Effective field theories and thermal effects for dark matter evolution in the Early Universe

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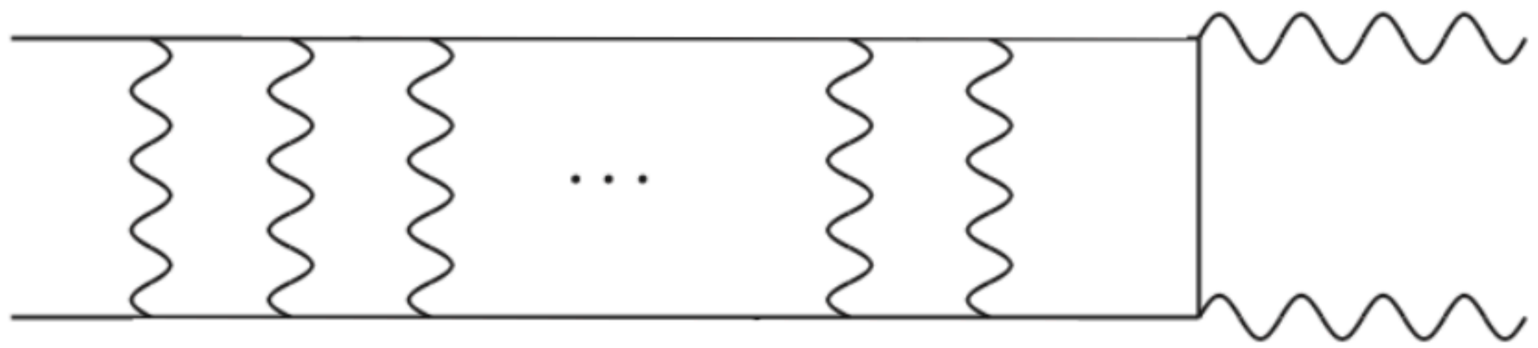
Introduction and motivation

- Explain the existence of the dark matter in the Universe:
  $\Omega_{DM} = 0.12 \pm 0.0012$

- Precise determination of the parameter space of the dark matter: (α, m)

- Develop a simple and systematic framework for the dark matter pair's description during freeze-out times (when particles are non-relativistic, i.e. cold dark matter):

$$(\partial_t + 3H)n = -\langle \sigma_{\mathrm{ann}} v_{\mathrm{rel}} \rangle (n^2 - n_{\mathrm{eq}}^2)$$

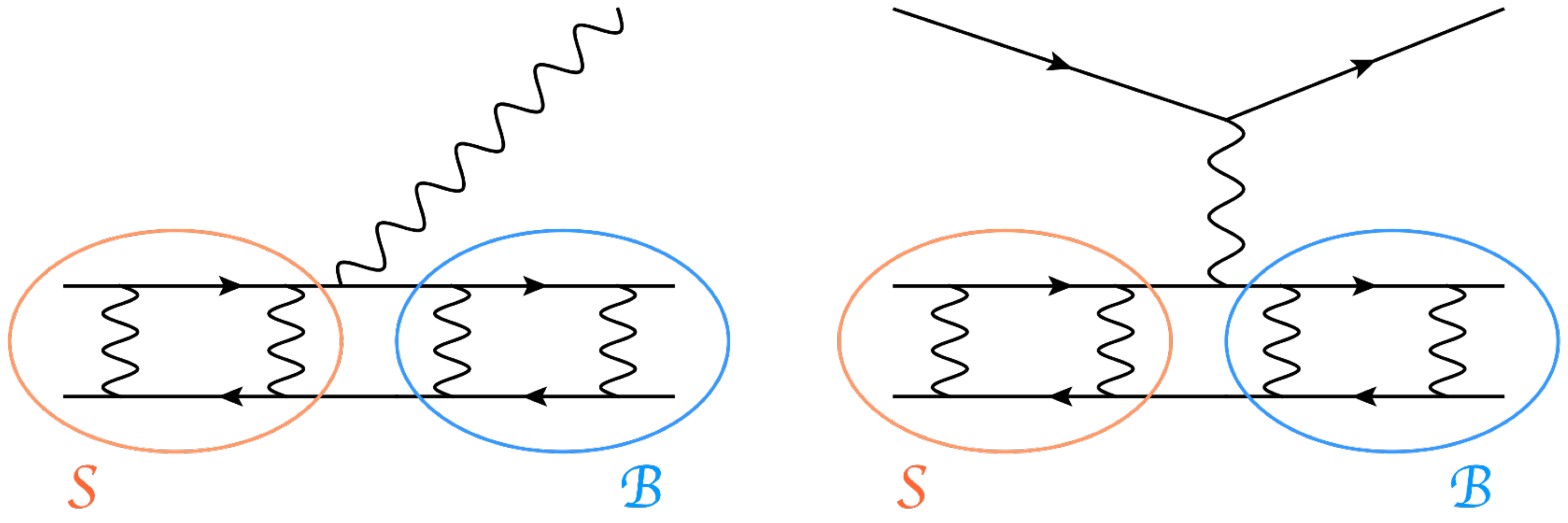


For instance, the accounting for the ladder resummation of soft gauge bosons lowers the relic DM density by 50% (Sommerfeld effect)

Introduction and motivation

- Determine the impact of the presence of bound states within the dark sector (wimponium),
  $U(1): S(\chi\bar{\chi}) \leftrightharpoons B(\chi\bar{\chi})$
  $SU(3): S(\chi\bar{\chi})_8 \leftrightharpoons B(\chi\bar{\chi})_1$

  when thermal effects are non-negligible



  Applications:

- Early universe evolution with primordial plasma

  (nucleosynthesis, relic abundances of DM)

- Heavy quarkonium production during heavy ion collision in QGP

  (properties of QGP, quarkonium suppression)

Model of the dark sector

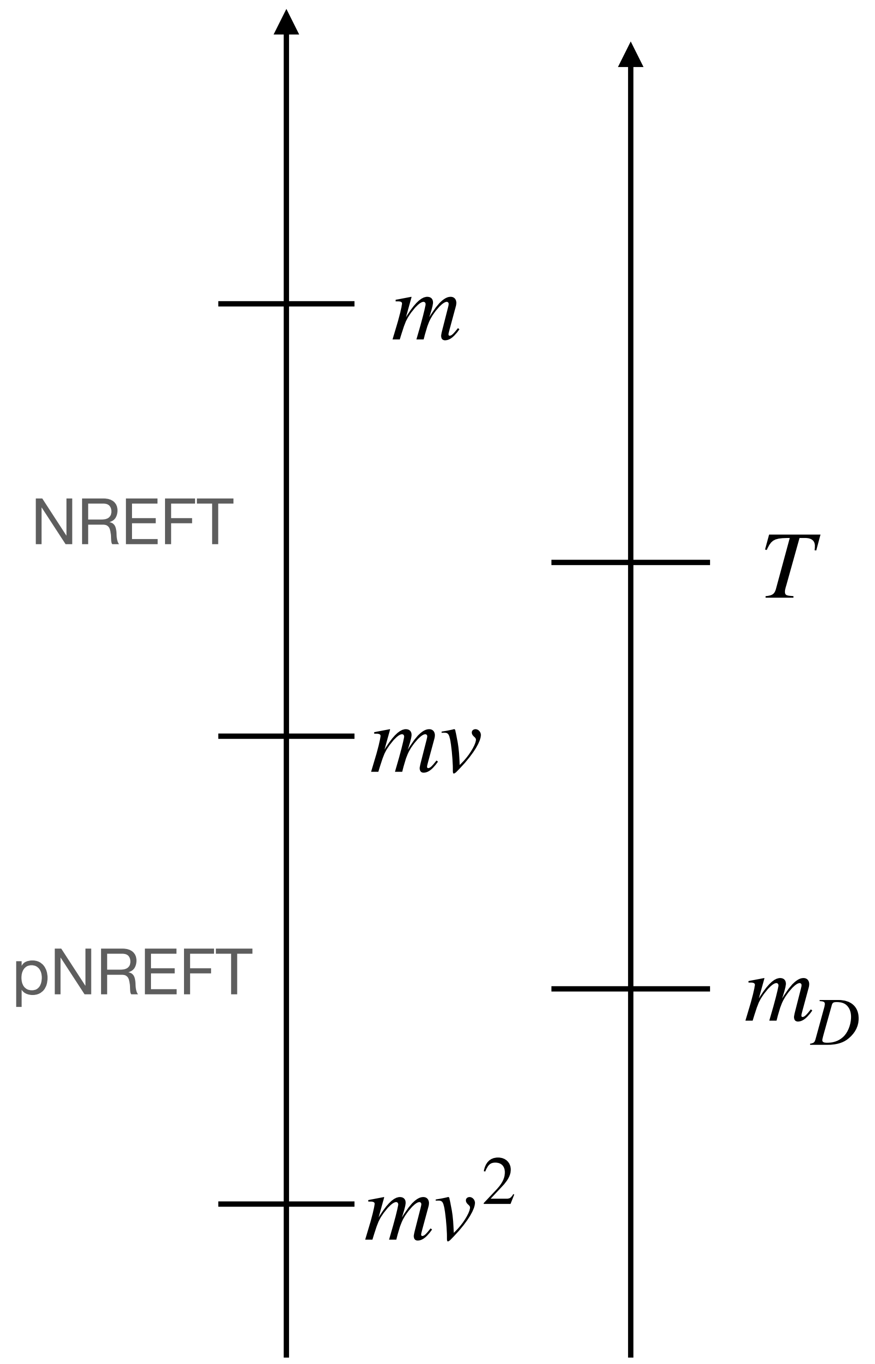
$$\mathcal{L}_{DM} = \bar{X}(i\not{D} - m)X - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{n_f} \bar{f}_i(i\not{D} - m_f)f_i,$$

- Heavy dark matter fermion
- Dark photons
- Light d.o.f, fermions - enable scale m_D to appear

* Debye mass: $m_D^2 = n_f \frac{g^2T^2}{3}$

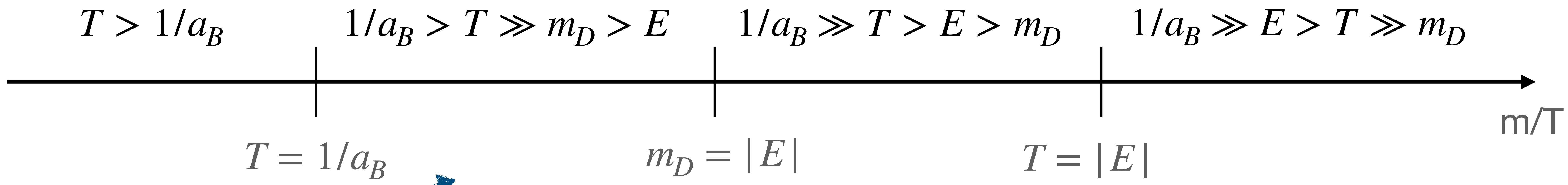
Energy scales present

- Two energy scales:
  - $T \gg m_D$ (thermal)
  - $m \gg mv \gg mv^2$ (non-relativistic)



- Cosmologically interesting: $m \gg T$;
  $T \gg m_D \sim gT$ (weakly interacting plasma)

- As the Universe expanse, temperature drops:



Decoupling from ionisation equilibrium usually occurs here

Universe expands

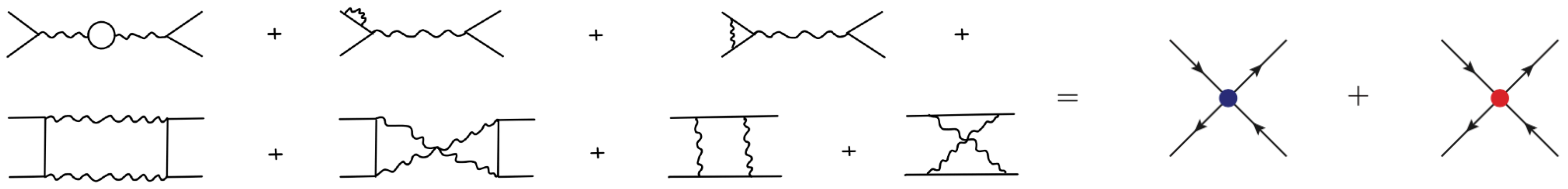
To get EFT for a specific energy hierarchy, we need to integrate out scale by scale, starting from the highest (m scale)

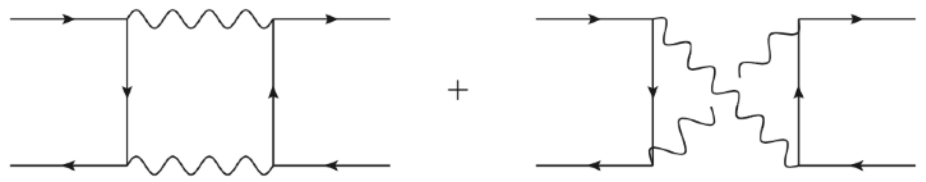
Pair description in EFT: NRQED

$$\mathcal{L}_{DM} = \bar{X}(i\not{D} - m)X - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{n_f} \bar{f}_i(i\not{D} - m_f)f_i,$$

Integrating out the hard scale $m \gg mv \sim p \sim \frac{1}{a_B}$

$$\begin{aligned}
\mathcal{L}_{NRQED_{DM}} =& \psi^\dagger \left(iD_0 + \frac{\vec{D}^2}{2m} + c_F \frac{\vec{\sigma} g\vec{B}}{2m} + c_D \frac{\vec{\nabla} g\vec{E}}{8m^2} + i c_S \frac{\vec{D}\times g\vec{E} - g\vec{E}\times\vec{D}}{8m^2} \right)\psi \\
&+ \chi^\dagger \left(iD_0 - \frac{\vec{D}^2}{2m} - c_F \frac{\vec{\sigma} g\vec{B}}{2m} + c_D \frac{\vec{\nabla} g\vec{E}}{8m^2} + i c_S \frac{\vec{D}\times g\vec{E} - g\vec{E}\times\vec{D}}{8m^2} \right)\chi \\
&- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{d_s}{m^2}F^{\mu\nu}\vec{D}^2F_{\mu\nu} + \frac{d_s}{m^2}\psi^\dagger\chi\chi^\dagger\psi + \frac{d_v}{m^2}\psi^\dagger\vec{\sigma}\chi\chi^\dagger\vec{\sigma}\psi \\
&+ \sum_{i=1}^{n_f} \bar{f}_i(i\not{D} - m_f)f_i + \mathcal{O}(1/m^3),
\end{aligned}$$





Pair description in EFT: pNRQED

Integrating out the soft scale $mv \gg mv^2 \sim E$

$$\mathcal{L}_{pNRQED_{DM}} = \int d^3r \phi^\dagger(t,\vec{r},\vec{R}) \left(i\partial_0 - H(\vec{r},\vec{p},\vec{P},\vec{s_1},\vec{s_2}) + g\vec{r}\vec{E}(\vec{R},t) \right) \phi(t,\vec{r},\vec{R}) - \\ -\frac{1}{4}F^{\mu\nu}(\vec{R},t)F_{\mu\nu}(\vec{R},t) + \mathcal{L}_{lightfermions},$$

Non-local in space
Local in time

$$H(\vec{r},\vec{p},\vec{P},\vec{s_1},\vec{s_2}) = \frac{\vec{p}^2}{m} + \frac{\vec{P}^2}{4m} + \mathcal{O}(1/m^3) + V(\vec{r},\vec{p},\vec{P},\vec{s_1},\vec{s_2});$$

$$V(\vec{r},\vec{p},\vec{P},\vec{s_1},\vec{s_2}) = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \mathcal{O}(1/m^3).$$

Bound state formation/dissociation from TFT

- The bound-state formation cross section, could be inferred as (i.e. optical theorem):

$$\sigma_{BSF} v_{rel} = < p, l | \frac{\Sigma_S^{>}}{i} | p, l > = -2 < p, l | \Im[\Sigma_S] | p, l >$$

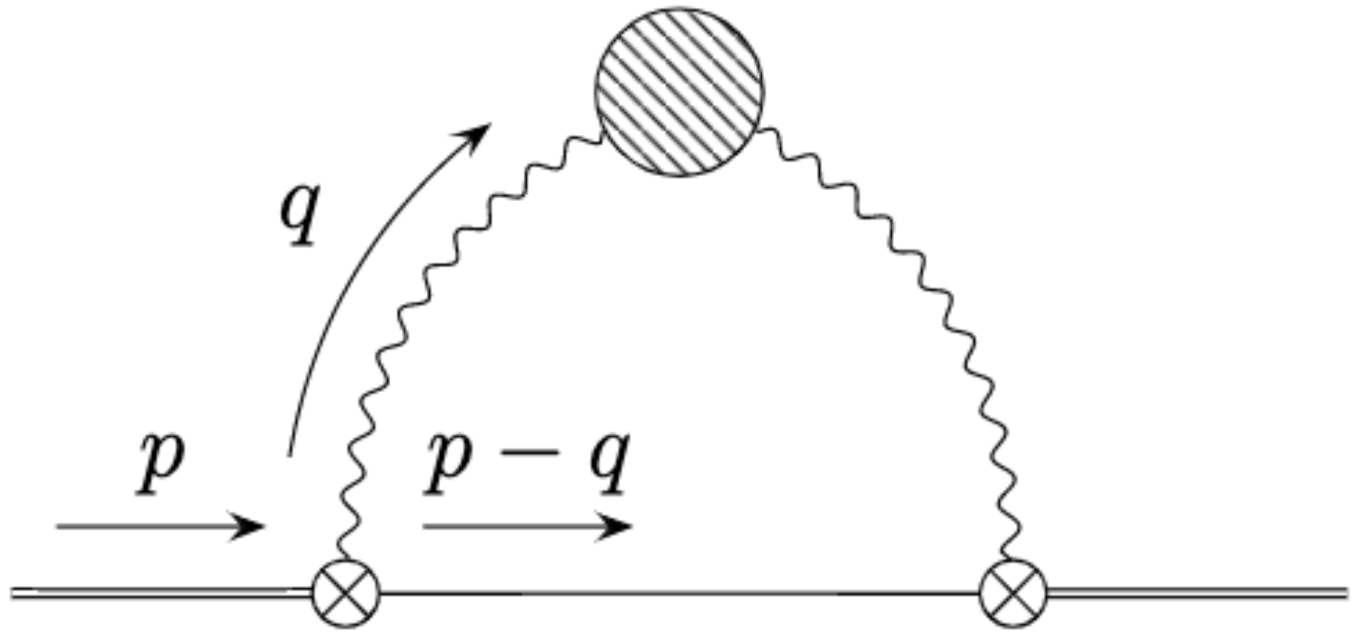
Pair's self-energy

Self-energy in pNRQED at lowest order operator expansion:

$$\Sigma_{S/B} = -ig^2 \int_0^\infty dt e^{it(p_0 - h^{(0)})} r^i r^j \left\langle E^i(t,0) E^j(0,0) \right\rangle =$$

$$= -ig^2 \frac{\mu^{4-d}}{d-1} r_i^2 \int \frac{d^d q}{(2\pi)^d} \frac{i}{p_0 - q_0 - h^{(0)} + i\epsilon} \left\langle \vec{E}(q) \vec{E}(0) \right\rangle$$

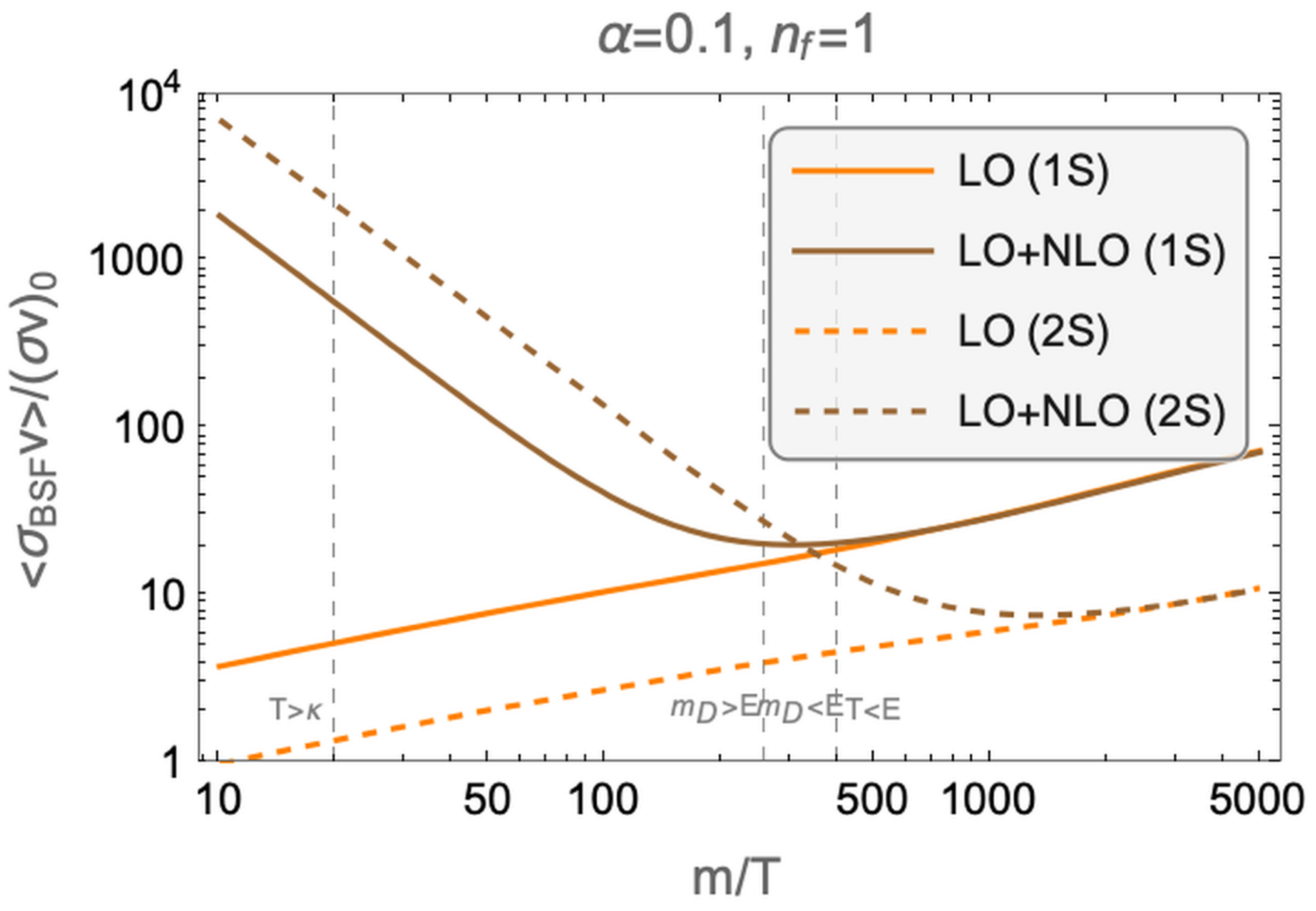
$$= -ig_d^2 \frac{\mu^{4-d}}{d-1} r_i^2 \int \frac{d^d q}{(2\pi)} \frac{i}{p_0 - q_0 - h^{(0)} + i\epsilon} [q_0^2 D_{ii}(q) + \vec{q}^2 D_{00}(q)]$$



