Universität Hamburg
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Master Colloquium

SUSY Parameter determination and dark matter phenomenology at future e^+e^--colliders

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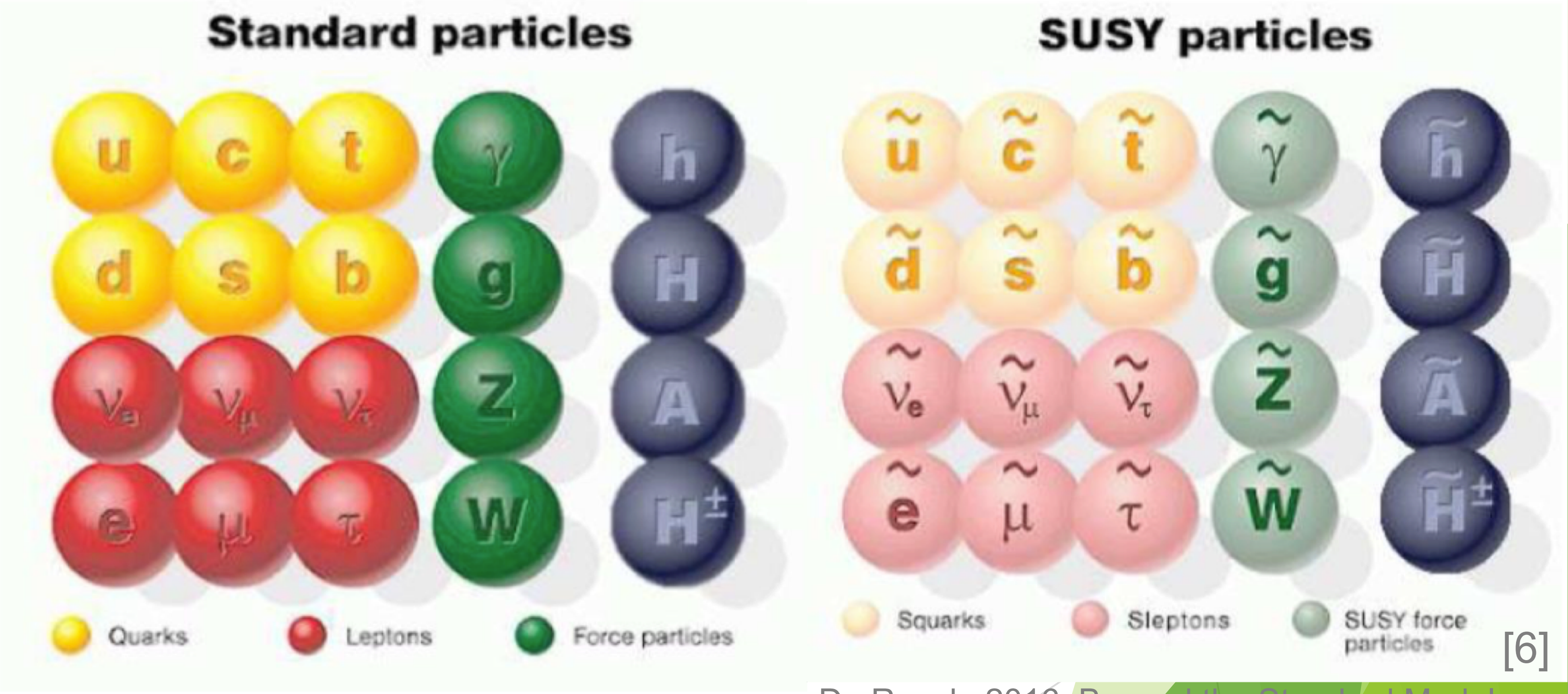
MSSM

- SM-particles get supersymmetric partners
- Additionally:
  - Neutralinos: $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$
  - Charginos: $\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$



[6]

De Roeck, 2016: Beyond the Standard Model…

- Dark matter candidate: $\tilde{\chi}_1^0$

Mass matrices in the MSSM

- We use the Bino-Wino basis $\{\tilde{B}, \widetilde{W}^0, \widetilde{H}_1^0, \widetilde{H}_2^0\}$
- Parameters:
  - M_1: Bino mass parameter
  - M_2: Wino mass parameter
  - μ: Higgsino mass parameter
  - $\tan\beta = \frac{v_2}{v_1}$
- Mixing angles $\phi_{L,R}$ used to diagonalize matrices

$$M_C = \begin{pmatrix} M_2 & m_W\sqrt{2}\sin\beta \\ m_W\sqrt{2}\cos\beta & \mu \end{pmatrix}$$

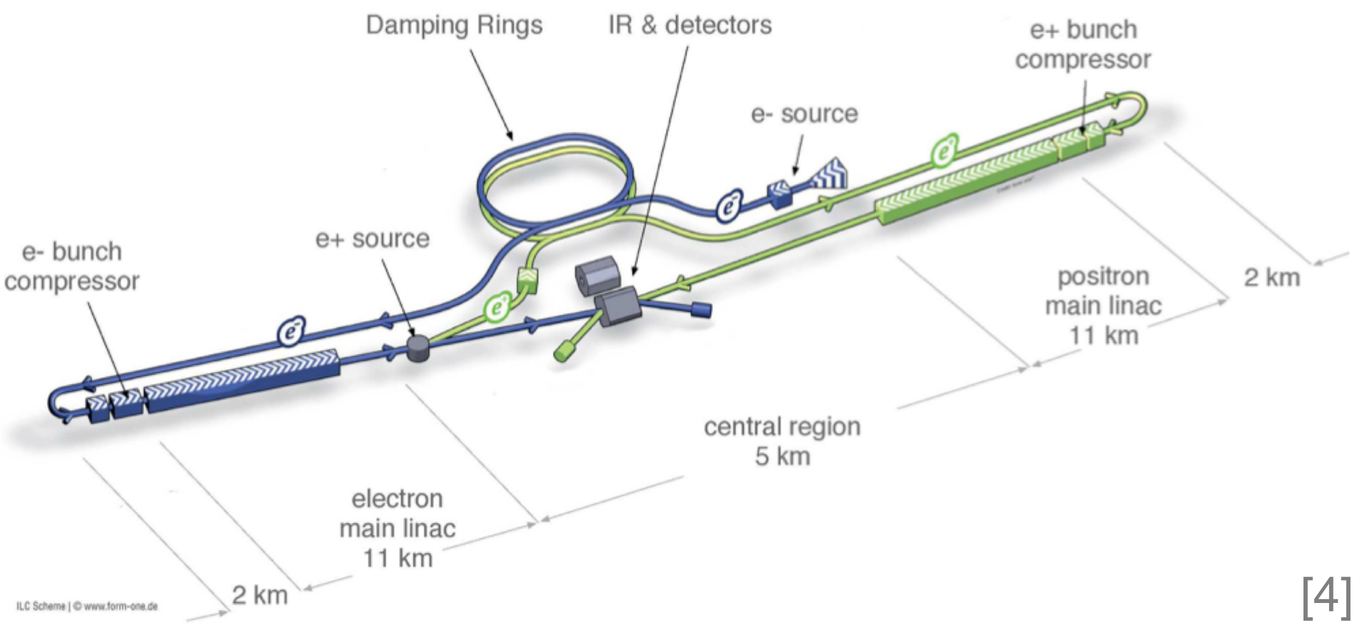
$$M_N = \begin{pmatrix} M_1 & 0 & -m_Z\cos\beta\sin\theta_W & m_Z\sin\beta\sin\theta_W \\ 0 & M_2 & m_Z\cos\beta\cos\theta_W & -m_Z\sin\beta\cos\theta_W \\ -m_Z\cos\beta\sin\theta_W & m_Z\cos\beta\cos\theta_W & 0 & -\mu \\ m_Z\sin\beta\sin\theta_W & -m_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix}$$

[1]

Desch et al.: arXiv:hep-ph/0607104v2

International Linear Collider

- Future e^+e^- linear collider with spin polarised beams
- Searches for signs of physics beyond the Standard Model
- Possible experiment:
  - $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$
  - Luminosity $L = 500 \text{ fb}^{-1}$
  - $\sqrt{s} = 500 \text{ GeV}$
    - $\sqrt{s} = 550 \text{ GeV}$ also possible



[4]

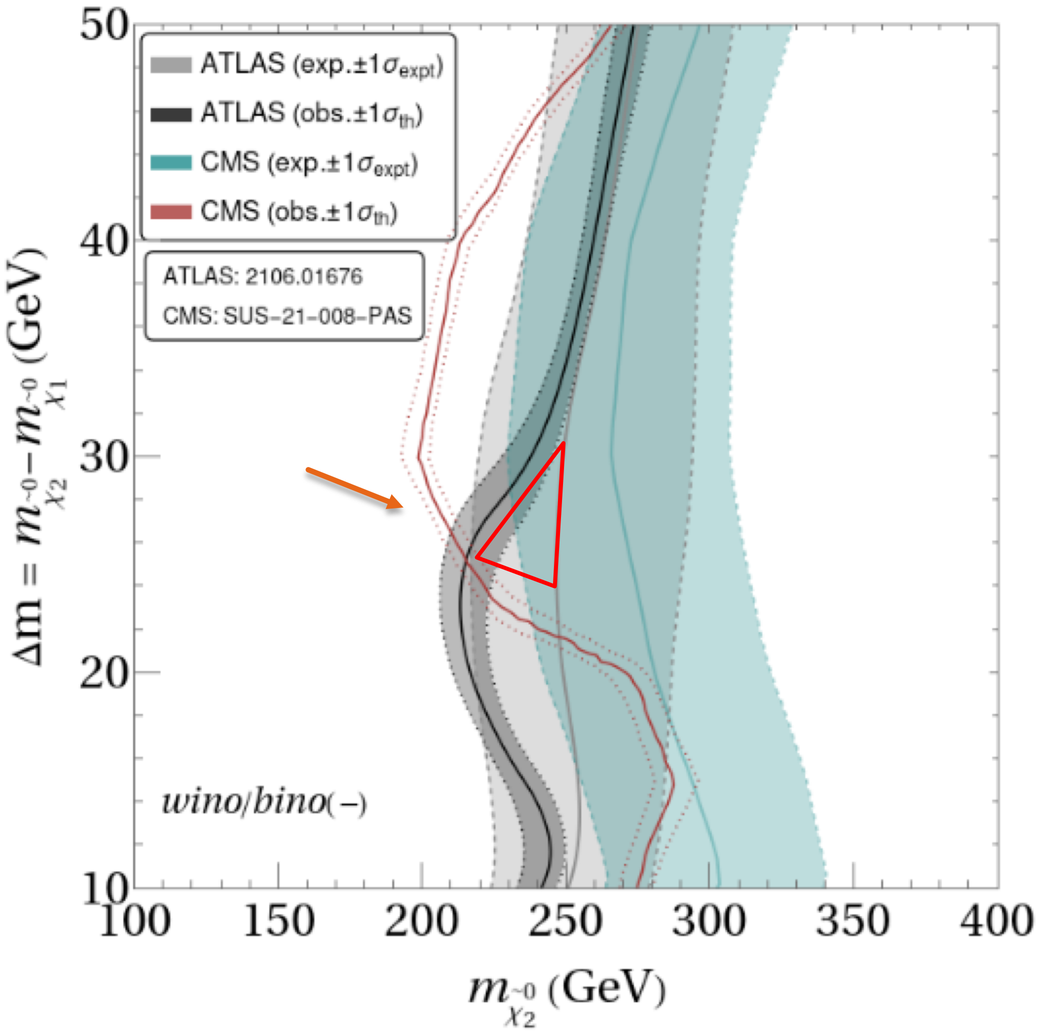
Zarnecki, 2020: On the physics potential…

[8] Abramowicz et al.: arXiv:2503.24049

Motivation of analysis scenario

- Excess in ATLAS & CMS searches in channel: $pp \to \tilde{\chi}_2^0 \tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 Z^{(*)} \tilde{\chi}_1^0 W^{\pm(*)}$
- Possible explanation: Bino-wino-like DM with $\mu \times M_1 < 0$ (Bino/Wino(–))

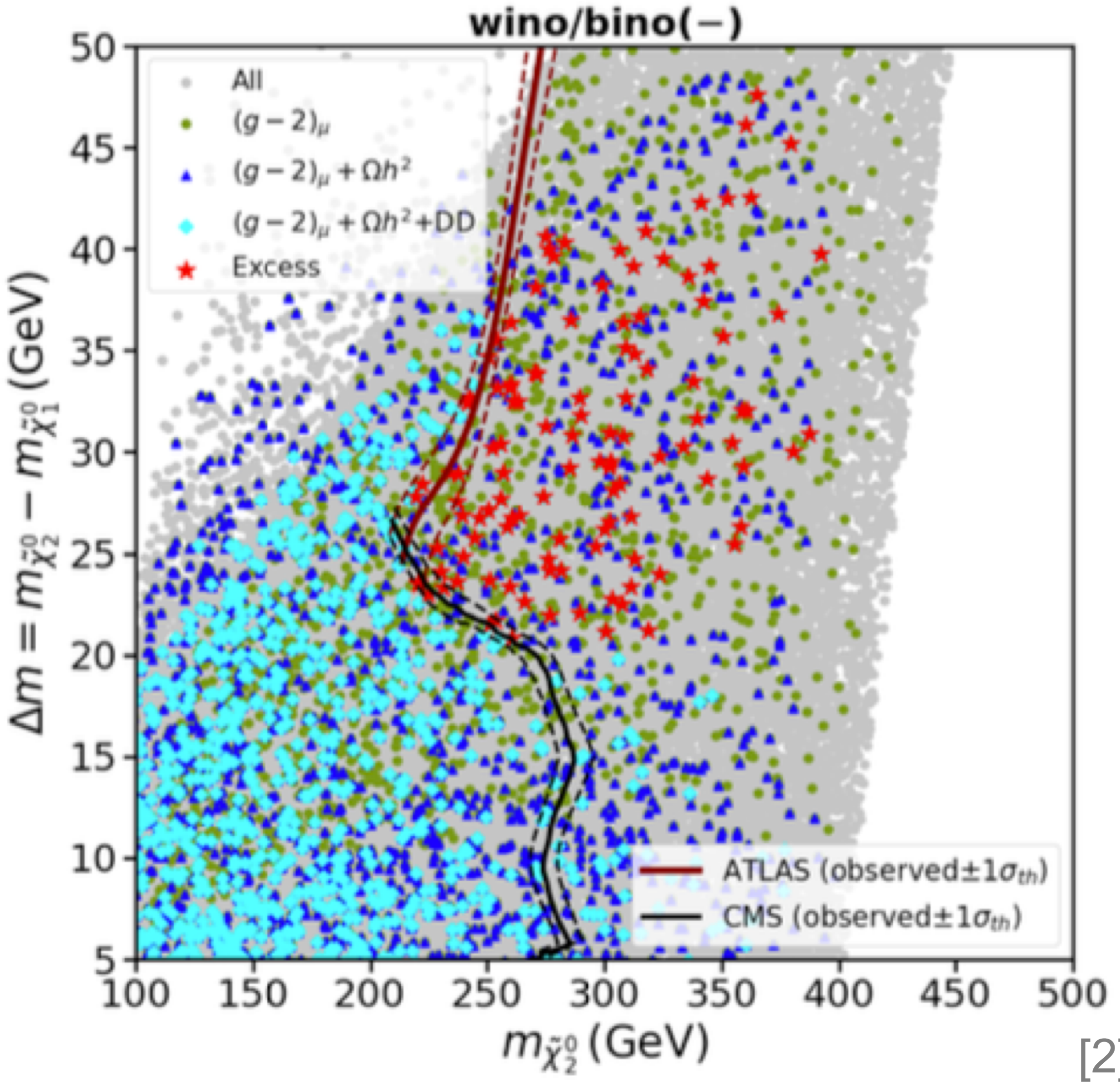
- Search for points in this excess was done in previous publication by Chakraborti et al. [2]



[2] Chakraborti et al.: arXiv:2403.14759v1

Motivation of analysis scenario

- Red points describe the excess
- Analysis is done for such parameter points!



