

# **Dark Matter and Extended Higgs Physics: A Muon Collider Perspective on the Singlet-Enhanced Two Higgs Doublet Model**

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## 2HDMS

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- We consider a CP conserving type II Two Higgs Doublet Model extended by complex scalar singlet (2HDMS)
- $\Phi_1$  couples to down-type quarks and  $\Phi_2$  couples to up-type quarks
- The scalar potential is,

$$V = V_{\text{2HDM}} + V_S$$

# 2HDMs

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$$\begin{aligned} V_{\text{2HDM}} = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left( m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) \\ & + \frac{\lambda_1}{2} \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) \\ & + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \left[ \frac{\lambda_5}{2} \left( \Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right] \end{aligned}$$

$$\begin{aligned} V_S = & m_S^2 S^* S + \left( \frac{m'_S^2}{2} S^2 + \text{h.c.} \right) \\ & + \left( \frac{\lambda''_1}{24} S^4 + \text{h.c.} \right) + \left( \frac{\lambda''_2}{6} (S^2 S^* S) + \text{h.c.} \right) + \frac{\lambda''_3}{4} (S^* S)^2 \\ & + S^* S \left[ \lambda'_1 \Phi_1^\dagger \Phi_1 + \lambda'_2 \Phi_2^\dagger \Phi_2 \right] + \left[ S^2 \left( \lambda'_4 \Phi_1^\dagger \Phi_1 + \lambda'_5 \Phi_2^\dagger \Phi_2 \right) + \text{h.c.} \right] \end{aligned}$$

# 2HDMs

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The singlet and doublet fields can be written as,

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\eta_i) \end{pmatrix} \quad \langle \Phi_i \rangle = \begin{pmatrix} 0 \\ \frac{v_i}{\sqrt{2}} \end{pmatrix},$$
$$S = \frac{1}{\sqrt{2}}(v_S + \rho_S + iA_S) \quad \langle S \rangle = \frac{v_S}{\sqrt{2}},$$

## Particle content

$h_1, h_2, h_3, A, H^\pm, A_S$

# 2HDMs

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The minimisation conditions are,

$$0 = \frac{\partial V}{\partial \Phi_1} \Big|_{\substack{\Phi_1=\langle \Phi_1 \rangle \\ \Phi_2=\langle \Phi_2 \rangle \\ S=\langle S \rangle}} = \frac{1}{\sqrt{2}} \left[ m_{11}^2 v_1 - m_{12}^2 v_2 + \frac{\lambda_1}{2} v_1^3 + \frac{\lambda_{345}}{2} v_1 v_2^2 + \left( \frac{\lambda'_1}{2} v_1 + \lambda'_4 v_1 \right) v_S^2 \right]$$

$$0 = \frac{\partial V}{\partial \Phi_2} \Big|_{\substack{\Phi_1=\langle \Phi_1 \rangle \\ \Phi_2=\langle \Phi_2 \rangle \\ S=\langle S \rangle}} = \frac{1}{\sqrt{2}} \left[ m_{22}^2 v_2 - m_{12}^2 v_1 + \frac{\lambda_2}{2} v_2^3 + \frac{\lambda_{345}}{2} v_1^2 v_2 + \left( \frac{\lambda'_2}{2} v_2 + \lambda'_5 v_2 \right) v_S^2 \right]$$

$$\begin{aligned} 0 = \frac{\partial V}{\partial S} \Big|_{\substack{\Phi_1=\langle \Phi_1 \rangle \\ \Phi_2=\langle \Phi_2 \rangle \\ S=\langle S \rangle}} &= \frac{1}{\sqrt{2}} \left[ m_S^2 v_S + m_S'^2 v_S + \frac{\lambda''_1}{12} v_S^3 + \frac{\lambda''_2}{3} v_S^3 + \frac{\lambda''_3}{4} v_S^3 \right. \\ &\quad \left. + \frac{v_S}{2} (\lambda'_1 v_1^2 + \lambda'_2 v_2^2) + v_S (\lambda'_4 v_1^2 + \lambda'_5 v_2^2) \right] \end{aligned}$$

# 2HDMs

- Eliminate the free parameters  $m_{11}^2, m_{22}^2, m_S^2$
- Set  $\lambda_1'' = \lambda_2''$