BSF at fixed order*

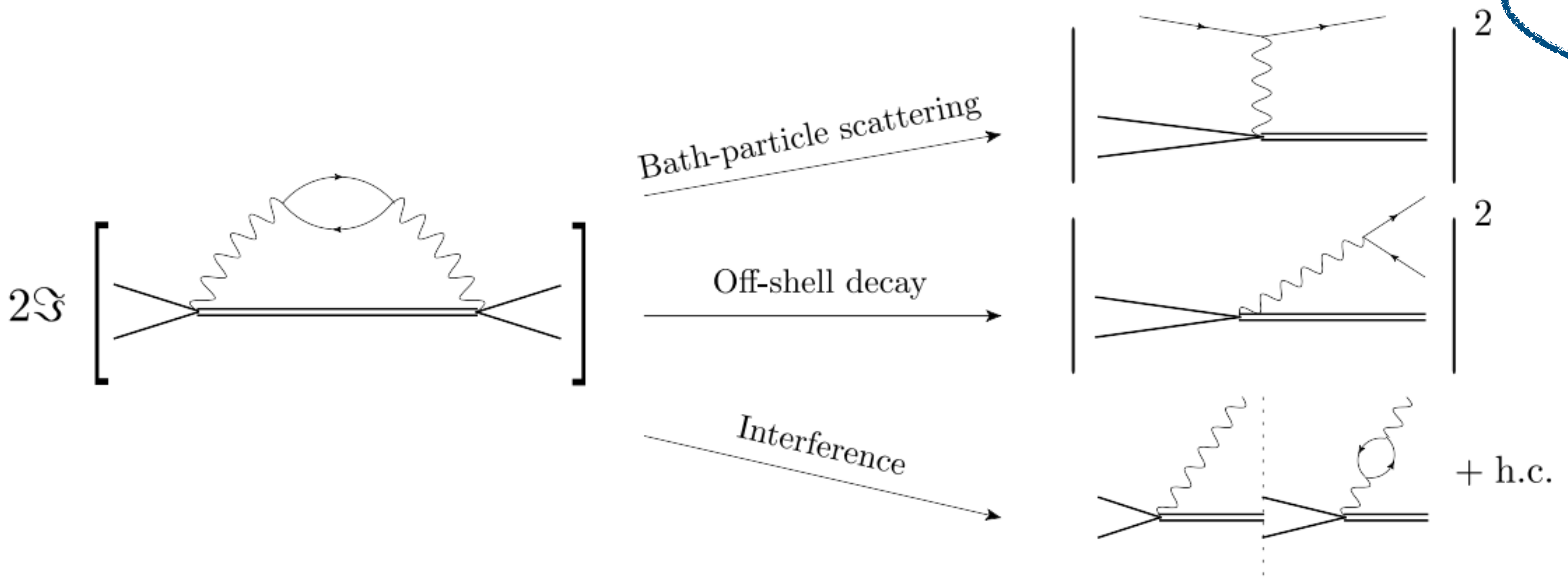
*in EE propagator

- LO (see 2304.00113)

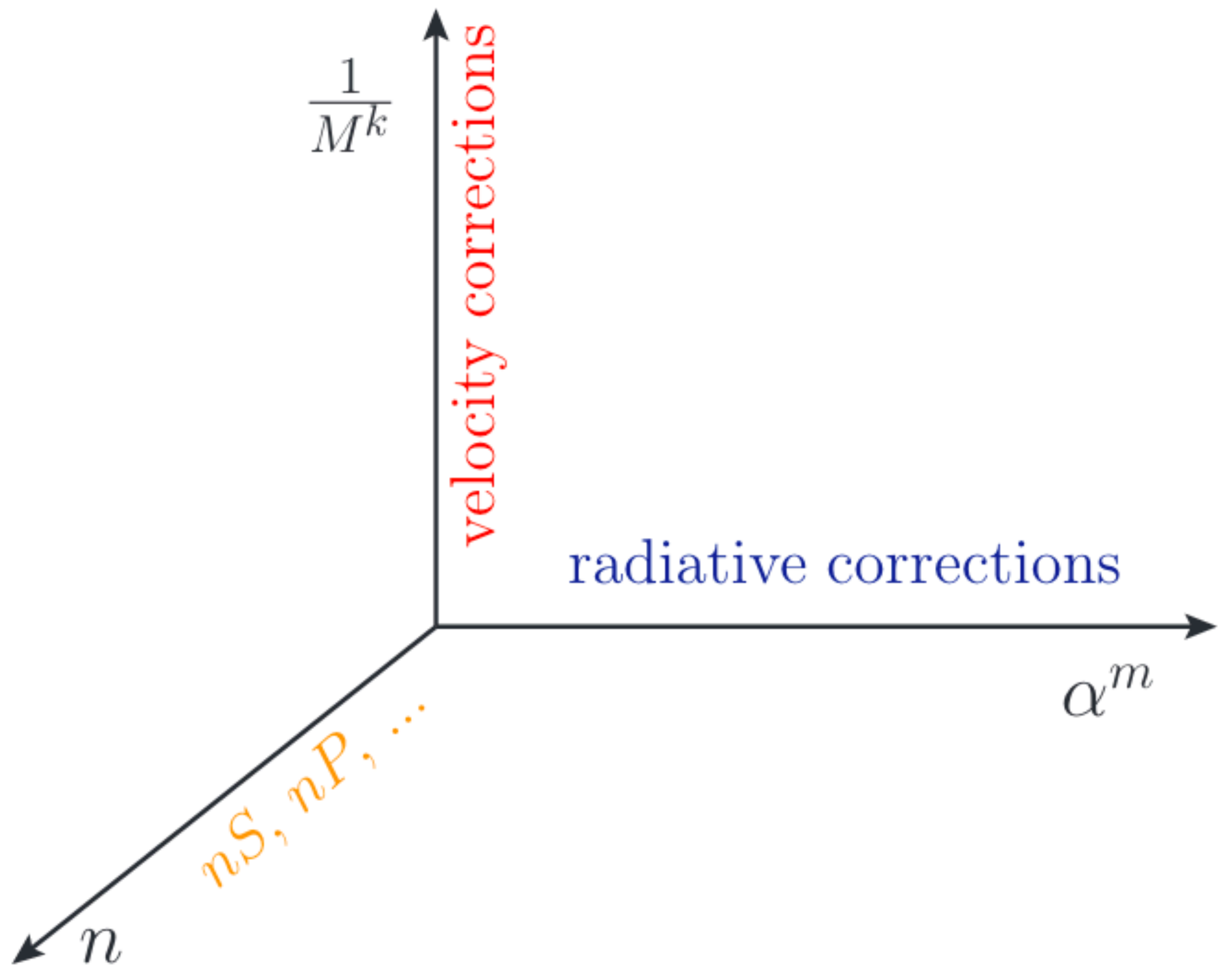


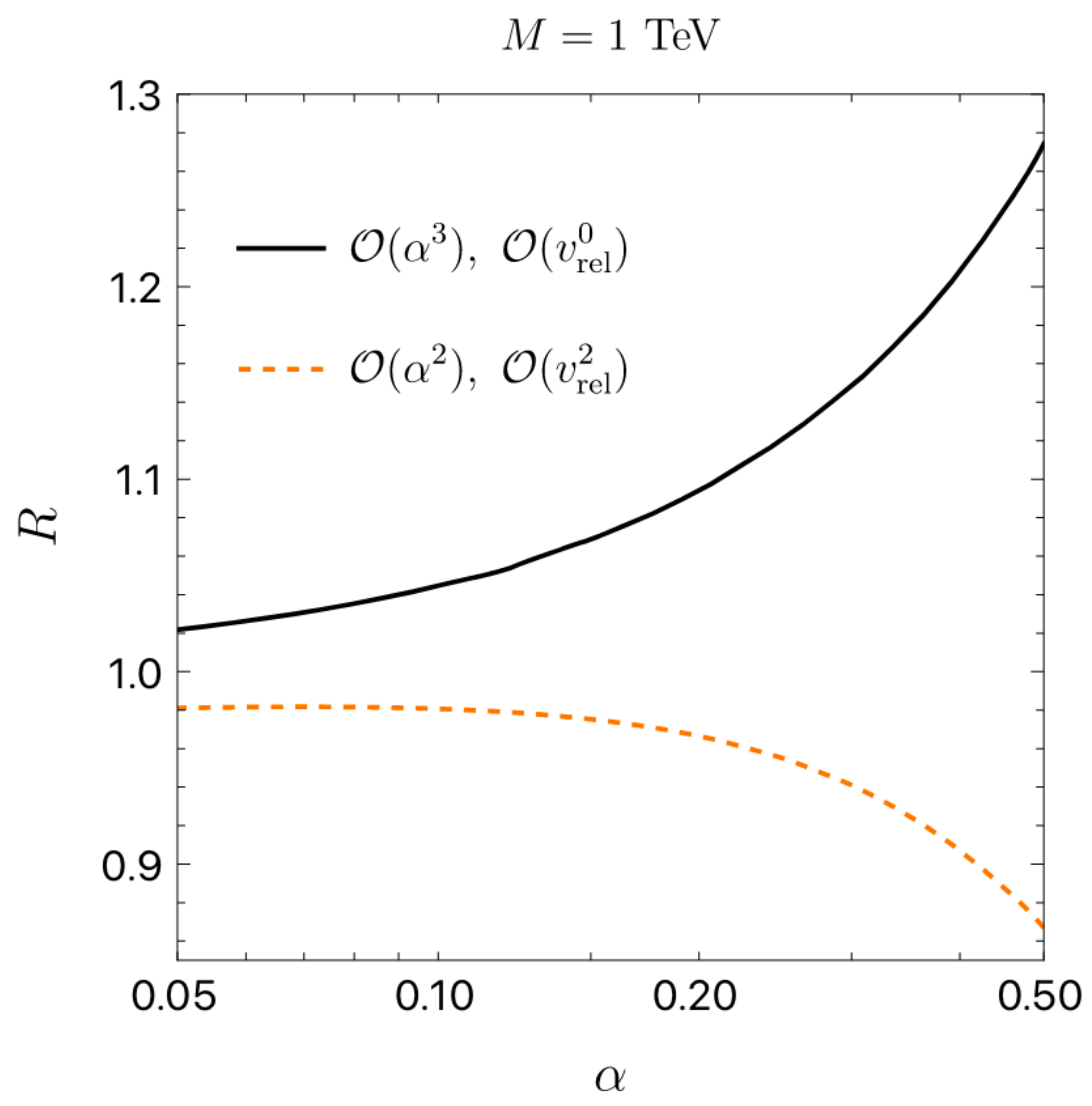
- NLO (see 2002.07145)