[2]

Chakraborti et al.: arXiv:2403.14759v1

Data

- SLHA files are taken from „Consistent Excesses in the Search for $\tilde{\chi}_2^0\tilde{\chi}_1^\pm$: Wino/bino vs. Higgsino Dark Matter“ by Chakraborti et al. [2]

- Constraints:
  - Vacuum stability constraints
  - DM relic density constraints
  - Constraints from LHC measurements
  - DM direct detection constraints

Steps of the analysis

Goal: Reconstruct DM with e^+e^- measurements

$$\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$$
$$m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^0}$$

$$A_{FB}^{prod.}, A_{FB}^{fin.}$$

$$\phi_L, \phi_R$$

$$\Delta M_{\tilde{\nu}}$$

Reconstructed via Scan with three different methods to get uncertainty in $\Delta M_{\tilde{\nu}}$

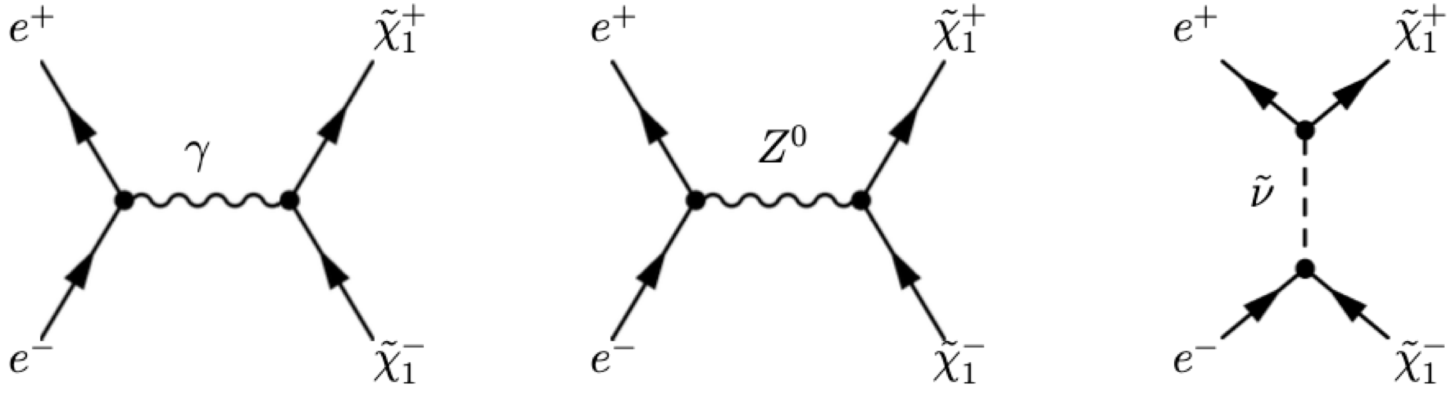
$$M_1, M_2, \mu$$

$$\Omega h^2_{true}$$

$$\Omega h^2_{scan}$$

Chargino pair production

- Contributing first order Feynman diagrams for chargino pair production:



[1]

Desch et al.: arXiv:hep-ph/0607104v2

- Crosssection calculated with mixing angles

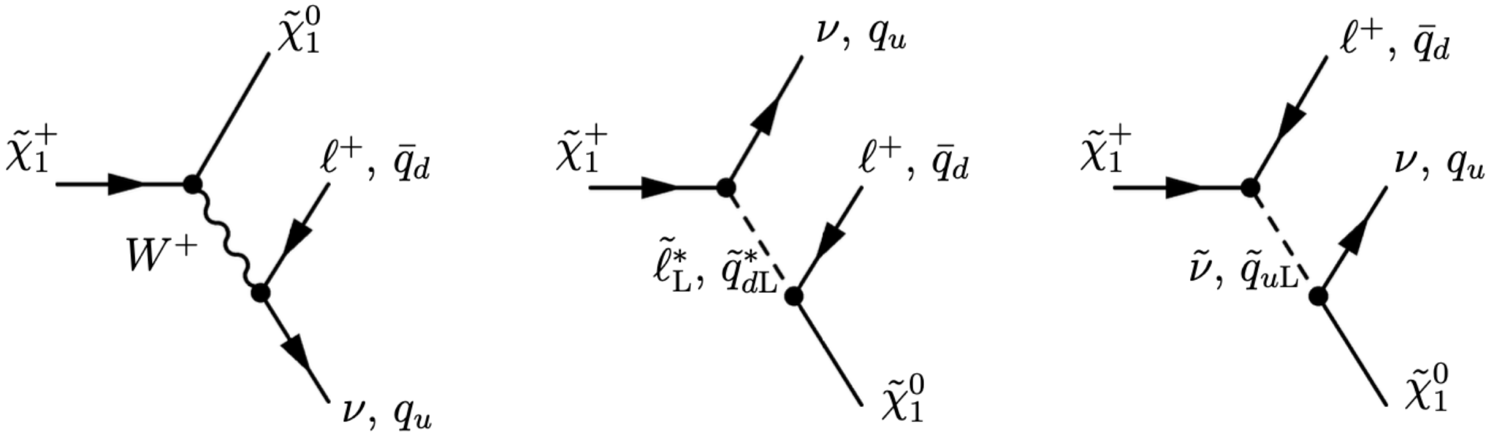
$$\sigma^{\pm}\{ij\} = c_1 \cos^2 2\Phi_L + c_2 \cos 2\Phi_L + c_3 \cos^2 2\Phi_R + c_4 \cos 2\Phi_R + c_5 \cos 2\Phi_L \cos 2\Phi_R + c_6$$

[5]

Desch et al.: arXiv:hep-ph/0312069v1

Decay modes

- Contributing first order Feynman diagrams for leptonic and hadronic decay of charginos:



[1]

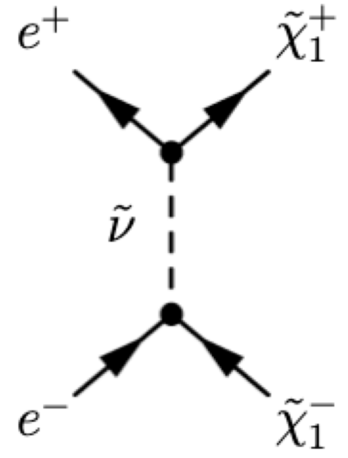
Desch et al.: arXiv:hep-ph/0607104v2

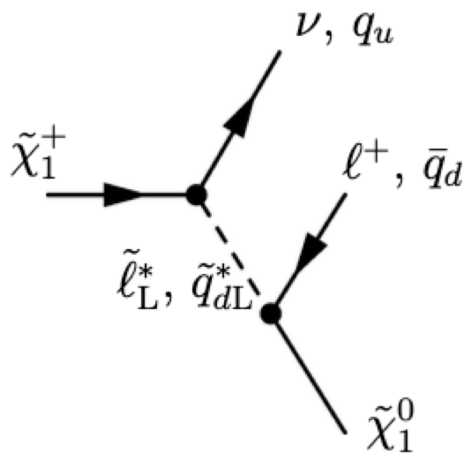
Forward-backward asymmetry

- Forward-backward asymmetry calculated with crosssections:

$$A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$$

- Different A_{FB} are used:
  - A_{FB} from production
  - Final state A_{FB}





[1]

Desch et al.: arXiv:hep-ph/0607104v2

Configurations

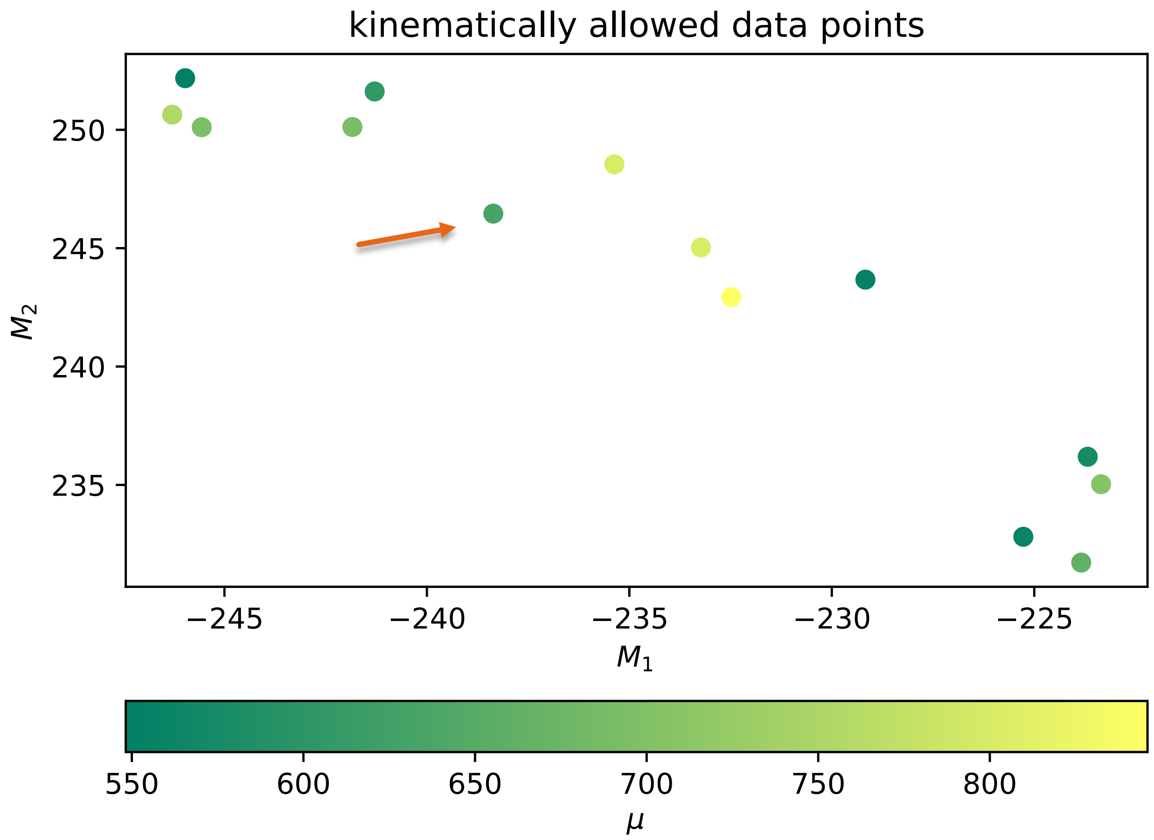
- We use four beam configurations to increase precision
- Consider the following uncertainties:

statistical uncertainty	$\sigma_{\text{stat}} = \frac{\sqrt{\sigma L}}{L}$
polarisation uncertainty	$P_{e^-} = \pm 0.804,\ P_{e^+} = \mp 0.603$ $P_{e^-} = \pm 0.796,\ P_{e^+} = \mp 0.597$
mass uncertainty	$\delta m_{\tilde{\chi}_1^\pm} = 0.005 m_{\tilde{\chi}_1^\pm}$

$\sqrt{s}$	P_{e^-}	P_{e^+}
500 GeV	-0.8	+0.6
500 GeV	+0.8	-0.6
550 GeV	-0.8	+0.6
550 GeV	+0.8	-0.6

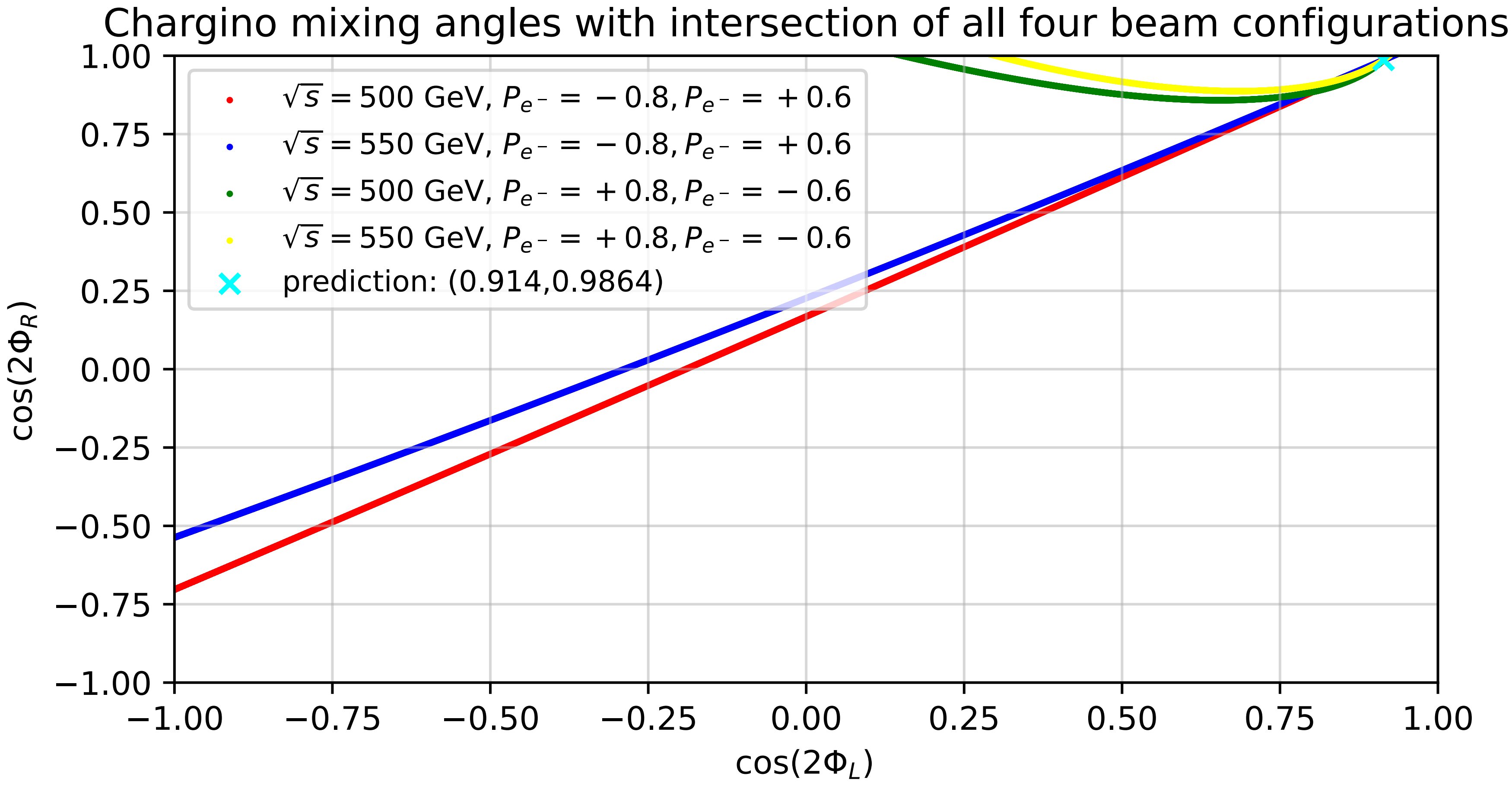
Example point

- Out of 14 kinematically allowed data points, one is taken as example point for following explanations