## Interaction basis

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, m_{12}^2, \tan \beta, v_S, m_S'^2, \lambda'_1, \lambda'_2, \lambda'_4, \lambda'_5, \lambda''_1 = \lambda''_2, \lambda''_3$$

- where  $\tan \beta = \frac{v_2}{v_1}$  and  $m_S'^2$  is the coefficient of the DM mass term
- Using the rotation matrix  $R$  in terms of the mixing angles  $\alpha_1, \alpha_2, \alpha_3$ , we go to the mass basis

$$R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -s_{\alpha_1} c_{\alpha_3} - c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ s_{\alpha_1} s_{\alpha_3} - c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & -c_{\alpha_1} s_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & c_{\alpha_2} c_{\alpha_3} \end{pmatrix}$$

## Mass basis

$$m_{h_1}, m_{h_2}, m_{h_3}, m_A, m_{A_S}, m_{H^\pm}, \lambda'_1 - 2\lambda'_4, \lambda'_2 - 2\lambda'_5,$$

$$\tan \beta, v_S, c_{h_1 bb}, c_{h_1 tt}, \tilde{\mu}^2, \lambda''_1 - \lambda''_3, \text{alignm}, \dots$$

$$c_{h_1 tt} = \frac{\sin(\alpha_1) \cos(\alpha_2)}{\sin(\beta)}$$

$$\Rightarrow \alpha_1 = \arctan \left( \frac{c_{h_1 tt}}{c_{h_1 bb}} \tan(\beta) \right),$$

$$c_{h_1 bb} = \frac{\cos(\alpha_1) \cos(\alpha_2)}{\cos(\beta)}$$

$$\Rightarrow \alpha_2 = \arccos \left( \cos(\beta) \cdot \sqrt{\tan(\beta)^2 c_{h_1 tt}^2 + c_{h_1 bb}^2} \right)$$

$$\text{alignm} = |\sin(\beta - (\alpha_1 + \alpha_3 \cdot \text{sgn}(\alpha_2)))|$$

$$\Rightarrow \alpha_3 = \frac{\beta - \alpha_1 - \arcsin(\text{alignm})}{\text{sgn}(\alpha_2)}$$

$$\tilde{\mu}^2 = \frac{m_{12}^2}{\sin(\beta) \cos(\beta)}$$

## Portal Couplings

$$\lambda'_1 - 2\lambda'_4,$$

$$\lambda'_2 - 2\lambda'_5$$

- Couples  $A_S$  to  $h_i$ ;
- Influence on DM observables

## Benchmarks

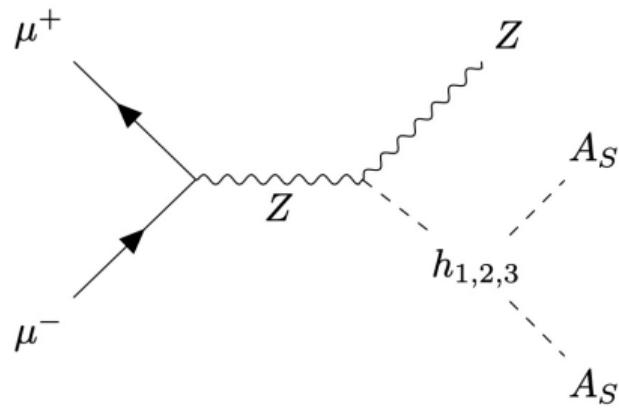
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	$m_{h_3}$ (GeV)	$m_\chi$ (GeV)	$\Omega h^2$	$BR(h_3 \rightarrow \chi\chi)$	$BR(h_2 \rightarrow \chi\chi)$
<b>BP3</b>	700	156.0	$1.61 \times 10^{-4}$	0.69	-
<b>BP55</b>	650	55.6	0.11	$3.81 \times 10^{-9}$	0.0199
<b>BP2900</b>	2900	1000	0.111	0.04	-

