Universität Hamburg
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Masters Thesis: Mid-Term Presentation | SARA CHOPRA

Sum rules for the Extended Higgs Models and their Phenomenological Applications:

A comparative analysis of 2HDM, GM and the Septet model

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15.05.2025

TABLE OF CONTENTS

Extended Higgs Models (BSM models):

1. Septet Model
2. Two Higgs Doublet Model
3. Georgie Mahachek Model

INTRODUCTION AND MOTIVATION

Rho - Parameter

$$\rho_{SM} = \frac{M_W^2}{M_Z^2 \cos(\theta_w)^2} = 1$$

$$\boxed{\rho_{exp} = 1.00040 \pm 0.00024}$$

Model	Hypercharge (Y)	Isospin (I)
2HDM	$Y = 1/2$	$I = 1/2$
GM	$Y_D = 1/2$ $Y_{RT} = 0$ $Y_{CT} = 1$	$I_D = 1/2$ $I_T = 1$
Septet	$Y = 2$	$I = 3$

$Y_D =$ For Doublet, $Y_{RT} =$ For Real triplet, $Y_{CT} =$ For Complex Triplet,

$I_D =$ For Doublet, $I_T =$ For Triplets

Hypercharge (Y_i)
& Isospin (I_i)

[5]

$$\rho = \frac{\sum_{i=1}^{n} v_i [4I_i(I_i+1) - Y_i^2]}{\sum_{i=1}^{n} 2Y_i^2 v_i}$$

Imp: All models - 2HDM, GM, Septet have **ρ = 1**

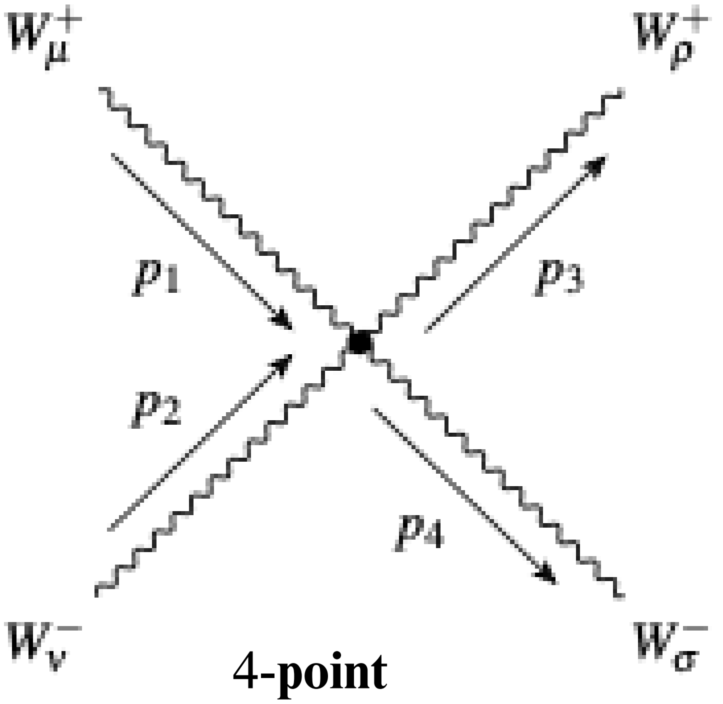
INTRODUCTION AND MOTIVATION

Sum Rules in BSM

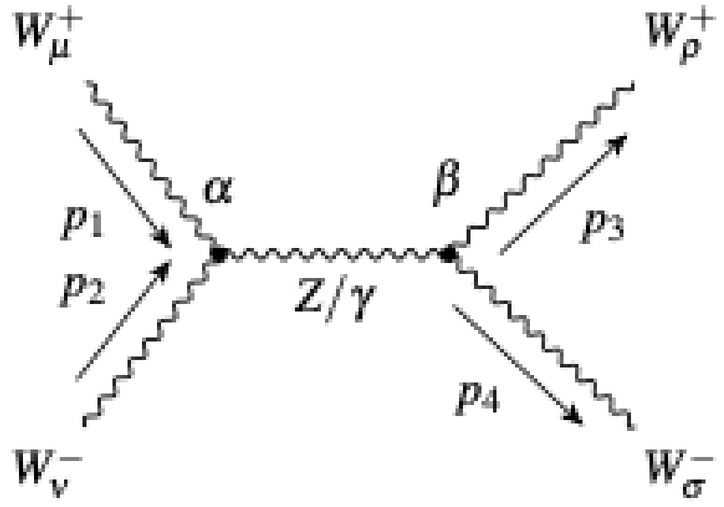
High Energy Vector Boson Scattering [1]