M_1	-238.36 GeV	$\sigma^{500}_{-.8,+.6}$	394.9612 fb
M_2	246.46 GeV	$\sigma^{500}_{+.8,-.6}$	11.0373 fb
μ	632.64 GeV	$\sigma^{550}_{-.8,+.6}$	519.8185 fb
$\tan\beta$	51.97	$\sigma^{550}_{+.8,-.6}$	14.4873 fb
$M_{\tilde{\nu}}$	496.78 GeV	$m_{\tilde{\chi}^{\pm}_1}$	241.48 GeV
Prod. $A^{500}_{-.8,+.6}$	0.1008	Fin. $A^{500}_{-.8,+.6}$	0.346
Prod. $A^{500}_{+.8,-.6}$	0.1012	Fin. $A^{500}_{+.8,-.6}$	0.342
Prod. $A^{550}_{-.8,+.6}$	0.2255	Fin. $A^{550}_{-.8,+.6}$	0.369
Prod. $A^{550}_{+.8,-.6}$	0.2262	Fin. $A^{550}_{+.8,-.6}$	0.367

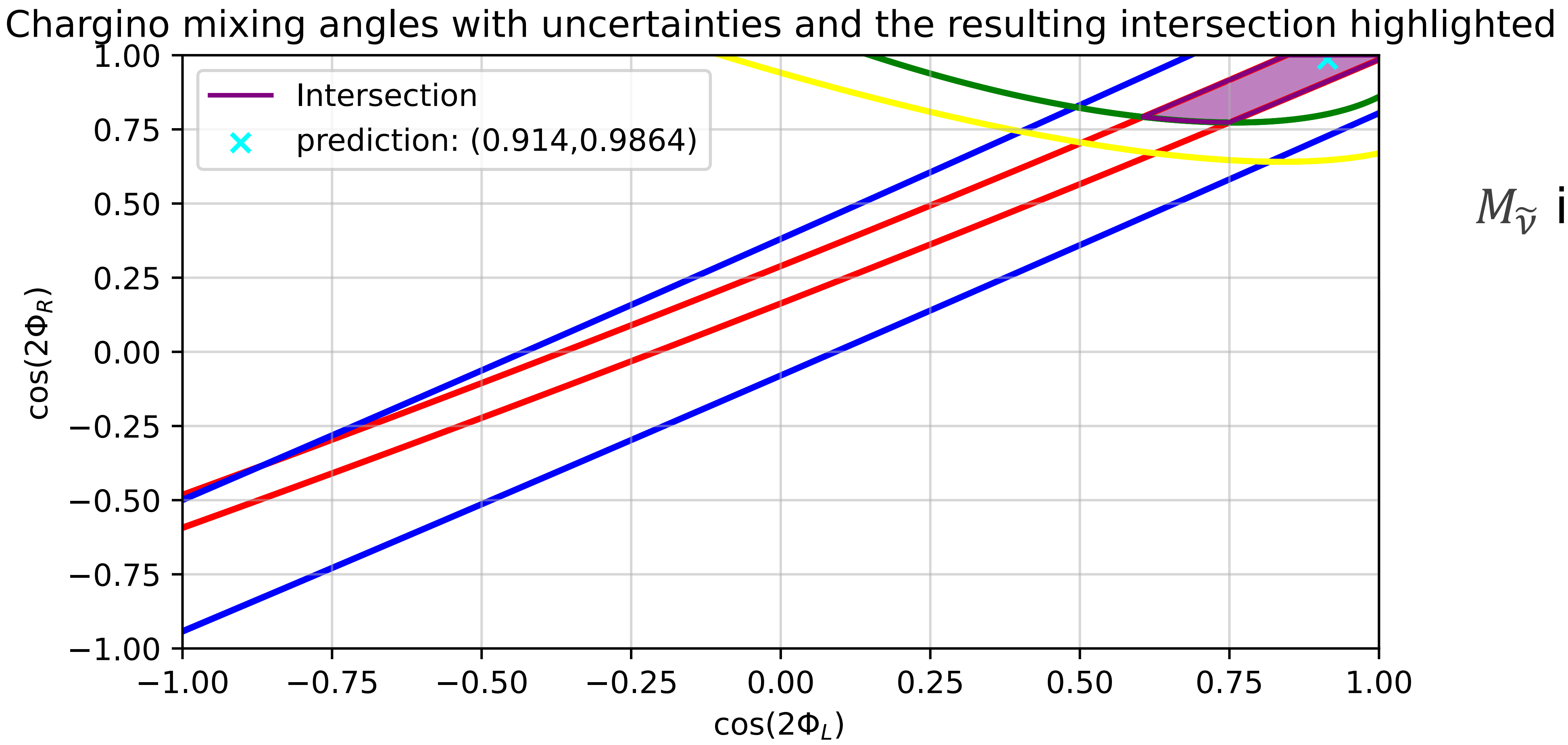
Mixing angles without uncertainties



$M_{\tilde{\nu}}$ is fixed!

$$\sigma^{\pm}\{ij\} = c_1 \cos^2 2\Phi_L + c_2 \cos 2\Phi_L + c_3 \cos^2 2\Phi_R + c_4 \cos 2\Phi_R + c_5 \cos 2\Phi_L \cos 2\Phi_R + c_6$$

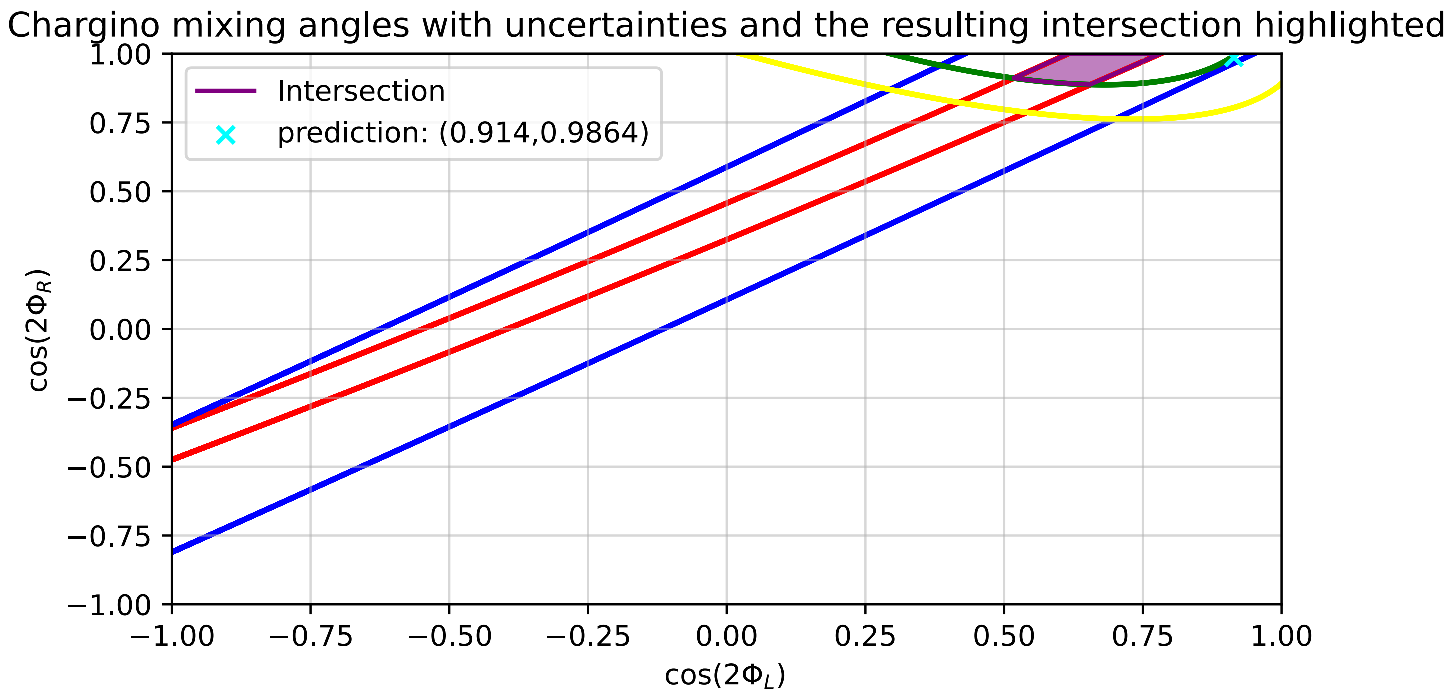
Valid mixing angles

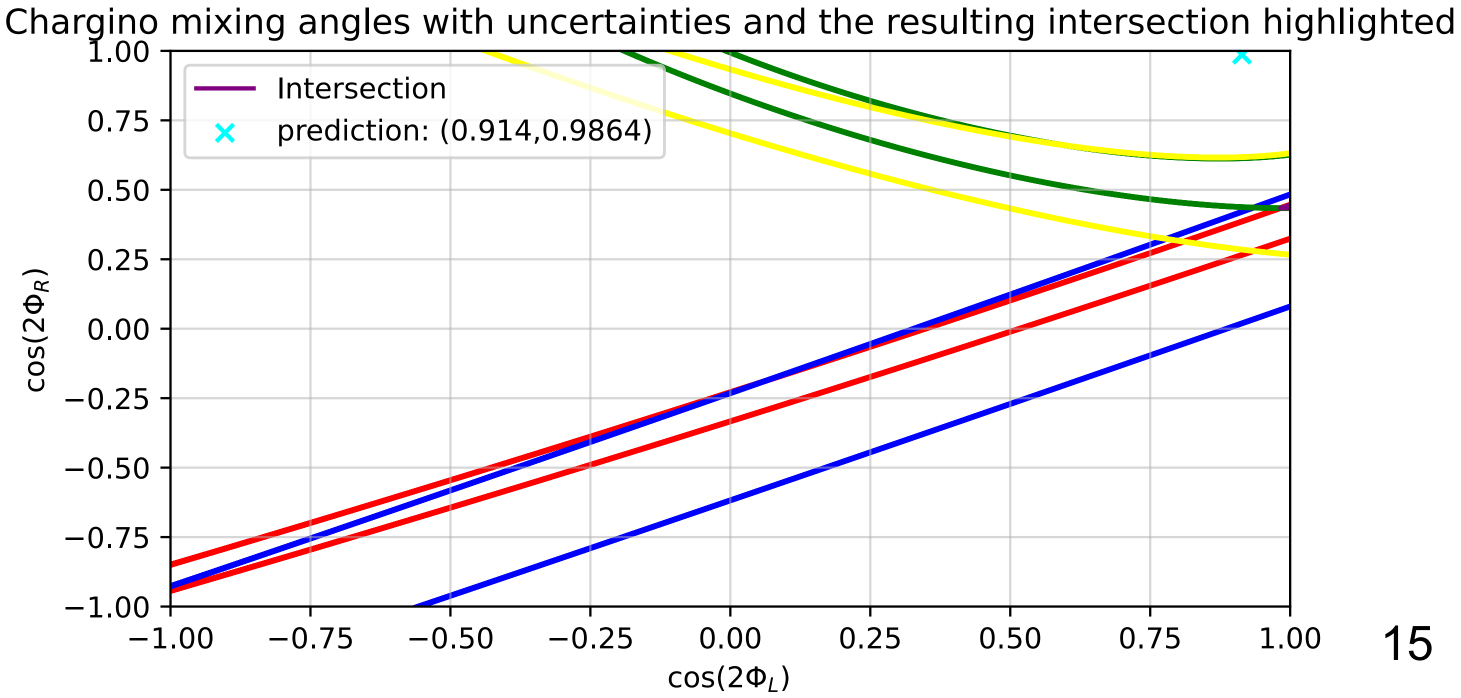


$M_{\tilde{\nu}}$ is fixed!

Uncertainty of $M_{\tilde{\nu}}$ without A_{FB}

- Variation in steps of 10 GeV until intersection area vanishes
- Variation for the whole data set: -140 to +110 GeV
- Variation for this point: -10 to +40 GeV

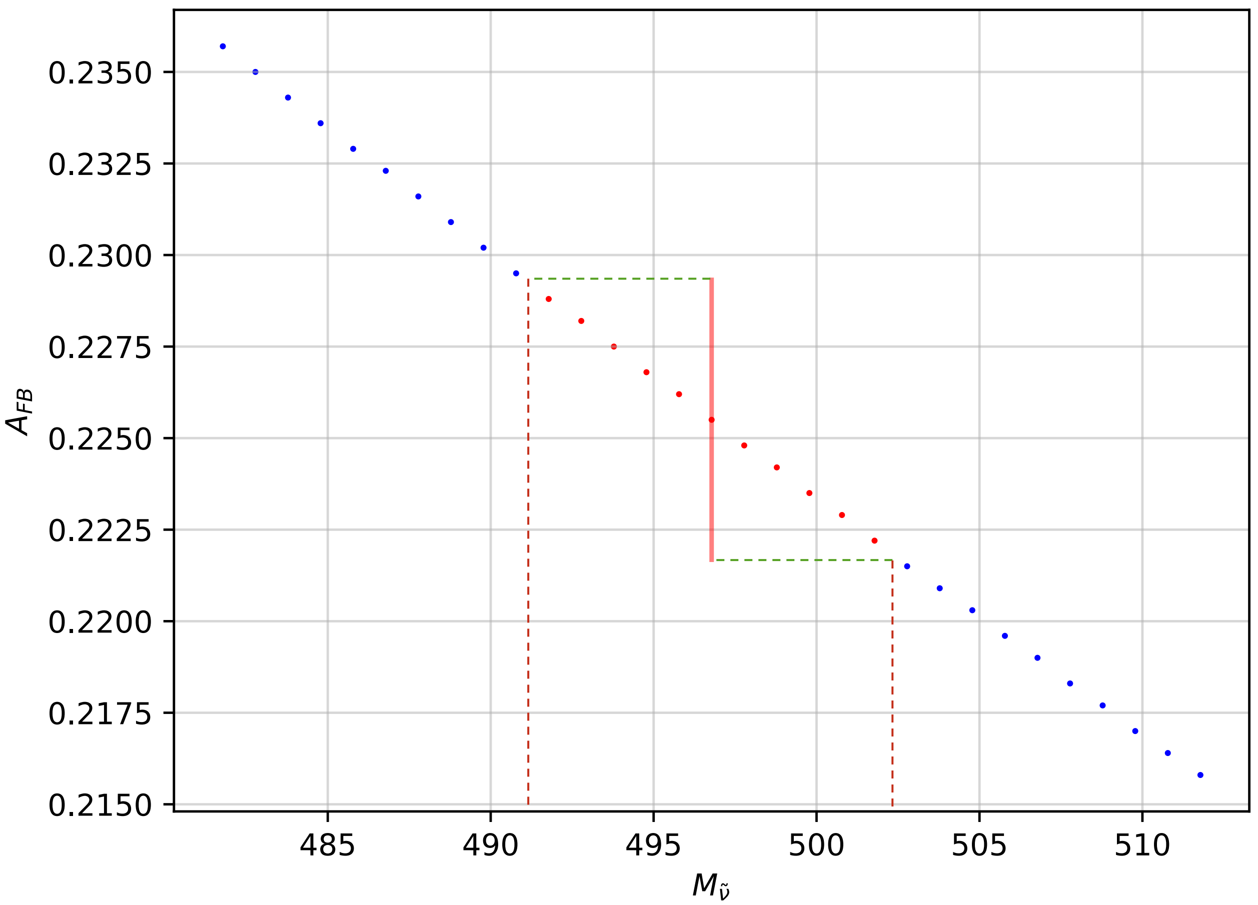




Uncertainty of $M_{\tilde{\nu}}$ via A_{FB}

- Derived via A_{FB} of the production channel and the final A_{FB}
- $\Delta A_{FB}^{stat} = 4\sqrt{\frac{\epsilon(1-\epsilon)}{N}}$ with $\epsilon = \frac{\sigma^F}{\sigma^F+\sigma^B}$ and N: number of events
- $M_{\tilde{\nu}}$ is varied and checked against the allowed A_{FB} within its statistical error
- Experimental error is not included! → will lead to relevant degradation in $M_{\tilde{\nu}}$ determination

