Impact of the different discrete symmetries in the 2HDM and N2HDM on domain wall formation and its phenomenological implications

Mastercolloqium

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Outline

1 Cosmic Domain walls

- Domain wall problem
- Solution and constraints
- Circumventing the DW problem for multiple discrete symmetries

2 Two Higgs doublet models

- 2HDM
- N2HDM
- Discrete symmetries

3 Solving the Domain wall EoMs

- Different DW configurations for different symmetries

4 Phenomenological impact of the different discrete symmetries

Domain wall problem

Domain walls are thought to be created when a discrete symmetry is spontaneously broken
⇒ After spontaneous symmetry breaking (SSB) there are regions of space, which live in different vacua
→ domains
⇒ Transition between these regions, where fields are trapped in false vacuum
→ **<u>Domain Wall</u>** (DW)

<u>Problem:</u> Stable DWs are cosmologically unfavourable

- Dynamics driven by tension force $p_T \sim \sigma$ → Enter „scaling“ regime $\rho_{DW} \sim \frac{\sigma}{t}$ σ: Surface energy density
  ⇒ $\rho_{DW} \sim t^{-1}$ while $\rho_m, \rho_r \sim t^{-2}$
  If existing, DWs would dominate the energy density of the universe (unless σ is very small)

- DWs imprint large scale density fluctuations which are constrained by CMB

  ⇒ Zel'dovich-Kobzarev-Okun (ZKO)-bound: $\sigma \lesssim \mathcal{O}(MeV^3)$

Solutions to the Domain wall problem

General idea to overcome the DW problem is to destabilize DWs

Two approaches:

a) Lift the energy degeneracy of the vacua → approximate discrete symmetry

b) Change initial probability to populate vacua

<u>Focus on option a):</u>

- Energy difference induces a vacuum pressure $p_V \sim V_{Bias}$, regions of true vacuum grow.
  → DWs become unstable if $p_V \sim p_T$

  Demanding DWs collapse before domination gives: $V_{bias} \gtrsim \frac{\sigma^2}{{M_{pl}}^2}$

- Relative population of the two vacua given by Boltzmann-equation: $\frac{p_-}{p_+} = e^{-\frac{\Delta F}{T}}$

  Kibble mechanism: After SSB thermal fluctuations between the vacua still possible, below the Ginzburg-temperature $T_G \approx V_0 \times V_\xi$ domains freeze out; Free energy $\Delta F = V_{Bias} \times V_\xi$

  Percolation theory gives a critical probability $p_c = 0.311$ up to which DW networks form:

$$\frac{V_{Bias}}{V_0} < \ln\left(\frac{1-p_c}{p_c}\right) = 0.795$$

V_0 : Potential barrier
V_{bias} : Energydiff. between vacua
V_ξ: Correlation volume
p_-: prob. of false vacuum
p_+: prob. of true vacuum

Circumventing the Domain wall problem for multiple discrete symmetries

2HDM and N2HDM scalar potential can exhibit several discrete symmetries
⇒ Can one overcome the DW problem without imposing an extra bias on the initial conditions ?

Consider a model with an approximate $\mathbb{Z}_2$ and an exact $\mathbb{Z}_2$ symmetry:

- If $\frac{V_{Bias}}{V_0} < 0.795$ is **not** fulfilled for the approximate symmetry
  → DW networks form between degenerate vacua corresponding to exact symmetry! → Stable DWs

- 3 potential barriers $V_{0,i}$ separating vacua
  ⇒ 3 different Ginzburg temperatures $T_{G,i} \approx V_{0,i} \times V_\xi$ until which thermal fluctuations between vacua can occur
  ⇒ suppression of stable DWs only possible if $T_{G,approx}$ (or $V_{0,approx}$) is the lowest :

$$\frac{T_{G,approx}}{T_{G,exact}} \leq 1$$

- Estimate relative probability of two DW states:

$$\frac{p_{exact}}{p_{approx}} = e^{-\frac{\Delta F}{T}} = e^{-\frac{\Delta\sigma \times A_\xi}{T_G}} \quad \text{with} \quad \Delta\sigma = \sigma_{exact} - \sigma_{approx}$$

2HDM

- Impose softly broken discrete $\mathbb{Z}_2$ symmetry to avoid tree-level FCNCs:

  $\Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2$

  $\to m_{12}^2$ soft breaking parameter
  $\to$ natural bias term!

$$V = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + (\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 - m_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.)$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1+\phi_1+i\eta_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2+\phi_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$$

- CP-conserving, all parameters and VEVs are real

$$tan(\beta) = \frac{v_2}{v_1} \qquad v_{SM} = \sqrt{v_1^2 + v_2^2} = 246\, GeV.$$

- Mixing angles α (CP-even) and β (CP-odd, charged)

$$[M_{ij}^2]^a = \frac{\partial^2 V}{\partial\phi_i^a \partial\phi_j^a} \quad with \quad a = 0, +; \quad and \quad i, j = 1, 2 \qquad R(\theta) = \begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix}$$

2HDM

- Use minimization conditions $\frac{\partial V}{\partial \phi_i}|_{\phi_i = v_i} = 0$ to eliminate m_{11}^2 and m_{22}^2

$$V = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + (\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 - m_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.)$$

- Express all quartic parameters λ_i in mass basis

$$\lambda_1 = \frac{-m_{12}^2 tan\beta + M_h^2 c_\alpha^2 + M_H^2 s_\alpha^2}{c_\beta^2 v_{SM}^2}$$
$$\lambda_2 = \frac{-m_{12}^2 cot\beta + M_h^2 s_\alpha^2 + M_H^2 c_\alpha^2}{s_\beta^2 v_{SM}^2}$$
$$\lambda_3 = \frac{-m_{12}^2 + 2M_{H\pm}^2 c_\beta s_\beta + (M_h^2 - M_H^2)c_\alpha s_\alpha}{c_\beta s_\beta v_{SM}^2}$$
$$\lambda_4 = \frac{m_{12}^2 + (M_A^2 - 2M_{H\pm}^2)c_\beta s_\beta}{c_\beta s_\beta v_{SM}^2}$$
$$\lambda_5 = \frac{m_{12}^2 - M_A^2 c_\beta s_\beta}{c_\beta s_\beta v_{SM}^2}$$

$\rightarrow$ 8 free real parameters:

$$M_h, \quad M_H, \quad M_A, \quad M_{H\pm}, \quad tan(\beta), \quad \alpha, \quad v_{SM} \quad \text{and} \quad m_{12}^2$$

N2HDM

- 2HDM + real scalar singlet Φ_S
- Mixing with CP-even doublet states
  $\Rightarrow$ 3 CP-even scalars H_1, H_2, H_3
  $\Rightarrow$ 3 mixing angles $\alpha_1, \alpha_2, \alpha_3$

$$V = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + (\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 - m_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.)$$
$$+ \frac{m_S^2}{2}\Phi_S^2 + \frac{\lambda_6}{8}\Phi_S^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger\Phi_1)\Phi_S^2 + \frac{\lambda_8}{2}(\Phi_2^\dagger\Phi_2)\Phi_S^2,$$

$$\Phi_S = \frac{v_S + \phi_S}{\sqrt{2}}$$

$$R = \begin{pmatrix} c_{\alpha_1}c_{\alpha_2} & s_{\alpha_1}c_{\alpha_2} & s_{\alpha_2} \\ c_{\alpha_1}s_{\alpha_2}s_{\alpha_3} + s_{\alpha_1}c_{\alpha_3} & c_{\alpha_1}c_{\alpha_3} - s_{\alpha_1}s_{\alpha_2}s_{\alpha_3} & c_{\alpha_2}s_{\alpha_3} \\ c_{\alpha_1}s_{\alpha_2}c_{\alpha_3} + s_{\alpha_1}s_{\alpha_3} & -(c_{\alpha_1}s_{\alpha_3} - s_{\alpha_1}s_{\alpha_2}c_{\alpha_3}) & c_{\alpha_2}c_{\alpha_3} \end{pmatrix}$$

N2HDM

$$V = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + (\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 - m_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.)$$
$$+ \frac{m_S^2}{2}\Phi_S^2 + \frac{\lambda_6}{8}\Phi_S^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger\Phi_1)\Phi_S^2 + \frac{\lambda_8}{2}(\Phi_2^\dagger\Phi_2)\Phi_S^2,$$