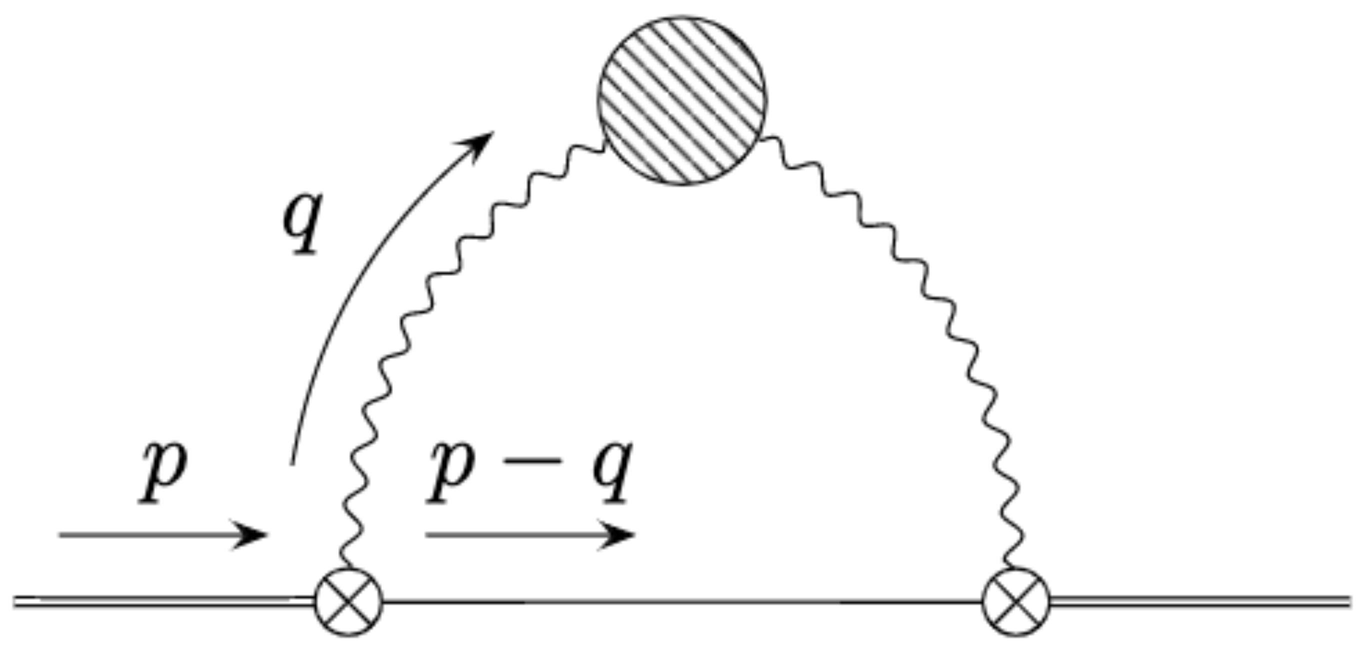
No hierarchy between scales E and T
m_D scale - not addressed



Higher order corrections







see 2304.00113
for discussion of corrections

Addressing the scale m_D

$m \gg mv \gg T \gg m_D, E$

- Presence of the scale m_D requires HTL resummation

$$\hat{D}_{\mu\nu} = \hat{D}^0_{\mu\nu} + \hat{D}^0_{\mu\lambda}(i\hat{\Pi}^{\lambda\rho})\hat{D}^0_{\rho\nu} + \ldots$$

$\Rightarrow$

$$D^{00}_{R/A} = \frac{i}{\vec{q}^2 + \Pi^{00}_{R/A}};$$

$$D^{ij}_{R/A} = \left(\delta_{ij} - \frac{q_i q_j}{\vec{q}^2}\right)\Delta_{R/A} = \frac{i\left(\delta_{ij} - \frac{q_i q_j}{\vec{q}^2}\right)}{(q_0 \pm i\epsilon)^2 - \vec{q}^2 + \Pi^T_{R/A}}$$

Exploiting the large scale separation

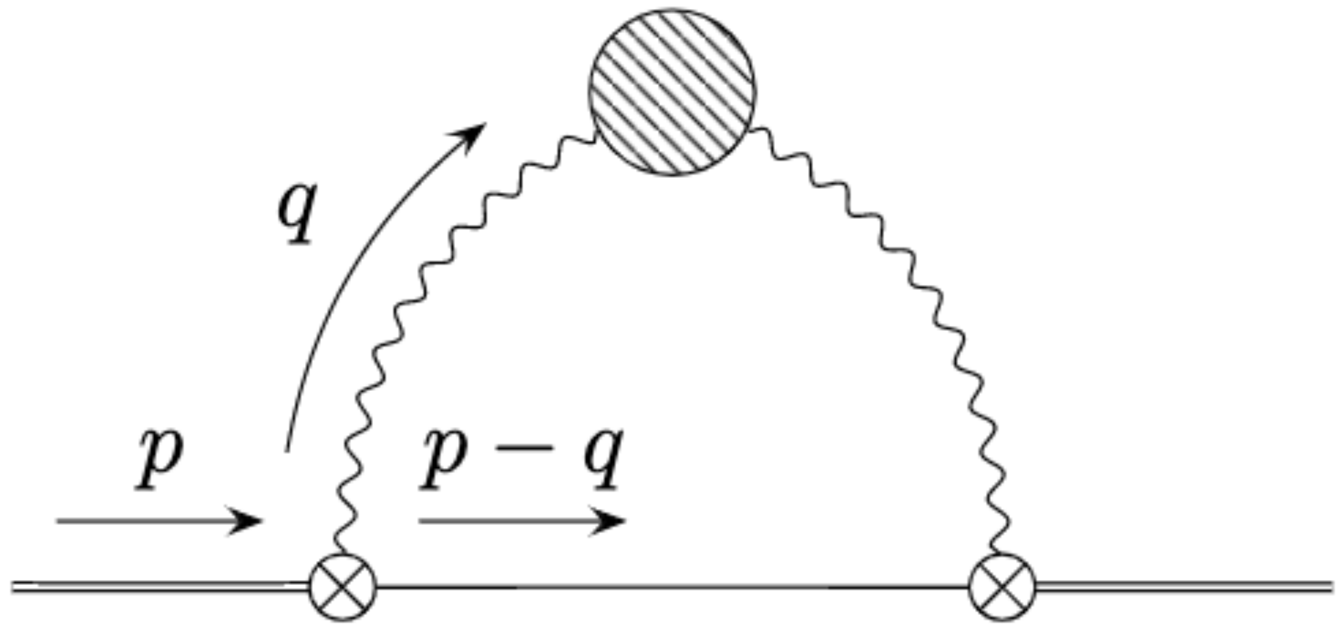
$m \gg mv \gg T \gg m_D, E$

- After integrating out scale M and Mv, we start with $pNRQED_{DM}$ at $T = 0$

$$\mathscr{L}_{pNREFT} = \mathscr{L}_{light} + \int d^3 r \phi^+ [i\partial_0 - \hat{h}^{(0)}]\phi + \phi^+ \vec{r} g \vec{E} \phi$$

- Integrate out scale T

  The photon sector is modified to HTL one.



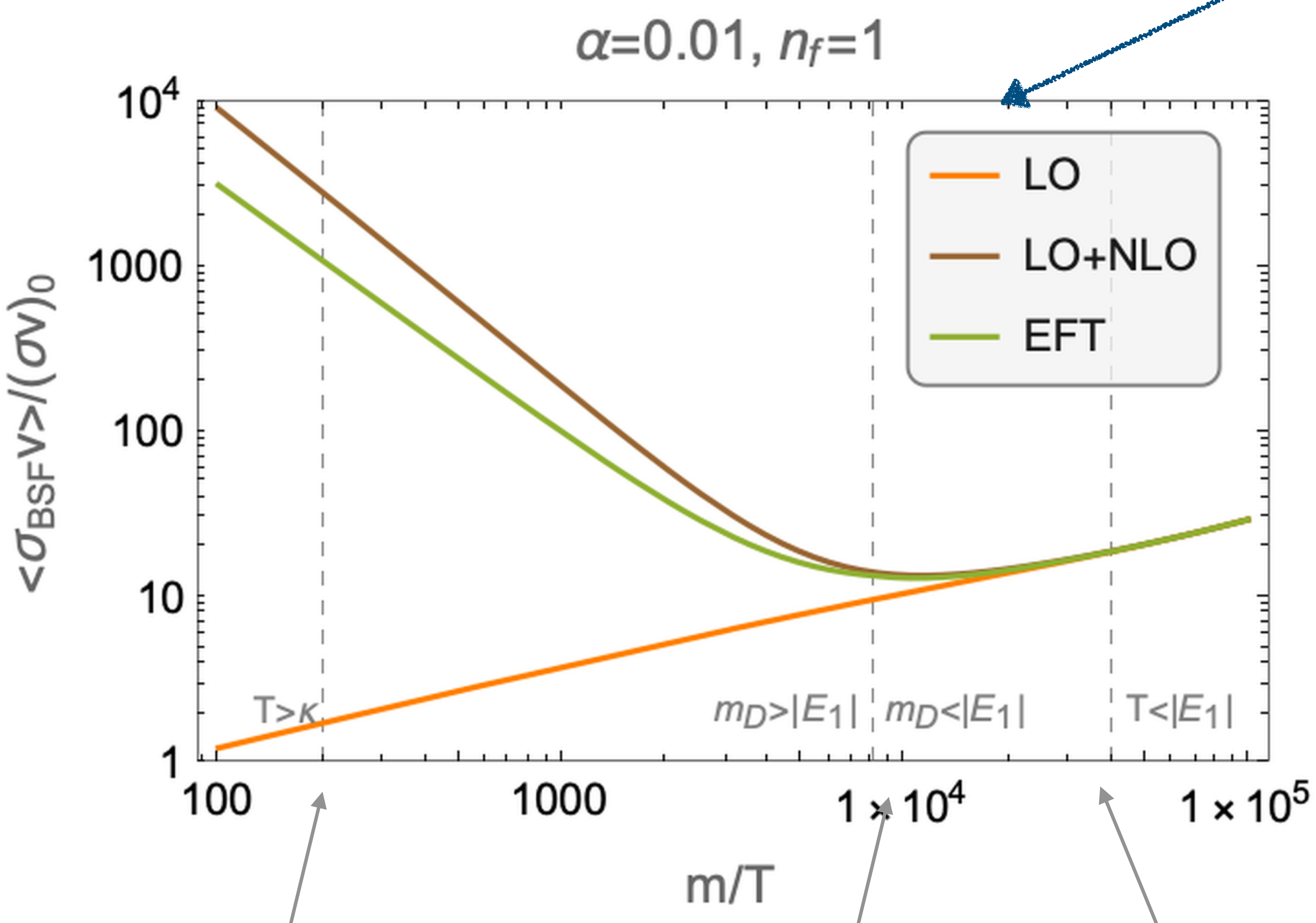
IR divergent,
will cancel-out with UV divergence from lower scale

