Classical dynamics from effective field theories

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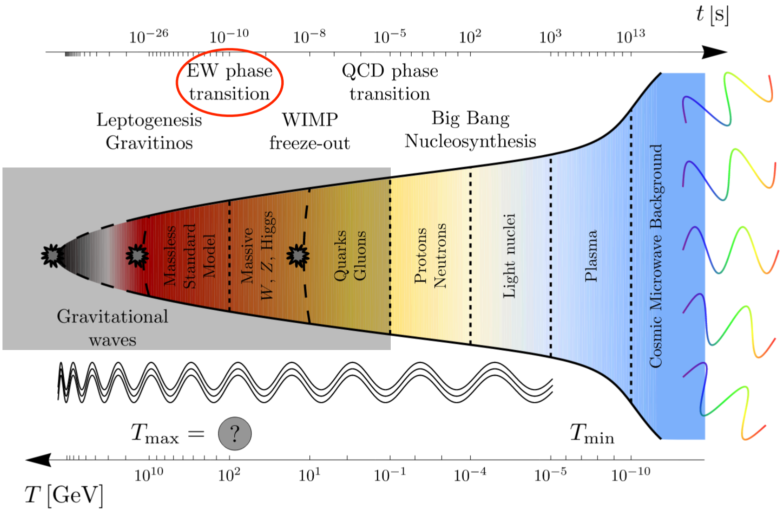
The Cosmological History

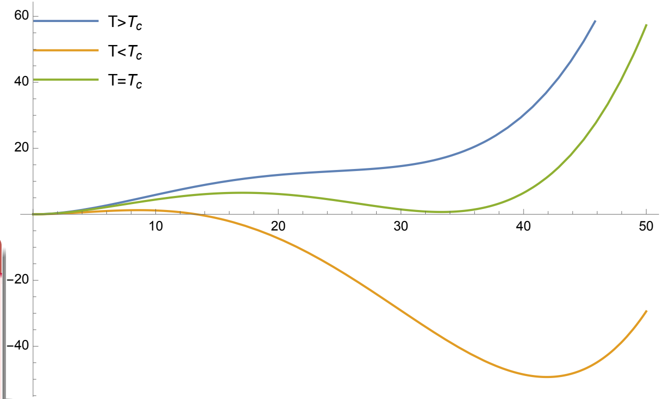


Adapted from 1307.3887

The Electroweak phase transition

The Electroweak symmetry was (**probably**) exact at $T \gtrsim 1$ TeV

A phase transition at $T_c \sim 100$ GeV.



What if the transition was first-order?

Electroweak Baryogenesis

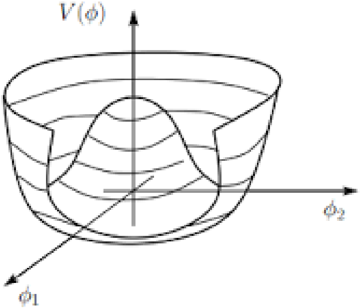
$\rightarrow$ Can explain the Baryon asymmetry

Gravitational waves

$\rightarrow$ Window into the early Universe

$\rightarrow$ Access to the Higgs potential

Parameters for the transition

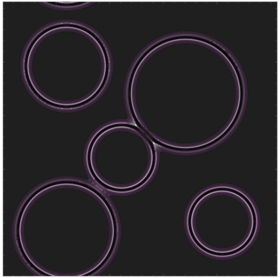


Equilibrium

Effective potential—$V(\phi, T)$

Critical temperature—T_c

Transition strength—$\alpha \sim \frac{d}{dT} V(\phi, T)$



Non-equilibrium

Nucleation rate—$\Gamma \sim e^{-S_3/T}$

Nucleation temperature—T_N, T_p

Inverse duration—$\beta \propto \frac{d}{d \log T} S_3/T$

Wall speed—$v \propto \dot{\phi}$

Theoretical uncertainties are **huge**

How bad is it?

