Two-loop corrections to the Higgs trilinear coupling in models with classical scale invariance

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Outline

- Why consider the Higgs trilinear coupling?
- Classical scale invariance
- One-loop result for λ_{hhh}
- Extension to two loops
- Theoretical & experimental constraints on CSI parameter space
- Numerical results
- Summary

Introduction/Motivation

Why investigate λ_{hhh}? 1/2

- **Probing the shape of the Higgs potential**: since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:

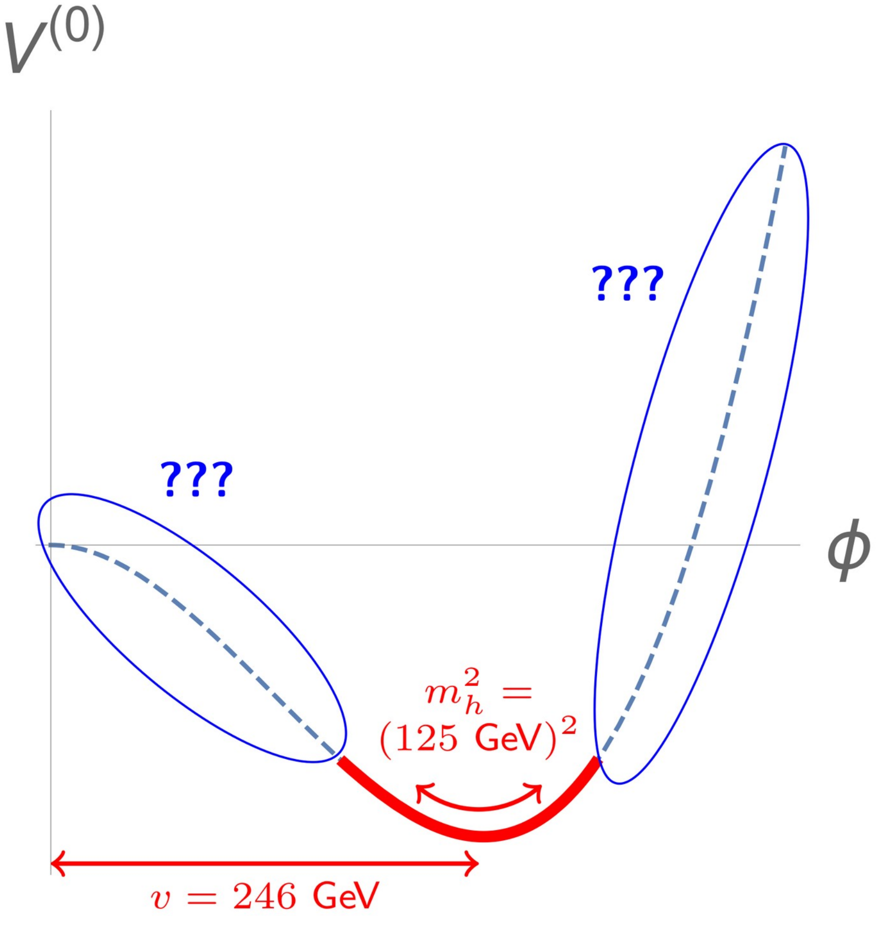
  → the location of the EW minimum: v = 246 GeV

  → the curvature of the potential around the EW minimum: m_h = 125 GeV

  However we still don't know the shape of the Higgs potential, which depends on λ_{hhh}

- **λ_{hhh} determines the nature of the EWPT**!

  ⇒ O(20 − 30%) deviation of λ_{hhh} from its SM prediction needed to have a strongly first-order EWPT → necessary for EWBG [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]



Why investigate λ_{hhh}? 2/2

- **Distinguish alignment with or without decoupling:**

➔ Aligned scenarios already seem to be favoured → Higgs couplings are SM-like at tree-level

➔ Non-aligned scenarios (e.g. in 2HDMs) could be almost entirely excluded in the close future using synergy of HL-LHC and ILC!

→ Alignment through decoupling? or alignment without decoupling?

➔ If alignment without decoupling, Higgs couplings like λ_{hhh} can still exhibit large deviations from SM predictions because of BSM loop effects

➔ Current best limit (at 95% CL): $-3.7 < \lambda_{hhh} /(\lambda_{hhh})^{SM} < 11.5$ [ATLAS-CONF-2019-049]

➔ Improvement at future colliders:

- ➔ HL-LHC: $\lambda_{hhh} /(\lambda_{hhh})^{SM}$ within $\sim 50 - 100\%$;
- ➔ At lepton colliders (ILC, CLIC) within some tens of %;
- ➔ At 100-TeV hadron collider, down to $5 - 7\%$

see e.g. [de Blas et al., 1905.03764], [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], etc.