Eg. $W^+W^- \xrightarrow{h^0/H^0/H^{++}} W^+W^-$

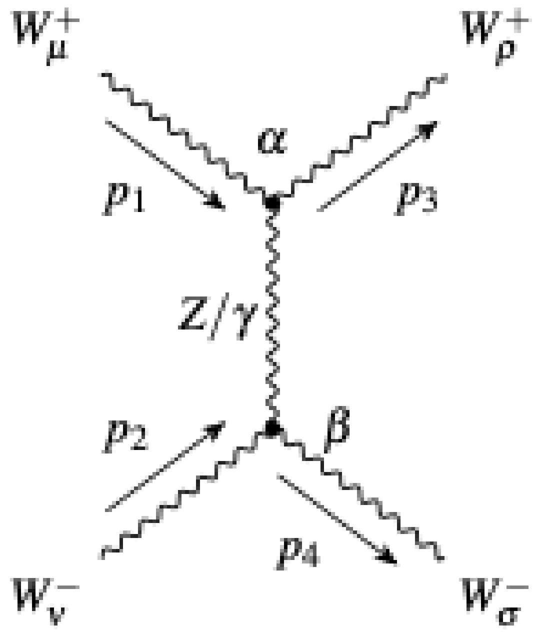
Figure: Standard Model Feynman Diagrams



4-point



s-channel



t-channel

E⁴ dependence

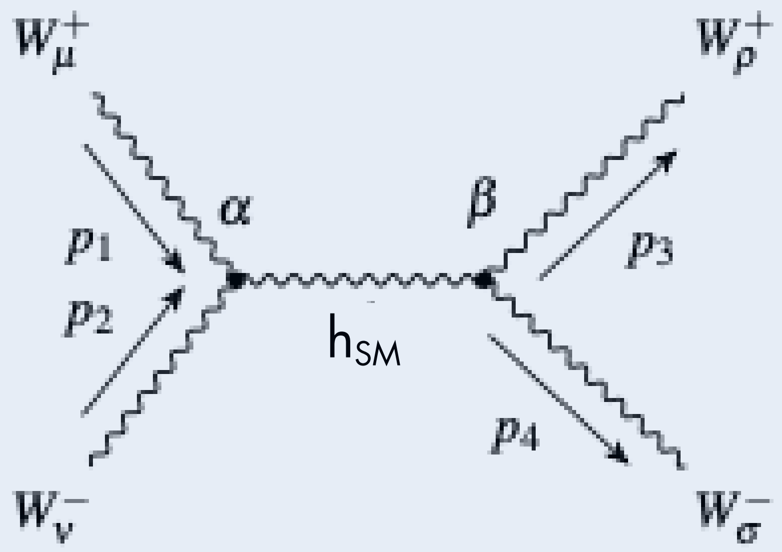
$$i\mathscr{M} \approx ig^2 \frac{E^4}{M_W^4}\left[..+\mathscr{O}\left(E^2\right)\right]$$