- Eliminating m_{11}^2, m_{22}^2 and m_S^2 via min. conditions
- Express all quartic parameters λ_i in mass basis

$$\lambda_1 = \frac{1}{v^2 c_\beta^2}\left(-\tilde{\mu}^2 s_\beta^2 + \sum_{i=1}^{3} m_{H_i}^2 R_{i1}^2\right),$$
$$\lambda_2 = \frac{1}{v^2 s_\beta^2}\left(-\tilde{\mu}^2 c_\beta^2 + \sum_{i=1}^{3} m_{H_i}^2 R_{i2}^2\right),$$
$$\lambda_3 = \frac{1}{v^2}\left(-\tilde{\mu}^2 + \frac{1}{s_\beta c_\beta}\sum_{i=1}^{3} m_{H_i}^2 R_{i1} R_{i2} + 2m_{H^\pm}^2\right),$$
$$\lambda_4 = \frac{1}{v^2}(\tilde{\mu}^2 + m_A^2 - 2m_{H^\pm}^2),$$
$$\lambda_5 = \frac{1}{v^2}(\tilde{\mu}^2 - m_A^2),$$
$$\lambda_6 = \frac{1}{v_S^2}\sum_{i=1}^{3} m_{H_i}^2 R_{i3}^2,$$
$$\lambda_7 = \frac{1}{v v_S c_\beta}\sum_{i=1}^{3} m_{H_i}^2 R_{i1} R_{i3},$$
$$\lambda_8 = \frac{1}{v v_S s_\beta}\sum_{i=1}^{3} m_{H_i}^2 R_{i2} R_{i3},$$

$$\tilde{\mu}^2 := \frac{m_{12}^2}{s_\beta c_\beta}.$$

$\rightarrow$ 12 free real parameters: $m_{H_{1,2,3}}$, m_A, $m_{H\pm}$, $\alpha_{1,2,3}$, $tan(\beta)$, v_{SM}, v_S and m_{12}^2

Discrete symmetries

Type	d_r	L	l_r	d-type	leptons
I	$-$	$+$	$-$	Φ_2	Φ_2
II	$+$	$+$	$+$	Φ_1	Φ_1
lepton-specific	$-$	$+$	$+$	Φ_2	Φ_1
flipped	$+$	$+$	$-$	Φ_1	Φ_2

- Softly broken $\mathbb{Z}_2$ symmetry imposed to avoid FCNCs at tree-level

  $\Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2.$

  Yukawa structure the same in 2HDM and N2HDM as singlet does not couple to fermions

- Exact $\widetilde{\mathbb{Z}}_2$ symmetry: $\Phi_1 \to -\Phi_1, \quad \Phi_2 \to -\Phi_2.$
  Yukawa Lagrangian invariant only for Type I 2HDM and N2HDM

For $\mathbb{Z}_2$ and $\widetilde{\mathbb{Z}}_2$: $\Phi_S \to \Phi_S$

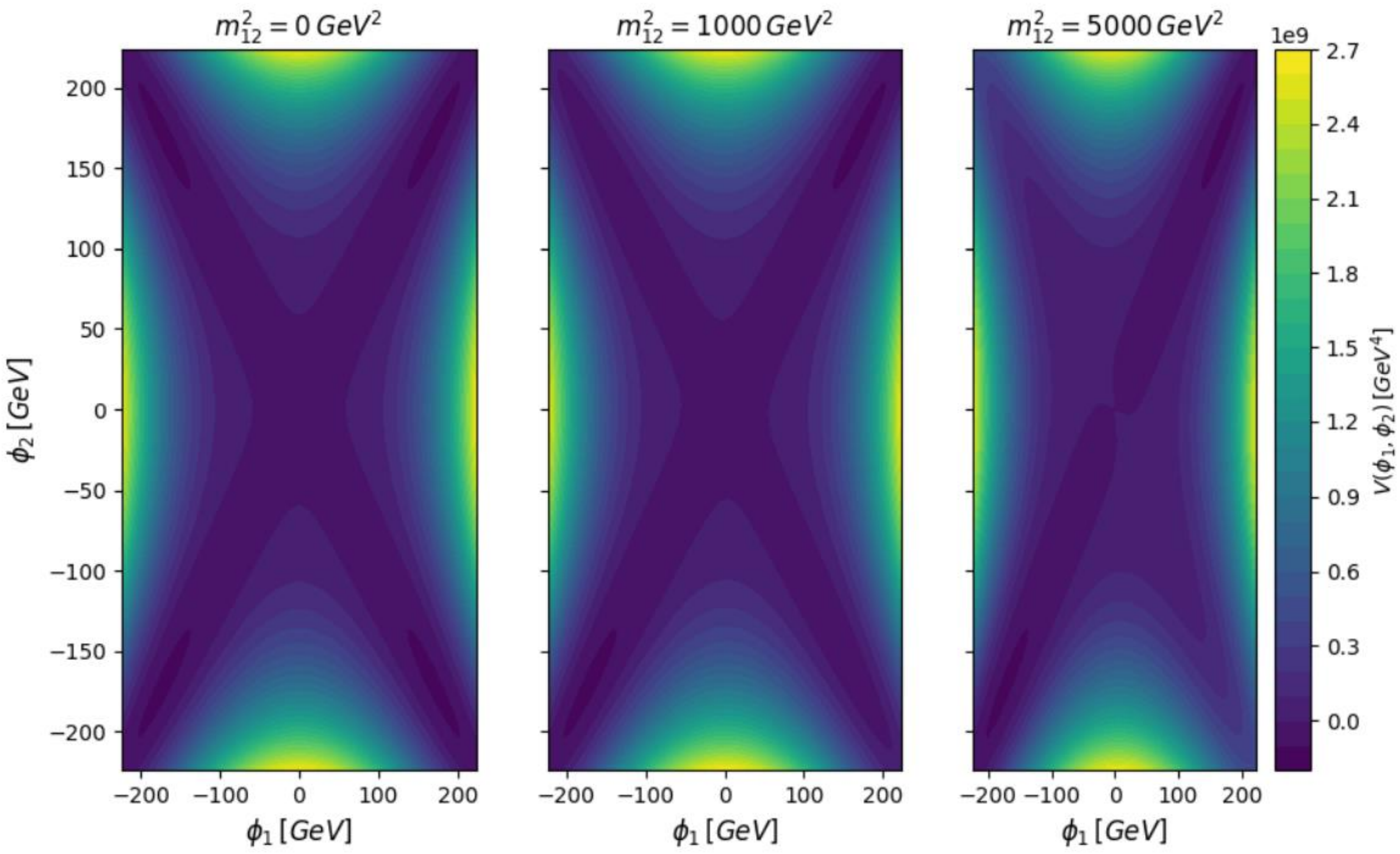
- Exact $\mathbb{Z}'_2$ symmetry: $\Phi_1 \to \Phi_1, \quad \Phi_2 \to \Phi_2, \quad \Phi_S \to -\Phi_S.$

  Present in N2HDM of all Yukawa types

  Note: In Dark matter models, where Φ_S acquires a vanishing VEV this symmetry is unbroken
  $\Rightarrow$ No DW formation

Discrete symmetries

Example: 2HDM, scalar potential for rising values of m_{12}^2



Solving the DW EoMs

2HDM, Type I

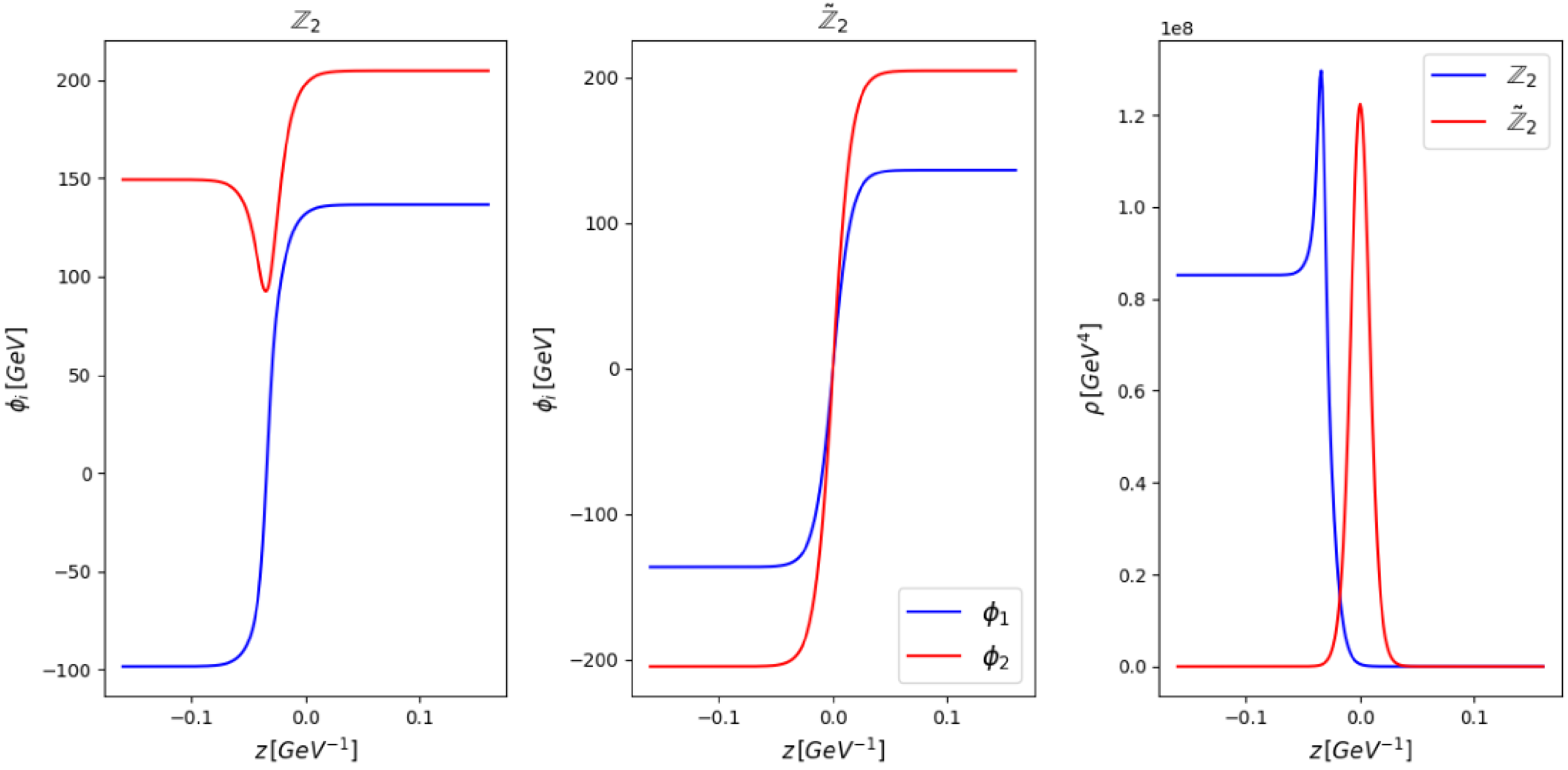
$\Phi_1 \to -\Phi_1, \quad \Phi_2 \to \Phi_2,$ $\qquad$ $\Phi_1 \to -\Phi_1, \quad \Phi_2 \to -\Phi_2.$