- Integrate out scales m_D, E

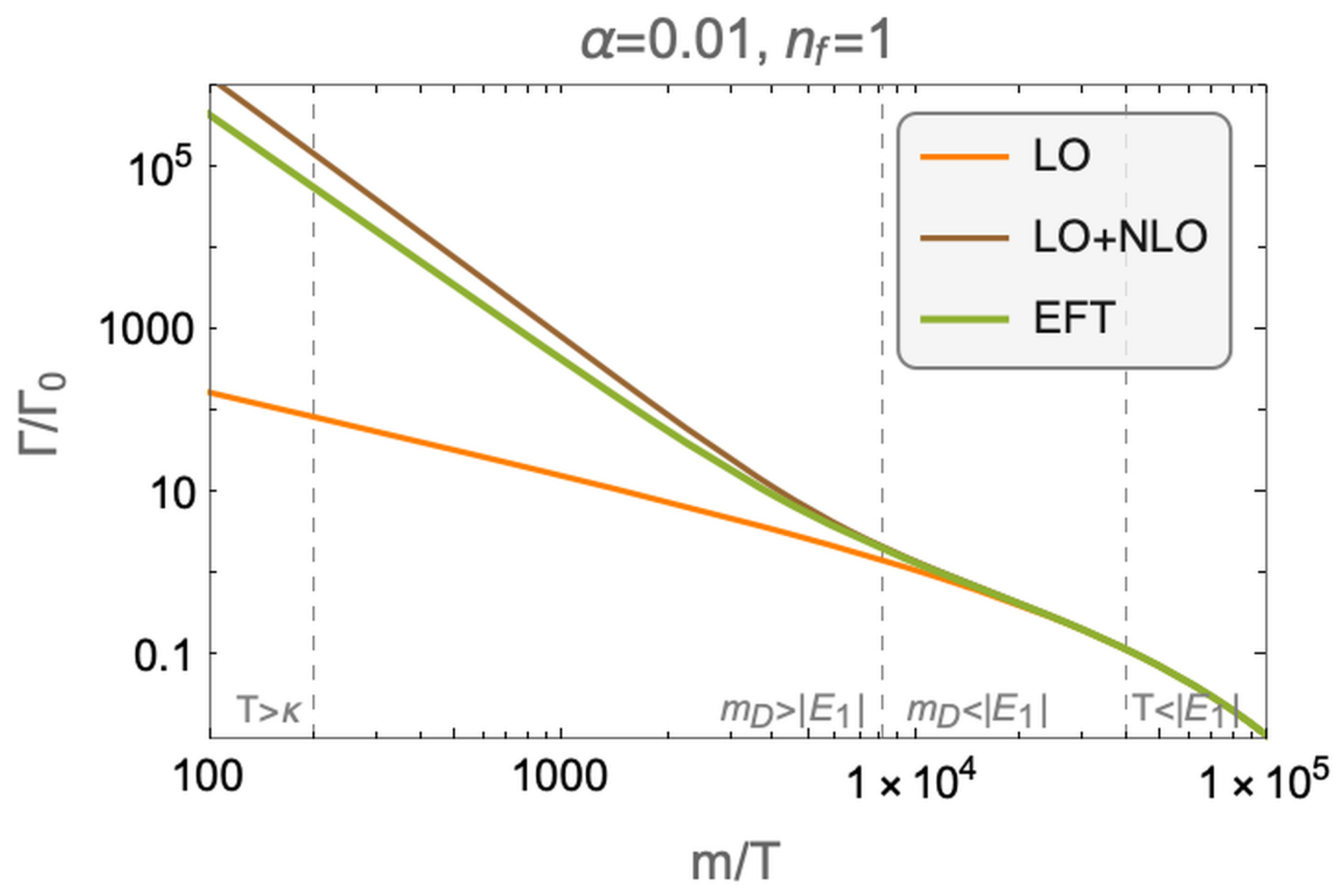
$$(\sigma_{BSF} v_{rel})_{T \gg m_D, E}$$

Bound-state formation: Results

In the same manner $E \gg m_D$ is treated

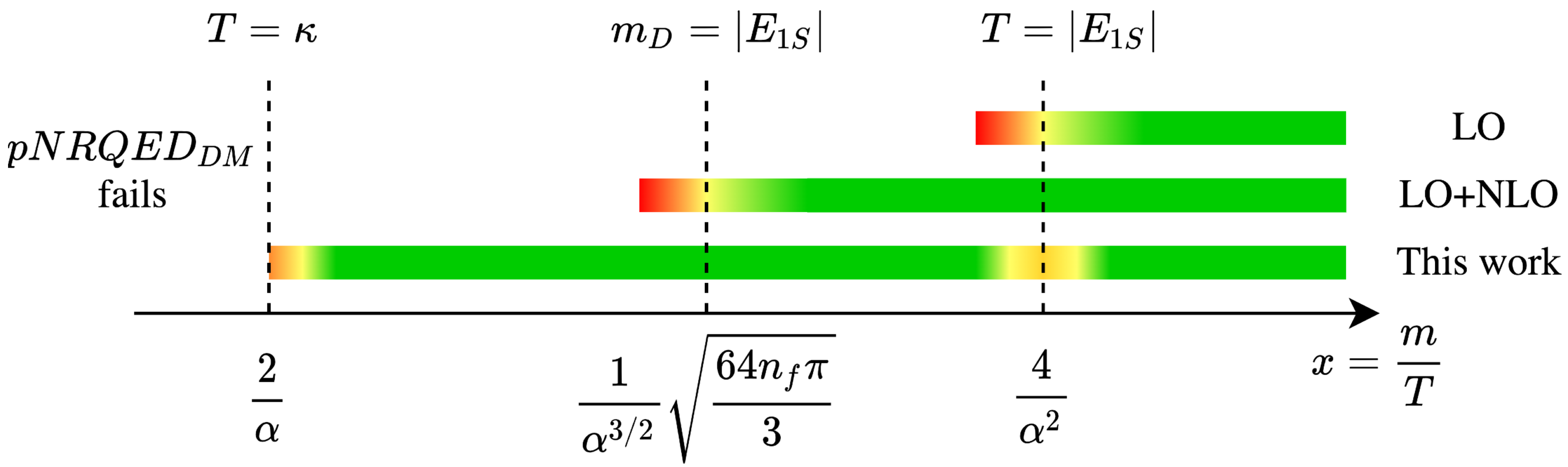


$T = 1/a_B$ $m_D = |E_{1S}|$ $T = |E_{1S}|$

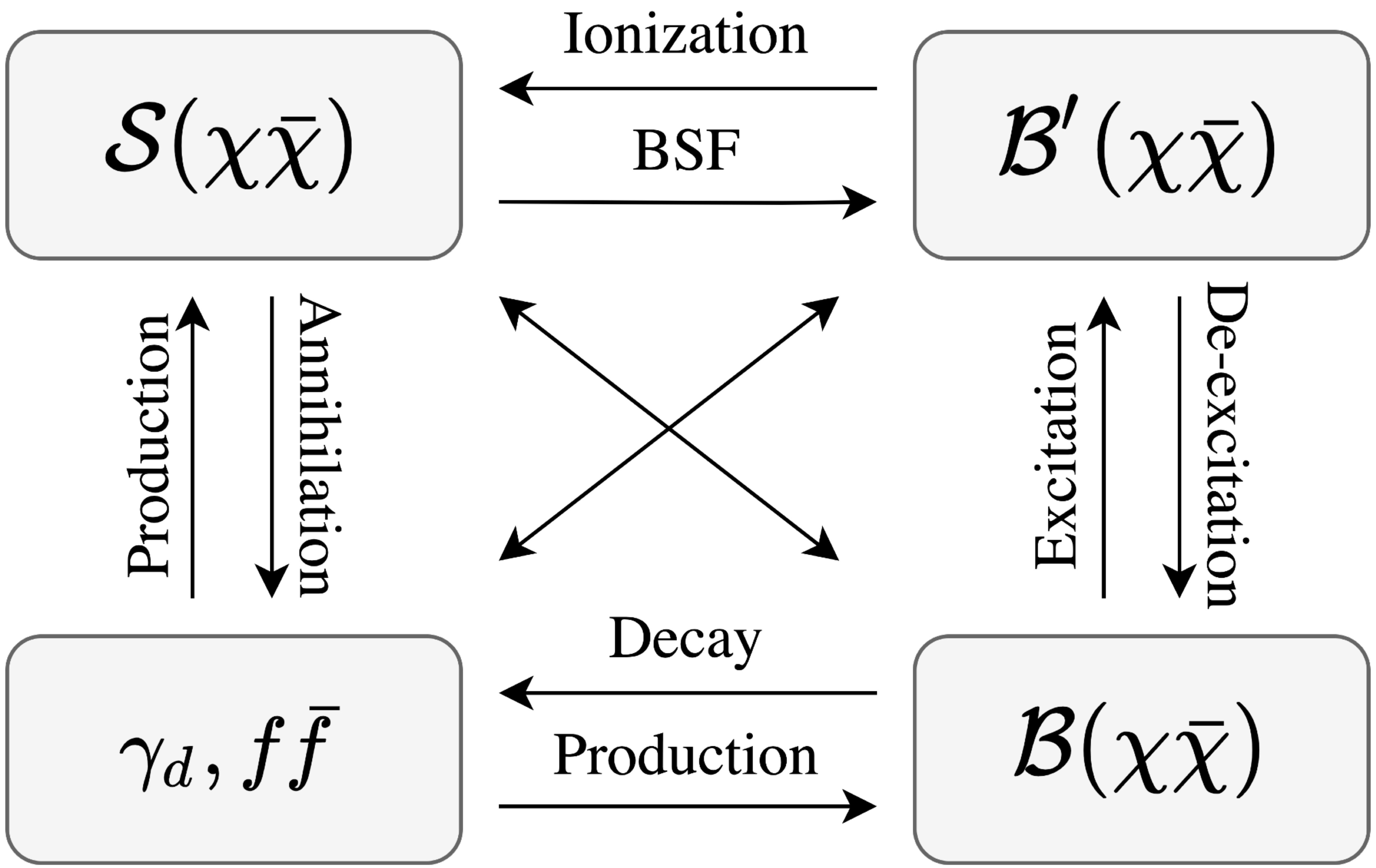


Higher n states are affected “longer” by effects of resummation

Regions of validity



Dark Matter Density Evolution



Dark Matter Density Evolution

Boltzmann equations - as the classic simplification of OQF

Unbound DM particles

$$\frac{dn_f}{dt} + 3Hn_f = -\left\langle \sigma_{ann} v_{rel} \right\rangle (n_f^2 - n_{f,eq}^2) - \sum_B \left(\left\langle \sigma_{BSF}^B v_{rel} \right\rangle n_f^2 - \Gamma_B^{ion} n_B \right)$$

Bound state DM

$$\frac{dn_B}{dt} + 3Hn_B = \left\langle \sigma_{BSF}^B v_{rel} \right\rangle n_f^2 - \Gamma^{ion} n_B - \Gamma_B^{dec}(n_B - n_{B,eq}) - \sum_{B \neq B'} (\Gamma_{B\to B'}^{exc} n_B - \Gamma_{B'\to B}^{exc} n_B')$$

Assumptions (1503.07142): $\Gamma_{dec} \gg H$
(Bound states - close to equilibrium)
+ detailed balance equation
+ neglecting (de)excitations

$$\frac{dn_f}{dt} + 3Hn_f = -\left(\left\langle \sigma_{ann} v_{rel} \right\rangle + \sum_B \left\langle \sigma_{BSF}^B v_{rel} \right\rangle \frac{\Gamma_B^{dec}}{\Gamma_B^{dec} + \Gamma_B^{ion}} \right) (n_f^2 - n_{f,eq}^2)$$

Effective cross-section

Dark Matter Density Evolution

Effective annihilation cross-section: asymptotic regimes

$$\langle \sigma_{eff} v_{rel} \rangle = \langle \sigma_{ann} v_{rel} \rangle + \sum_B \langle \sigma^B_{BSF} v_{rel} \rangle \frac{\Gamma^{dec}_B}{\Gamma^{dec}_B + \Gamma^{ion}_B}$$

- Ionisation equilibrium: $T \gg E, \Gamma_{ion} \gg \Gamma_{dec}$

$$\langle \sigma_{eff} v_{rel} \rangle \approx \langle \sigma_{ann} v_{rel} \rangle + \sum_B \Gamma^{dec}_B \frac{n^{eq}_B}{(n^{eq}_f)^2}$$

- “No back-reaction”: $T < E, \Gamma_{dec} \gg \Gamma_{ion}$

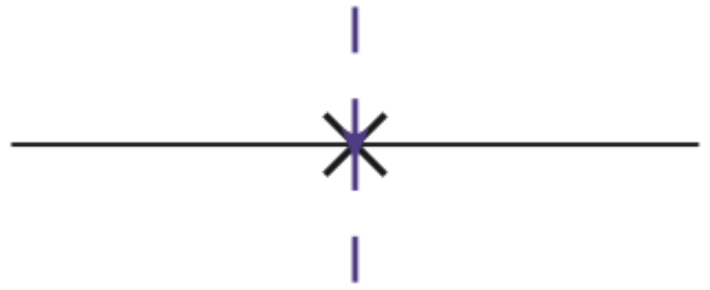
$$\langle \sigma_{eff} v_{rel} \rangle \approx \langle \sigma_{ann} v_{rel} \rangle + \sum_B \langle \sigma^B_{BSF} v_{rel} \rangle$$