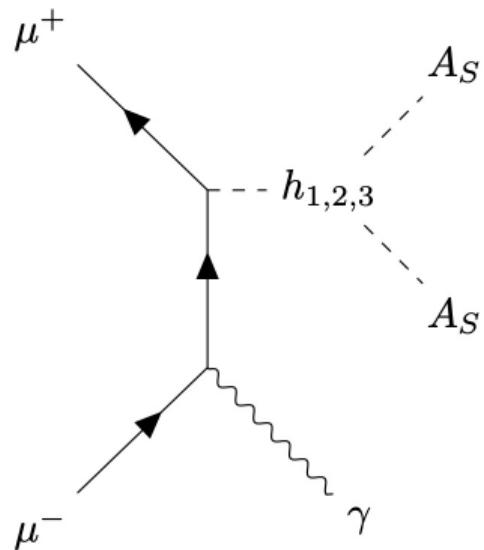
**Table:** Benchmarks **BP3**, **BP55**, **BP2900** with  $m_{h_1} = 95.4$  GeV,  $m_{h_2} = 125.09$  GeV consistent with theoretical as well as experimental constraints

# DM Production Processes

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(a)  $\mu^-\mu^+ \rightarrow ZA_S A_S$



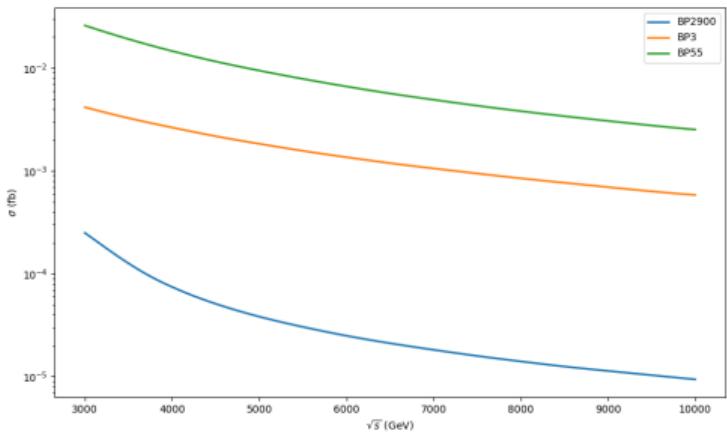
(b)  $\mu^-\mu^+ \rightarrow A_S A_S \gamma$

# Analysis

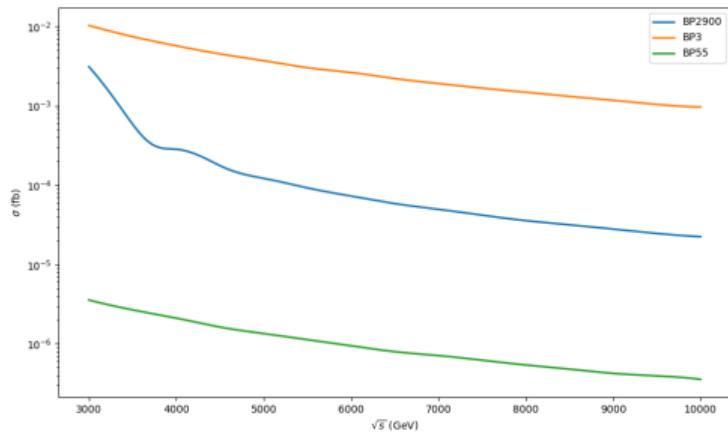
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- Unpolarised cross sections  $\sigma$  against  $\sqrt{s}$
- Polarised cross sections against  $\sqrt{s}$  for different  $P_{eff}$  [-100%, +100%]
- Angular distribution for  $Z A_s A_s$
- All analysis done using WHIZARD

# Cross Sections at the Muon Collider



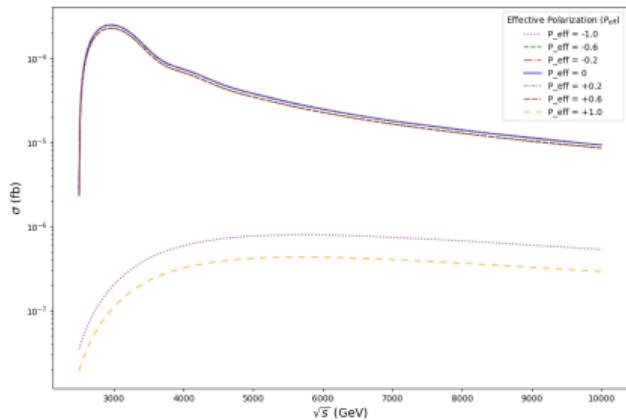
(a) Unpolarised cross sections for  $\mu^- \mu^+ \rightarrow Z A_s A_s$