Assume planar DWs perpendicular to z-axis in Minkowski-space → EoM:

$$\frac{d^2\phi_i}{dz^2} = \frac{dV}{d\phi_i}$$

With boundary values

$$\phi_i \to v_i \quad for \quad z \to \infty,$$
$$\phi_i \to \chi_i(v_i) \quad for \quad z \to -\infty.$$



$$\rho(z) = \sum_i \left|\frac{d\phi_i}{dz}\right|^2 + V(\phi_i) - V(v_i) \quad \to \quad \sigma = \int \rho(z)\,\mathrm{d}z$$

$$m_H = 715\,GeV,\, m_{H\pm} = 350\,GeV,\, m_A = 350\,GeV,$$
$$m_{12}^2 = 2000\,GeV^2,\, tan(\beta) = 1.5,\, cos(\beta-\alpha) = 1$$

Use Gradient flow method to solve EoMs

Solving the DW EoMs

N2HDM

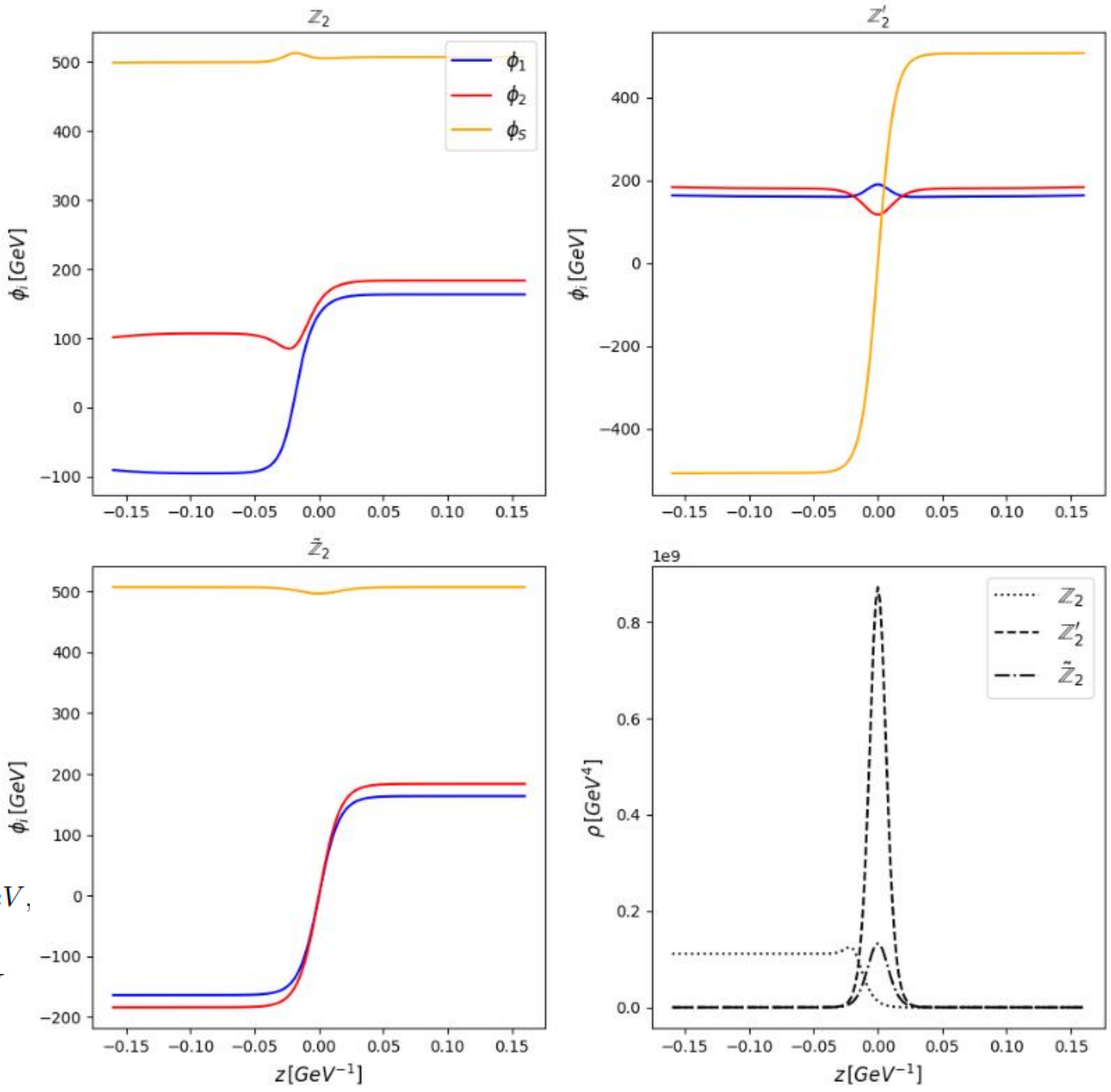
In the N2HDM there are 3 discrete symmetries present: $\mathbb{Z}_2$, $\widetilde{\mathbb{Z}}_2$ and $\mathbb{Z}'_2$

Boundary values are given by:

Symmetry	$\phi_1(z \to \pm\infty)$	$\phi_2(z \to \pm\infty)$	$\phi_S(z \to \pm\infty)$
$\mathbb{Z}_2$	$v_1/\pm v_1$	$\pm v_2/v_2$	v_S
$\tilde{\mathbb{Z}}_2$	$\pm v_1$	$\pm v_2$	v_S
$\mathbb{Z}'_2$	v_1	v_2	$\pm v_S$

$$tan(\beta) = 1.12,\ \alpha_1 = 0.91,\ \alpha_2 = 0.31,\ \alpha_3 = 1.29,\ m_{H_2} = 157\, GeV,$$

$$m_{H_3} = 495\, GeV,\ m_{H^\pm} = m_A = 415\, GeV,\ m_{12}^2 = 2839\, GeV^2 \text{ and } v_S = 507\, GeV$$



Phenomenological impact of the different symmetries

2HDM, Type I

Apply different theoretical constraints:

Grey: Bounded from below + global minimum

Yellow: Perturbative unitarity

Cyan: Percolation constraint

$$\frac{V_{Bias}}{V_0} < 0.795$$

Blue: All constraints fulfilled

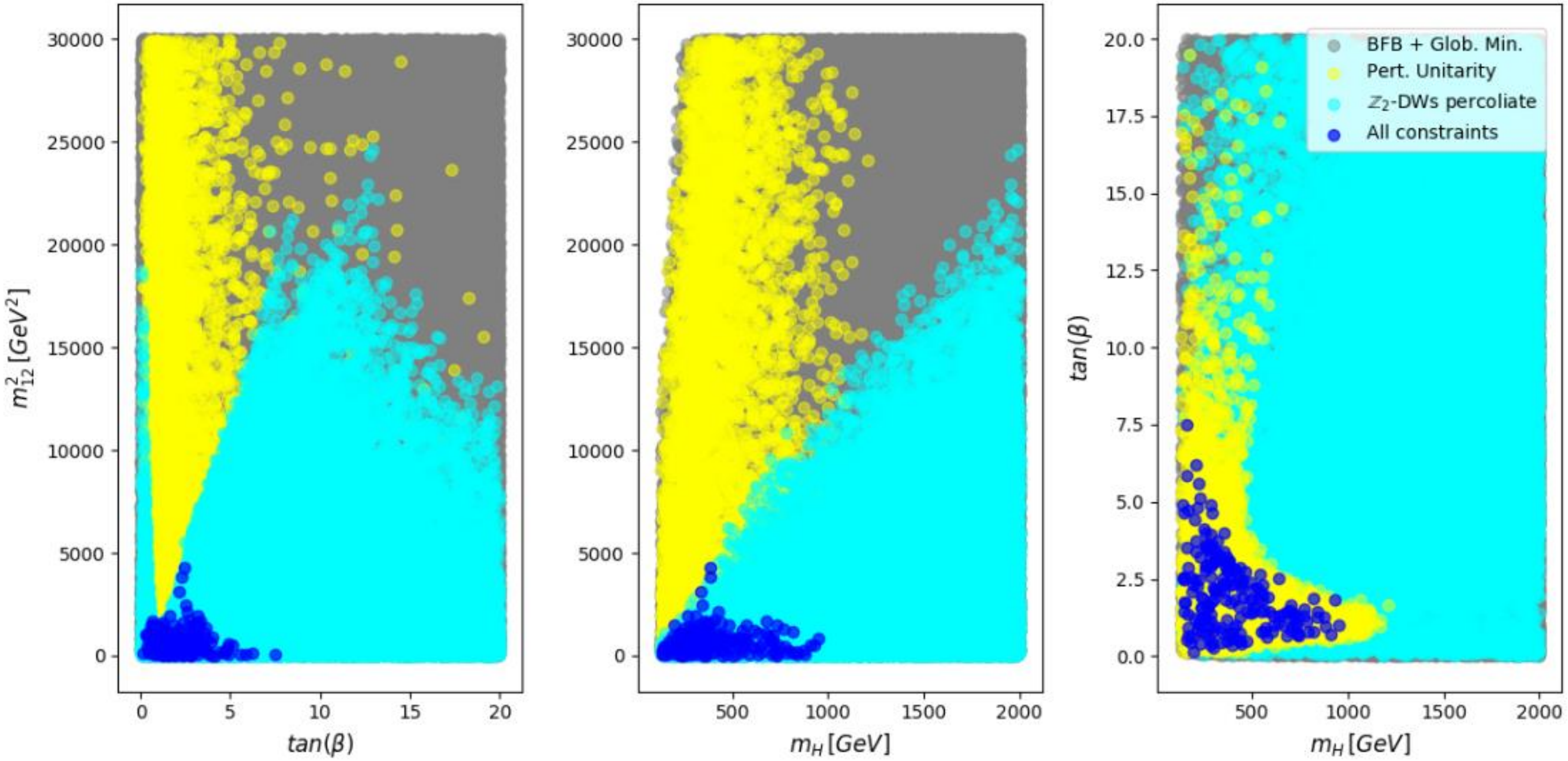
$$0 \leq tan(\beta) \leq 20,$$

$$\alpha = \beta \pm 0.5,$$

$$m_h \leq m_H \leq 2000\, GeV,$$

$$80\, GeV \leq m_{H^\pm}, m_A \leq 2000\, GeV,$$

$$10^{-3} \leq m_{12}^2 \leq 30000\, GeV^2.$$



Phenomenological impact of the different symmetries

2HDM, Type I

Next step: What is the influence of suppression constraints?

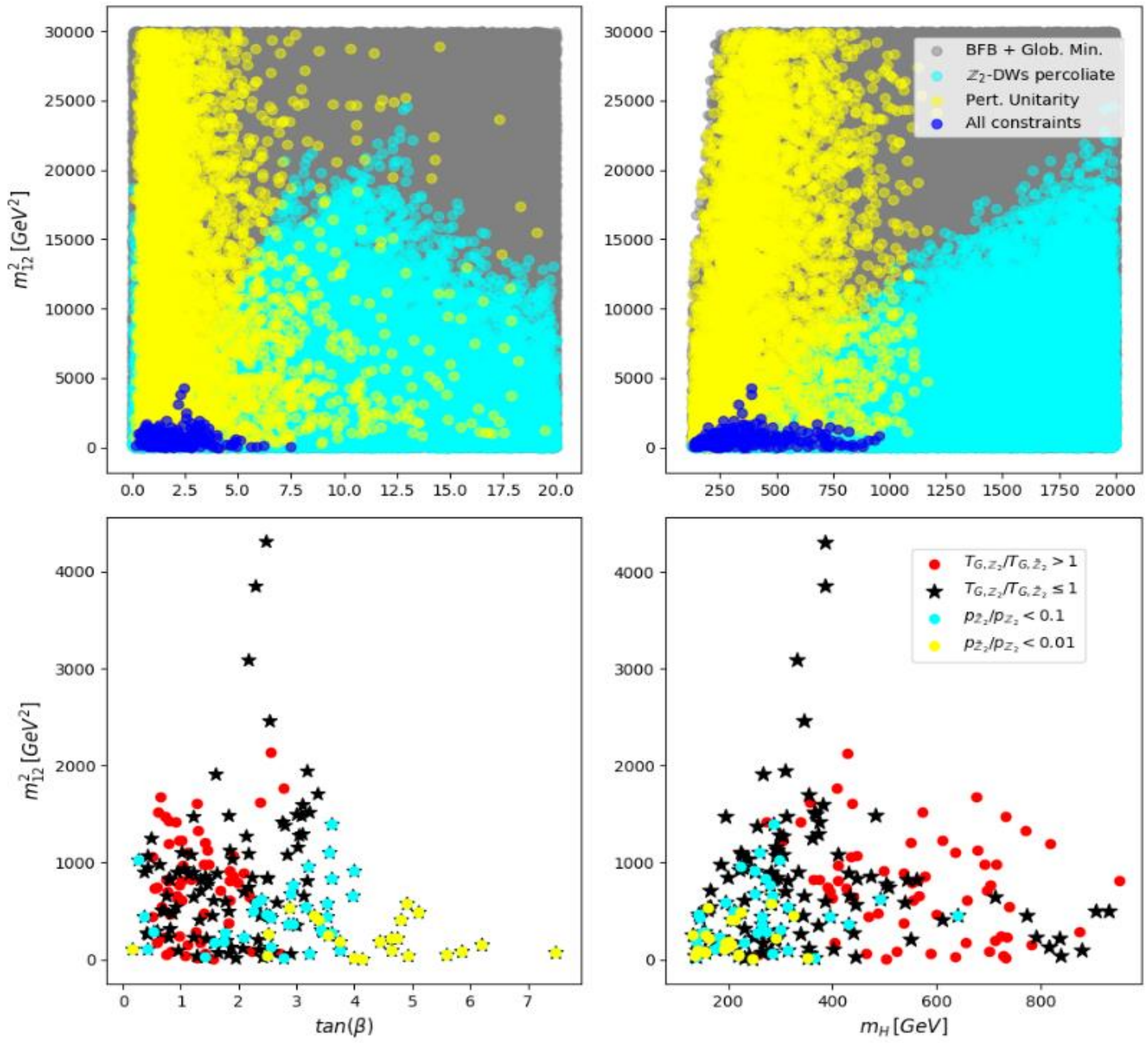
Bottom plots:

Red: $$\frac{T_{G,\mathbb{Z}_2}}{T_{G,\tilde{\mathbb{Z}}_2}} > 1$$

Black stars: $$\frac{T_{G,\mathbb{Z}_2}}{T_{G,\tilde{\mathbb{Z}}_2}} \leq 1$$

Cyan: $$\frac{p_{\tilde{\mathbb{Z}}_2}}{p_{\mathbb{Z}_2}} = e^{-\frac{\Delta\sigma\times A_\xi}{T_G}} < 0.1$$

Yellow: $$\frac{p_{\tilde{\mathbb{Z}}_2}}{p_{\mathbb{Z}_2}} < 0.01$$



Phenomenological impact of the different symmetries

2HDM, Type I

Comparing two cases

a) $m_H = m_{H^\pm}$ (green)

b) $m_A = m_{H^\pm}$ (blue)

- Percolation constraint restricts for case
  a) $m_H < 700\ GeV$, b) $m_H < 1000\ GeV$