Classical scale invariance (CSI)

- Forbid mass-dimensionful parameters at classical (= tree) level → tree-level potential: $V^{(0)} = \Lambda_{ijkl}\varphi_i\varphi_j\varphi_k\varphi_l$

- *Explicitly* broken by radiative corrections

- EW symmetry breaking: (c.f. [Coleman, Weinberg '73], [Gildener, Weinberg '76])
  - Must occur along a flat direction of $V^{(0)}$ (= Higgs/scalon direction)
  - EW sym. broken à la **Coleman-Weinberg** along flat direction
  - EW scale generated by **dimensional transmutation**

Classically scale invariant models

- If CSI assumed at Planck scale $\rightarrow$ possible solution to hierarchy problem

  (see e.g. [Bardeen ‘95])

- Here: **CSI assumed around EW scale**, for phenomenology
  - Higgs (scalon) automatically aligned at tree level
  - BSM states can’t be decoupled (no BSM mass term!)

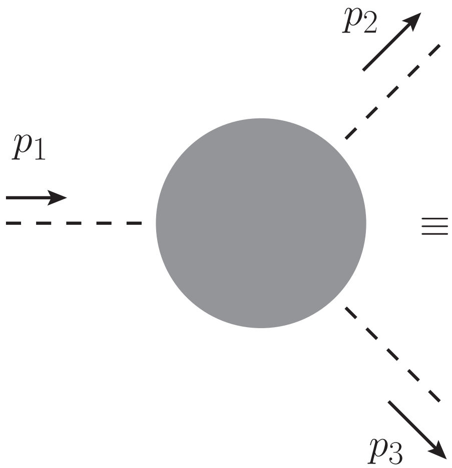
  ➔ CSI scenarios: **alignment with decoupling**

- CSI can (arguably) help lessen the hierarchy problem, even for scenarios that don’t extend up to Planck scale because of Landau poles

Calculational setup

An effective Higgs trilinear coupling

- In principle: consider 3-pt. function Γ_{hhh} but this is momentum dependent → very difficult beyond one loop



$$\equiv \Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$$

- Instead, consider an **effective trilinear coupling**

$$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min}}$$

- Momentum effects are neglected, but are expected to be *sub-leading* anyway
  - At one loop [Kanemura, Okada, Senaha, Yuan ‘04]: effects of a few % (away from thresholds)
  - At two loops, no study for 3-pt. functions but experience from Higgs mass calculations

One-loop effective potential

- Only source of mass: coupling to Higgs and its VEV

$$m_i^2(h) = m_i^2 \times \left(1 + \frac{h}{v}\right)^2$$

- Greatly simplifies the one-loop potential along Higgs (scalon) direction:

$$\mathrm{V}^{(1)} = \frac{1}{64\pi^2}\mathrm{STr}\left[m_X^4(h)\left(\log\frac{m_X^2(h)}{Q^2} - c_X\right)\right] = A(v+h)^4 + B(v+h)^4\log\frac{(v+h)^2}{Q^2}$$

with

$$A \equiv \frac{1}{64\pi^2 v^4}\left\{\mathrm{tr}\left[M_S^4\left(\log\frac{M_S^2}{v^2} - \frac{3}{2}\right)\right] - 4\mathrm{tr}\left[M_f^4\left(\log\frac{M_f^2}{v^2} - \frac{3}{2}\right)\right] + 3\mathrm{tr}\left[M_V^4\left(\log\frac{M_V^2}{v^2} - \frac{5}{6}\right)\right]\right\}$$

$$B \equiv \frac{1}{64\pi^2 v^4}\left(\mathrm{tr}\left[M_S^4\right] - 4\mathrm{tr}\left[M_f^4\right] + 3\mathrm{tr}\left[M_V^4\right]\right)$$

One-loop λ_{hhh}

- Taking derivatives of the potential:
  - 1^{st} derivative: tadpole equation → eliminate A

$$\left.\frac{\partial V^{(1)}}{\partial h}\right|_{\mathrm{min}} = 0 \quad \Rightarrow \quad A = -B\left(\frac{1}{2} + \log\frac{v^2}{Q^2}\right)$$

  - 2^{nd} derivative: Higgs (curvature) mass → fix B

$$\left.\frac{\partial^2 V^{(1)}}{\partial h^2}\right|_{\mathrm{min}} = [M_h^2]_{V_{\mathrm{eff}}} \quad \Rightarrow \quad B = \frac{[M_h^2]_{V_{\mathrm{eff}}}}{8v^2}$$