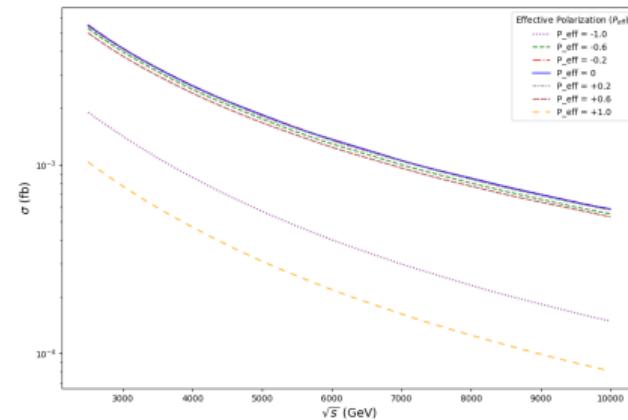
(b) Unpolarised cross sections for  $\mu^- \mu^+ \rightarrow A_s A_s \gamma$

# Polarised Cross Sections: $\mu^- \mu^+ \rightarrow Z A_s A_s$

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(a)  $\sigma$  against  $\sqrt{s}$  for BP2900



(b)  $\sigma$  against  $\sqrt{s}$  for BP3

# Polarised Cross Sections: $\mu^-\mu^+ \rightarrow ZA_sA_s$

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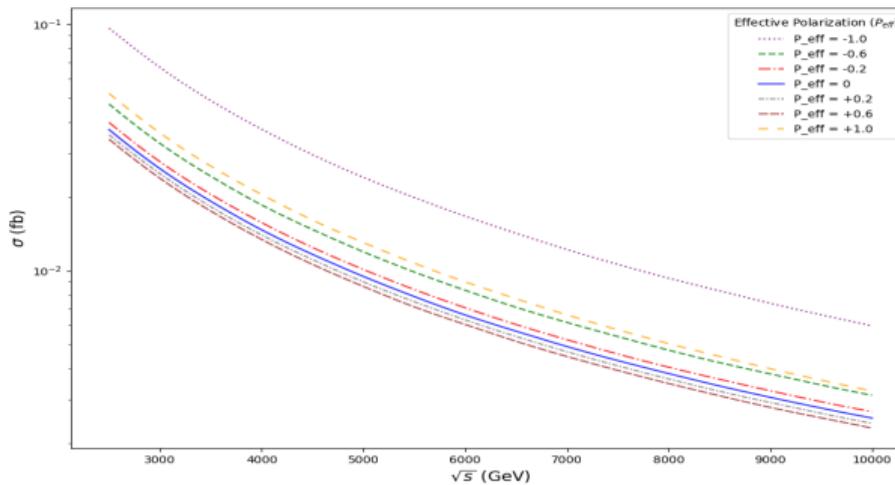