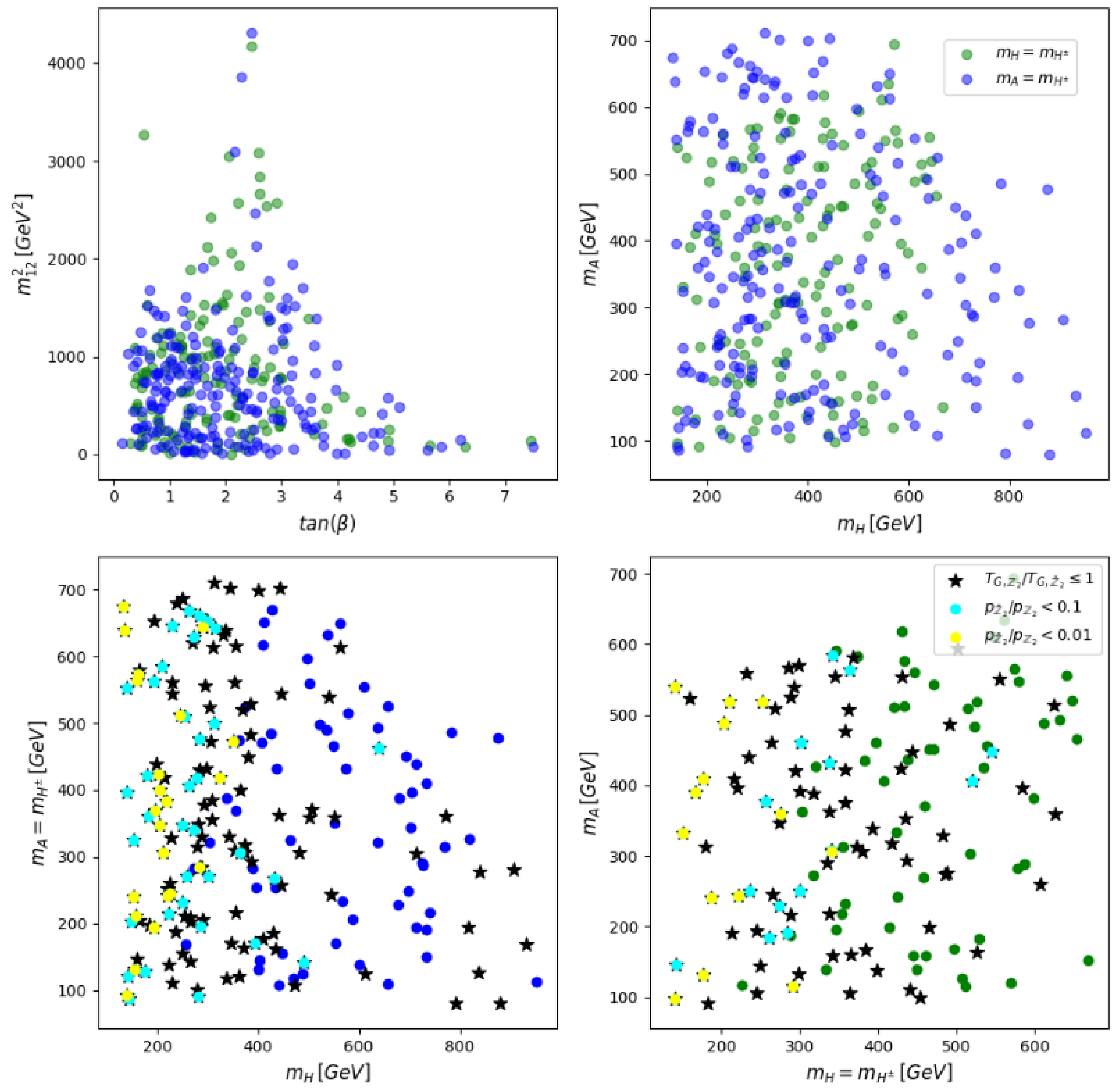
- Suppression of $\frac{p_{\tilde{\mathbb{Z}}_2}}{p_{\mathbb{Z}_2}} < 0.01$ only possible for

$$m_H < 400\ GeV$$

Remaining parameters behave similar for both cases

- Percolation:
  $\tan(\beta) < 8, m_{12}^2 < 5000\ GeV^2, m_A < 700\ GeV$

- Suppression of $\frac{p_{\tilde{\mathbb{Z}}_2}}{p_{\mathbb{Z}_2}} < 0.01$:
  $\tan(\beta) \in (3{,}8), m_{12}^2 \lesssim 1000\ GeV^2, m_A < 700\ GeV$



Phenomenological impact of the different symmetries

N2HDM

Same procedure as for the 2HDM, but due to $\mathbb{Z}'_2$ symmetry the percolation constraint is valid for all Yukawa types!

$$10^{-3} \leq m_{12}^2 \leq 20000\, GeV^2$$

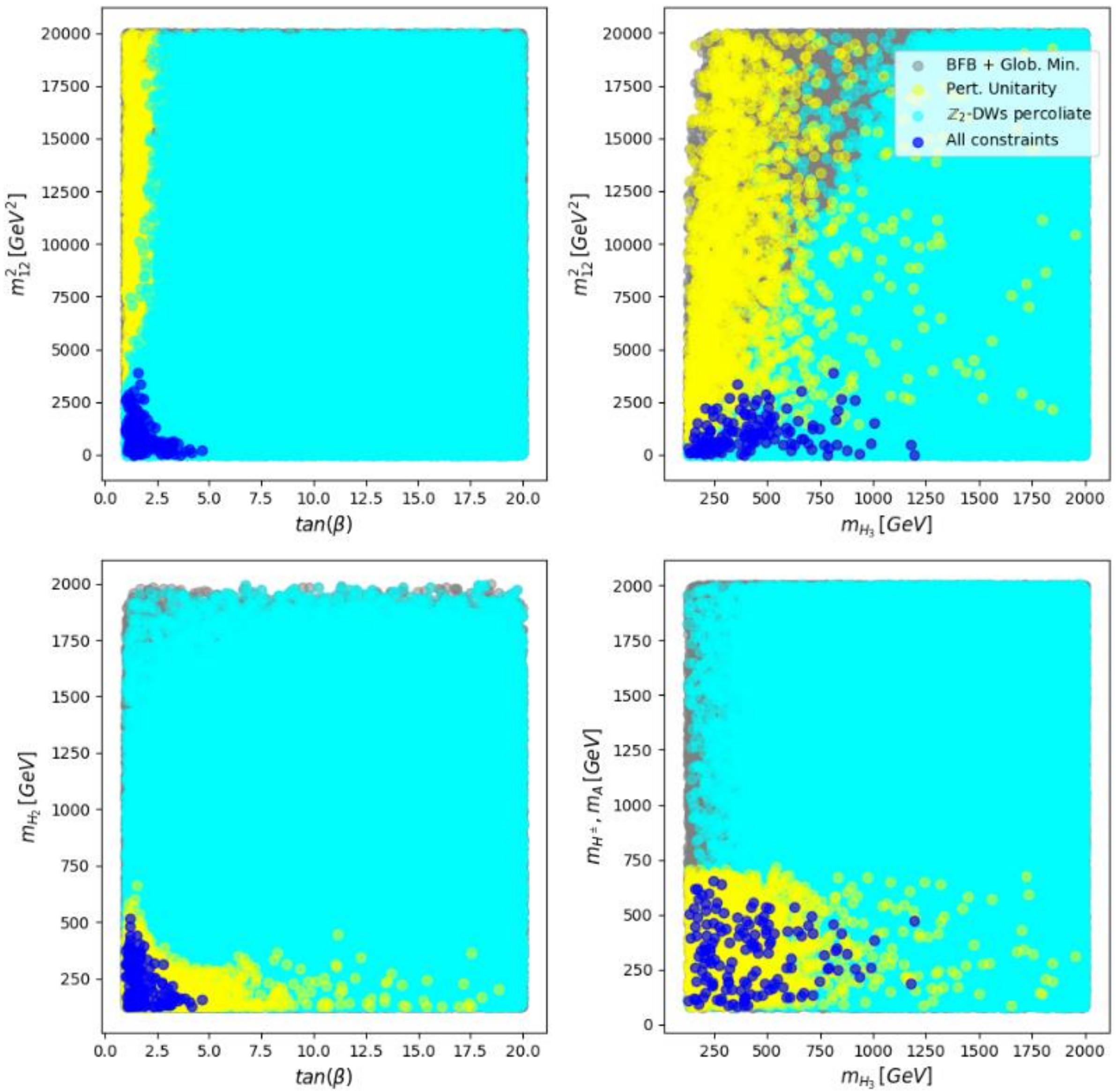
$$m_{H_1} < m_{H_2} < m_{H_3} \leq 2000\, GeV$$

$$80\, GeV \leq m_{H\pm}, m_A \leq 2000\, GeV,$$

$$1 \leq tan(\beta) \leq 20$$

$$-\frac{\pi}{2} \leq \alpha_{2,3} < \frac{\pi}{2} \qquad \Sigma_{H_1} = |R_{1,3}|^2 \leq 20\%$$