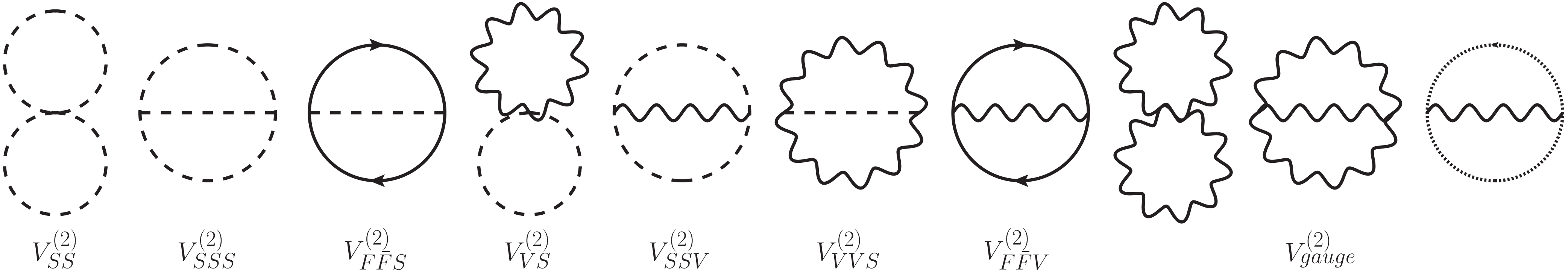
  - 3^{rd} derivative: λ_{hhh} → no further free parameter

$$\lambda_{hhh} = \left.\frac{\partial^3 V^{(1)}}{\partial h}\right|_{\mathrm{min}} = \frac{5[M_h^2]_{V_{\mathrm{eff}}}}{v} = \frac{5}{3}(\lambda_{hhh}^{(0)})^{\mathrm{SM}}$$

- **<u>Universal</u>** result for CSI models (w/o mixing)!

Two-loop effective potential

- Once one includes two-loop corrections, the form of the effective potential is changed



- New form:

$$V_{\text{eff}} = A(v+h)^4 + B(v+h)^4 \log\frac{(v+h)^2}{Q^2} + C(v+h)^4 \log^2\frac{(v+h)^2}{Q^2}$$

Two-loop λ_{hhh} in CSI models

[JB, Kanemura, Shimoda '20]

- Follow same procedure as at one loop:
  - ➔ Eliminate A with tadpole eq., B with Higgs mass
  - ➔ Still, C remains!

- One finds:

$$\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min}} = \frac{5[M_h^2]_{V_{\text{eff}}}}{v} + 32Cv$$

- ➔ Deviation in λ_{hhh} depends on log^2 term in V_{eff}
- ➔ **Universality found at one loop is lost at two loops!**

Computing λ_{hhh} in non-CSI models

- Massive models → V_{eff} has a more complicated form

- Define a differential operator D_3 to compute eff. Higgs trilinear coupling, including tadpole eq. and Higgs mass

$$\lambda_{hhh} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \mathcal{D}_3 \Delta V_{\text{eff}}\Big|_{\text{min.}}, \qquad \text{with } \mathcal{D}_3 \equiv \frac{\partial^3}{\partial h^3} - \frac{3}{v}\left[-\frac{1}{v}\frac{\partial}{\partial h} + \frac{\partial^2}{\partial h^2}\right]$$

- Same overall procedure for scheme conversion (c.f. next slide) in CSI/non-CSI models

$\overline{\text{MS}}$ to OS scheme conversion

- V_{eff}: we use expressions in $\overline{\text{MS}}$ scheme hence results for λ_{hhh} also in $\overline{\text{MS}}$ scheme

- We include finite counterterms to express the Higgs trilinear coupling in terms of physical quantities

$$\underbrace{m_X^2}_{\overline{\text{MS}}} = \underbrace{M_X^2}_{\text{pole}} - \mathfrak{Re}[\Pi_{XX}^{\text{fin.}}(p^2 = M_X^2)], \qquad v^2 = \underbrace{(\sqrt{2}G_F)^{-1}}_{\equiv v_{\text{OS}}^2} + \frac{3M_t^2}{16\pi^2}\left(2\log\frac{M_t^2}{Q^2} - 1\right) + \cdots$$

- Also we include finite WFR effects → OS scheme

$$\underbrace{\hat{\lambda}_{hhh}}_{\text{OS}} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{finite WFR}} \underbrace{\lambda_{hhh}}_{\overline{\text{MS}}} = -\underbrace{\Gamma_{hhh}(0,0,0)}_{\text{3-pt. func.}}$$

Numerical analysis

Our questions:

- how large can two-loop effects be?
- can they allow distinguishing CSI vs non-CSI?