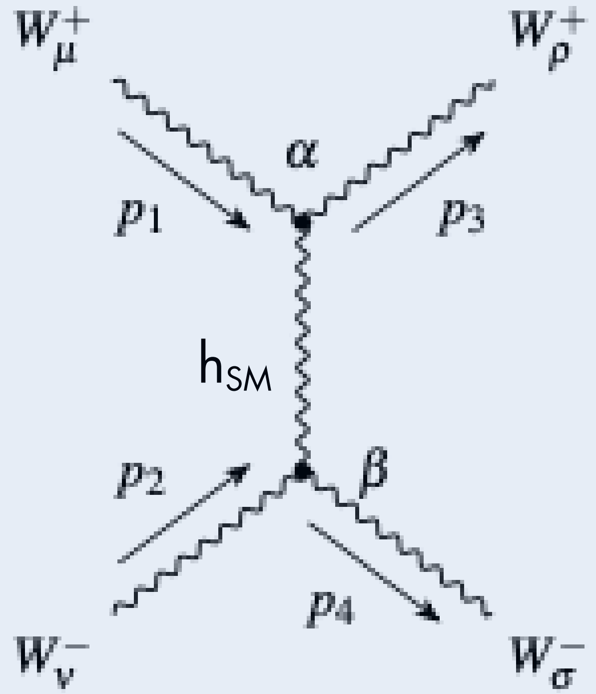
$$i\mathscr{M} = \frac{ig^2}{4M_{W^2}}\left(s+t+\mathscr{O}\left(M_W^2\right)\right)$$

$$i\mathscr{M} \approx \frac{i}{v^2}(s+t)$$

E^4 cancels out but E^2 remains

SM: h_{SM} = 125 Gev





t-channel

$$i\mathscr{M} = -\frac{i}{v^2}\left(\frac{s^2}{s-m_h^2}+\frac{t^2}{t-m_h^2}+..\right)$$

$$i\mathscr{M} = -\frac{i}{v^2}\left(s+t+2m_h^2\right)$$

E^2 cancels due to Higgs

INTRODUCTION AND MOTIVATION

Rho - Parameter

- ρ parameter is an important experimental parameter which preserves the custodial symmetry at tree level and for Standard Model $\rho = 1$.
- We preserve this quantity in our models through Hypercharge and Isospin combinations.

Sum Rules in BSM

- To ensure no bad high energy behaviour exists in our tree level vector boson calculations, we obtain sum rules for our BSM models.

THEORETICAL AND ANALYTIC COMPUTATIONS

SEPTET MODEL

SEPTET MODEL

1 Higgs Doublet (ϕ) & 1 Septet (X) [Z]

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$X = \begin{pmatrix} \chi^{+5} \\ \chi^{+4} \\ \chi^{+3} \\ \chi^{+2} \\ \chi_1^{+1} \\ \chi^0 \\ \chi_2^{-1} \end{pmatrix}$$

note: $(\chi_1^{+1})^* \neq (\chi_2^{-1})$

Using Rotation matrix, ***R***, we obtain physical Higgs fields

GAUGE FIELDS:

Higher charged states	$\chi^{\pm5}$, $\chi^{\pm4}$, $\chi^{\pm3}$, $\chi^{\pm\pm}$
Singly charged states	$\phi^{\pm}$, $\chi_1^{\pm}$, $\chi_2^{\pm}$
CP-even states:	$\phi^{0,r}$, $\chi^{0,r}$
CP-odd states:	$\phi^{0,i}$, $\chi^{0,i}$

PHYSICAL FIELDS:

Higher charged states	$H^{\pm\pm}$, $H^{\pm3}$, $H^{\pm4}$, $H^{\pm5}$
Singly charged States	G^+, H_1^+, H_2^+
Intermediate States	H_v^+ (vector boson) H_f^+ (fermion)
CP-even states:	h^0, H^0
CP-odd states:	G^0, A^0

$$\phi^0 = \frac{v_\phi + \phi^{0,r} + i\phi^{0,i}}{\sqrt{2}},$$

$$\chi^0 = \frac{v_\chi + \chi^{0,r} + i\chi^{0,i}}{\sqrt{2}}.$$

neutral state decompsition

attain vevs as

$$\boxed{v_\phi^2 + 16 v_\chi^2 = v^2 \approx (246\ \mathrm{GeV})^2}$$

SEPTET MODEL

Hyper Charges:

$Y_\phi = 1/2$,

$Y_\chi = 2$,

T^3(Isospin) = 3

$Q = T^3 + Y$

Rotation Matrices, R

[2]