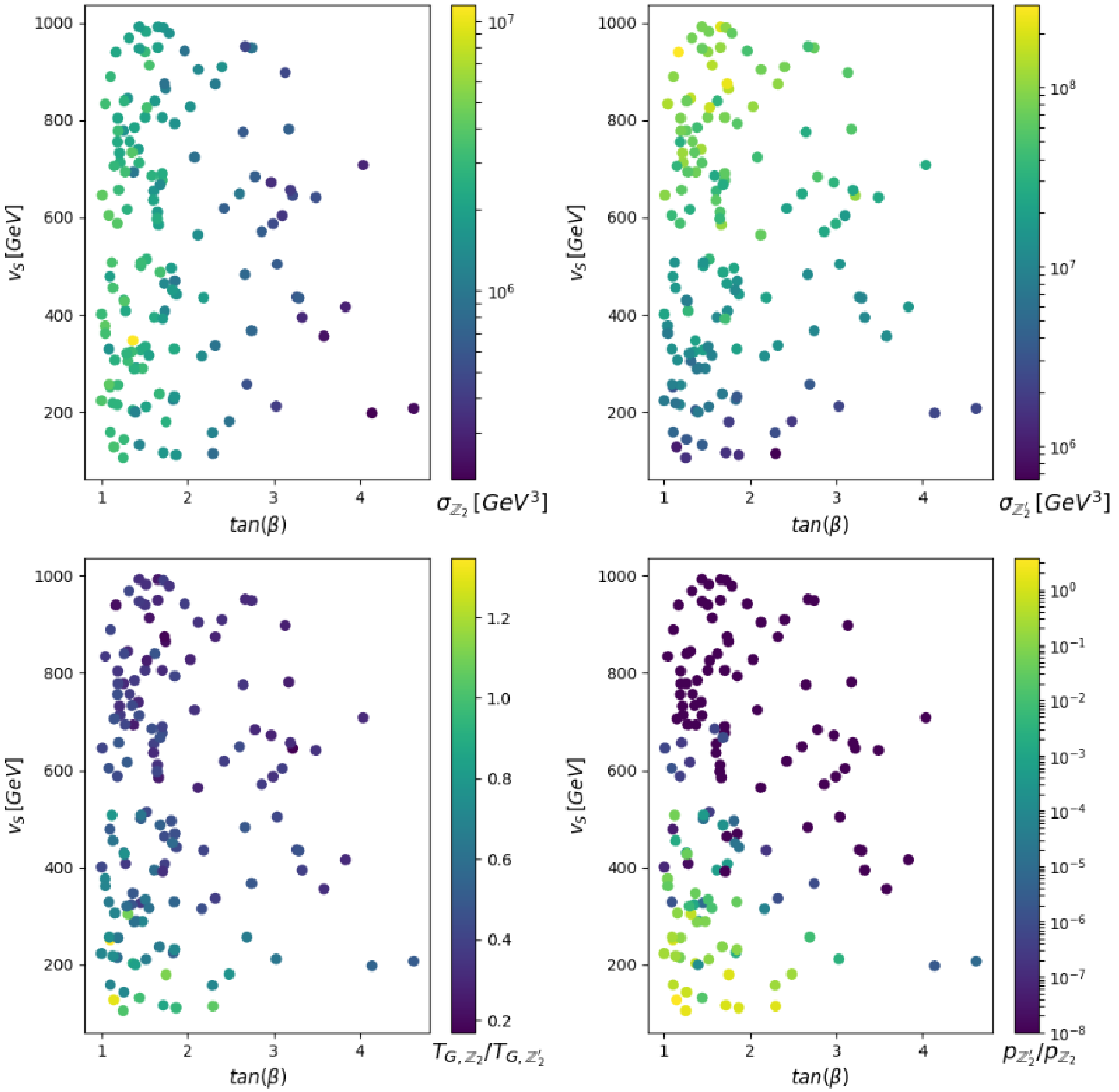
$$cos(\beta - \alpha_1) cos(\alpha_2) = 1$$



Phenomenological impact of the different symmetries

N2HDM, $\mathbb{Z}'_2$ suppression

- Only two parameters which can be constrained by suppression of $\mathbb{Z}'_2$ DWs are v_S and $\tan(\beta)$
- Good agreement between ratios $\frac{T_{G,\mathbb{Z}_2}}{T_{G,\mathbb{Z}_2'}}$ and $\frac{p_{\mathbb{Z}_2'}}{p_{\mathbb{Z}_2}}$
- $v_S \gtrsim 500\ GeV$ to achieve $\frac{p'_{\mathbb{Z}_2}}{p_{\mathbb{Z}_2}} < 0.01$ for higher $\tan(\beta)$ also lower values of v_S possible

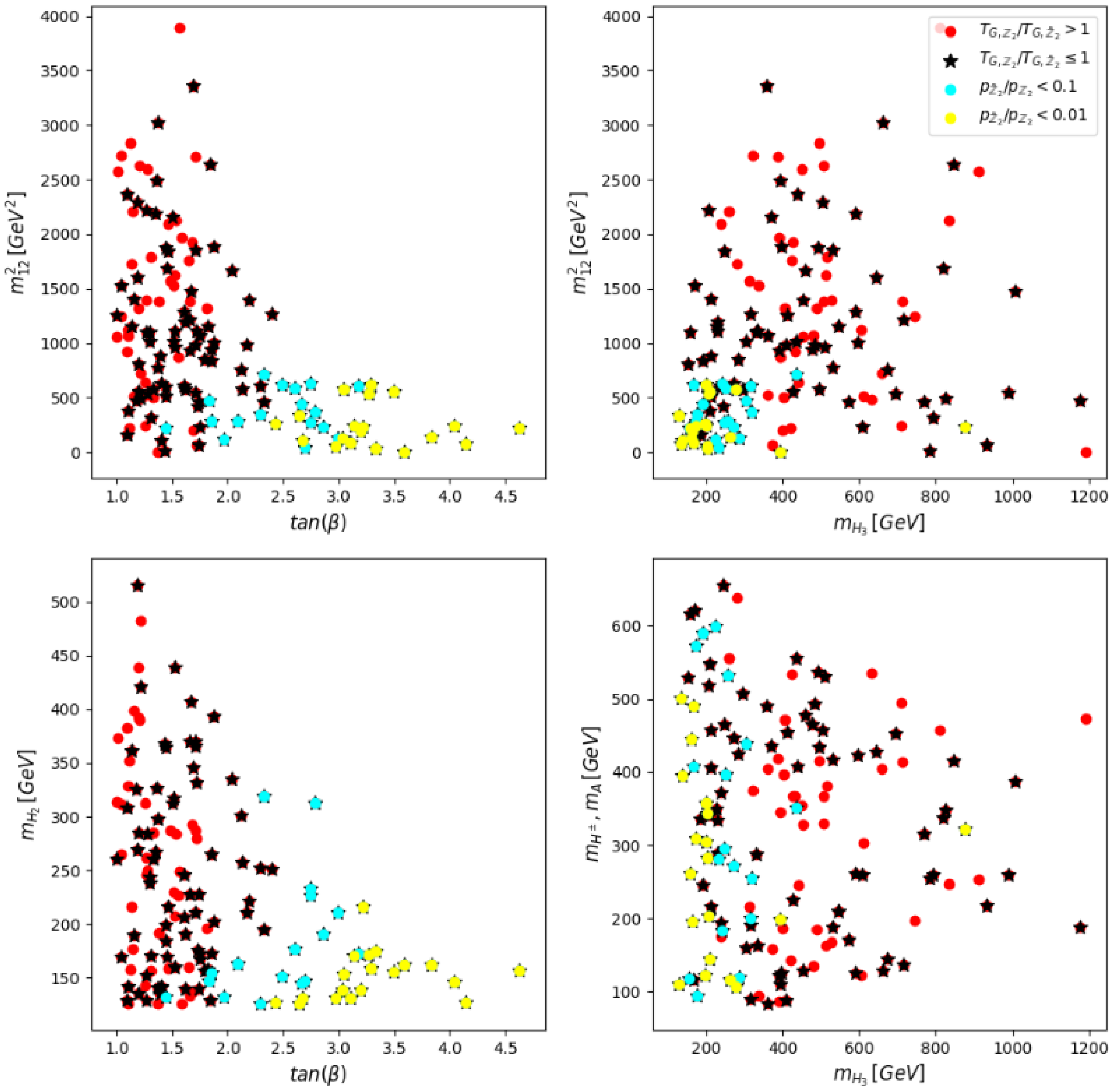


Phenomenological impact of the different symmetries

N2HDM, Type I, $\tilde{\mathbb{Z}}_2$ suppression

- Similar dependencies and tendecies as in the 2HDM
- Upper bounds on all Higgs masses
- $m_{H_2} < 500\ GeV$ (percolation constraint) and $m_{H_2} < 200\ GeV$ (suppression constr. <1%)
- All other masses

  $$m_i < 500\ GeV \text{ for } \frac{p'_{\mathbb{Z}_2}}{p_{\mathbb{Z}_2}} < 0.01$$