CSI-2HDM

(See e.g. [Lee, Pilaftsis '12]*)*

$$V^{(0)} = \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5\left((\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right)$$

- Similar conventions to usual 2HDM, but no mass terms m_{11}, m_{22}, m_{12}
- CP-conserving case, Z_2 symmetry imposed to avoid tree-level FCNCs

$$\Phi_1 \to \Phi_1\,, \qquad \Phi_2 \to -\Phi_2$$

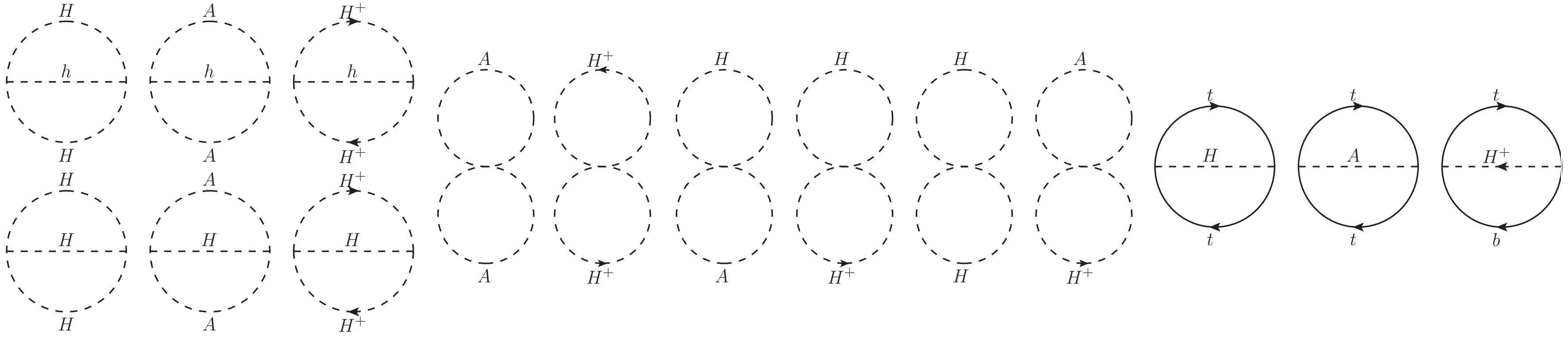
- At tree-level, 4 free parameters: 3 scalar masses + tanβ

$$m_H^2 = -(\lambda_3+\lambda_4+\lambda_5)v^2\,, \quad m_A^2 = -\lambda_5 v^2\,, \quad m_{H^\pm}^2 = -\frac{1}{2}(\lambda_4+\lambda_5)v^2\,, \quad \tan\beta = v_2/v_1$$

- Dominant corrections to $V^{(2)}$:
  - diagrams involving BSM scalars (H,A,H^+) and top quark
  - neglect corrections to CP-even mixing angle α (=β-π/2 at tree level) as sub-leading effect on λ_{hhh} + exp. results indicate near alignment

Steps of calculation in the CSI-2HDM

- Compute V_{eff} ($\overline{\text{MS}}$ scheme)



- Derive C and then λ_{hhh} → $\overline{\text{MS}}$ scheme result

- Convert BSM scalar (H,A,H⁺) and top quark masses + include finite WFR → OS scheme result

$$\hat{\lambda}_{hhh} = \frac{5M_h^2}{v_{\text{OS}}} + \frac{1}{16\pi^2}\frac{5M_h^2}{v_{\text{OS}}}\left[\frac{7}{2}\frac{M_t^2}{v_{\text{OS}}^2} - \frac{2}{3}\frac{M_\Phi^2}{v_{\text{OS}}^2}\right] + \frac{1}{(16\pi^2)^2}\left[\frac{768 g_3^2 M_t^4}{v_{\text{OS}}^3} - \frac{288 M_t^6}{v_{\text{OS}}^5} + \frac{32 M_\Phi^6}{v_{\text{OS}}^5}(4 + 21\cot^2 2\beta) + \frac{192 M_t^2 M_\Phi^4 \cot^2\beta}{v_{\text{OS}}^5}\left(1 - \frac{3M_t^2}{M_\Phi^2} - \frac{3M_t^4}{2M_\Phi^4}\right)\right].$$

(*Blue: SM*; *red: BSM*)

Theoretical and experimental constraints

- **Perturbative unitarity**: we constrain parameters entering only at two loops
  → tree-level perturbative unitarity suffices [Kanemura, Kubota, Takasugi '93]

- EW vacuum must be **true minimum of V_{eff}**, i.e. check that

$$\underbrace{V_{\text{eff}}(v+h=0)}_{=0} - V_{\text{eff}}(h=0) > 0 \quad \Rightarrow \quad V_{\text{eff}}(h=0) < 0$$