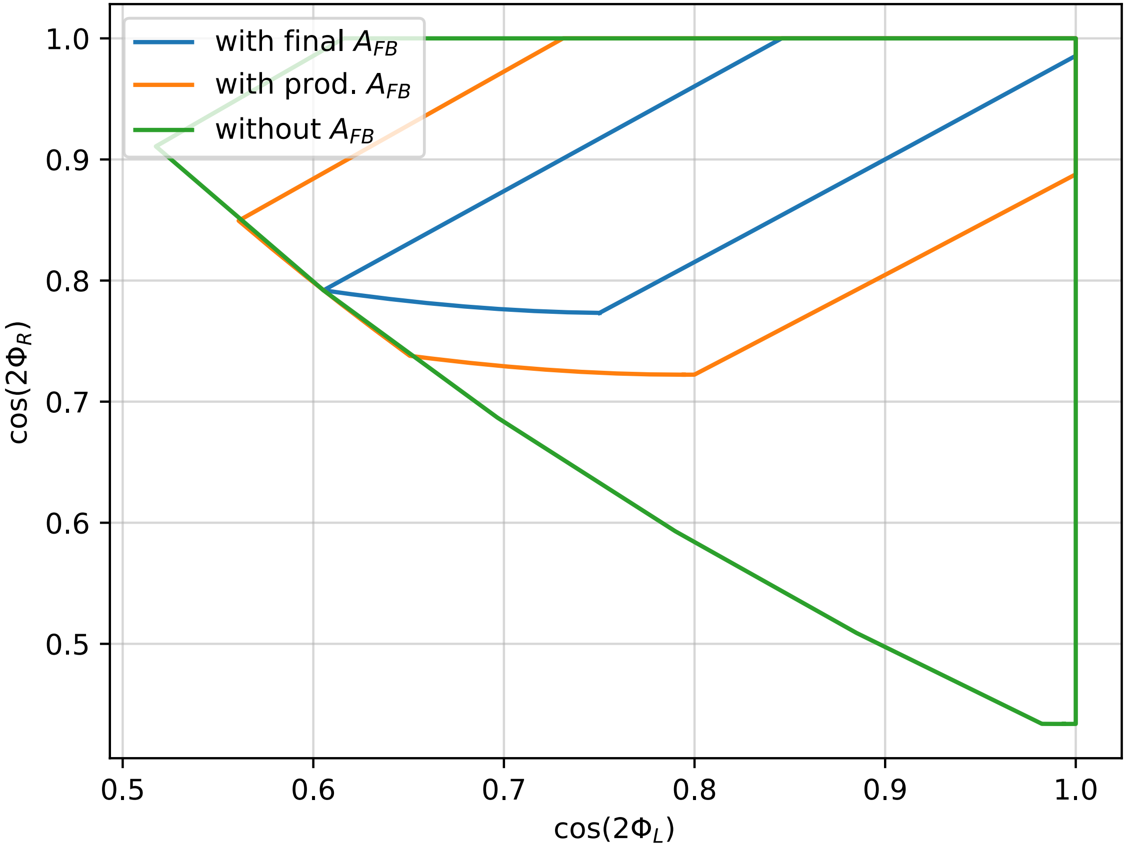
Derivation of $M_{\tilde{\nu}}$ uncertainty via A_{FB} (here for prod. $A^{550}_{-.8,+.6}$)



[1] Desch et al.: arXiv:hep-ph/0607104v2

Comparison

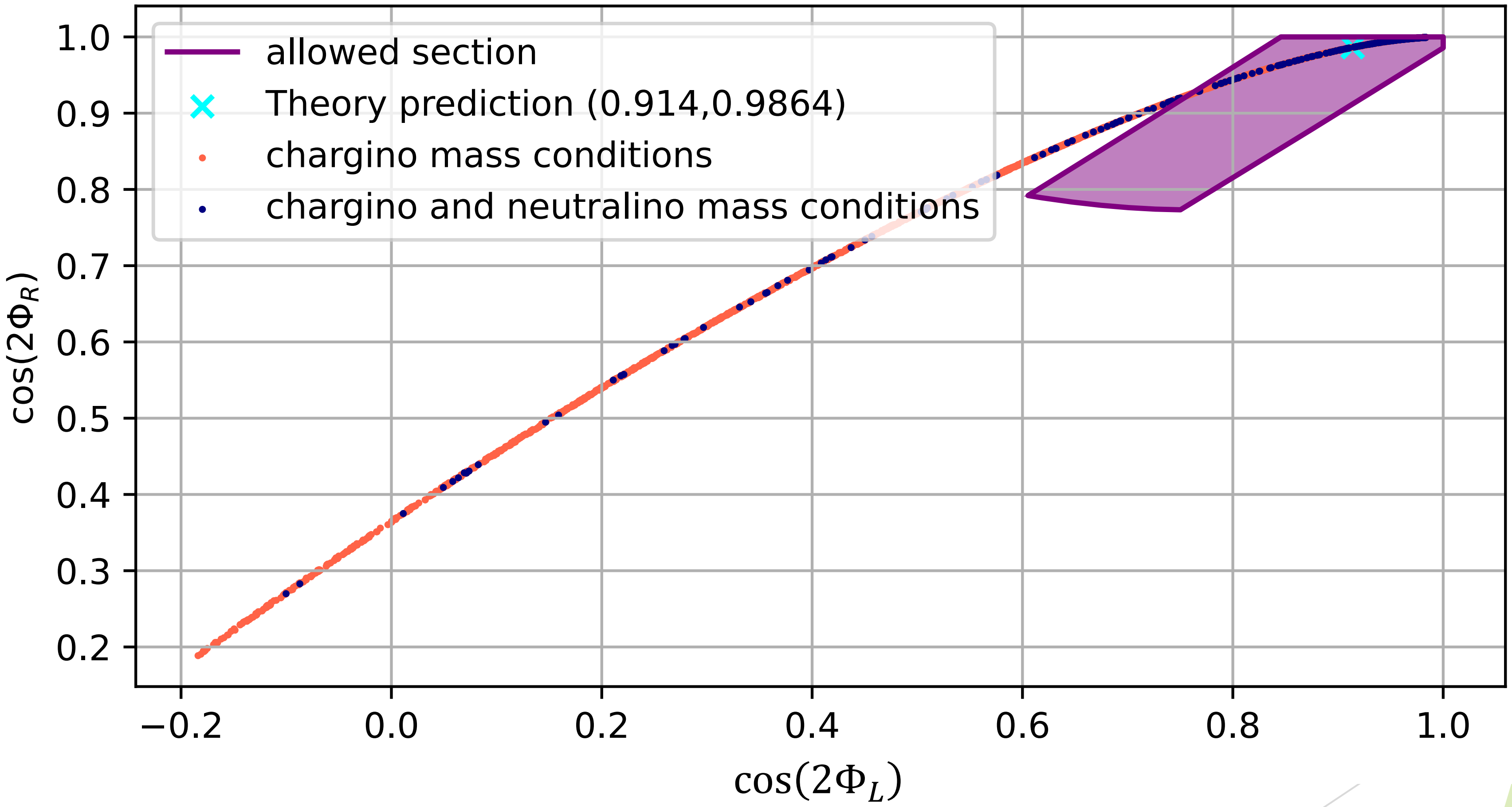
- Improvement in accuracy
  - Especially in comparison to the method without A_{FB}
- Relevant degradation expected by including experimental error on A_{FB}



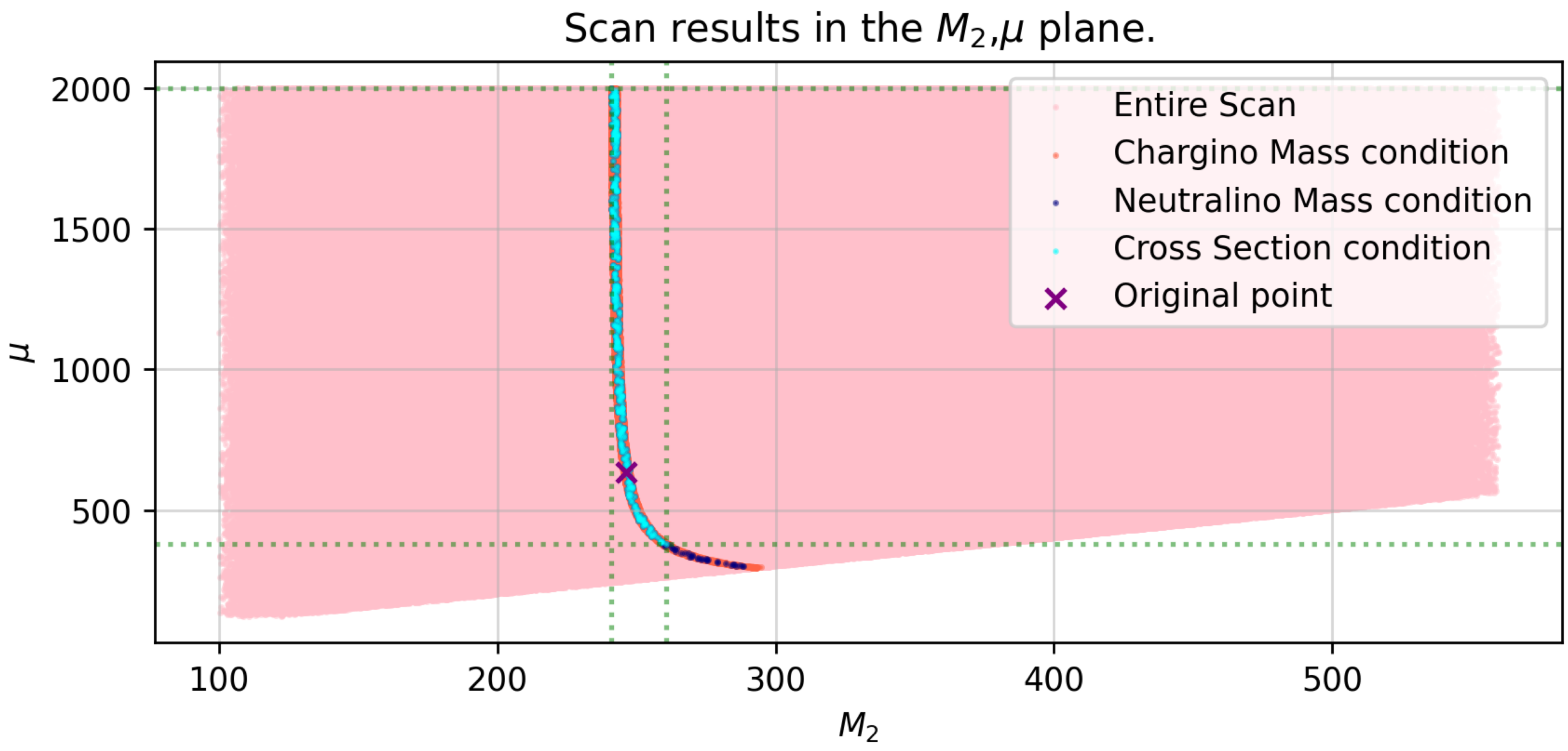
	Minimal variation of $M_{\tilde{\nu}}$	Maximal variation of $M_{\tilde{\nu}}$
Without A_{FB}	~ -10 GeV	~ +40 GeV
A_{FB} from production	~ -5 GeV	~ +5 GeV
A_{FB} final	>-1 GeV	<+1 GeV

Scanning for valid points

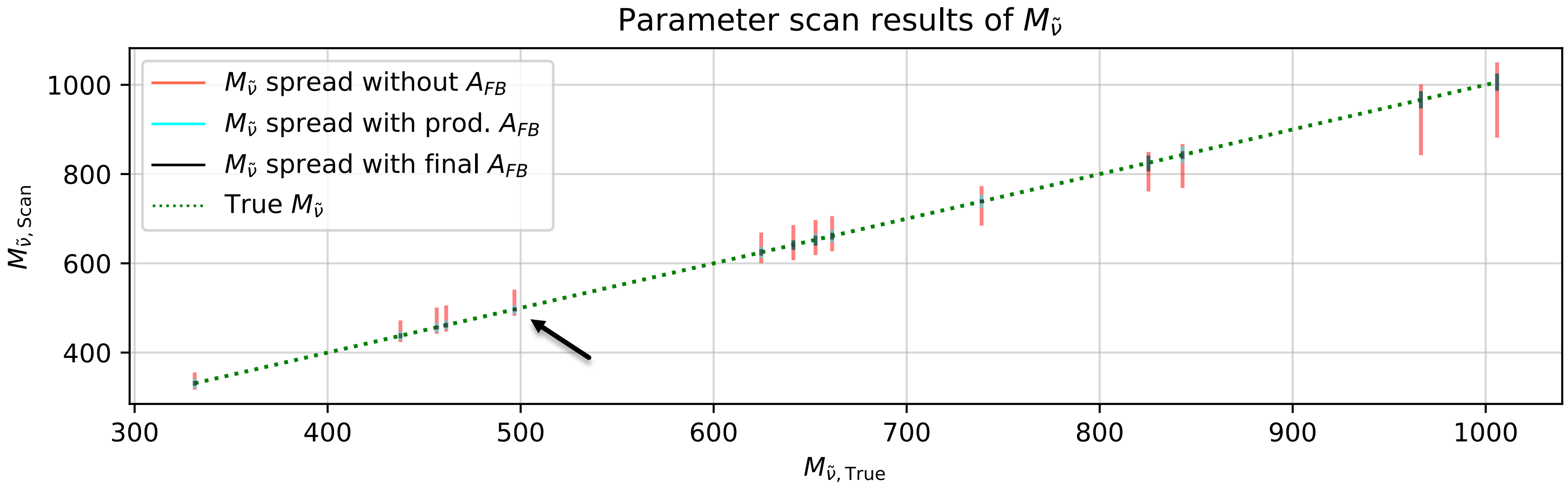
Scan results with cross-section condition visualised using the intersection surface.