CP-even

$$R_\alpha = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$$

CP-odd & Charged

$$R_7 = \begin{pmatrix} c_7 & -s_7 \\ s_7 & c_7 \end{pmatrix}$$

Singly charged

$$R_\gamma = \begin{pmatrix} c_\gamma & -s_\gamma \\ s_\gamma & c_\gamma \end{pmatrix}$$

$$c_7 \equiv \cos\theta_7 = \frac{v_\phi}{v},$$

$$s_7 \equiv \sin\theta_7 = \frac{4v_\chi}{v}.$$

$$M_W^2 = \frac{g^2(v_\phi^2 + 16v_\chi^2)}{4}$$

SEPTET MODEL

$$g_{h^0VV} = g_{h_{SM}VV} \cdot \kappa_{VV}^{h^0}$$

Interaction	Effective Coupling
$\kappa_{WW}^{h^0} = \kappa_{ZZ}^{h^0}$	$c_7 c_\alpha - 4 s_7 s_\alpha \equiv \kappa_{VV}^{h^0}$
$\kappa_{WW}^{H^0} = \kappa_{ZZ}^{H^0}$	$c_7 s_\alpha + 4 s_7 c_\alpha \equiv \kappa_{VV}^{H^0}$
$\kappa_{WZ}^{H_v^+}$	$-\sqrt{15} s_7$
$\kappa_{WW}^{H^{++}}$	$\sqrt{15} s_7$
$\kappa_{WZ}^{H_1^+}$	$\sqrt{15} s_7 s_\gamma$
$\kappa_{WZ}^{H_2^+}$	$-\sqrt{15} s_7 c_\gamma$

Table: Septet Model: Effective coupling

$$g_{h^0VV}^2 + g_{H^0VV}^2 = g_{h_{SM}VV}^2$$

$$\Rightarrow \quad (\kappa_{VV}^{h^0})^2 + (\kappa_{VV}^{H^0})^2 = (\kappa_{VV}^{h_{SM}})^2 = 1$$

SEPTET CONTRIBUTIONS TO WW → WW :