- M_h, generated at loop level, must be **125 GeV**
  → imposes a relation between SM parameters, M_H, M_A, M_{H^+}, tanβ, e.g. we can extract:

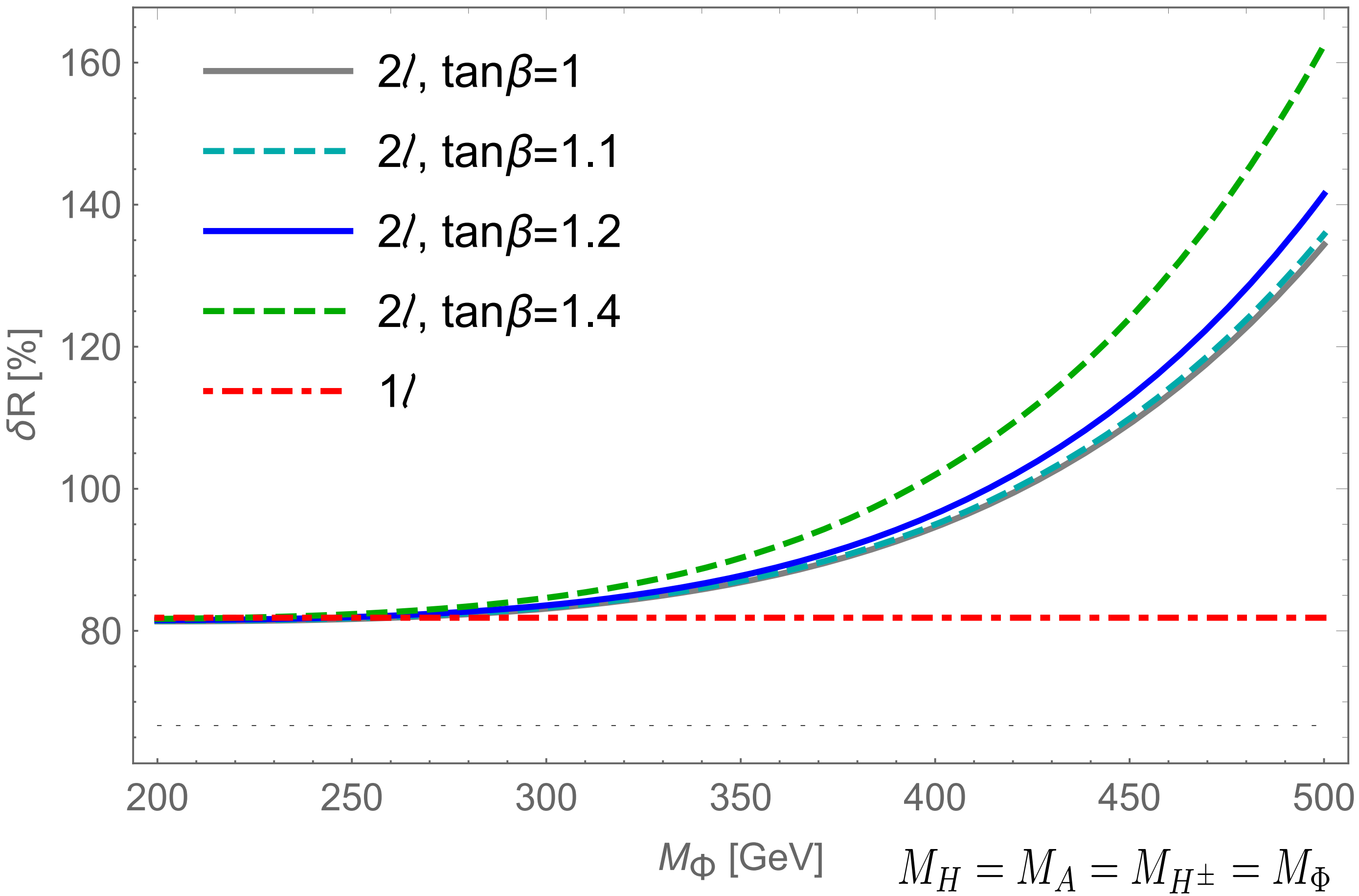
$$[M_h^2]_{V_{\text{eff}}} = \frac{\partial^2 V_{\text{eff}}}{\partial h^2}\bigg|_{\text{min}} \quad \Rightarrow \quad \tan\beta = \tan\beta(\underbrace{M_h, M_t, \cdots}_{\text{measured SM values}}, M_H, M_A, M_{H^\pm})$$

