frac{m_a^2+q_\perp^2}{2q^0} \\
&= \frac{1}{2p^0}(4k^0p^0 - \frac{m_{b,\text{outside}}^2+k_\perp^2}{x} + \underbrace{\frac{q_\perp^2}{1-x}}_{q_\perp^2=k_\perp^2} + \underbrace{\frac{m_a^2}{1-x} - m_a^2)}_{\approx 0} \\
&\approx \frac{A_{\text{inside,backward}} - m_{b,\text{outside}}^2/x}{2p^0}
\end{aligned}
$$

we then get for the term $(\frac{A_{\text{inside}}-A_{\text{outside}}}{A_{\text{inside}}A_{\text{outside}}})^2$:

$$
(\frac{A_{\text{inside}}-A_{\text{outside}}}{A_{\text{inside}}A_{\text{outside}}})^2|_{\text{backward}} = x^2 \frac{m_{b,\text{outside}}^4}{(\underbrace{4xk^0p^0-k_\perp^2}_{(2k^0)^2})^2(\underbrace{4xk^0p^0}_{(2k^0)^2}-k_\perp^2-m_{b,\text{outside}}^2)^2} \qquad (3.16)
$$

Backup, symmetry restoring, NLO

	forward scattering, $m_{b,\text{inside}} = m_{b,s} = 0$	backward scattering, $m_{b,\text{outside}} = m_{b,h} \neq 0$
transverse vector boson	$\lvert V\rvert^2 = 4g^2 C_2[R] \frac{1}{x^2} k_\perp^2$ $(\frac{A_{in} - A_{out}}{A_{in} A_{out}})^2 \approx x^2 \frac{m_{b,out}^4}{k_\perp^4 (k_\perp^2 + m_{b,out}^2)^2}$ $\Delta p_{1\to 2} \approx -\frac{m_a^2}{2p^0} + \frac{k_\perp^2}{2k^0}$	$\lvert V\rvert^2 = 4g^2 C_2[R] \frac{1}{x^2} k_\perp^2$ $(\frac{A_{in} - A_{out}}{A_{in} A_{out}})^2 \approx x^2 \frac{m_{b,out}^4}{((2k^0)^2 - k_\perp^2)^2 ((2k^0)^2 - k_\perp^2 - m_{b,out}^2)^2}$ $\Delta p_{1\to 2} \approx 2k^0 - \frac{m_a^2}{2p^0} - \frac{m_{b,out}^2 + k_\perp^2}{2k^0}$
longitudinal vector boson	$\lvert V\rvert^2 = 0$	$\lvert V\rvert^2 = 4g^2 C_2[R] \frac{1}{x^2} m_{b,out}^2$ $(\frac{A_{in} - A_{out}}{A_{in} A_{out}})^2 \approx x^2 \frac{m_{b,out}^4}{((2k^0)^2 - k_\perp^2)^2 ((2k^0)^2 - k_\perp^2 - m_{b,out}^2)^2}$ $\Delta p_{1\to 2} \approx 2k^0 - \frac{m_a^2}{2p^0} - \frac{m_{b,out}^2 + k_\perp^2}{2k^0}$

Backup, symmetry breaking, LO

$$\mathcal{P}_{1\to 1} = \sum_a \nu_a \int \frac{d^3p}{(2\pi)^3} f_a(p) \underbrace{\Delta p_{1\to 1}}_{\approx \frac{m^2_{\text{inside}} - m^2_{\text{outside}}}{2E}}$$

$$\approx \sum_a \nu_a \int \frac{d^3p}{(2\pi)^3 2E} f_a(p)(m^2_{\text{inside}} - \underbrace{m^2_{\text{outside}}}_{\approx 0})$$

$$\sim m^2_{\text{inside}} T^2$$

Backup, symmetry breaking, NLO

- particle b moves forward inside the bubble (see middle image of Figure 3). The momentum transfer $\Delta p_{1\to 2,\text{forward}}$ on the wall is[1]:

$$\begin{aligned}
\Delta p_{1\to 2,\text{forward}} = \Delta p_{z,\text{forward}} &= \underbrace{p_z}_{\approx p^0 - \frac{m_a^2+p_\perp^2}{2p^0}} - \underbrace{k_{z,\text{forward}}}_{\approx k^0 - \frac{m_{b,\text{inside}}^2+k_\perp^2}{2k^0}} - \underbrace{q_z}_{\approx q^0 - \frac{m_c^2+q_\perp^2}{2q^0}} \\
&\approx \underbrace{p^0 - k^0 - q^0}_{=0} - \frac{1}{2p^0}(m_a^2 + \underbrace{p_\perp^2}_{=0}) + \frac{1}{2k^0}(m_{b,\text{inside}}^2 + k_\perp^2) + \underbrace{\frac{1}{2q^0}}_{q^0\approx p^0}(m_c^2 + \underbrace{q_\perp^2}_{=k_\perp^2}) \\
&\approx -\underbrace{\frac{m_a^2}{2p^0}}_{\approx 0} + \frac{m_{b,\text{inside}}^2 + k_\perp^2}{2k^0} + \frac{m_c^2}{2p^0} + \underbrace{\frac{k_\perp^2}{2p^0}}_{\approx 0} \approx \frac{m_{b,\text{inside}}^2 + k_\perp^2}{2k^0} + \frac{m_c^2}{2p^0}
\end{aligned} \tag{4.2}$$

- particle b moves backwards outside the bubble (see right image of Figure 3), hence k_z has a negative sign and m_b is zero. The momentum transfer $\Delta p_{1\to 2,\text{backward}}$ on the wall is:

$$\begin{aligned}
\Delta p_{1\to 2,\text{backward}} = \Delta p_{z,\text{backward}} &= \underbrace{p_z}_{\approx p^0 - \frac{m_a^2+p_\perp^2}{2p^0}} - \underbrace{k_{z,\text{backward}}}_{\approx -k^0 + \frac{m_{b,\text{outside}}^2+k_\perp^2}{2k^0}} - \underbrace{q_z}_{\approx q^0 - \frac{m_c^2+q_\perp^2}{2q^0}} \\
&\approx \underbrace{p^0 - q^0}_{=k^0} + k^0 - \frac{1}{2p^0}(m_a^2 + \underbrace{p_\perp^2}_{=0}) - \frac{1}{2k^0}(\underbrace{m_{b,\text{outside}}^2}_{=0} + k_\perp^2) + \underbrace{\frac{1}{2q^0}}_{q^0\approx p^0}(m_c^2 + \underbrace{q_\perp^2}_{=k_\perp^2}) \\
&\approx 2k^0 - \underbrace{\frac{m_a^2}{2p^0}}_{\approx 0} - \frac{k_\perp^2}{2k^0} + \frac{m_c^2}{2p^0} + \underbrace{\frac{k_\perp^2}{2p^0}}_{\approx 0} \approx 2k^0 - \frac{k_\perp^2}{2k^0} + \frac{m_c^2}{2p^0}
\end{aligned} \tag{4.3}$$

Backup, symmetry breaking, NLO

	forward scattering, $m_{b,\text{inside}} = m_{b,\text{h}} \neq 0$	backward scattering, $m_{b,\text{outside}} = m_{b,\text{s}} = 0$
transverse vector boson	$\lvert V\rvert^2 = 4g^2 C_2[R]\frac{1}{x^2}k_\perp^2$ $(\frac{A_{\text{in}}-A_{\text{out}}}{A_{\text{in}}A_{\text{out}}})^2 \approx x^2 \frac{m_{b,\text{in}}^4}{k_\perp^4(k_\perp^2+m_{b,\text{in}}^2)^2}$ $\Delta p_{1\to 2} \approx \frac{m_{b,\text{in}}^2+k_\perp^2}{2k^0} + \frac{m_c^2}{2p^0}$	$\lvert V\rvert^2 = 4g^2 C_2[R]\frac{1}{x^2}k_\perp^2$ $(\frac{A_{\text{in}}-A_{\text{out}}}{A_{\text{in}}A_{\text{out}}})^2 \approx x^2 \frac{m_{b,\text{in}}^4}{((2k^0)^2-k_\perp^2)^2((2k^0)^2-k_\perp^2-m_{b,\text{in}}^2)^2}$ $\Delta p_{1\to 2} \approx 2k^0 - \frac{k_\perp^2}{2k^0} + \frac{m_c^2}{2p^0}$
longitudinal vector boson	$\lvert V\rvert^2 = 4g^2 C_2[R]\frac{1}{x^2}m_{b,\text{in}}^2$ $(\frac{A_{\text{in}}-A_{\text{out}}}{A_{\text{in}}A_{\text{out}}})^2 \approx x^2 \frac{m_{b,\text{in}}^4}{k_\perp^4(k_\perp^2+m_{b,\text{in}}^2)^2}$ $\Delta p_{1\to 2} \approx \frac{m_{b,\text{in}}^2+k_\perp^2}{2k^0} + \frac{m_c^2}{2p^0}$	$\lvert V\rvert^2 = 0$