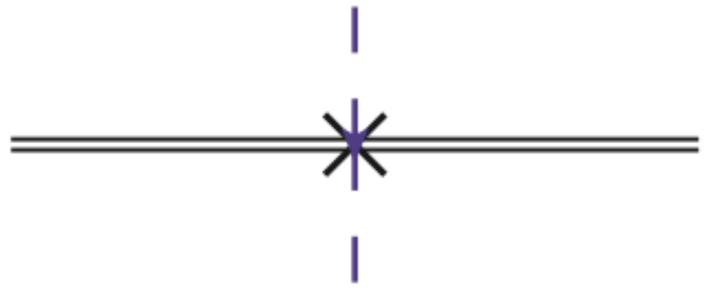
Decay and Annihilation

<u>Decay and Annihilation:</u> Directly from $pNRQED_{DM}$ Lagrangian (imaginary part)

$$\delta\mathcal{L}^{annih}_{pNRQED_{DM}} = \frac{i}{m^2}\int d^3r\,\phi^\dagger\delta(\vec{r})\left(2\Im[d_s] - \vec{S}^2\left(\Im[d_s] - \Im[d_v]\right)\right)\phi,$$

2 spin states:
para-, orthodarkonium





$$\langle\sigma_{ann}v_{rel}\rangle = \frac{Im(d_s) + 3Im(d_v)}{m^2}S(\alpha/v_{rel}) = /LO\ in\ \alpha/ = \frac{\alpha^2\pi(1+n_f)}{m^2}\frac{2\pi\alpha/v_{rel}}{1-e^{2\pi\alpha/v_{rel}}}$$

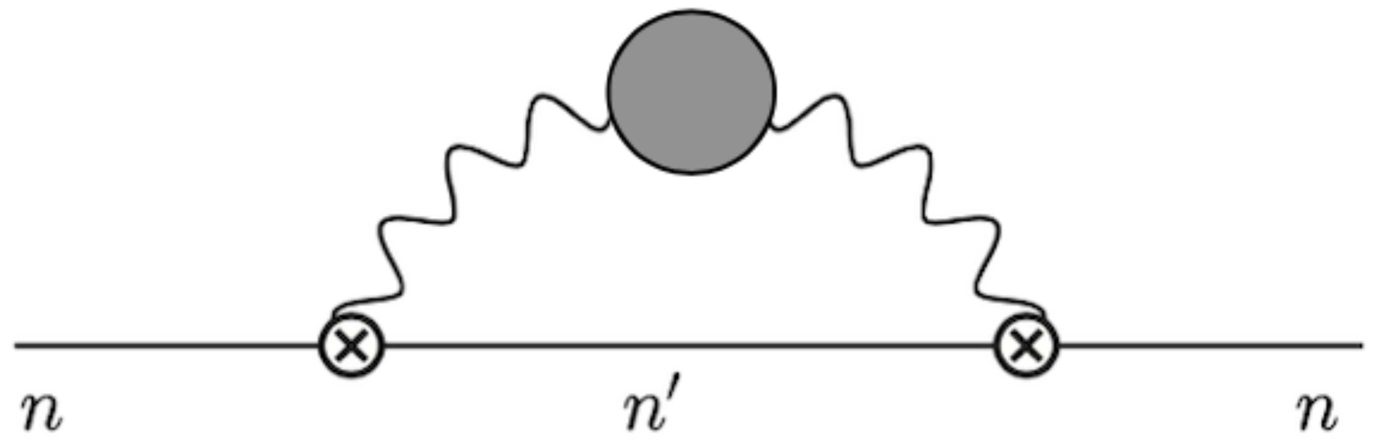
$$\Gamma^{dec}_{1S,pd} = \frac{4\mathfrak{I}[d_s]}{m^2}|\psi_{100}(0)|^2 = \frac{m\alpha^5}{2} + \mathcal{O}(\alpha^6)$$

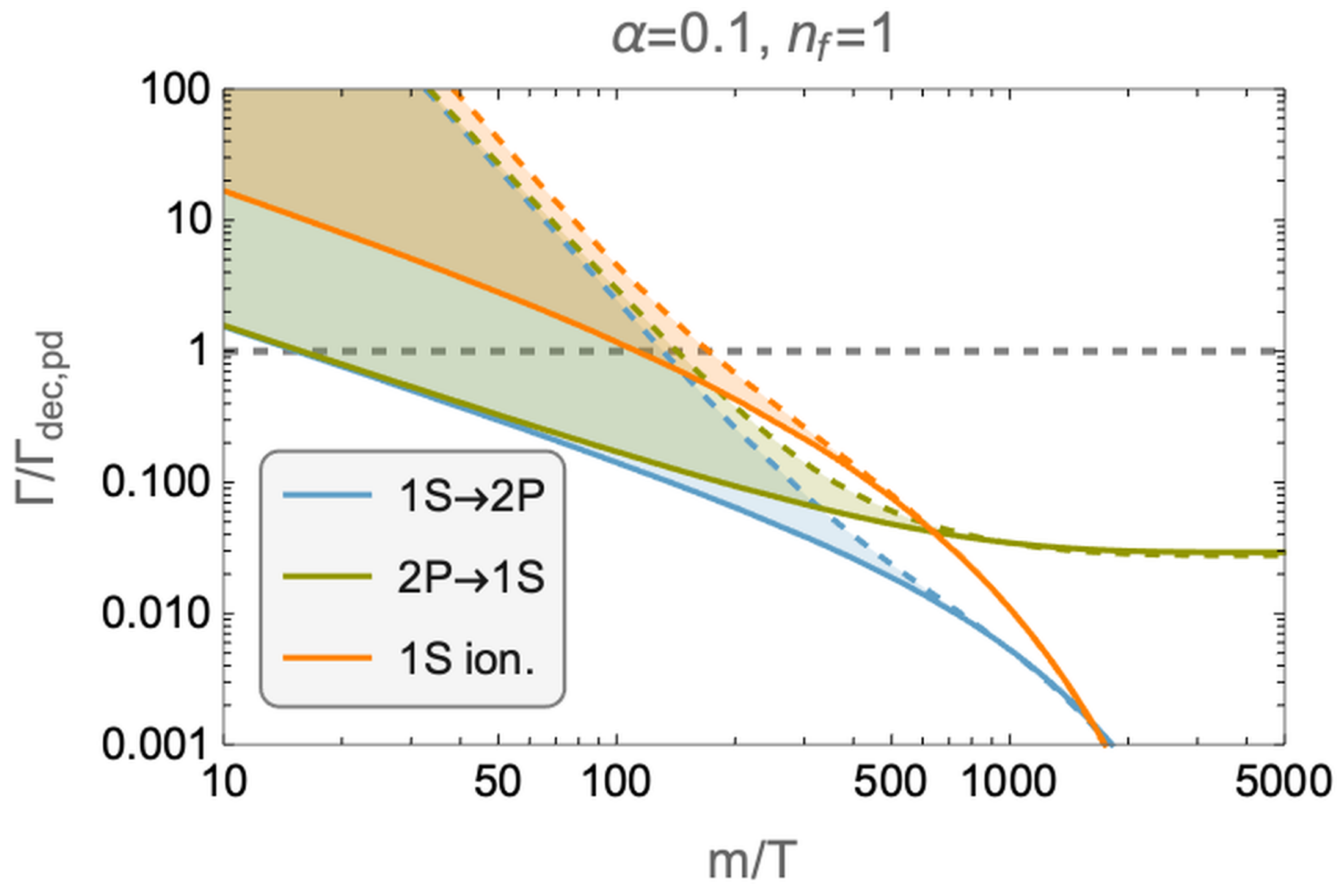
$$\Gamma^{dec}_{1S,od} = \frac{4\mathfrak{I}[d_v]}{m^2}|\psi_{100}(0)|^2 = \frac{n_f}{3}\frac{m\alpha^5}{2} + \mathcal{O}(\alpha^6)$$