- Limits from **collider searches** with HiggsBounds

Numerical results

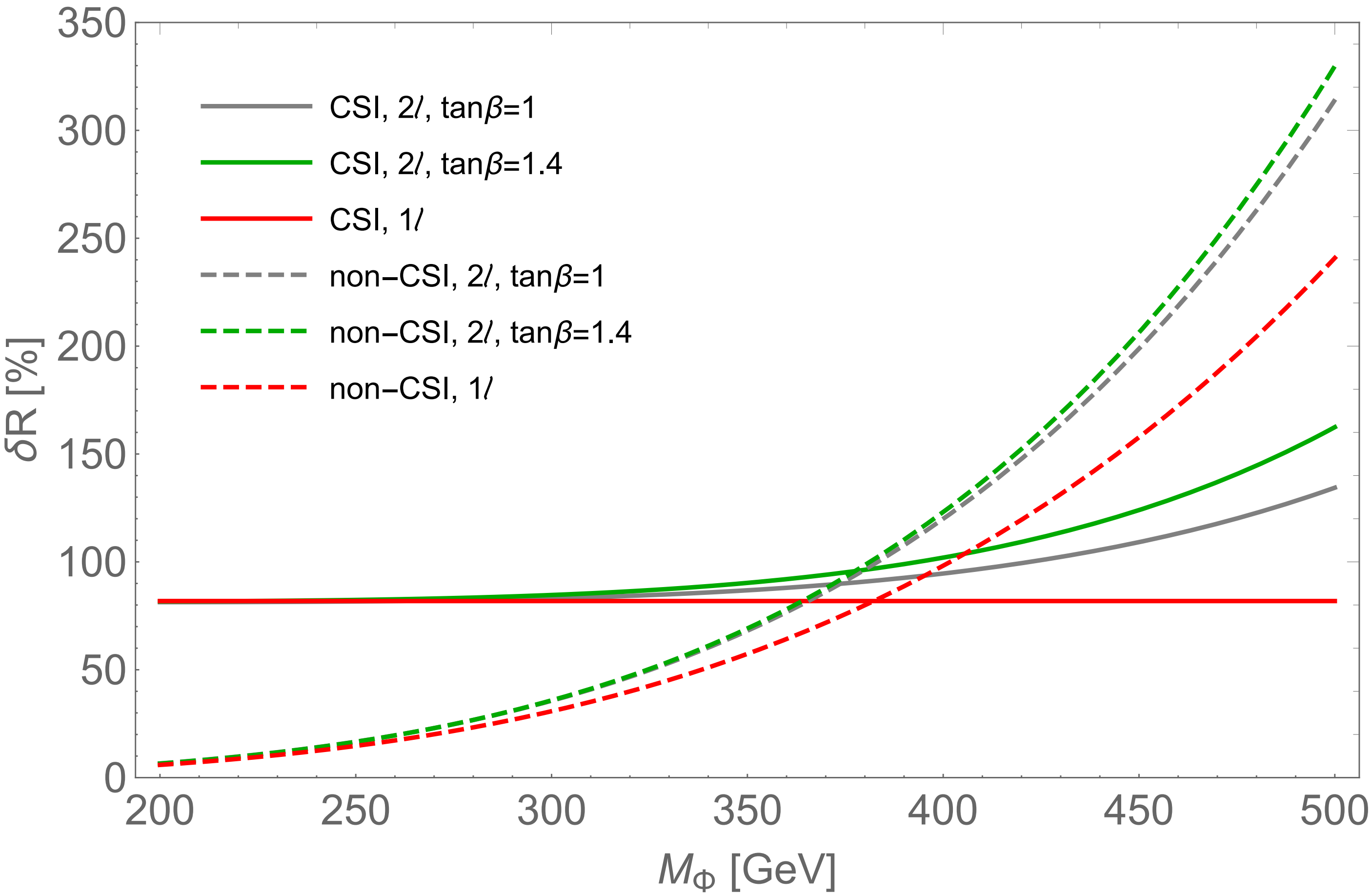
$$\delta R \equiv \frac{\hat{\lambda}_{hhh}^{\text{CSI-2HDM}} - \hat{\lambda}_{hhh}^{\text{SM}}}{\hat{\lambda}_{hhh}^{\text{SM}}}$$