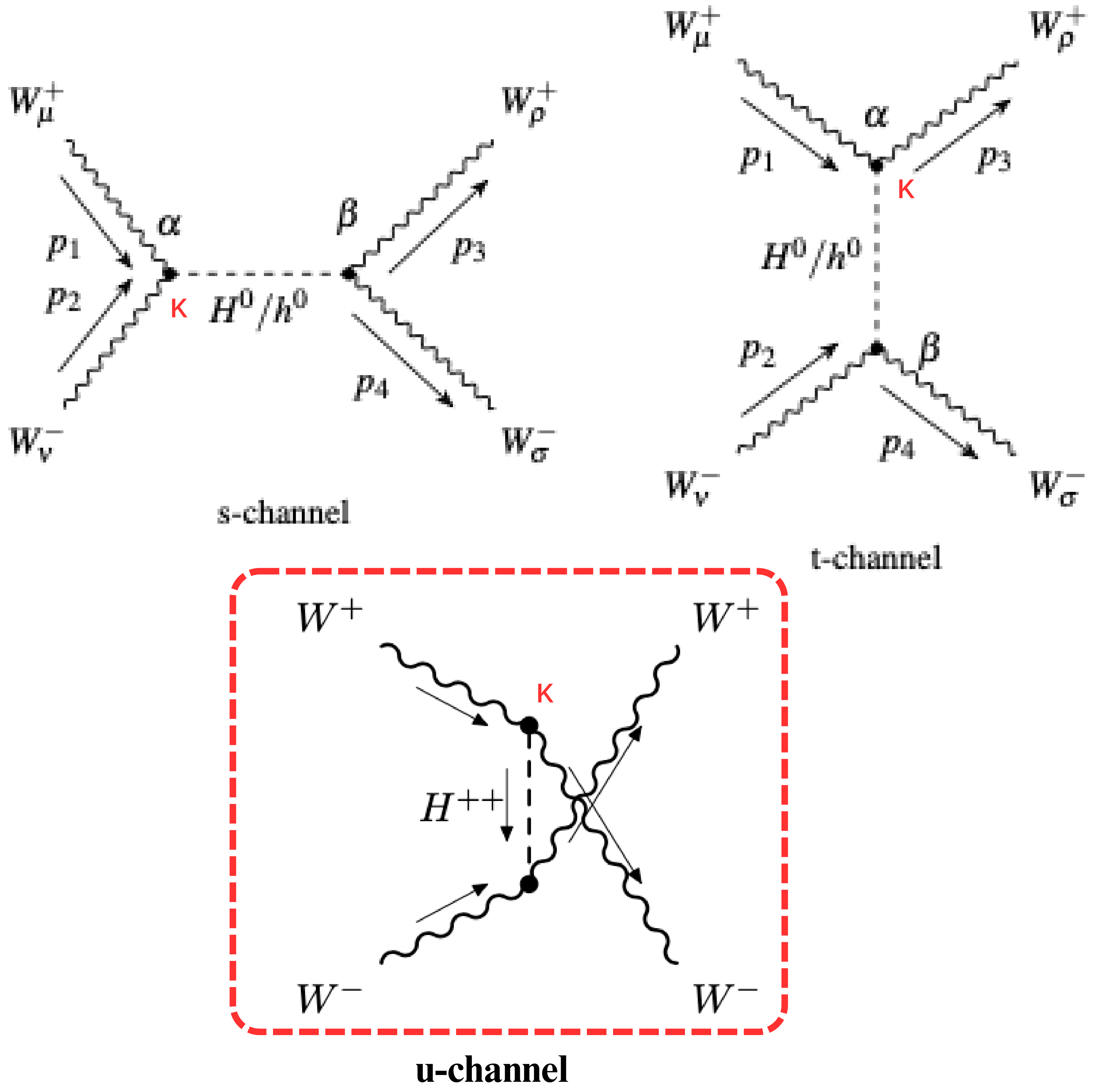


Figure: Septet Model: Feynman Diagrams

[2]

SEPTET MODEL

High Energy Vector Boson Scattering

1. $W^+W^- \xrightarrow{h^0/H^0/H^{++}} W^+W^-$
2. $W^\pm Z \xrightarrow{h^0/H^0/H^{+}} W^\pm Z$

Matrix Element and 1st Sum Rule:

$$\mathcal{M} \simeq -\frac{1}{v^2}\left[\left(\left(\kappa_{WW}^{h^0}\right)^2+\left(\kappa_{WW}^{H^0}\right)^2\right)(s+t)+\left(\kappa_{WW}^{H^{++}}\right)^2 u\right.$$

$s+t \simeq -u$

$$\left.+2\left(\kappa_{WW}^{h^0}\right)^2 m_{h^0}^2+2\left(\kappa_{WW}^{H^0}\right)^2 m_{H^0}^2+\left(\kappa_{WW}^{H^{++}}\right)^2 m_{H^{++}}^2\right].$$

1st Sum Rule:

$$\boxed{\left(\kappa_{WW}^{h^0}\right)^2+\left(\kappa_{WW}^{H^0}\right)^2-\left(\kappa_{WW}^{H^{++}}\right)^2=1.}$$

Theoretical Check: SUCCESS!!

2nd Sum Rule: $W^\pm Z \longrightarrow W^\pm Z$

$$\boxed{(\kappa_V^h)^2+(\kappa_V^H)^2-(\kappa_{WZ}^{H_1^+})^2-(\kappa_{WZ}^{H_2^+})^2=1}$$

[2]

SEPTET MODEL

Mass bound: 1st Sum Rule

$$\boxed{\left(\kappa_{WW}^{h^0}\right)^2 + \left(\kappa_{WW}^{H^0}\right)^2 - \left(\kappa_{WW}^{H^{++}}\right)^2 = 1.}$$

Assuming $\kappa_V^H = 0$

$$\xrightarrow[\text{Decoupling limit}]{} \quad (\kappa_V^h)^2\big|_{\max} = 1 + (\kappa_{WW}^{H^{++}})^2.$$

Perturbative Unitarity: Mass bounds

$$(\kappa_{WW}^{H^{++}})^2 \leq \frac{8\pi v^2 - 2m_h^2}{m_{H^{++}}^2 + 2m_h^2}.$$