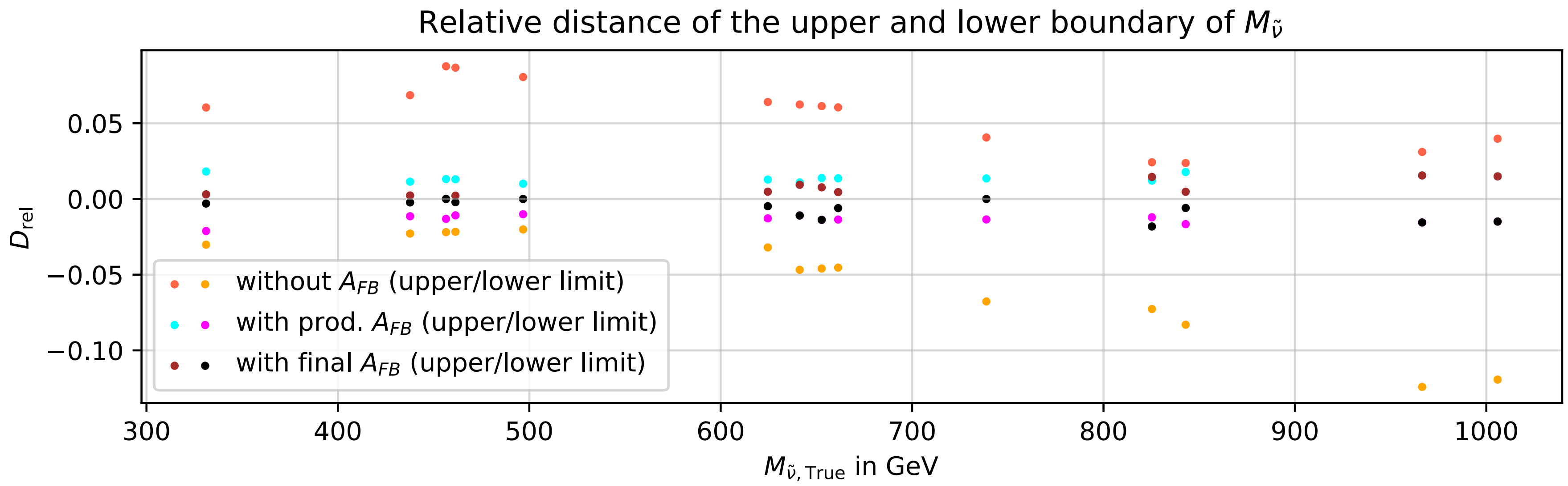


Scanning for valid points



Results for $M_{\tilde{\nu}}$

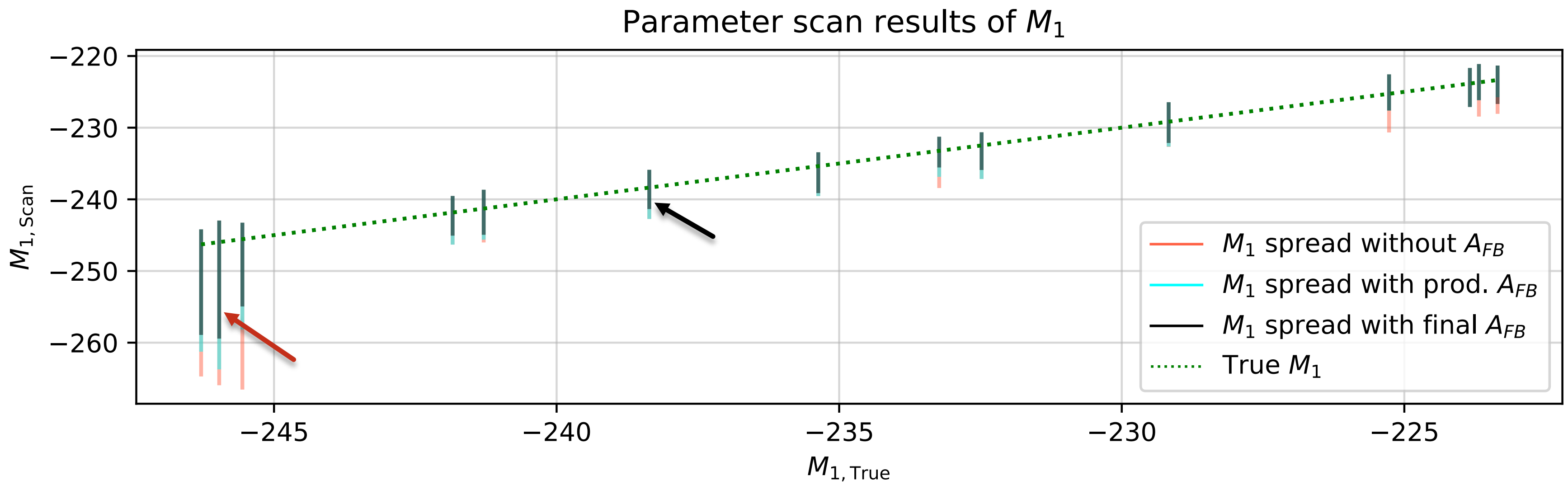


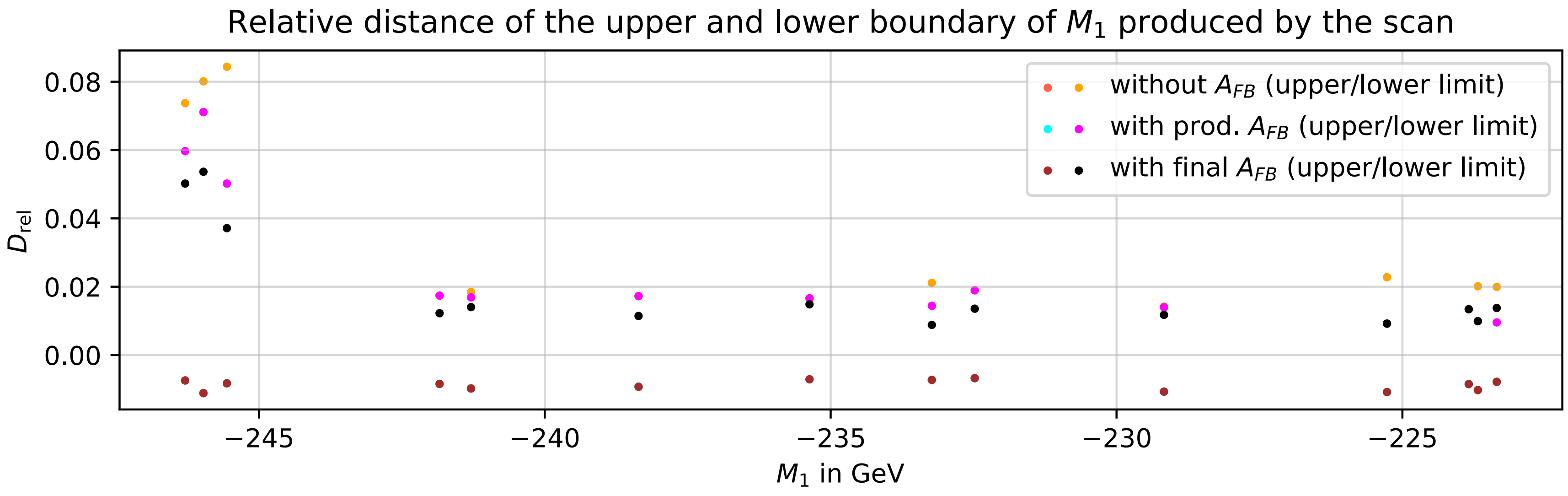


From now on, all points are included!

$$D_{rel} = \frac{M_{\tilde{\nu},scan} - M_{\tilde{\nu},true}}{M_{\tilde{\nu},true}}$$

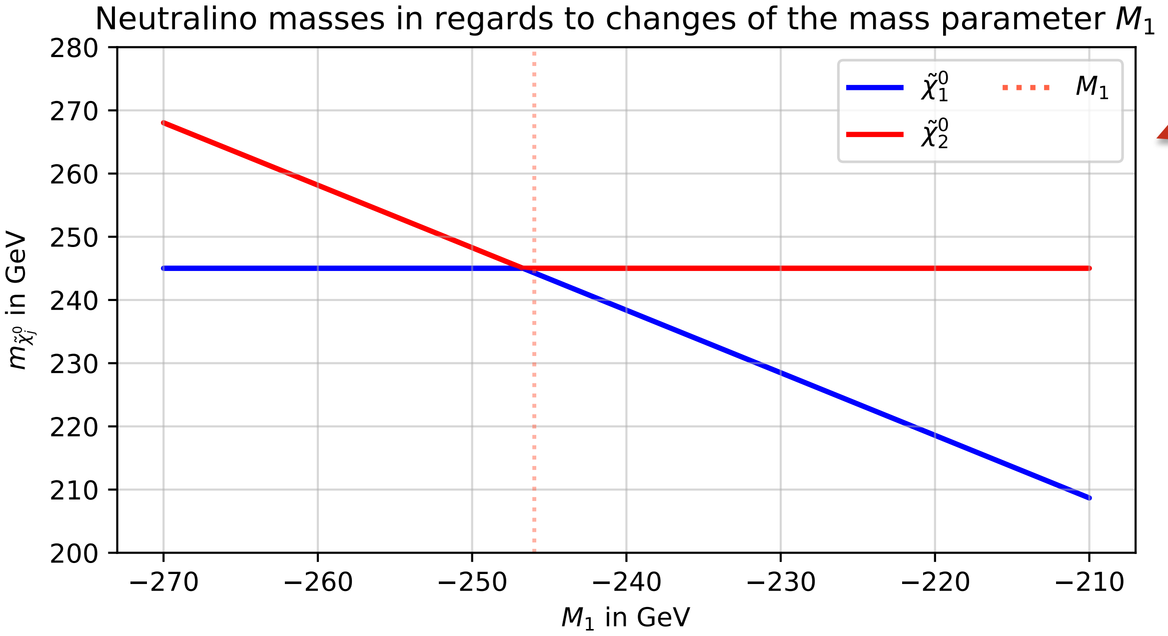
Results for M_1



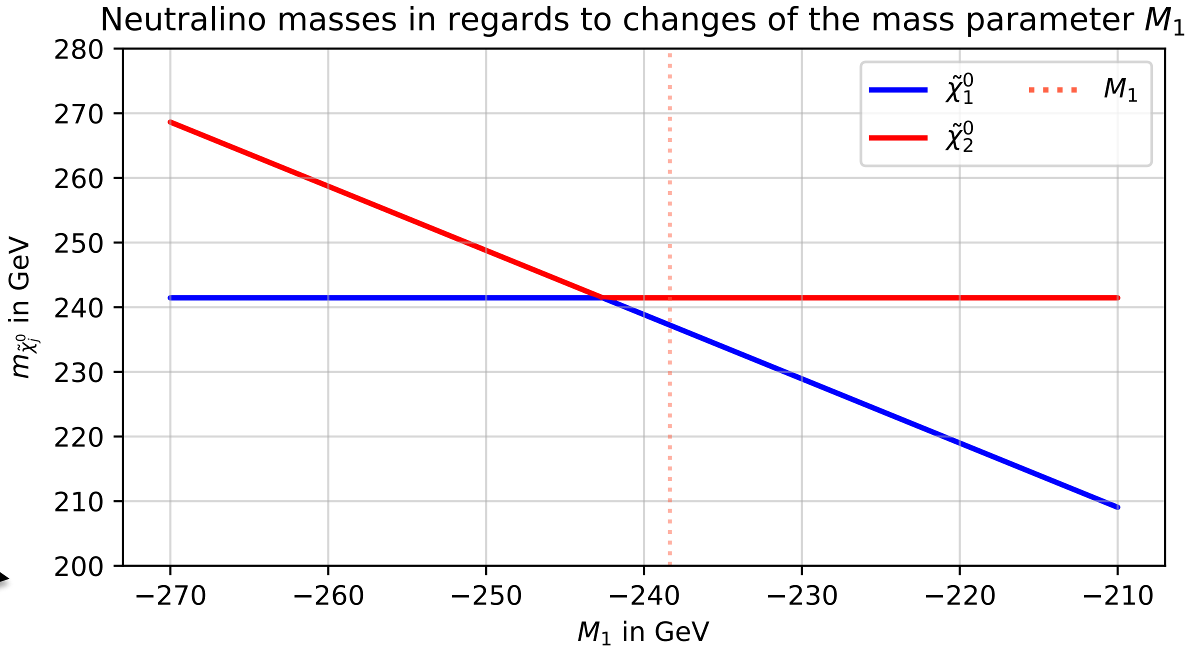


$$D_{rel} = \frac{M_{1,scan} - M_{1,true}}{M_{1,true}}$$

Neutralinos & M_1

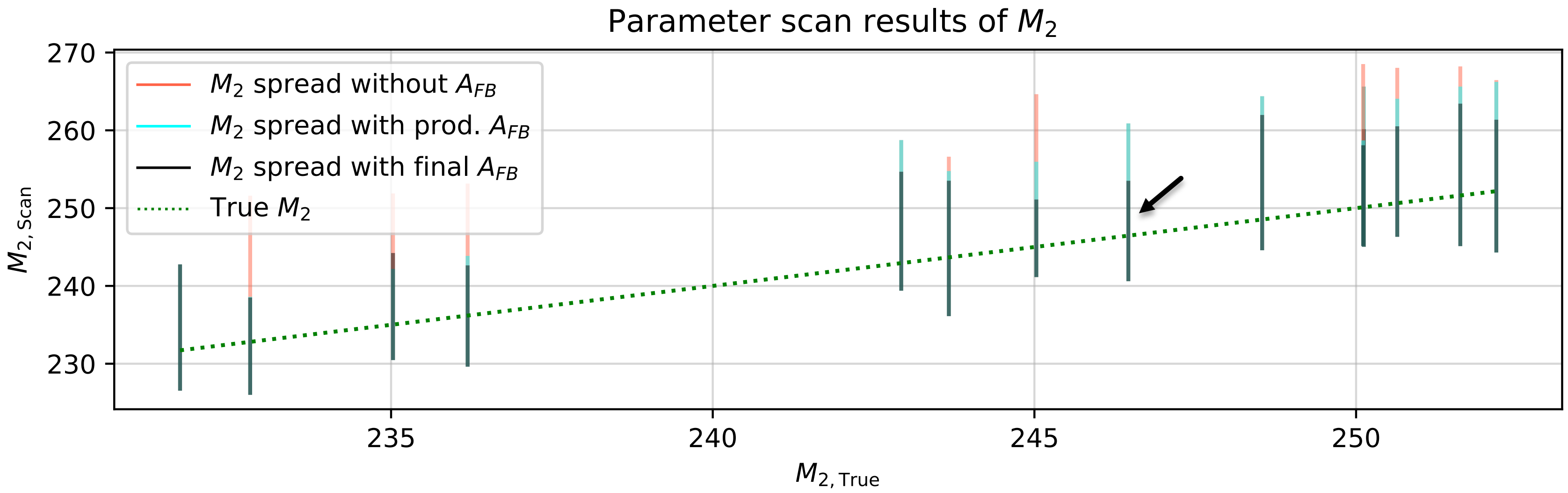


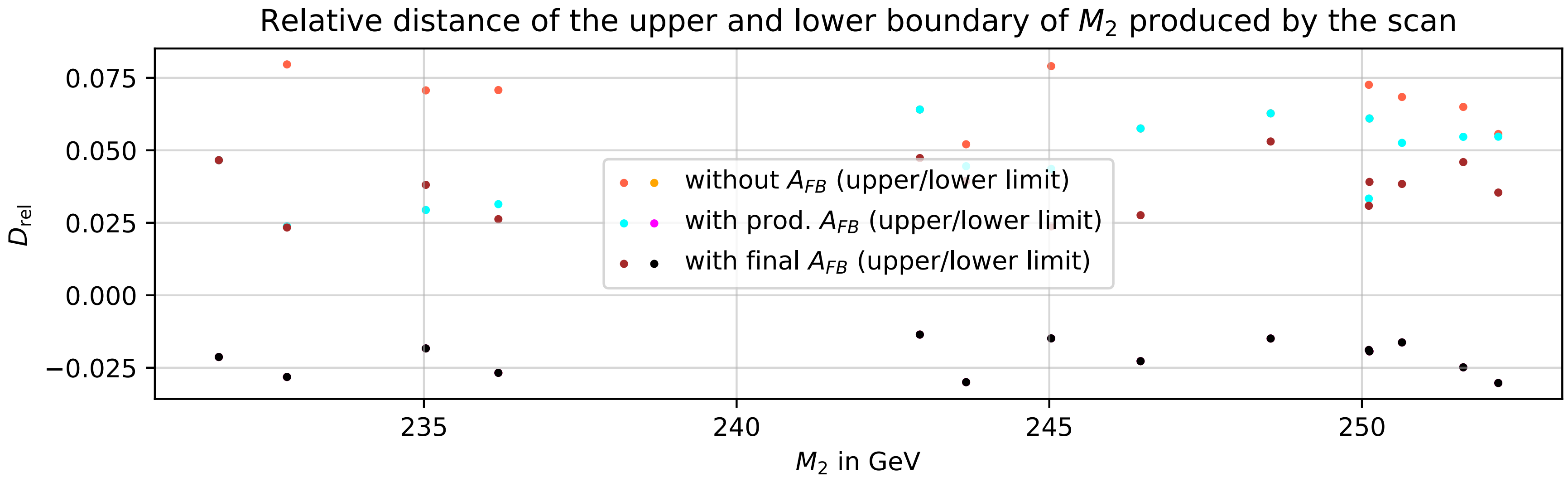
Badly reconstructed point (red arrow): M_1 where neutralino masses similar to each other → larger error