No constraints



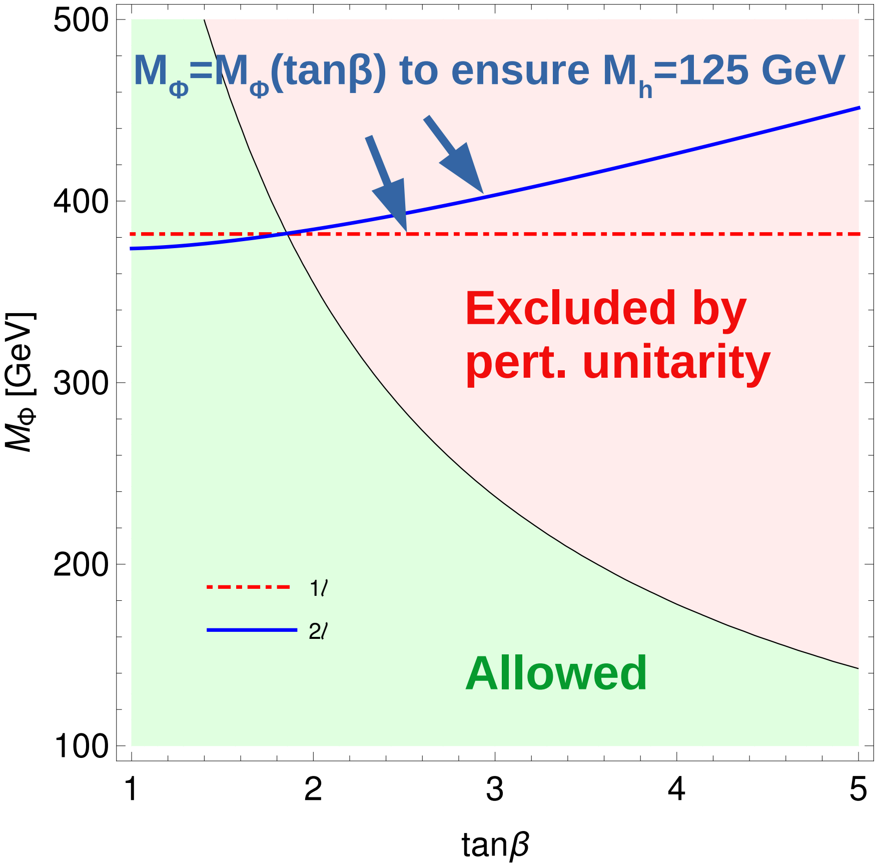
$M_H = M_A = M_{H^\pm} = M_\Phi$

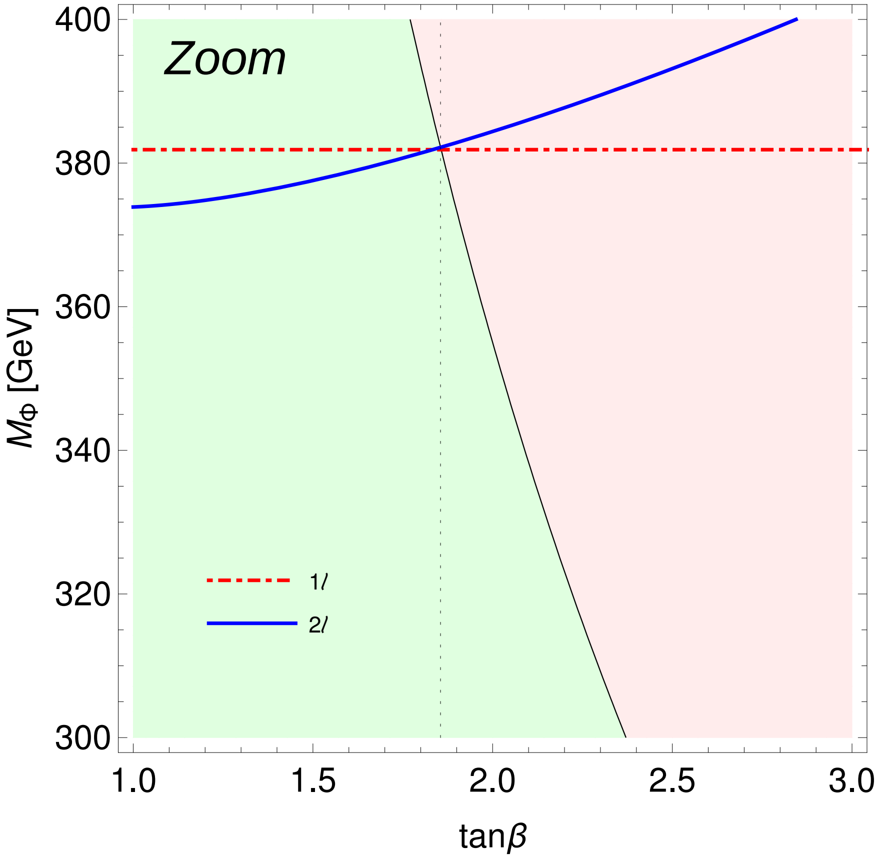
CSI vs non-CSI 2HDMs



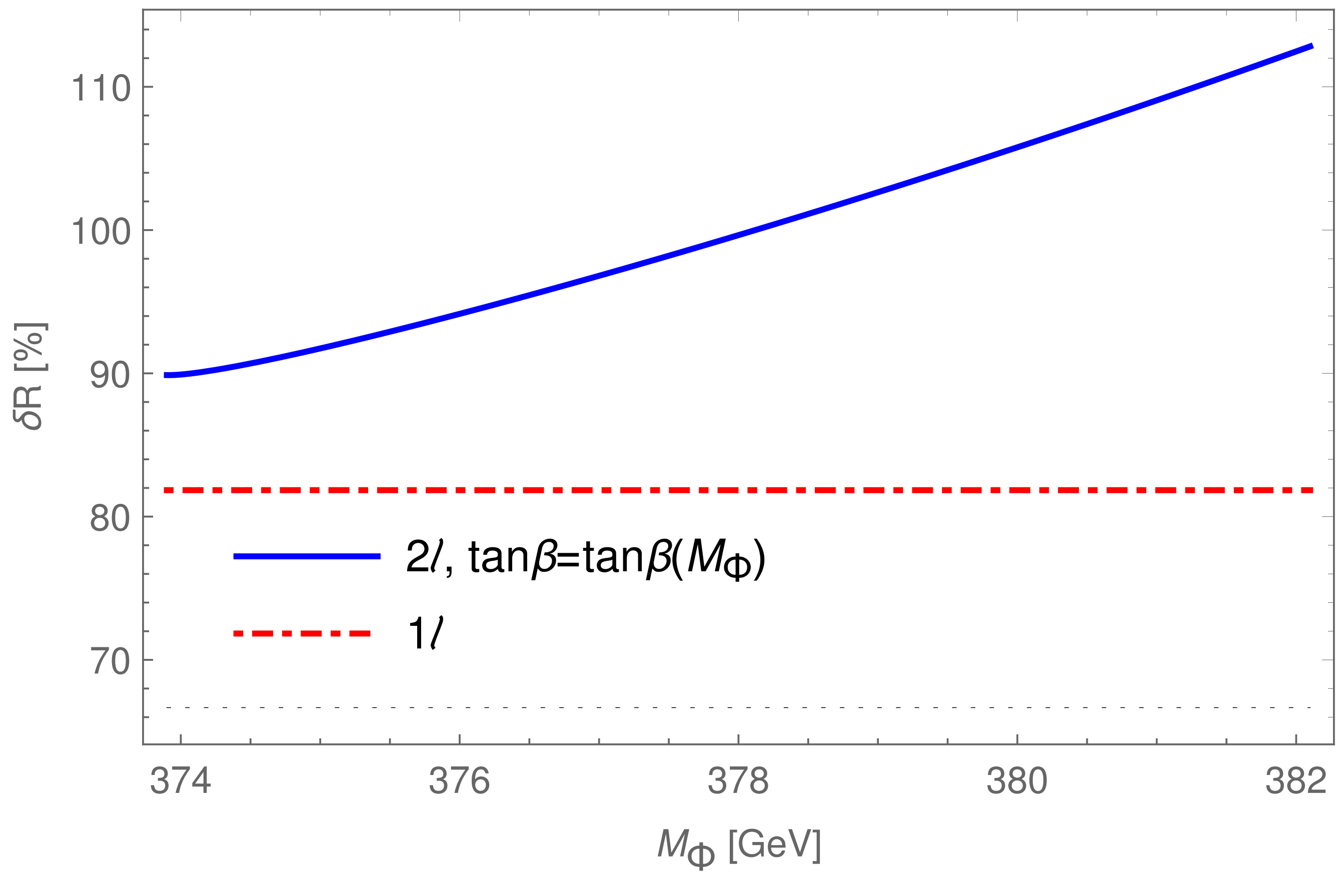
- *Solid*: CSI-2HDM [JB, Kanemura, Shimoda ‘20]
- *Dashed*: normal 2HDM, in maximal non-decoupling limit M=0 [JB, Kanemura ‘19]

Unitarity and constraint from M_h





Once all constraints are included



Summary

- First two-loop calculation of Higgs trilinear coupling in theories with CSI
  - Matches level of accuracy for non-CSI, non-SUSY, extensions of SM in [JB, Kanemura '19]
  - Two-loop corrections allow distinguishing different scenarios with CSI
  - Separate models w. or w/o. CSI difficult with only λ_{hhh}, but possible with **synergy** of λ_{hhh} and either collider or GW signals (see e.g. [Hashino, Kakizaki, Kanemura, Matsui '16])

- Appendix includes results for generic CSI theories (adapted from Steve Martin's expressions for V_{eff} in [hep-ph/0111209])

Thank you very much for your attention!