$h^0 \longrightarrow h_{SM}$

$$\xrightarrow[\text{Alignment limit}]{} \quad s_7^2 \leq \frac{1}{15}\frac{8\pi v^2 - 2m_h^2}{m_{H^{++}}^2 + 2m_h^2}$$

SM:

m_h = 125 Gev

[2]

Mass bound: 2nd Sum Rule

$$(\kappa_V^h)^2 + (\kappa_V^H)^2 - (\kappa_{WZ}^{H_1^+})^2 - (\kappa_{WZ}^{H_2^+})^2 = 1 \longrightarrow s_7^2 \leq \frac{1}{15}\frac{8\pi v^2 - m_h^2}{2s_\gamma^2 m_{H_1^+}^2 + 2c_\gamma^2 m_{H_2^+}^2 + m_h^2}$$

SEPTET MODEL

Mass bound obtained using sum rule

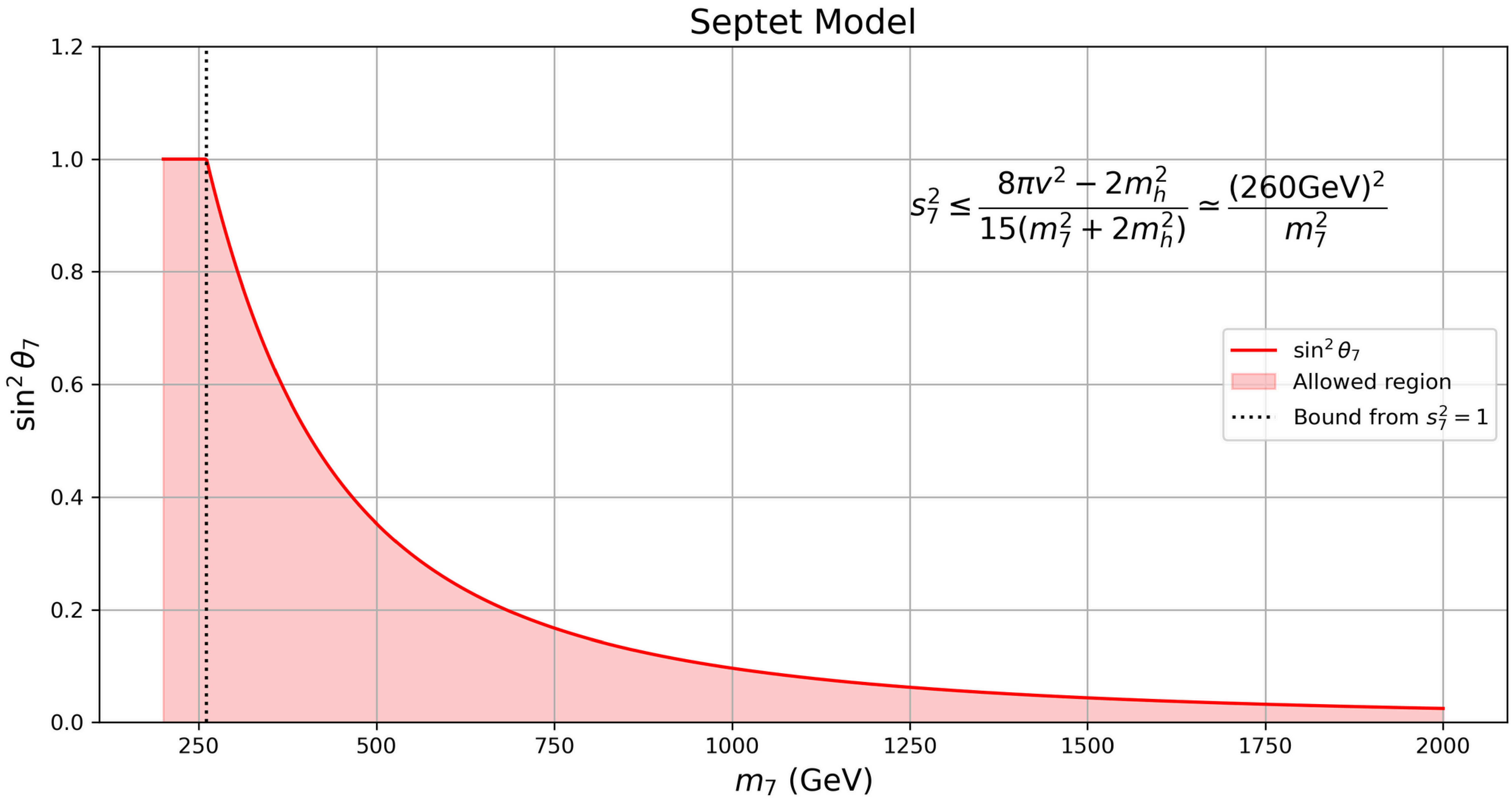


Figure: Septet: The relation sets a bound on s_7 that falls with increasing m_7

THEORETICAL AND ANALYTIC COMPUTATIONS

TWO HIGGS DOUBLET MODEL (THDM)

THE TWO HIGGS DOUBLET MODEL (THDM)

Two Complex SU(2)L Doublets : Φ_i (i = 1,2) [5]

$$\boxed{\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}}$$

Two CP – Conserving Neutral minima

attain vevs as → $\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}$

GAUGE FIELDS:

$(i = 1,2)$

Charged states:	$\omega_i^\pm$
CP-even states:	ρ_i
CP-odd states:	η_i

→

PHYSICAL FIELDS:

Charged states:	$H^\pm$ $G^\pm$
CP-even states:	h^0 H^0
CP-odd states:	G^0 A^0

2HDM Coupling:

$$g_{h^0W^+W^-} = g_{H_{SM}W^+W^-} \cdot \kappa^{h^0}_{W^+W^-}$$