Example point (black arrow): M_1 where neutralino masses can be distinguished very well → smaller error

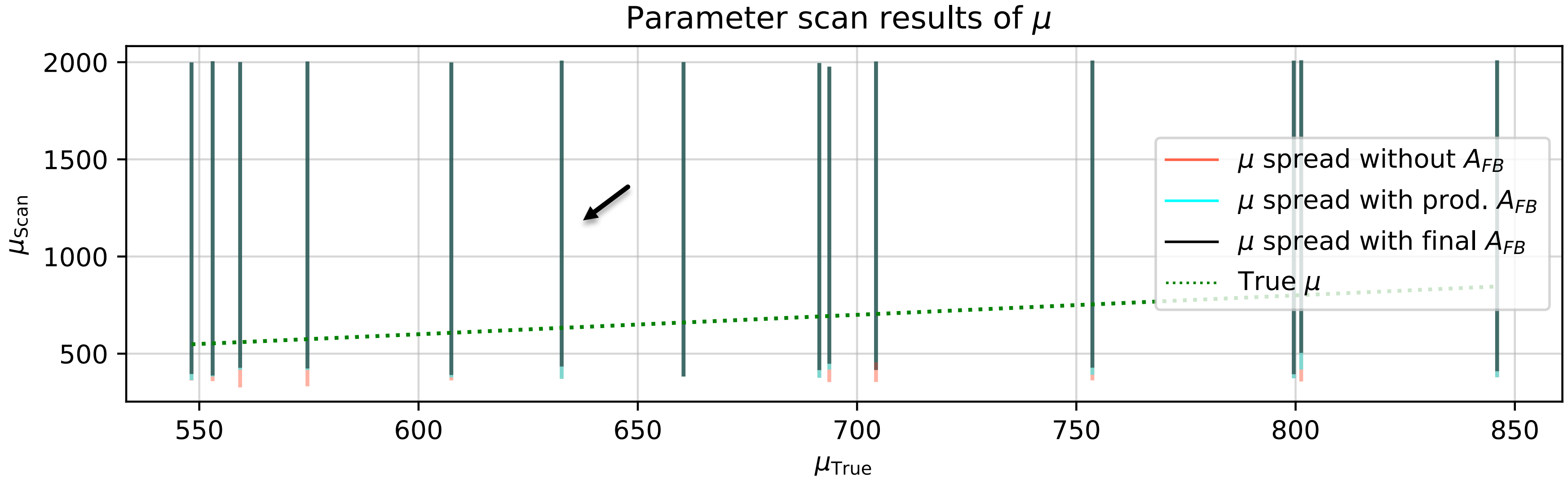
Results for M_2

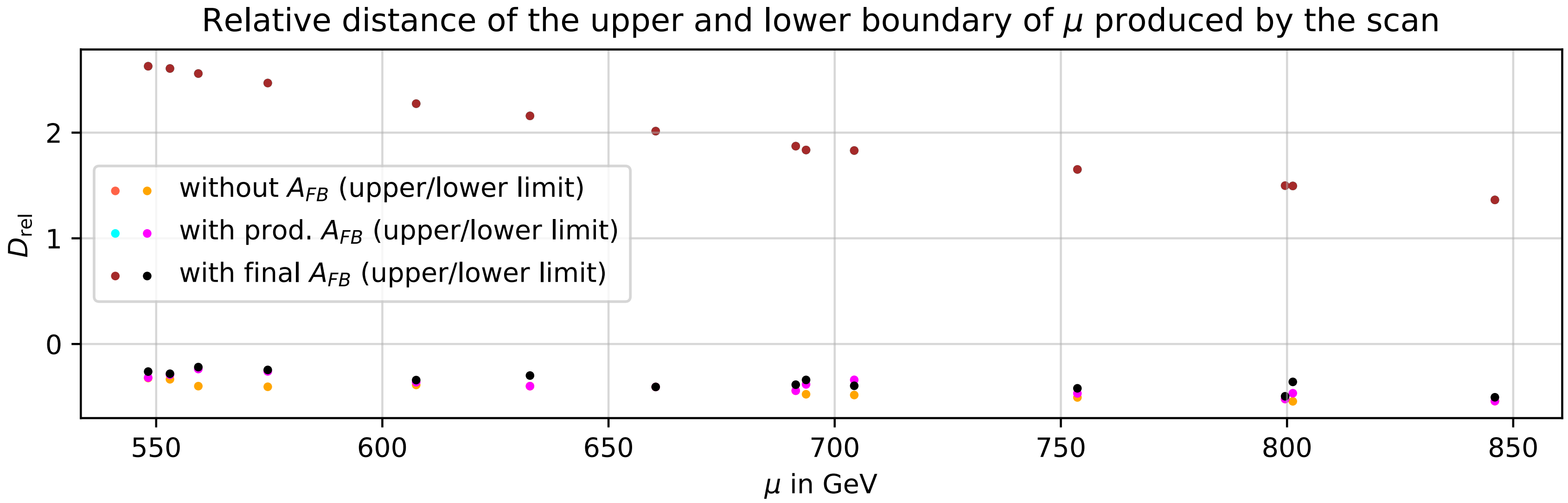




$$D_{rel} = \frac{M_{2,scan} - M_{2,true}}{M_{2,true}}$$

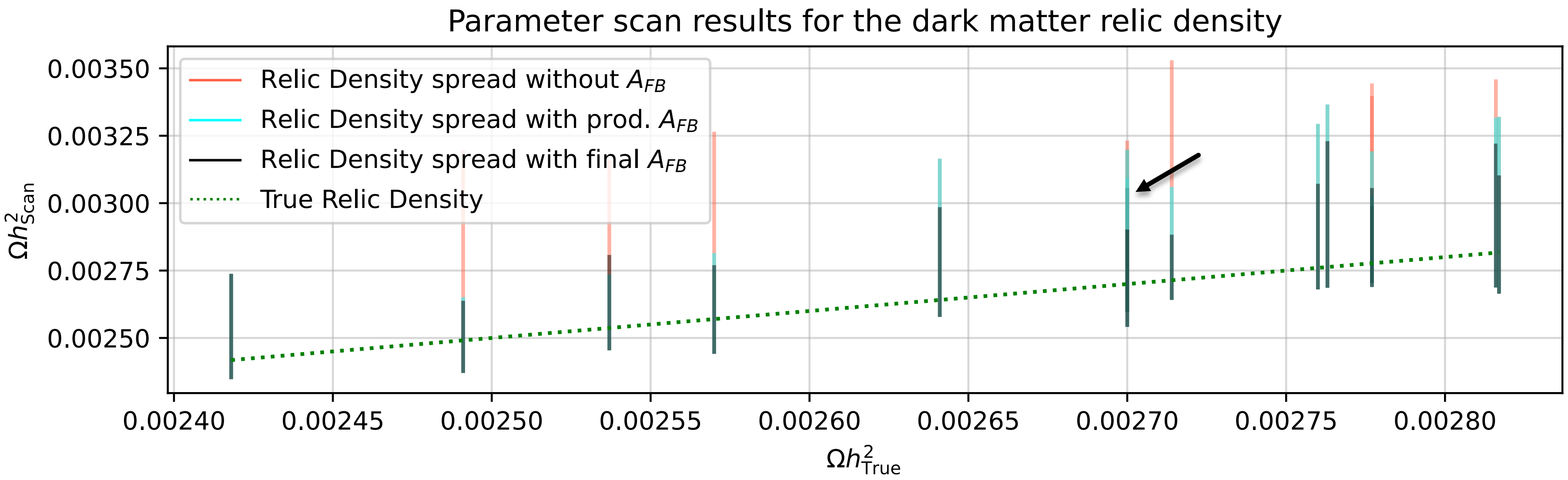
Results for μ

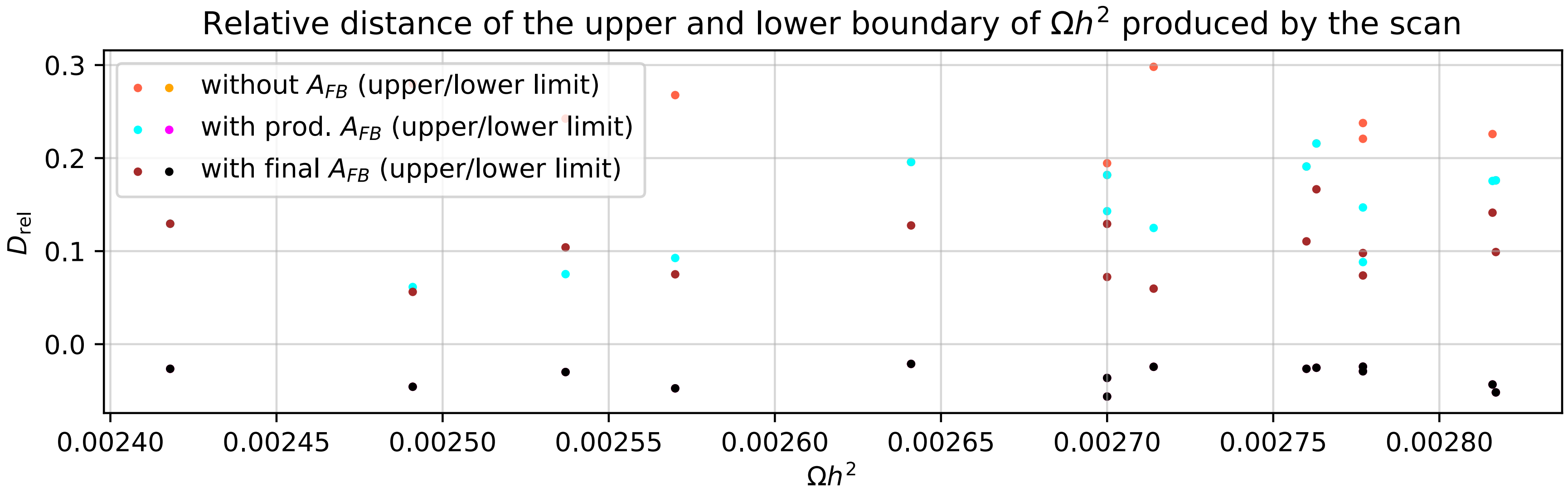




$$D_{rel} = \frac{\mu_{scan} - \mu_{true}}{\mu_{true}}$$

Results for DM relic density





$$D_{rel} = \frac{\Omega h^2{}_{scan} - \Omega h^2{}_{true}}{\Omega h^2{}_{true}}$$

Conclusion & Outlook

- Goal: Reconstruct DM with e^+e^- measurements
- Relevant parameters reconstructed with accuracy of ~ 5 %
- DM reconstructed with accuracy of ~ 15 %
- Including an additional measurement leads to improved accuracy
- Next steps:
  - Study other datasets or other scenarios
  - Improve $\sqrt{S}$ to get a higher crosssection and thus an even higher accuracy
  - For reliable interpretation of analysis, experimental uncertainty on A_{FB} is needed

Thank you for your attention! ☺

Special thank you to Gudi, Sven, Florian, Robin, Laurenz and Nele

Questions?