Figure:  $\sigma$  against  $\sqrt{s}$  for BP55

# Angular Distribution for Outgoing Z: $\mu^-\mu^+ \rightarrow Z A_s A_s$

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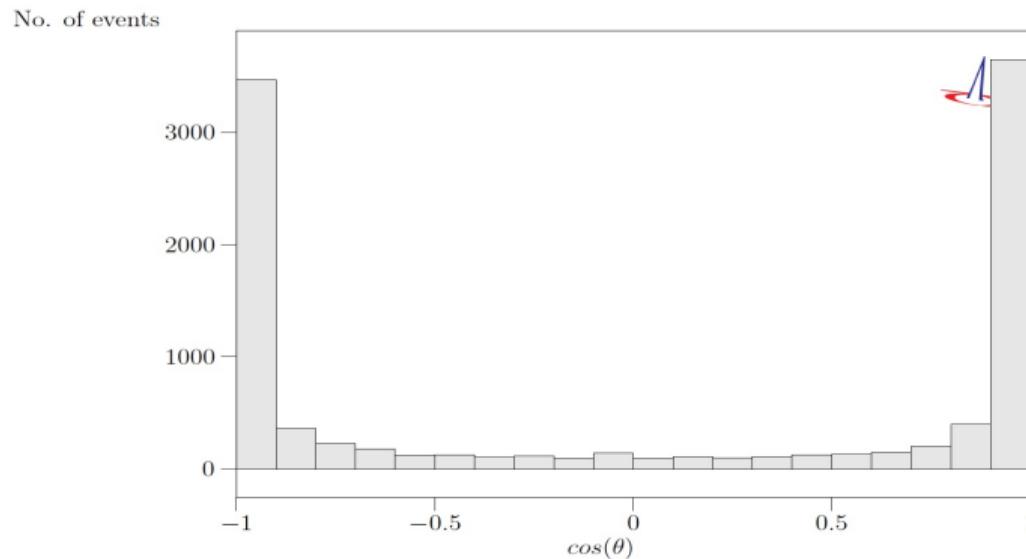
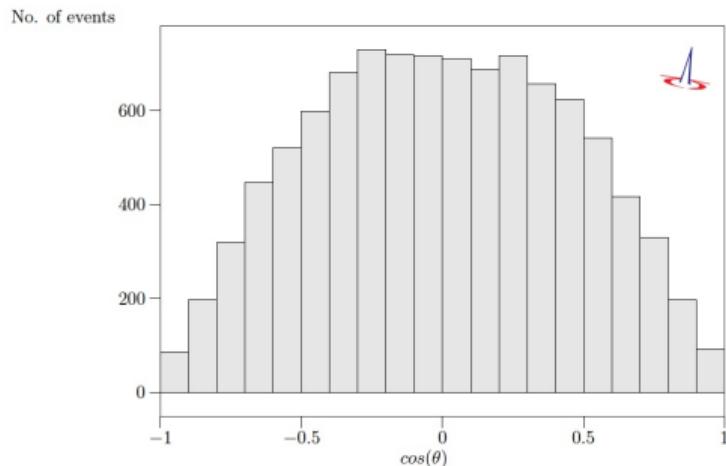


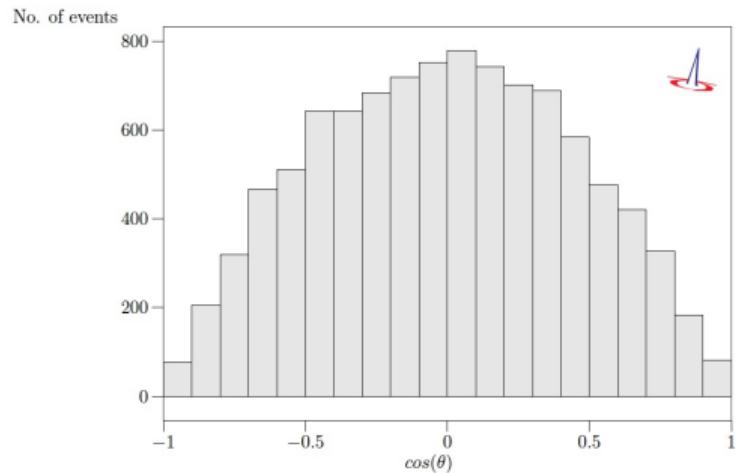
Figure: Unpolarised angular distribution for BP3 at  $\sqrt{s} = 3$  TeV

# Angular Distribution of Outgoing Z: $\mu^-\mu^+ \rightarrow ZA_sA_s$

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(a) Angular distribution for **BP3** at  $\sqrt{s} = 3$  TeV for  $P_{\text{eff}} = +100\%$



(b) Angular distribution for **BP3** at  $\sqrt{s} = 3$  TeV for  $P_{\text{eff}} = -100\%$