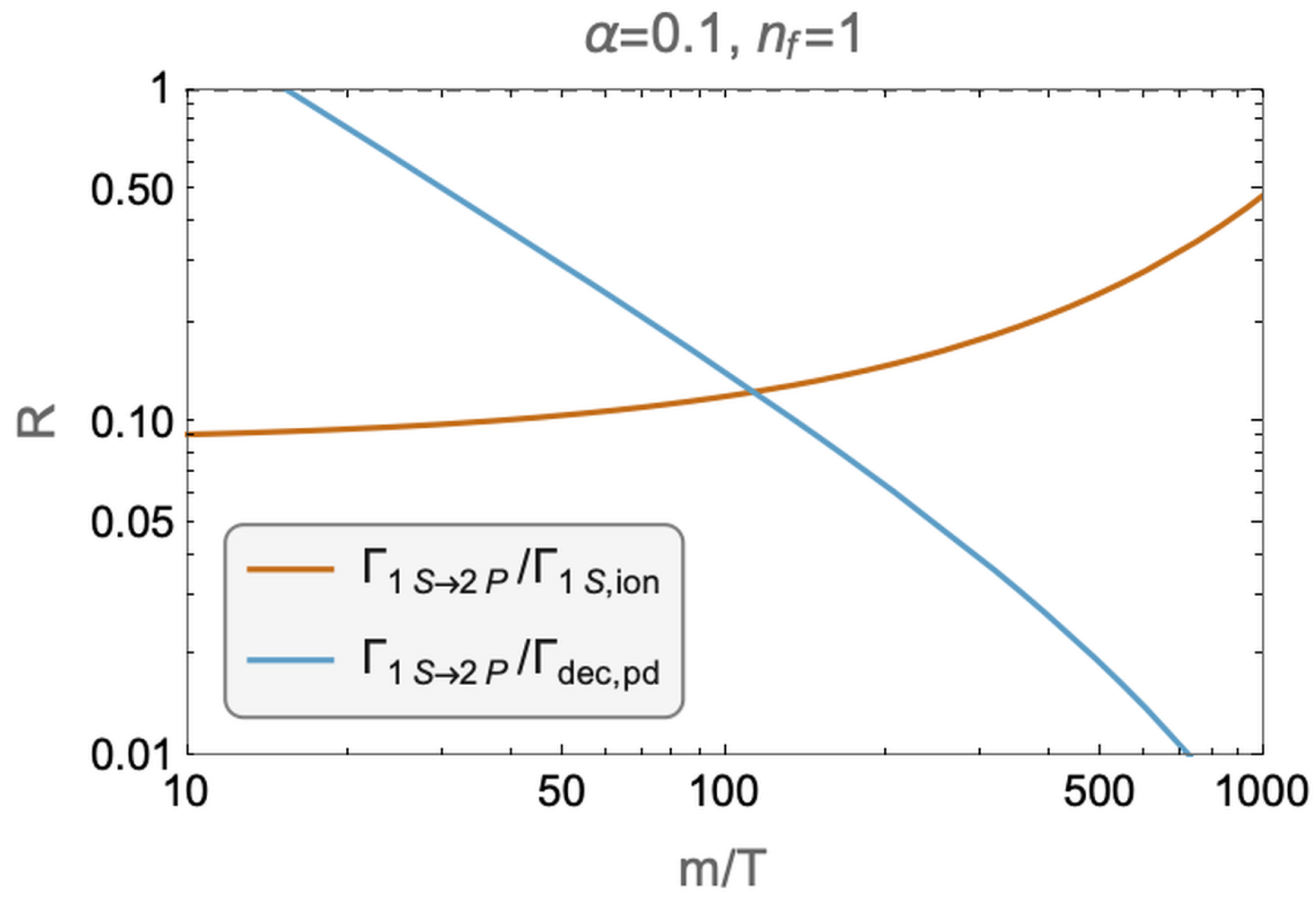
Sommerfeld enhancement factor

Transitions between bound states

De-, Excitations



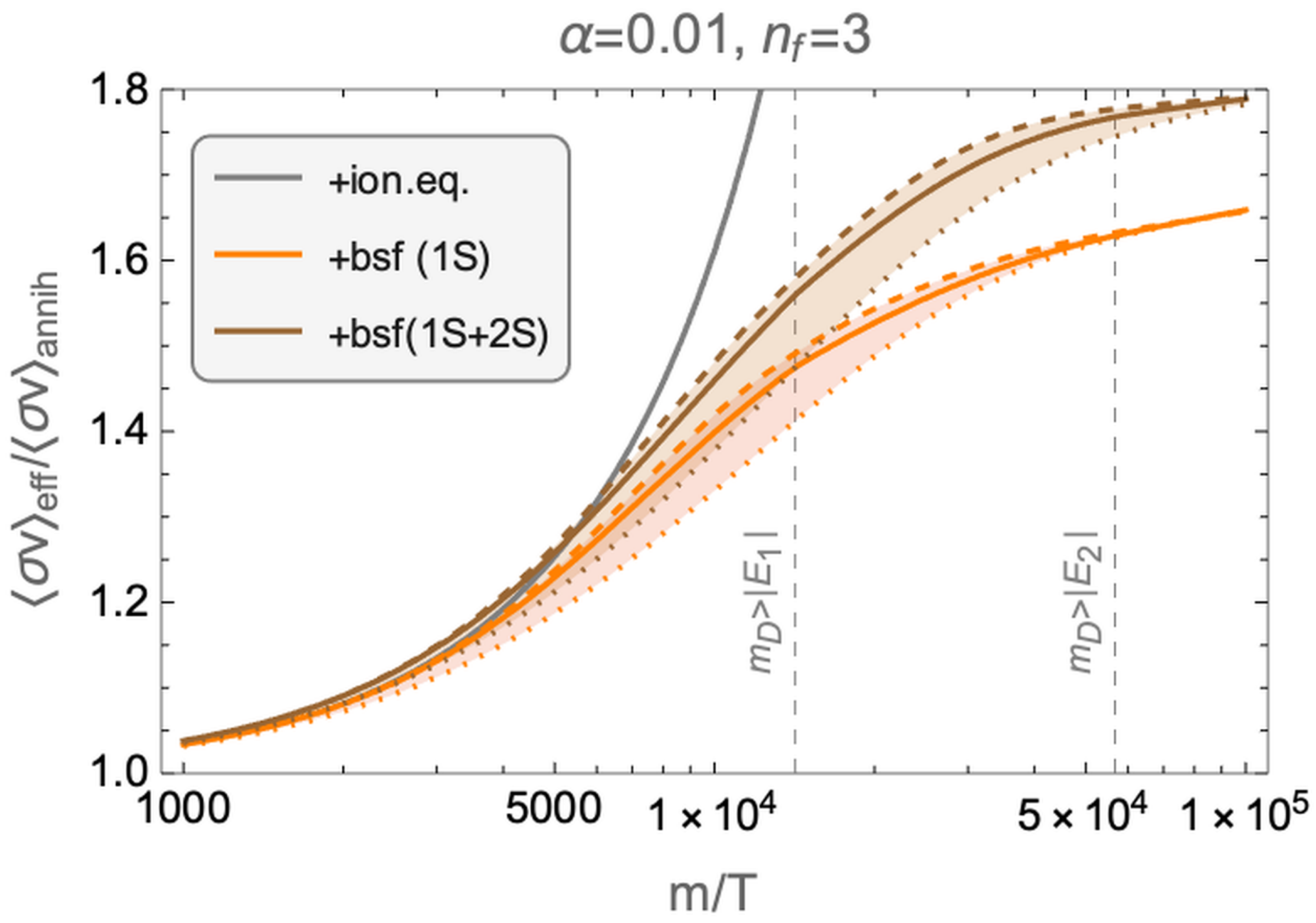


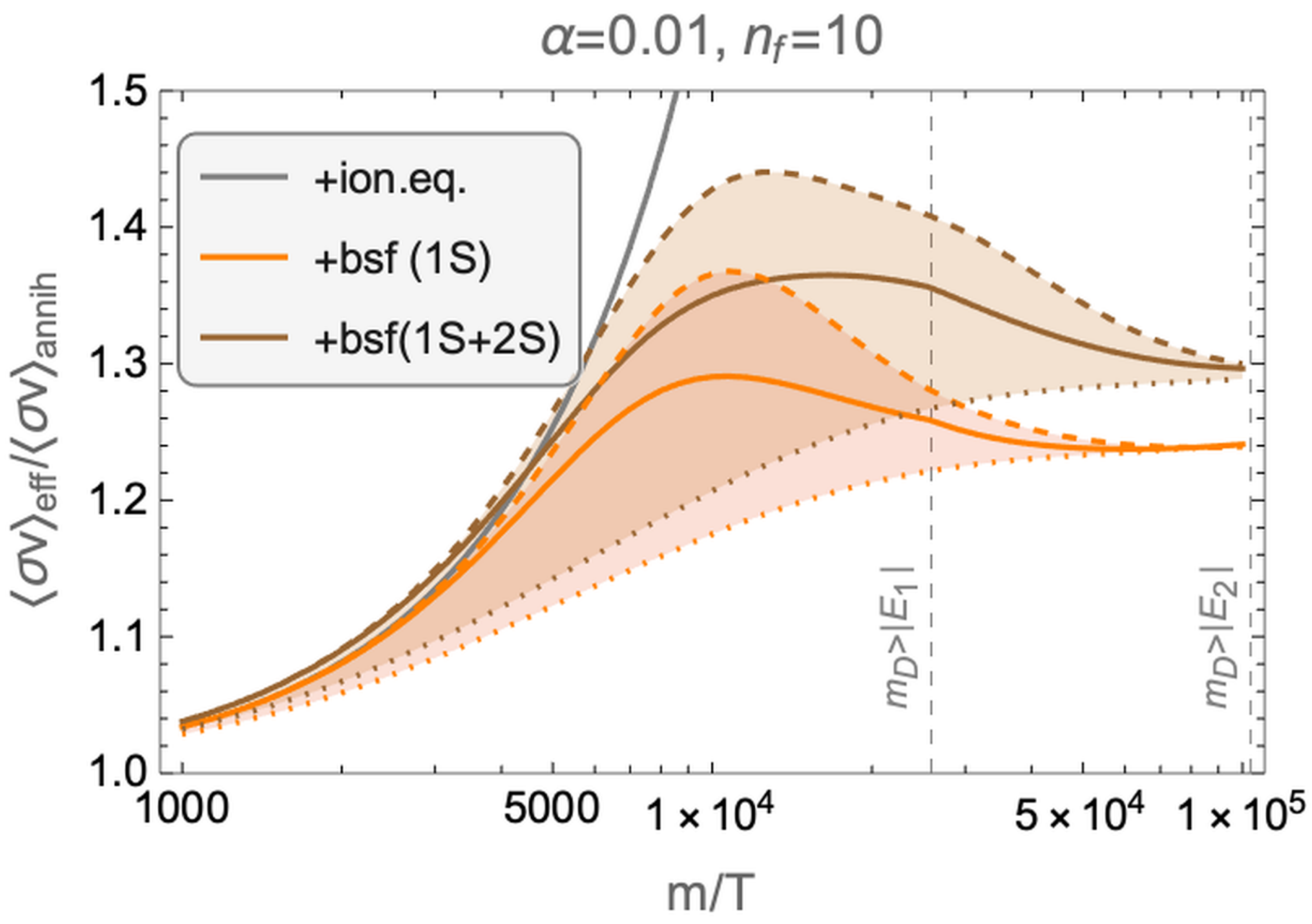


Dark Matter Density Evolution

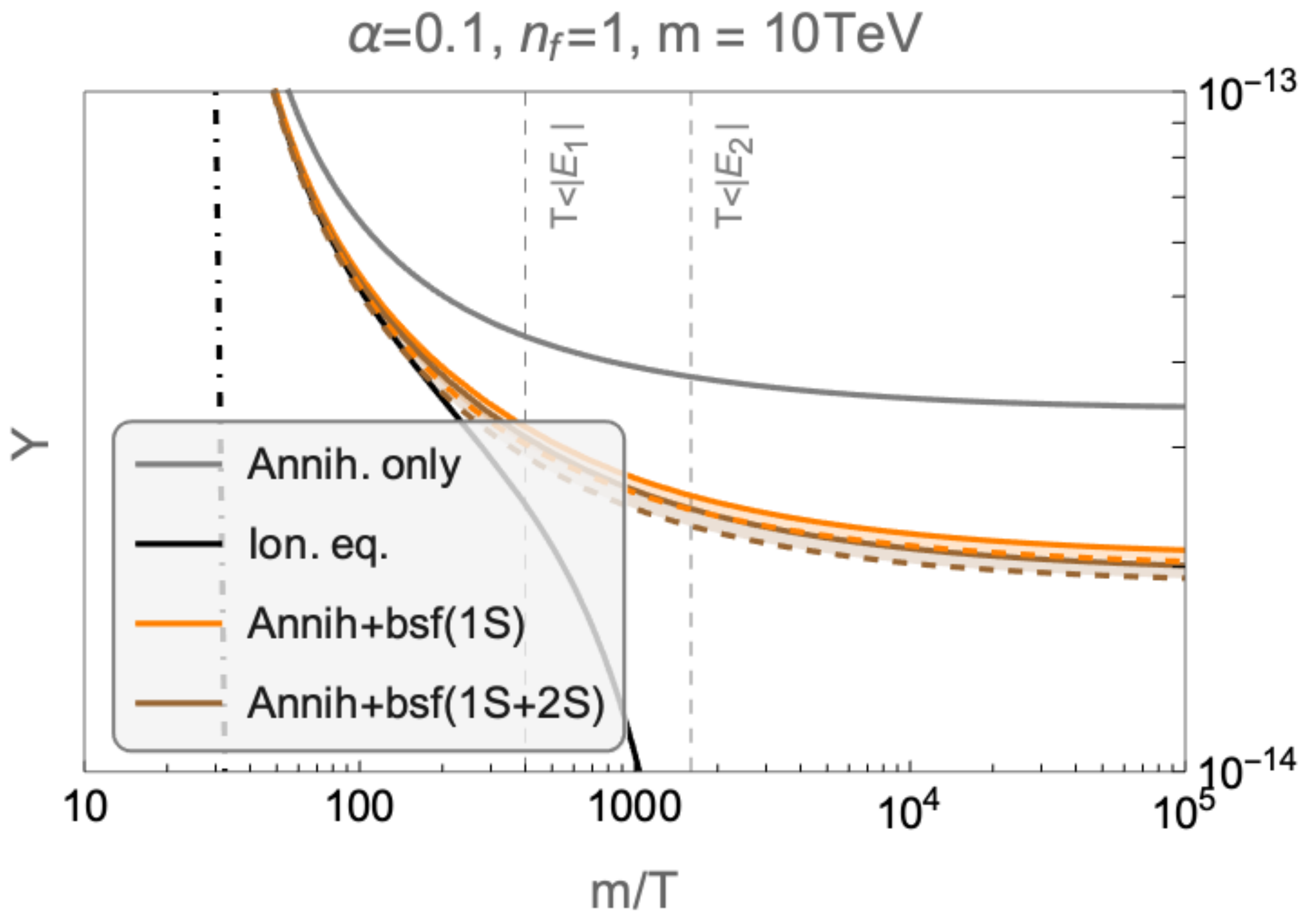
Effective cross-section

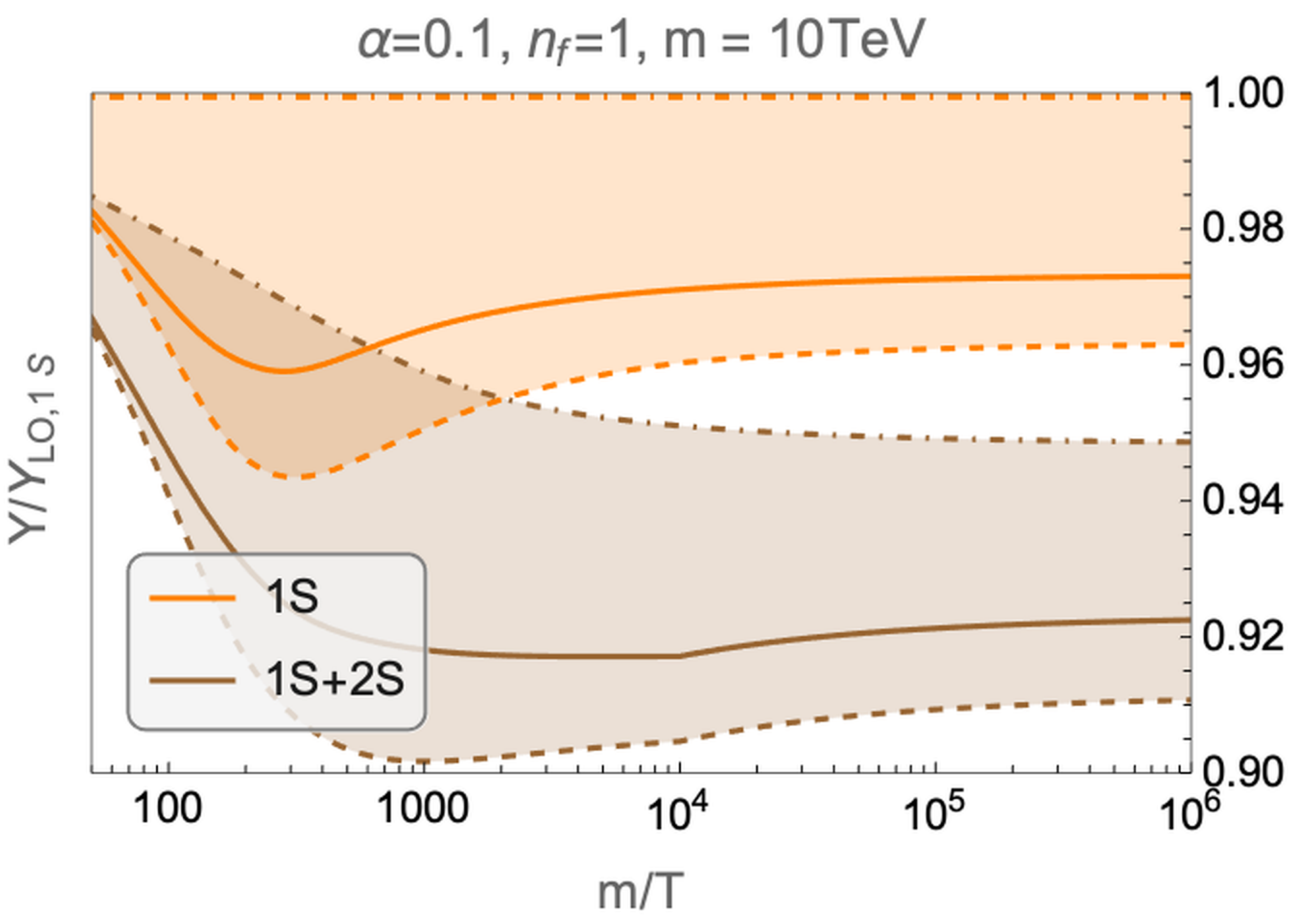
$$\langle \sigma_{eff} v_{rel} \rangle = \langle \sigma_{ann} v_{rel} \rangle + \sum_B \langle \sigma_{BSF}^B v_{rel} \rangle \frac{\Gamma_B^{dec}}{\Gamma_B^{dec} + \Gamma_B^{ion}}$$



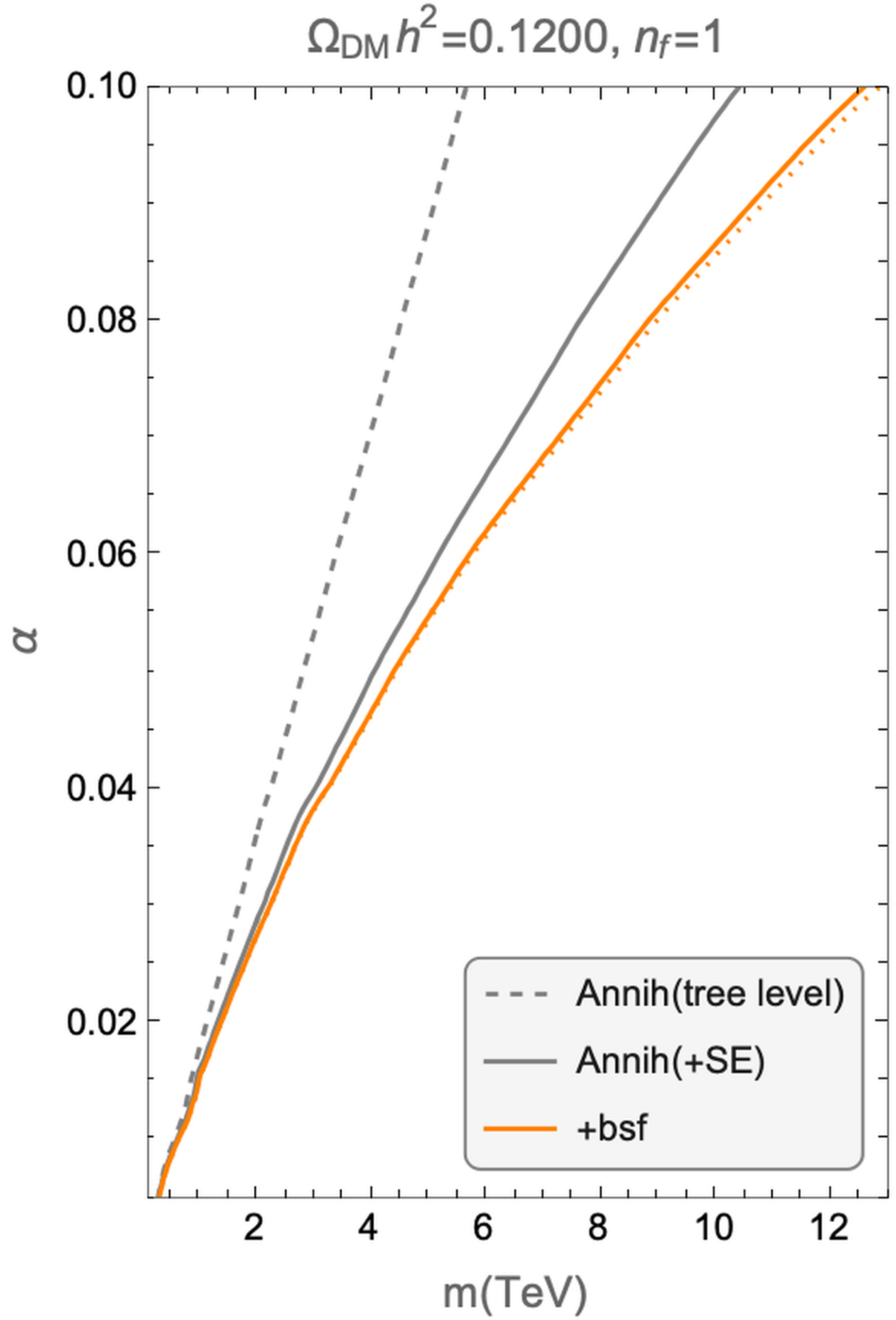