T_c, T_N can change by factor of **2**

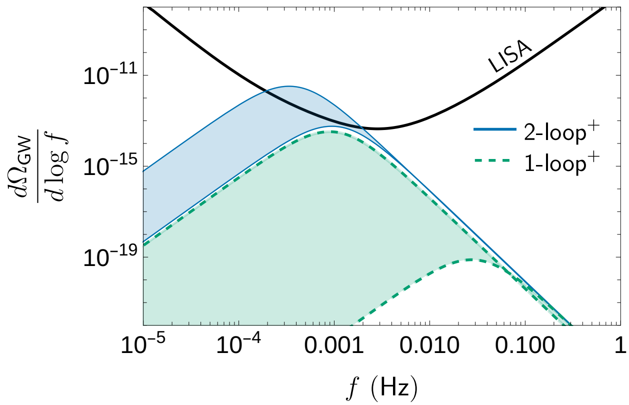
β can change by an **order** of magnitude

α can change by an **order** of magnitude

$v \in \text{Random}[0, 1]$

The predicted amplitude can change by 10 orders of magnitude

See also 1911.10206, 210.16305, 2210.07075 for recent studies

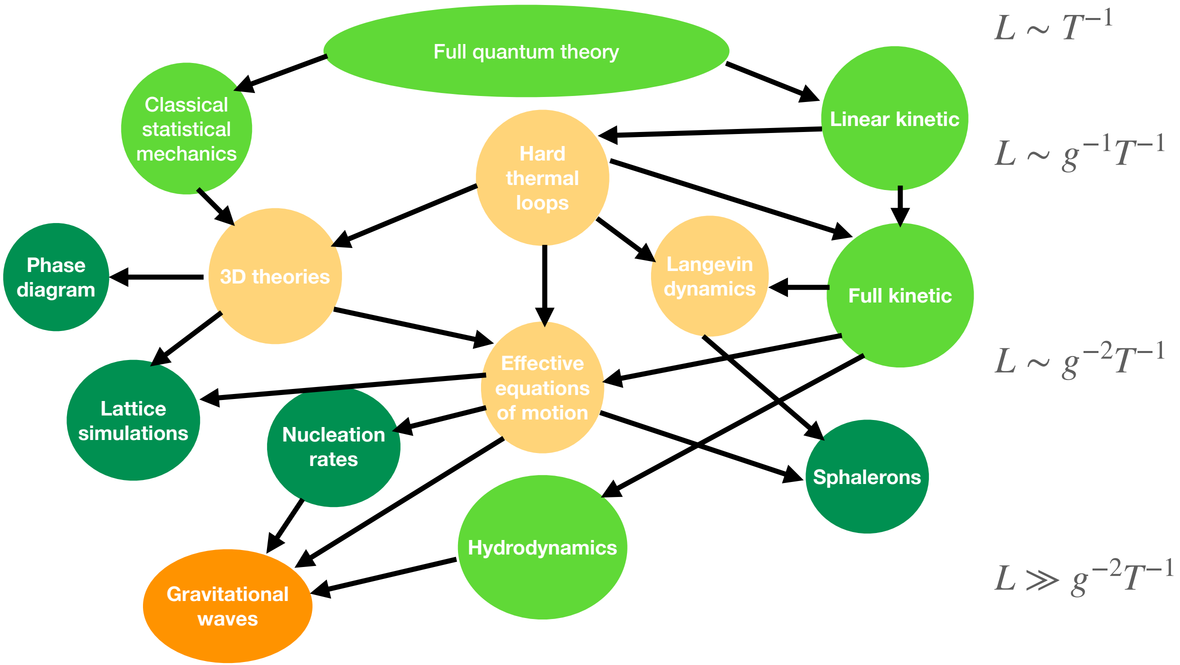


Gould & Tenkanen 2104.04399

State-of-the art: Where are we at?

Ongoing flurry of work targeted at smashing uncertainties

- $\Delta\Omega_{\mathrm{GW}}/\Omega$ down from 10 orders to **2-4** — 2104.04399, 2009.10080,
- β known within a factor of **2** — 2205.07238, 2205.05145
- T_N, T_c known within $1-10\%$ — 2005.11332, 2104.04399
- α known within $1-10\%$ — 2005.11332, 2104.04399
- c_s^2 known to 1% — 2206.01130



Full quantum theory
Classical statistical mechanics
Linear kinetic
Hard thermal loops
3D theories
Phase diagram
Langevin dynamics
Full kinetic
Effective equations of motion
Lattice simulations
Nucleation rates
Sphalerons
Hydrodynamics
Gravitational waves
$L \sim T^{-1}$
$L \sim g^{-1}T^{-1}$
$L \sim g^{-2}T^{-1}$
$L \gg g^{-2}T^{-1}$

How do these classical equations of motion typically look like?