Sources

- [1] Desch et al.: Combined LHC/ILC analysis of a SUSY scenario with heavy sfermions, 2018, arXiv:hep-ph/0607104v2
- [2] Chakraborti et al.: Consistent Excesses in the Search for $\tilde{\chi}_2^0\tilde{\chi}_1^\pm$: Wino/bino vs. Higgsino Dark Matter, 2024, arXiv:2403.14759v1
- [3] Master thesis of Florian Lika: SUSY Parameter determination within Dark Matter Phenomenology at future e+e− Colliders, 2023
- [4] On the physics potential of ILC and CLIC, A.F.Zarnecki, 2020
- [5] Desch et al.: SUSY Parameter Determination in Combined Analyses at LHC/LC, 2003, arXiv:hep-ph/0312069v1
- [6] Beyond the standard model seraches at the LHC, 2016, https://indico.cern.ch/event/555909/contributions/2265363/attachments/1325399/1989408/deroeck_Islamabad2016_2_v1.pdf
- [7] Ushakov et al.: Simulations of the ILC positron source with 120 GeV electron drive beam, 2013
- [8] Abramowicz et al. The Linear Collider Facility (LCF) at CERN, 2025, arXiv:2503.24049

Backup

- Coefficients from crosssection formula:

$$c_1 = \int_C |Z|^2\{c_{LR}L^2 f_2 + c_{RL}R^2 f_1\}$$

$$\begin{aligned} c_2 = & \int_C |Z|^2\{c_{LR}L^2(1-4L)(2f_2+f_3) + c_{RL}R^2(1-4R)(2f_1+f_3)\} \\ & - \int_C G\tilde{N}4\{c_{LR}L(2f_2+f_3) + c_{RL}R(2f_1+f_3)\} - \int_C Re(Z)\tilde{N}c_{LR}Lf_3 \end{aligned}$$

$$c_3 = \int_C |Z|^2(c_{LR}L^2 f_1 + c_{RL}R^2 f_2) - \int_C Z\tilde{N}2c_{LR}Lf_1 + \int_C \tilde{N}^2 c_{LR}f_1$$

$$\begin{aligned} c_4 = & \int_C |Z|^2(1-4L)\{c_{LR}L^2(2f_1+f_3) + c_{RL}R^2(2f_2+f_3)\} + \int_C \tilde{N}^2 2c_{LR}f_1 \\ & + \int_C Re(Z)\tilde{N}c_{LR}L\{-4f_1 - f_3 + 4L(2f_1+f_3)\} + \int_C G\tilde{N}4c_{LR}(2f_1+f_3) \\ & - \int_C GRe(Z)4\{c_{LR}L(2f_1+f_3) + c_{RL}R(2f_2+f_3)\} \end{aligned}$$

$$c_5 = \int_C |Z|^2(c_{LR}L^2 + c_{RL}R^2)f_3 - \int_C Re(Z)\tilde{N}c_{LR}Lf_3$$

$$\begin{aligned} c_6 = & \int_C |Z|^2\{c_{LR}L^2(1-8L) + c_{RL}R^2(1-8L) + 16L^2(c_{LR}L^2 + c_{RL}R^2)\}(f_1+f_2+f_3) \\ & - \int_C Re(Z)\tilde{N}c_{LR}L(1-4L)(2f_1+f_3) + \int_C G^2(c_{LR}+c_{RL})(f_1+f_2+f_3) \\ & - \int_C Re(Z)G8\{c_{RL}R + c_{LR}L(1-4L)\}(f_1+f_2+f_3) + \int_C \tilde{N}^2 c_{LR}f_1 \\ & + \int_C G\tilde{N}4c_{LR}(2f_1+f_3) \end{aligned}$$

[5]

Backup

$$m^2_{\tilde{\chi}^{\pm}_{1,2}} = \frac{1}{2}(M_2^2 + \mu^2 + 2m_W^2 \mp \Delta_C)$$

$$\Delta c = \left[(M_2^2 - \mu^2)^2 + 4m_W^4 \cos^2 2\beta + 4m_W^2(M_2^2 + \mu^2) + 8m_W^2 M_2 \mu \sin 2\beta\right]^{\frac{1}{2}}$$

Table 5.1: **Comparison of the average errors of the reconstruction of $m_{\tilde{\nu}}$, M_1, M_2, μ and Ωh^2 of the three different methods.** The average errors of the upper and lower bound of $m_{\tilde{\nu}}$, M_1, M_2, μ and Ωh^2 from the three different methods are shown. The relative improvement of the methods with the A_{FB} in comparison to the method without A_{FB} is shown as well. All values are given in percent.

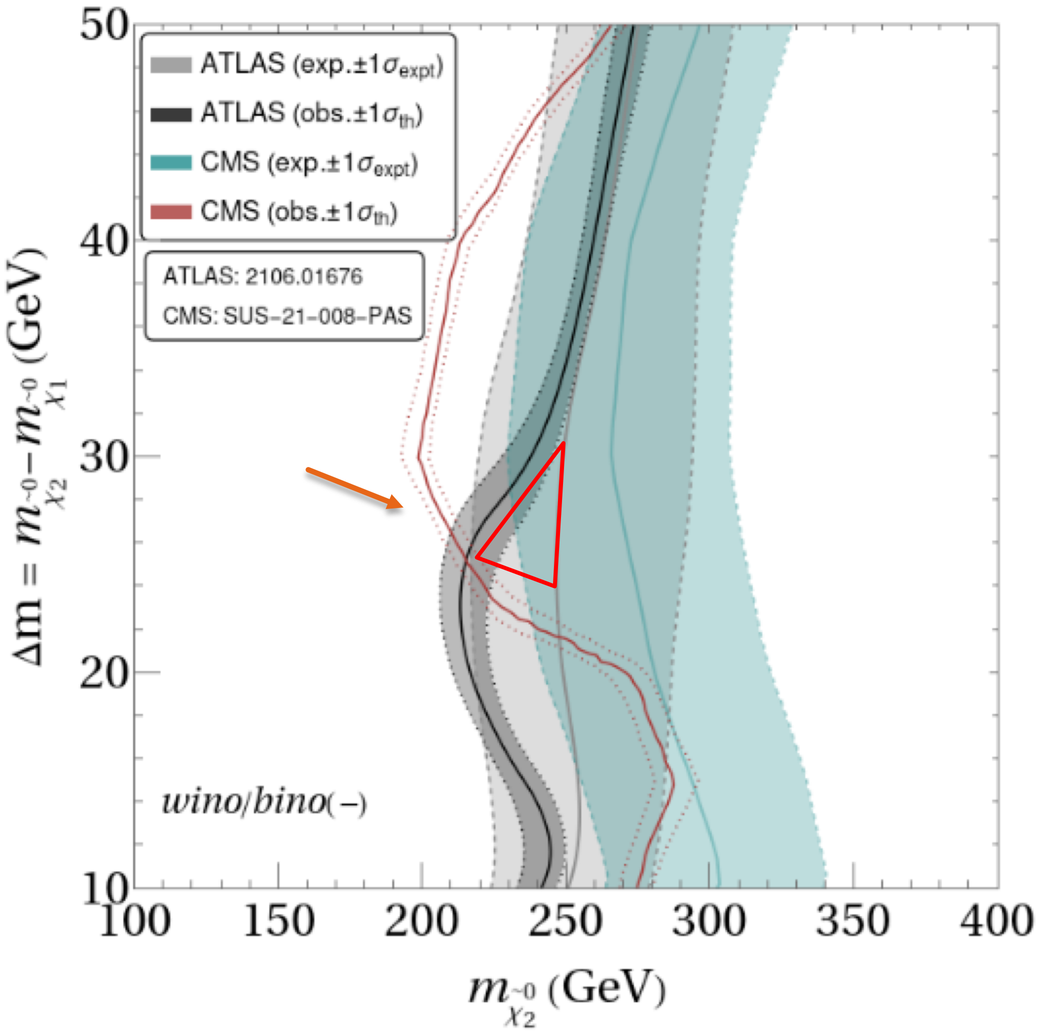
		without A_{FB}	with prod. A_{FB}	rel. improv.	with final A_{FB}	rel.improv.
$m_{\tilde{\nu}}$	upper	5.7	1.4	75.4	0.6	89.5
	lower	5.4	1.4	74.1	0.7	87.0
M_1	upper	0.9	0.9	0	0.9	0
	lower	3.1	2.4	22.6	2.0	35.5
M_2	upper	6.5	4.7	27.7	3.7	43.1
	lower	2.1	2.1	0	2.1	0
μ	upper	201.9	201.9	0	201.9	0
	lower	44.0	38.8	11.8	35.3	19.8
Ωh^2	upper	21.8	14.3	34.4	10.3	52.8
	lower	3.5	3.5	0	3.5	0

Motivation of analysis scenario

- Bino/Wino(–): Bino-wino DM with $\mu \times M_1 < 0$

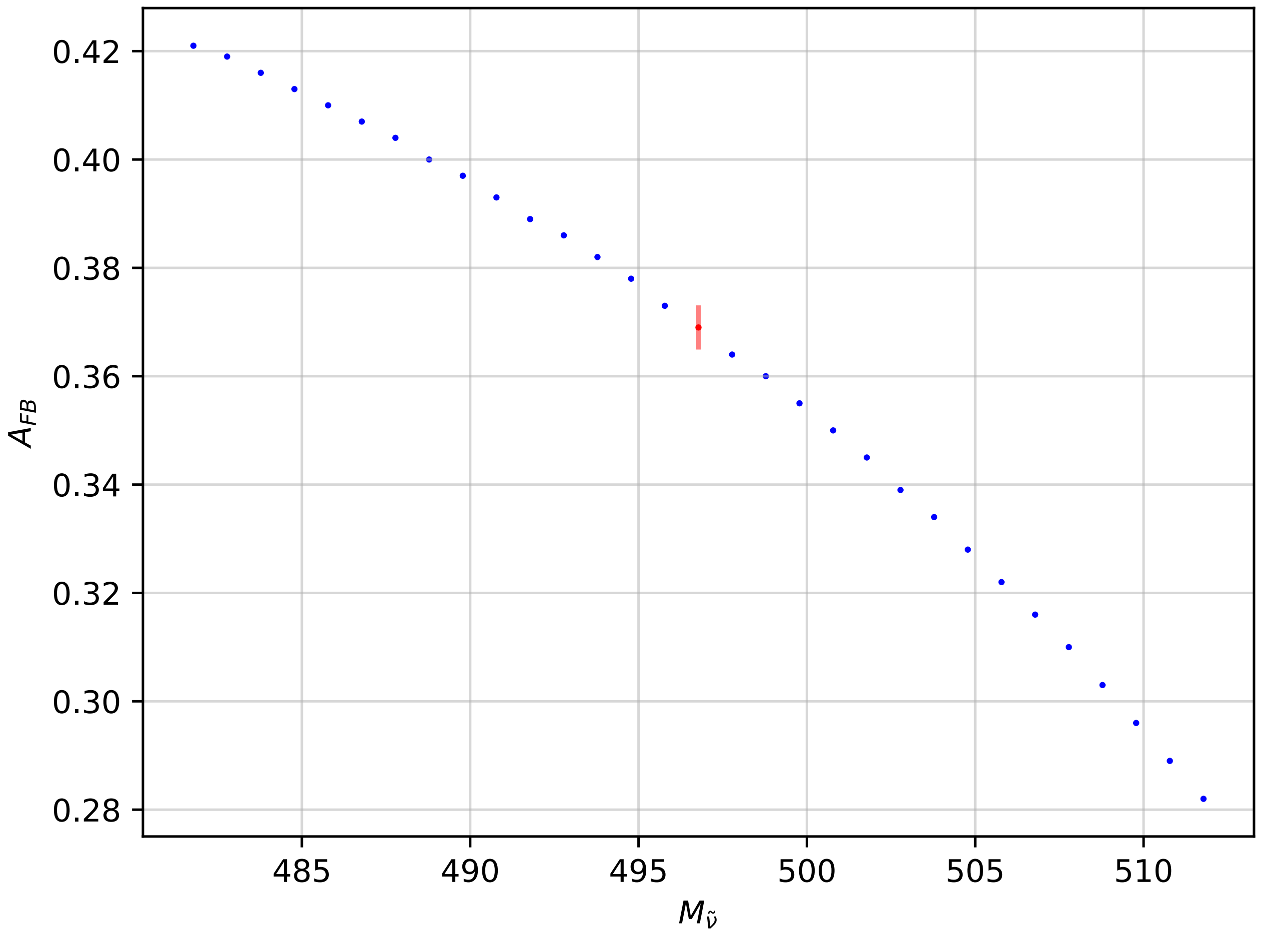
$$100\ \mathrm{GeV} \le -M_1 \le 400\ \mathrm{GeV}\,, \quad |M_1| \le M_2 \le 1.4\,|M_1|$$
$$1.2\,|M_1| \le \mu \le 2\ \mathrm{TeV}\,, \quad 2 \le \tan\beta \le 60$$
$$100\ \mathrm{GeV} \le m_{\tilde{l}_L} = m_{\tilde{l}_R} \le 1.5\ \mathrm{TeV}\,, \quad M_A = 3\ \mathrm{TeV}$$