# Angular Distribution of Outgoing Z: $\mu^-\mu^+ \rightarrow ZA_sA_s$

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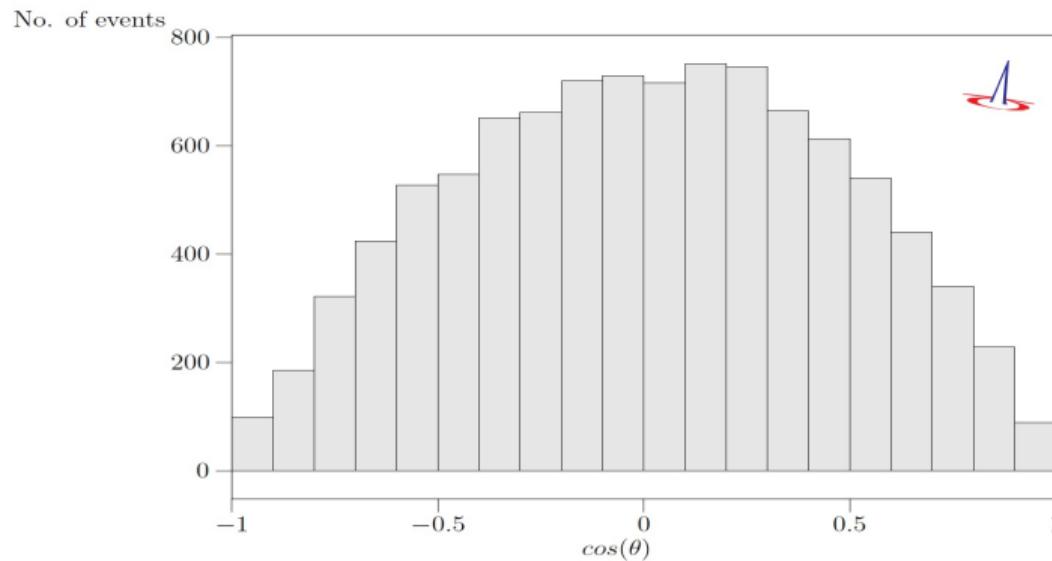
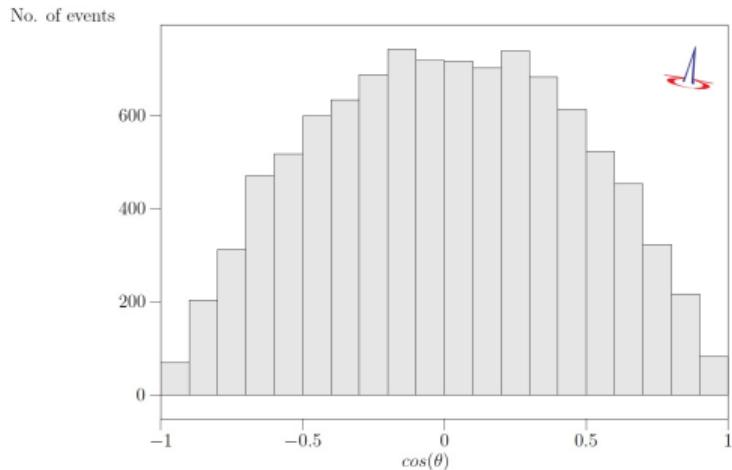


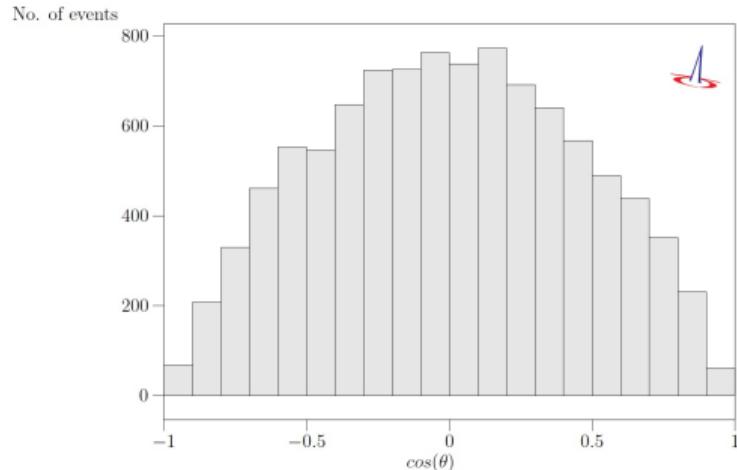
Figure: Unpolarised angular distribution for **BP55** at  $\sqrt{s} = 3$  TeV