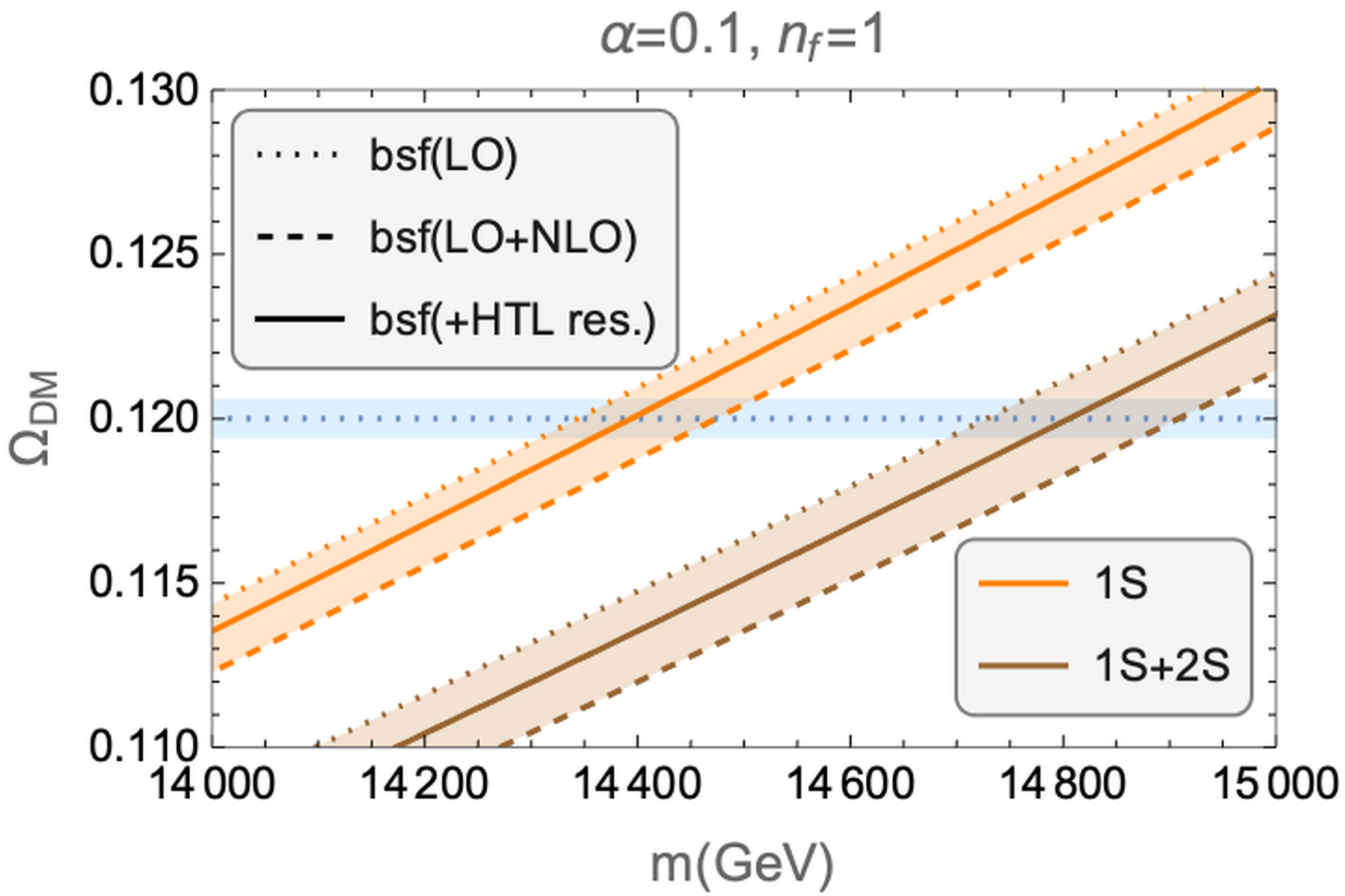
Results: yield





Results: parameters space(s)





Conclusions and Future Work

- EFT framework allows for a rigorous derivation and a systematic inclusion of corrections to the relevant observables

- The presence of Debye mass scale affects the evolutions of the dark matter in the Early Universe.

- The effects are of the same order as the NLO (electric correlator) corrections.

- Explore $T \approx m_D$ (exactly where we expect the effect to be the strongest).

- Study Open Quantum System formalism.

Thank you for your attention