Scalars

$$\ddot{\phi} - \vec{\nabla}^2\phi + V'_{3\mathrm{d}}(\phi) + \eta\dot{\phi} = \xi(x), \quad \langle\xi(x)\rangle = 0 \quad \& \quad \langle\xi(x)\xi(y)\rangle = 2T\eta\delta^4(x-y)$$

Gauge bosons

$$\sigma\dot{\vec{A}} = \vec{v}\times\vec{B} + g_w^2\phi^2\vec{A} + \vec{\zeta}(x), \quad \left\langle\vec{\zeta}(x)\right\rangle = 0 \quad \& \quad \left\langle\zeta^i(x)\zeta^j(y)\right\rangle = 2T\sigma\delta^{ij}\delta^4(x-y)$$

Roughly $\eta \sim \phi^2(T\log g_s^{-1})^{-1}$ and $\sigma \sim T(\log g_w^{-1})^{-1}$ 9905239, 9506475, 9503296

Effective field theories at high temperatures

Phase transitions in a nutshell

Effective mass:
$m_{\text{eff}}^2 = (m^2 + \underbrace{aT^2}_{\text{Thermal Mass}}) \ll m^2$

RG $\implies \mu \frac{d}{d\log\mu} m_{\text{eff}}^2 \approx m_{\text{eff}}^2$

Fine-tuning $\implies \underbrace{bT^2}_{\text{2-loop Mass}} \approx m_{\text{eff}}^2$

Logarithms $\implies \log T^2/m_{\text{eff}}^2 \gg 1$

Extreme uncertainties for $\Omega_{\text{GW}} \implies$ Can we trust theoretical calculations?

Solution: Integrate out $E \sim T$ modes (9508379,2104.04399)

No more large logs: $\log T^2/m_{\text{eff}}^2 \to \underbrace{\log T^2/\mu^2}_{\text{Match at } \mu\sim T} + \underbrace{\log \mu^2/m_{\text{eff}}^2}_{\text{RG-evolution in the EFT}}$ ✓

Two-loop thermal masses → From matching ✓

Thermally resummed couplings → From matching ✓

Simpler calculations $V_{\text{1-Loop}} \to -m_{\text{eff}}^3, \quad V_{\text{2-Loop}} \to m_{\text{eff}}^2 \log \mu^2/m_{\text{eff}}^2$ ✓

Integrating out heavy "particles"

In equilibrium we can view temperature effects through Matsubara modes:

$$\partial_\mu \phi(x) \partial^\mu \phi(x) \to \vec{\nabla}\phi(\vec{x}) \cdot \vec{\nabla}\phi(\vec{x}) + \sum_{n=-\infty}^{\infty} (2\pi n T)^2 \phi(\vec{x})^2$$

In essence an infinite tower of heavy particles$\sim T \gg m$

What do we do with heavy particles? $\rightarrow$ Integrate them out

In practice: Write down the most general 3d-spatial Lagrangian
$\rightarrow$ **match** the coefficients

$$\frac{1}{2} m^2 \phi^2 + \frac{1}{4}\lambda \phi^4 \to \frac{1}{2} m_{3\mathrm{d}}^2 \phi^2 + \frac{1}{4} \lambda_{3\mathrm{d}} \phi^4$$