Conclusion and outlook

Conclusion

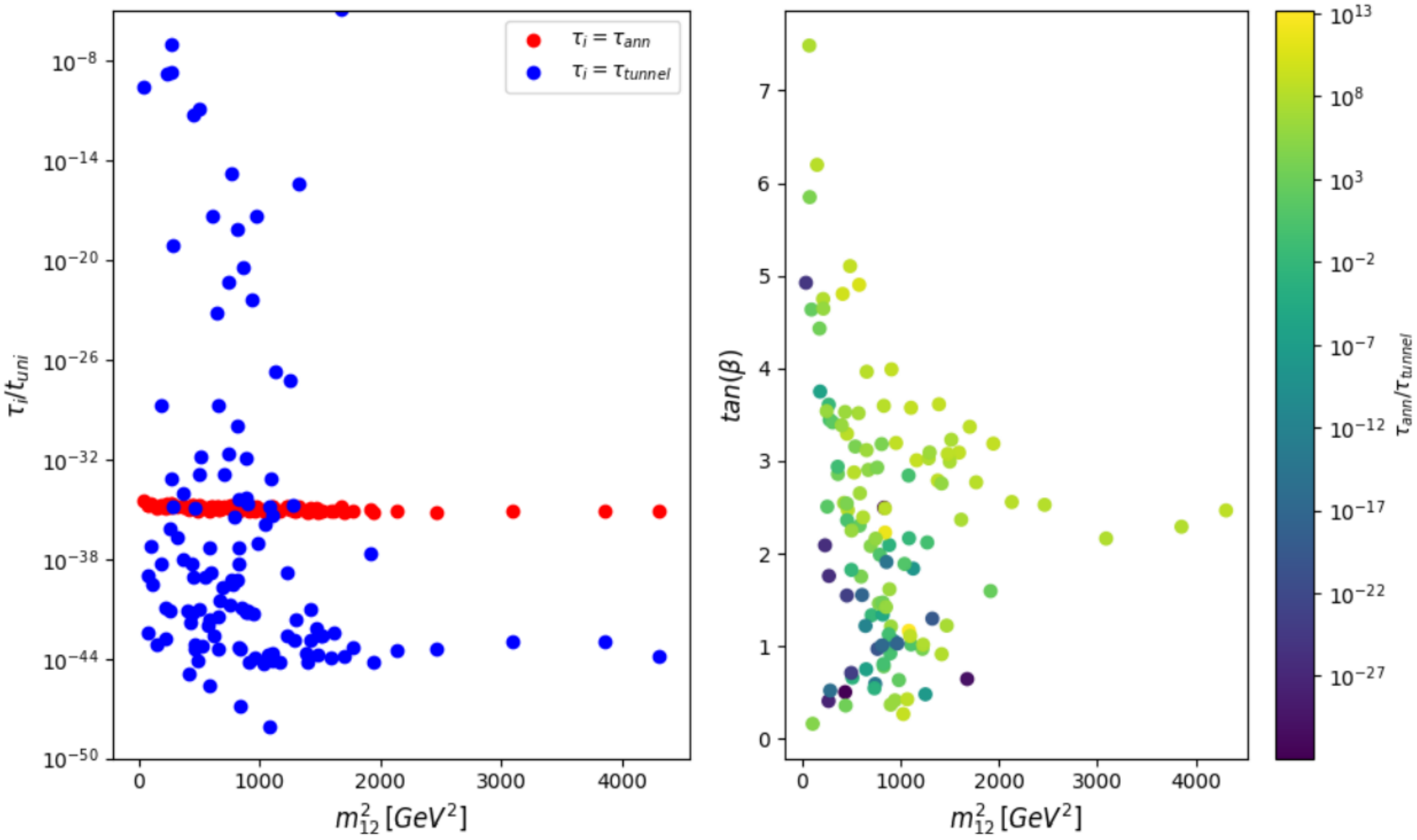
- 2HDM and N2HDM scalar sectors exhibit multiple discrete symmetries, which can lead to DW formation
- For multiple symmetries, the DW problem can only be circumvented if DWs form for approximate symmetry.
- We applied a new method to estimate suppression of formation of stable DWs
  $\Rightarrow$ Upper bounds on all scalar masses, soft breaking parameter m_{12}^2 and $\tan(\beta)$

Outlook

- Finite temperature effects
- Dynamical simulations needed to verify the approach and to find limits of sufficient suppression
- Apply this method to CP-symmetries $\rightarrow$ Baryogenesis, Strong first order PT
- Include collider data into parameter scans