# Angular Distribution for Outgoing Z: $\mu^-\mu^+ \rightarrow Z A_s A_s$

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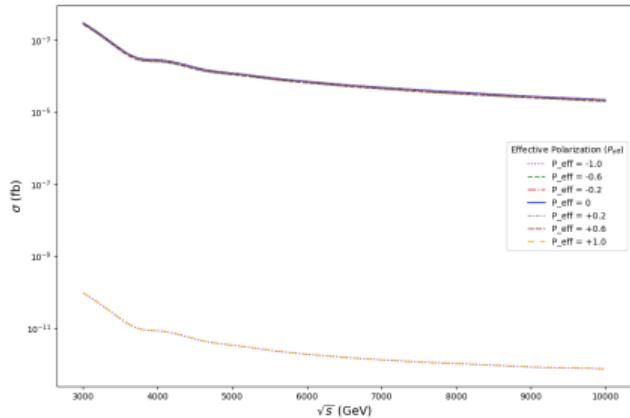
(a) Angular distribution for **BP55** at  $\sqrt{s} = 3$  TeV  
for  $P_{\text{eff}} = +100\%$



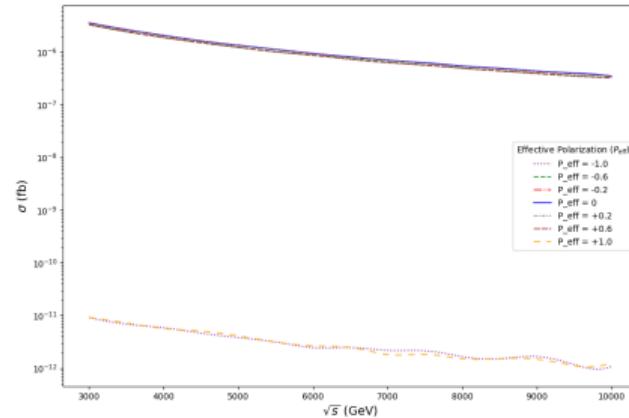
(b) Angular distribution for **BP55** at  $\sqrt{s} = 3$  TeV  
for  $P_{\text{eff}} = -100\%$

# Polarised Cross Sections: $\mu^- \mu^+ \rightarrow A_s A_s \gamma$

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(a)  $\sigma$  against  $\sqrt{s}$  for BP2900



(b)  $\sigma$  against  $\sqrt{s}$  for BP55

# Polarised Cross Sections: $\mu^-\mu^+ \rightarrow A_s A_s \gamma$

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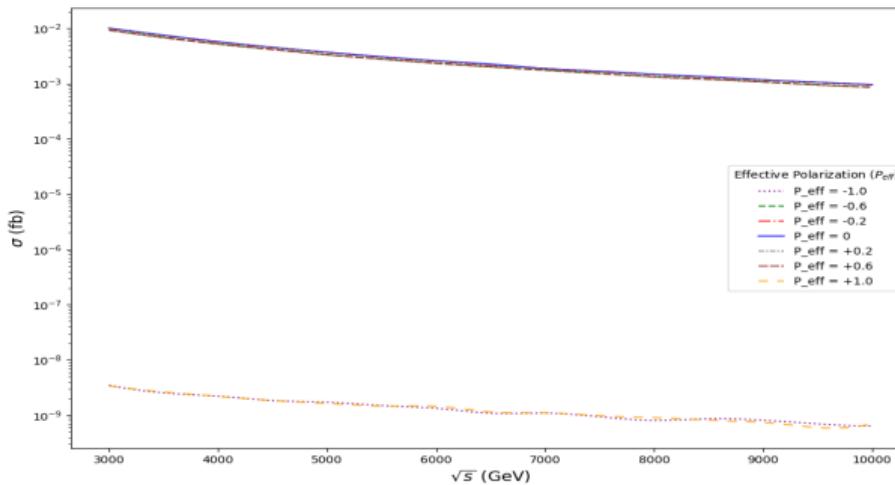


Figure:  $\sigma$  against  $\sqrt{s}$  for BP3

## Takeaways

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For  $\mu^-\mu^+ \rightarrow ZA_sA_s$ ,

- BP3 gives the best unpolarised cross sections
- Cross sections heavily suppressed at extreme polarisations for BP3 and BP2900 but amplified for BP55

For  $\mu^-\mu^+ \rightarrow A_sA_s\gamma$

- BP3 gives the best unpolarised cross sections
- Cross sections heavily suppressed at extreme polarisations for all three benchmarks