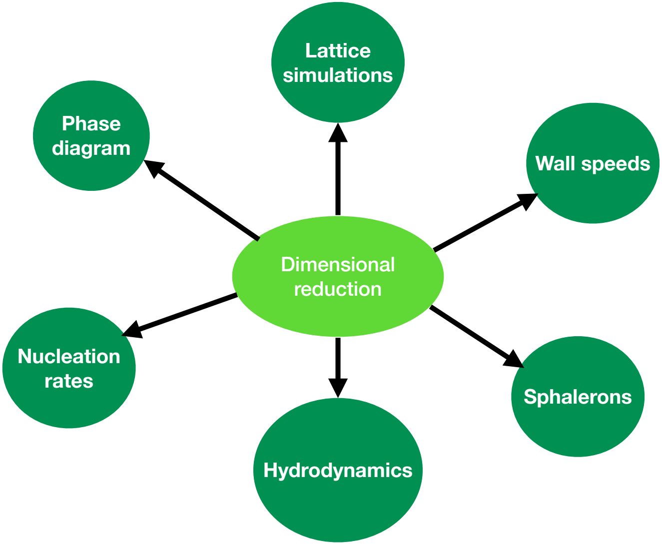
Example for a quartic coupling measured at $\mu_0 \sim m_Z$: $\lambda_{4\mathrm{d}}(\mu_0)$

$$\lambda_{3\mathrm{d}}(\mu) = T\left[\lambda_{4\mathrm{d}}(\mu) + \lambda_{4\mathrm{d}}^2(\mu)\left(a\log\frac{\mu}{T} + b\right)\right]$$

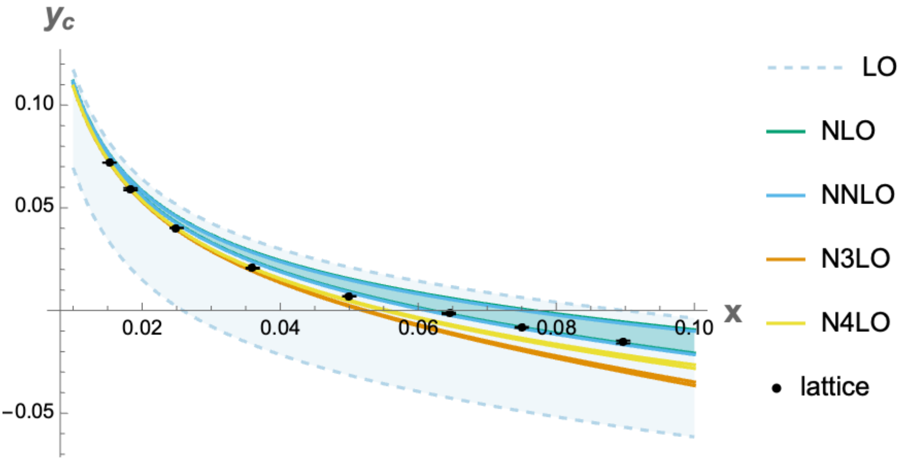
$$T^{-1}\frac{d}{d\log\mu}\lambda_{3\mathrm{d}}(\mu) \underbrace{= \beta_\lambda + a\lambda_{4\mathrm{d}}^2}_{=0} + \mathcal{O}(\lambda_{4\mathrm{d}}^4) = \mathcal{O}(\lambda_{4\mathrm{d}}^4)$$

How it works in practice

- Evolve $\lambda_{4\mathrm{d}}$ from $\mu = \mu_0$ to $\mu = T$
- Plug the result into $\lambda_{3\mathrm{d}}$ with $\mu = T$ (makes the logarithm small)
- Calculate the effective potential
- Change T until the two minima coincide at T_c
- Calculate observables at T_c


Lattice simulations
Phase diagram
Wall speeds
Dimensional reduction
Nucleation rates
Sphalerons
Hydrodynamics

Lattice versus perturbation theory: Radiative barriers at three loops



y_c is a dimensionless version of T_c, and $x = \frac{\lambda}{g^2}$, $\quad x \sim 0.1 \to m_H \approx 72$ GeV

Backup slides

Why classical?

High-temperature limit: $n = (e^{E/T} - 1)^{-1} \sim \frac{T}{E} \gg 1$

Many particles in each state $\rightarrow$ Decoherence

Fields commute:

$\langle \phi(p)\phi(q) \rangle \sim n(E)$ & $\langle \phi(q)\phi(p) \rangle \sim 1 + n(E)$

$\rightarrow [\phi(p), \phi(q)] \approx 0$

Equipartition of energy: $\langle E \rangle / N = T$

$\rightarrow$ Rayleigh-Jeans divergence for blackbody radiation

$\rightarrow$ Kinetic theory