Vacuum Stability in the 2HDM and Beyond

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The Most General Scalar Potential

$$V(\phi_a) = \lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + A_{abc}\phi_a\phi_b\phi_c + m^2_{ab}\phi_a\phi_b + t_a\phi_a + c$$

Expand around EW vacuum $\vec{\phi} \to \vec{v} + \vec{\varphi}$:

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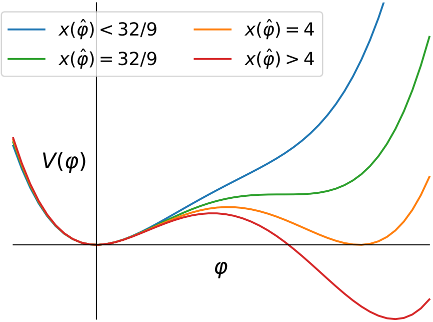
Introduce polar coordinates $\vec{\varphi} \to \varphi\hat{\varphi}$:

$$V(\varphi, \hat{\varphi}) = \lambda(\hat{\varphi})\varphi^4 - A(\hat{\varphi})\varphi^3 + m^2(\hat{\varphi})\varphi^2$$

- $\lambda(\hat{\varphi}) > 0 \Leftrightarrow$ *boundedness from below*
- $A(\hat{\varphi}) > 0$ by choice $(\varphi \leftrightarrow -\varphi)$
- $m^2(\hat{\varphi}) > 0$ if the EW vacuum is a local minimum

Stability of Fieldspace Directions



Possibilities depending on

$$x(\hat{\varphi}) = \frac{A(\hat{\varphi})^2}{m^2(\hat{\varphi})\lambda(\hat{\varphi})}\,.$$

- at most one additional minimum for each $\hat{\varphi}$
- the additional minimum is deeper if $A(\hat{\varphi})^2 > 4m^2(\hat{\varphi})\lambda(\hat{\varphi})$

Lifetime and Vacuum Decay

The vacuum tunneling decay width is given by [Coleman 1977]

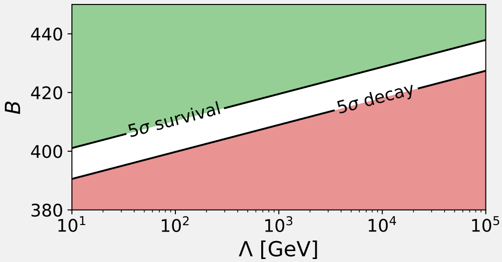
$$\frac{\Gamma}{V} = Ke^{-B}$$

The bounce action B

- analytic solution in the straight path approximation [Adams 1993]
- associated uncertainty of $\mathcal{O}(10\,\%)$

[Masoumi et al. 2017]

The prefactor $K \sim \Lambda^4$



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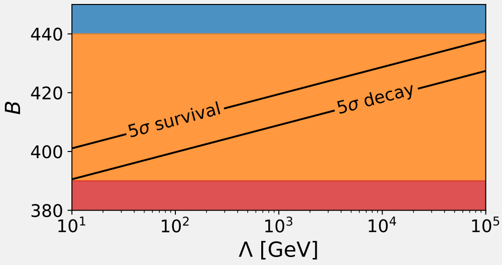

Boundedness from Below

The potential is unbounded in a direction $\hat{\varphi}$ if $V(\varphi, \hat{\varphi}) \to -\infty$ as $\varphi \to \infty$.

Bounded *in the strong sense*

$$\lambda(\hat{\varphi}) > 0 \;\forall\; \hat{\varphi}$$

Bounded *in the weak sense*

$$\lambda(\hat{\varphi}) \geq 0 \;\forall\; \hat{\varphi}$$

$$\left.\begin{aligned} A(\hat{\varphi}) &= 0 \\ m^2(\hat{\varphi}) &> 0 \end{aligned}\right\} \;\forall\; \hat{\varphi} \text{ with } \lambda(\hat{\varphi}) = 0$$

Example: Boundedness in the 2HDM

$$V_4 = \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.} \geq 0$$

$$\text{for} \quad \Phi_1^\dagger\Phi_2 = 0 \quad V_4 = \frac{1}{2}\begin{pmatrix}|\Phi_1|^2 & |\Phi_2|^2\end{pmatrix}\begin{pmatrix}\lambda_1 & \lambda_3 \\ \lambda_3 & \lambda_2\end{pmatrix}\begin{pmatrix}|\Phi_1|^2 \\ |\Phi_2|^2\end{pmatrix} > 0$$

Definitely holds if the matrix is positive definite

$$\det = \lambda_1\lambda_2 - \lambda_3^2 > 0 \Rightarrow \lambda_1\lambda_2 > 0 \text{ with } \mathrm{Tr} = \lambda_1 + \lambda_2 > 0 \Rightarrow \lambda_1, \lambda_2 > 0$$

$$\Rightarrow \sqrt{\lambda_1\lambda_2} > \lambda_3 > -\sqrt{\lambda_1\lambda_2}$$

Only copositivity needed since $|\Phi_1|^2, |\Phi_2|^2 > 0 \Rightarrow \lambda_1, \lambda_2 > 0$ and $\lambda_3 > -\sqrt{\lambda_1\lambda_2}$.

$$\text{for } \Phi_1^\dagger\Phi_1 \neq 0 \text{ additionally } \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2}$$

Example: Boundedness in the MSSM

MSSM Higgs sector maps to the 2HDM as

$$\lambda_1 = \lambda_2 = \frac{g^2 + g'^2}{4}, \; \lambda_3 = \frac{g^2 - g'^2}{4}, \; \lambda_4 = -\frac{g^2}{2}, \; \lambda_5 = 0$$

meaning for boundedness

$$\begin{array}{ccc} \lambda_1, \lambda_2 > 0, & \lambda_3 > -\sqrt{\lambda_1 \lambda_2} & \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1 \lambda_2} \\ g^2 + g'^2 > 0, & g^2 - g'^2 > -g^2 - g'^2, & g^2 - g'^2 - 2g^2 = -g^2 - g'^2 \end{array}$$

$\Rightarrow$ The MSSM Higgs sector is only bounded in the weak sense.

- Long-lived Unbounded Directions?
- Analytic conditions for Boundedness
- Numerical approaches for Boundedness
- Stability at Finite Field Values
- Phases of the 2HDM
- Bilinear Formalism
- Analytic Absolute Stability
- Beyond the 2HDM