- Some excess of events → Search for points in this excess

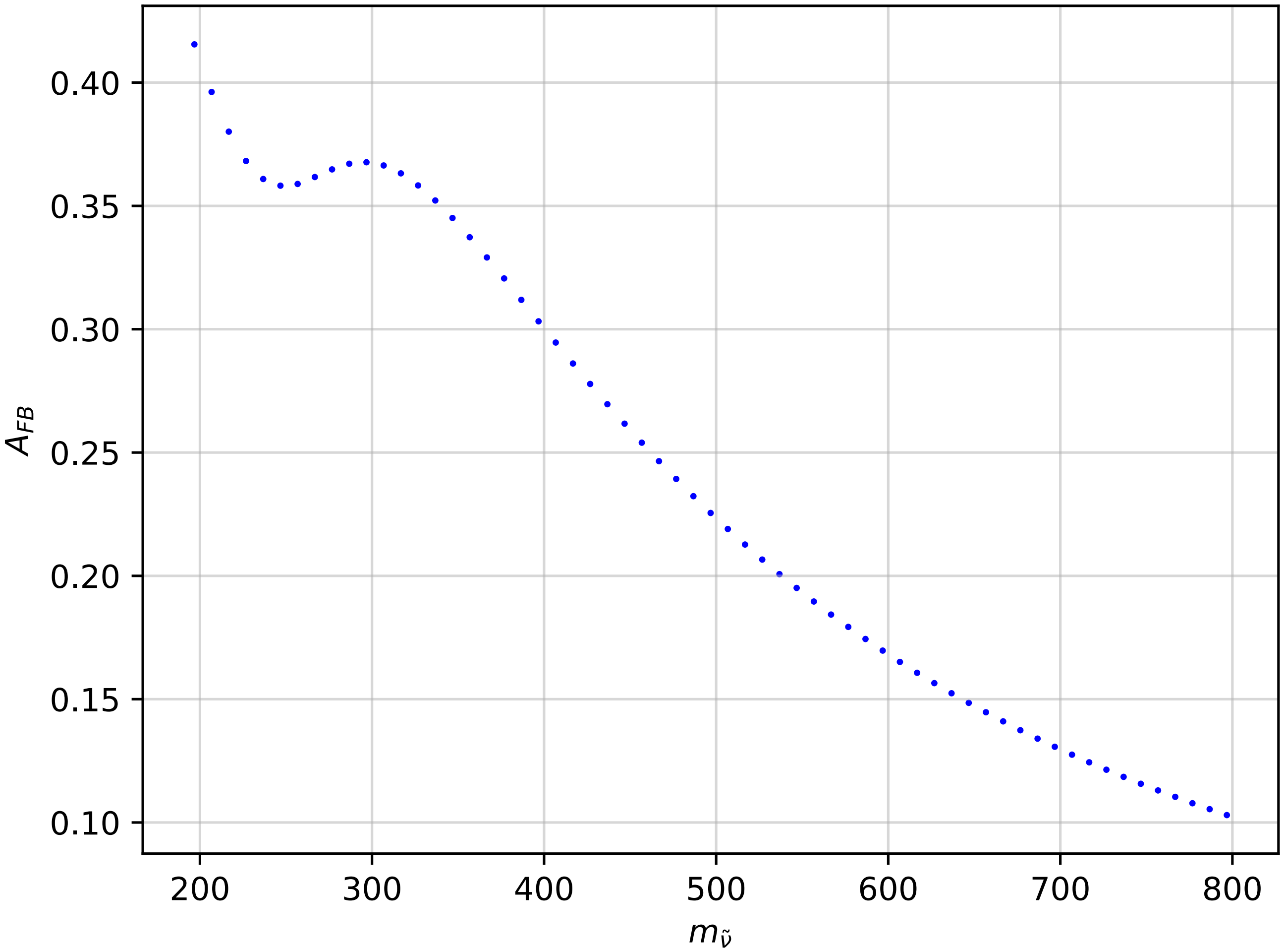


[2] Chakraborti et al.: arXiv:2403.14759v1

Final Asymmetry, 550, -.8,+.6

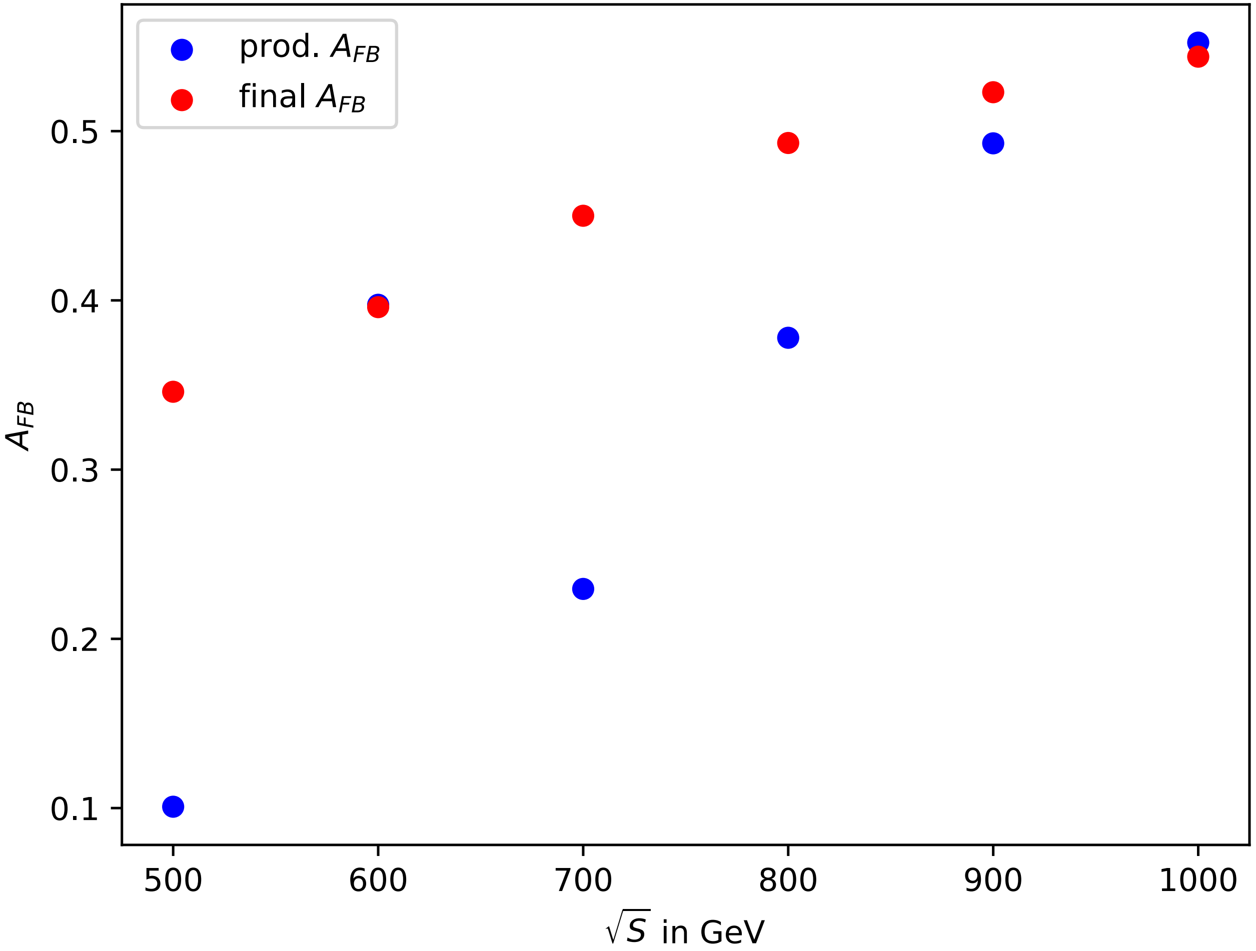


Asymmetry overview

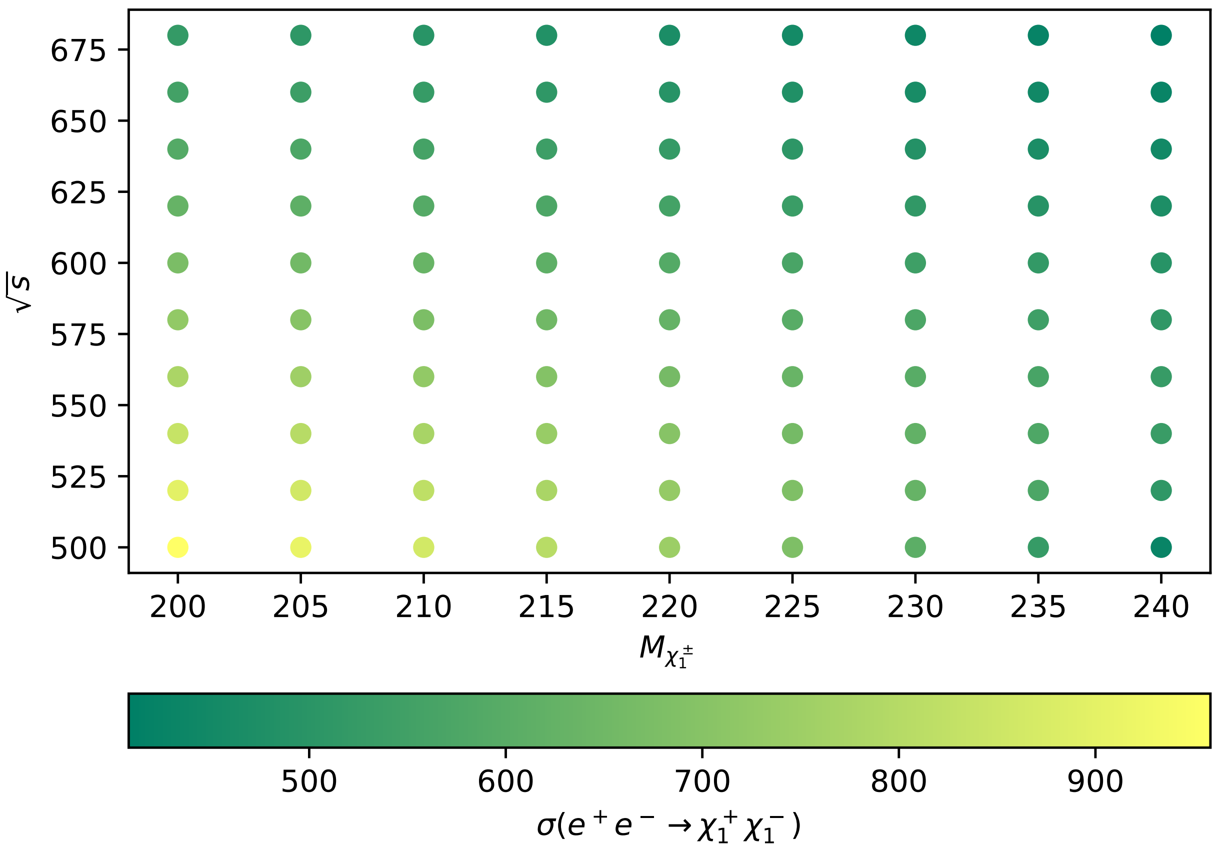


Backup

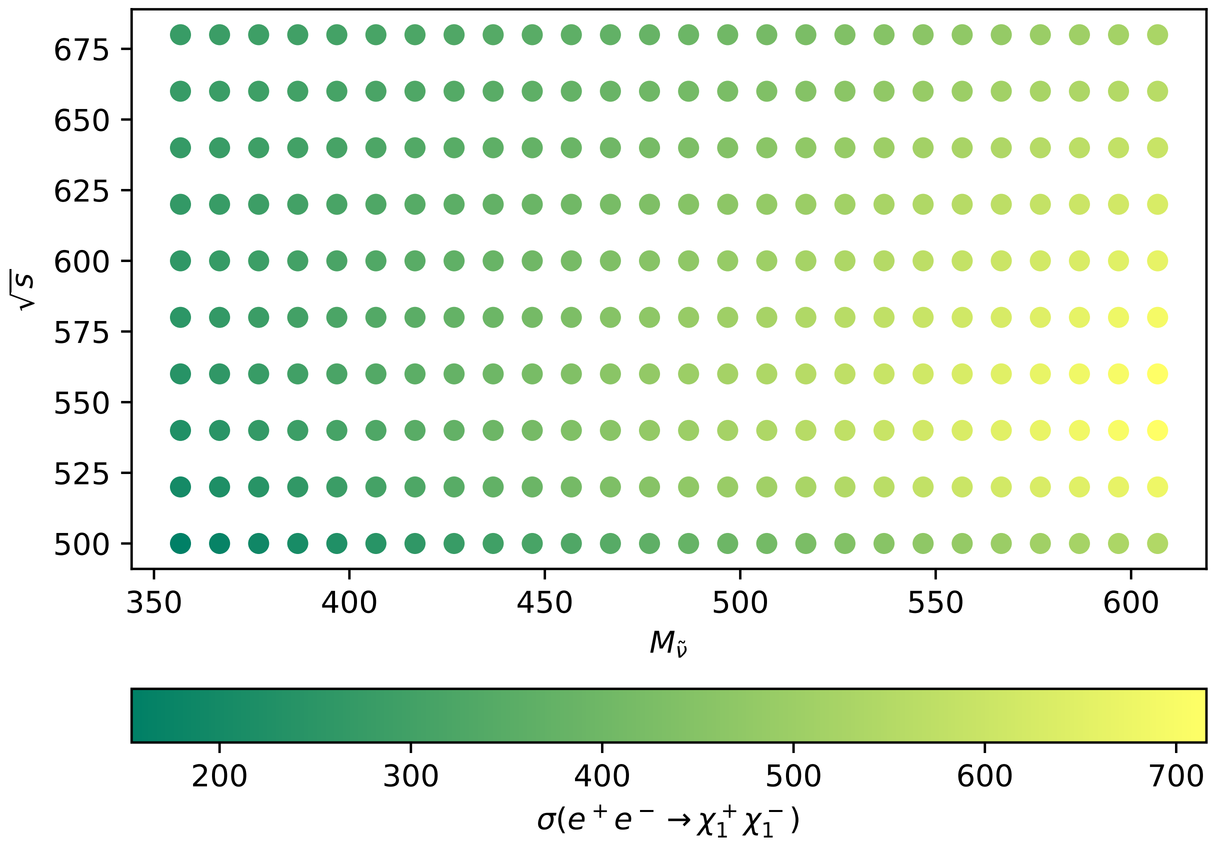
- The prod. A_{FB} converges to the final A_{FB} with higher $\sqrt{S}$



Backup



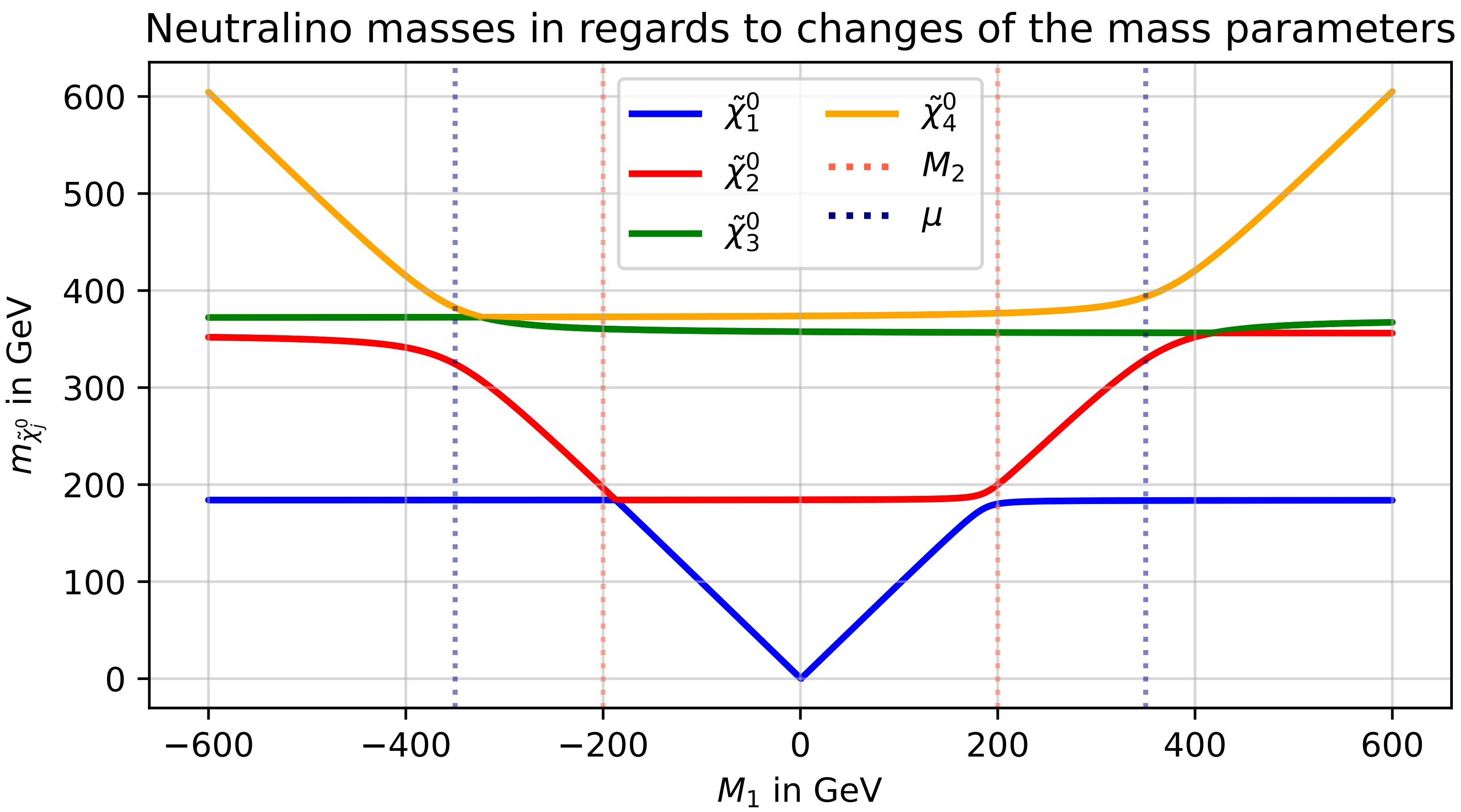
675
650
625
600
575
550
525
500
$\sqrt{s}$
200
205
210
215
220
225
230
235
240
$M_{\chi_1^\pm}$
500
600
700
800
900
$\sigma(e^+e^- \to \chi_1^+\chi_1^-)$



675
650
625
600
575
550
525
500
$\sqrt{s}$
350
400
450
500
550
600
$M_{\tilde{\nu}}$
200
300
400
500
600
700
$\sigma(e^+e^- \to \chi_1^+\chi_1^-)$

Mass hierarchy

- Particles are assigned by mass hierarchy
- Hierarchy determines sensitivity
- $\tilde{\chi}_1^0$ is a good candidate for DM



Steps of the analysis

- Chargino production crosssection $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ and chargino and neutralino masses $(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^0})$ are measured for the allowed parameter points
- Left and right chargino mixing angles $\phi_{L,R}$ are determined
- The parameters (M_1, M_2, μ) are reconstructed with uncertainties via a scan over the parameter space
  - Uncertainty in $M_{\tilde{\nu}}$ is determined via A_{FB}
- DM relic density (Ωh^2) is calculated and compared to the relic density gained from the true values

Muster

- blabla