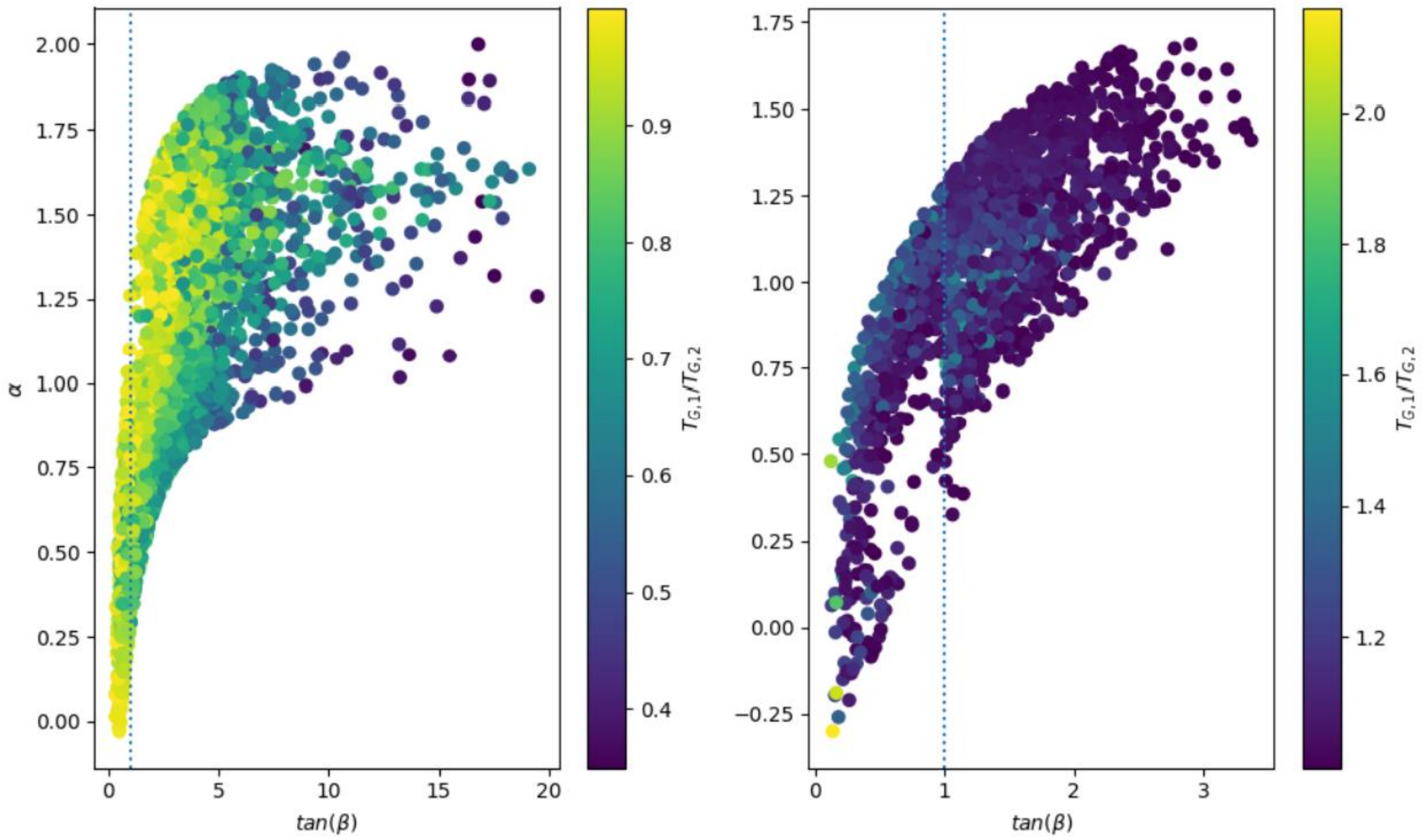
Thank you

Backup



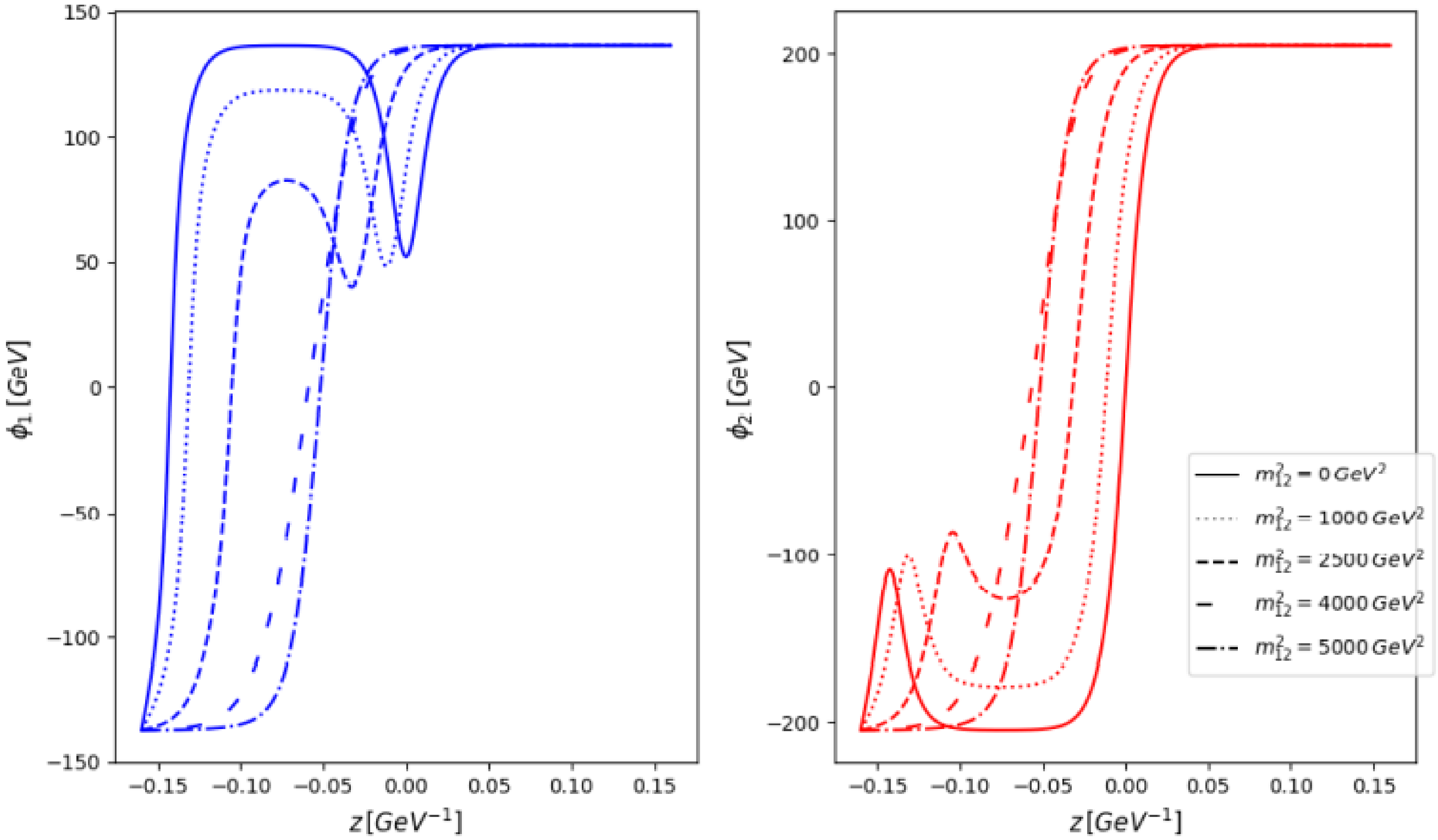
$\tau_i = \tau_{ann}$
$\tau_i = \tau_{tunnel}$
τ_i / t_{uni}
10^{-8}
10^{-14}
10^{-20}
10^{-26}
10^{-32}
10^{-38}
10^{-44}
10^{-50}
0
1000
2000
3000
4000
$m_{12}^2\,[GeV^2]$
$tan(\beta)$
0
1
2
3
4
5
6
7
0
1000
2000
3000
4000
$m_{12}^2\,[GeV^2]$
10^{13}
10^{8}
10^{3}
10^{-2}
10^{-7}
10^{-12}
10^{-17}
10^{-22}
10^{-27}
τ_{ann}/τ_{tunnel}

Backup



2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
α
0
5
10
15
20
tan(β)
0.9
0.8
0.7
0.6
0.5
0.4
$T_{G,1}/T_{G,2}$
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
0
1
2
3
tan(β)
2.0
1.8
1.6
1.4
1.2
$T_{G,1}/T_{G,2}$

Backup



ϕ_1 [GeV]
$z[GeV^{-1}]$
ϕ_2 [GeV]
$z[GeV^{-1}]$
$m_{12}^2 = 0\,GeV^2$
$m_{12}^2 = 1000\,GeV^2$
$m_{12}^2 = 2500\,GeV^2$
$m_{12}^2 = 4000\,GeV^2$
$m_{12}^2 = 5000\,GeV^2$

Backup – Perturbative unitarity

- 2HDM:

$$
\begin{aligned}
|\lambda_3 - \lambda_4| &< 8\pi \\
|\lambda_3 + 2\lambda_4 \pm 3\lambda_5| &< 8\pi \\
\left|\frac{1}{2}\left(\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}\right)\right| &< 8\pi \\
\left|\frac{1}{2}\left(\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}\right)\right| &< 8\pi
\end{aligned}
\tag{41}
$$

[10][20].

- For the N2HDM one has additionally:

$$
\begin{aligned}
|\lambda_7|,\ |\lambda_8| &< 8\pi \\
\frac{1}{2}|a_{1,2,3}| &< 8\pi
\end{aligned}
\tag{42}
$$

with $a_{1,2,3}$ being the real roots of the cubic polynomial:

$$
\begin{aligned}
&4(-27\lambda_1\lambda_2\lambda_6 + 12\lambda_3^2\lambda_6 + 12\lambda_3\lambda_4\lambda_6 + 3\lambda_4^2\lambda_6 + 6\lambda_2\lambda_7^2 - 8\lambda_3\lambda_7\lambda_8 \\
&- 4\lambda_4\lambda_7\lambda_8 + 6\lambda_1\lambda_8^2) + x(36\lambda_1\lambda_2 - 16\lambda_3^2 - 16\lambda_3\lambda_4 - 4\lambda_4^2 + 18\lambda_1\lambda_6 + 18\lambda_2\lambda_6 \\
&- 4\lambda_7^2 - 4\lambda_8^2) + x^2(-6(\lambda_1 + \lambda_2) - 3\lambda_6) + x^3.
\end{aligned}
\tag{43}
$$

Backup – Bounded from below

$$\lambda_1 > 0\,, \quad \lambda_2 > 0\,, \quad \lambda_3 > -\sqrt{\lambda_1\lambda_2}\,, \quad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2}$$

$$D = min(\lambda_4 - |\lambda_5|, 0).$$

Using the definition of D, one can define the two sets Ω_1 and Ω_2:

$$\Omega_1 = \left(\lambda_1, \lambda_2, \lambda_6 > 0;\ \sqrt{\lambda_1\lambda_6} + \lambda_7 > 0;\ \sqrt{\lambda_2\lambda_6} + \lambda_8 > 0\right.$$

$$\left.\sqrt{\lambda_1\lambda_2} + \lambda_3 + D > 0;\ \lambda_7 + \sqrt{\frac{\lambda_1}{\lambda_2}}\lambda_8 > 0\right),$$

$$\Omega_2 = \left(\lambda_1, \lambda_2, \lambda_6 > 0;\ \sqrt{\lambda_2\lambda_6} \geq \lambda_8 > -\sqrt{\lambda_2\lambda_6};\ \sqrt{\lambda_1\lambda_6} > -\lambda_7 \geq \sqrt{\frac{\lambda_1}{\lambda_2}}\lambda_8;\right.$$

$$\left.\sqrt{(\lambda_7^2 - \lambda_1\lambda_6)(\lambda_8^2 - \lambda_2\lambda_6)} > \lambda_7\lambda_8 - (D + \lambda_3)\lambda_6\right).$$

Backup – Papers

- “A review of gravitational waves from cosmic domain walls” , K. Saikawa, 1703.02576v2
- Initial condition bias: „Evading the Cosmological Domain Wall Problem”, Larsson et al.,
- SM DWs “Domain walls and gravitational waves in the Standard Model”, Krajewski et al., 1608.05719
-

Solving the DW EoMs

Gradient flow method

$$\vec{x}_{n+1} = \vec{x}_n - \epsilon \nabla f(\vec{x}_n).$$

Looking for minimum energy configurations of the fields

- Introduce an artificial time parameter t=nε → Integration time
- Gradient flow is limit of gradient descent method with step-size close to zero

$$g(t+\epsilon) = \vec{x}_{n+1} = \vec{x}_n - \epsilon \nabla f(\vec{x}_n) = g(t) - \epsilon \nabla f(g(t))$$

$$\Rightarrow \dot{g}(t) = -\nabla f(g(t)).$$

$$\frac{\partial \phi_n}{\partial t} = -\frac{\delta E}{\delta \phi_n} = \frac{\partial}{\partial x}\left(\frac{\partial \mathcal{E}}{\partial\left(\partial \phi_n / \partial x\right)}\right) - \frac{\partial \mathcal{E}}{\partial \phi_n}.$$

$$\frac{\partial \phi_n}{\partial t} = \frac{\partial^2 \phi_n}{\partial z^2} - \frac{dV}{d\phi_n}$$

$$\lim_{t \to \infty} \frac{\partial \phi_n}{\partial t} = 0,$$