Coupling	Effective Coupling $\kappa^{h^i}_{VV}$
$\kappa^{h^0}_{W^+W^-}$	$\sin(\beta-\alpha)$
$\kappa^{H^0}_{W^+W^-}$	$\cos(\beta-\alpha)$
$\kappa^{h^0}_{ZZ}$	$\sin(\beta-\alpha)$
$\kappa^{H^0}_{ZZ}$	$\cos(\beta-\alpha)$
$\kappa^{h^0}_{H^+W^-}$	$\cos(\beta-\alpha)$
$\kappa^{H^0}_{H^+W^-}$	$\sin(\beta-\alpha)$
$\kappa^{h^0}_{A^0Z}$	$\cos(\beta-\alpha)$
$\kappa^{H^0}_{A^0Z}$	$\sin(\beta-\alpha)$

Table: 2HDM Model: Effective coupling

THE TWO HIGGS DOUBLET MODEL

Sum Rules in 2HDM

[5]

1. W^+W^- —— h^0/H^0 ——→ W^+W^-
2. $Z\,Z$ —— h^0/H^0 ——→ $Z\,Z$

$$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 = 1,\ V = W, Z \text{ or}$$

$$\sum_i g_{h_iVV}^2 = g_{H_{SM}VV}^2, \quad h_i = h^0, H^0$$

3. $W^\pm Z$ —— $h^0/H^0/H^+$ ——→ $W^\pm Z$

$$\left(\kappa_{WW}^{h^0}\kappa_{ZZ}^{h^0}\right) + \left(\kappa_{WW}^{H^0}\kappa_{ZZ}^{h^0}\right) = 1 \quad \text{,or}$$

$$\sum_i g_{h_iWW}\, g_{h_iZZ} = g_{H_{SM}WW}\, g_{H_{SM}ZZ}, \quad h_i = h^0, H^0$$

Mass bounds from Perturbative unitarity

$$\cos^2(\beta-\alpha) \leq \frac{4\pi v^2 - m_h^2}{m_{H^0}^2 - m_h^2} \simeq \left(\frac{880\ \text{GeV}}{m_{H^0}}\right)^2,$$

$$\boxed{m_{H^0}^2 \lesssim \left(\frac{880\ \text{GeV}}{\kappa_{WW}^{H^0}}\right)^2}$$

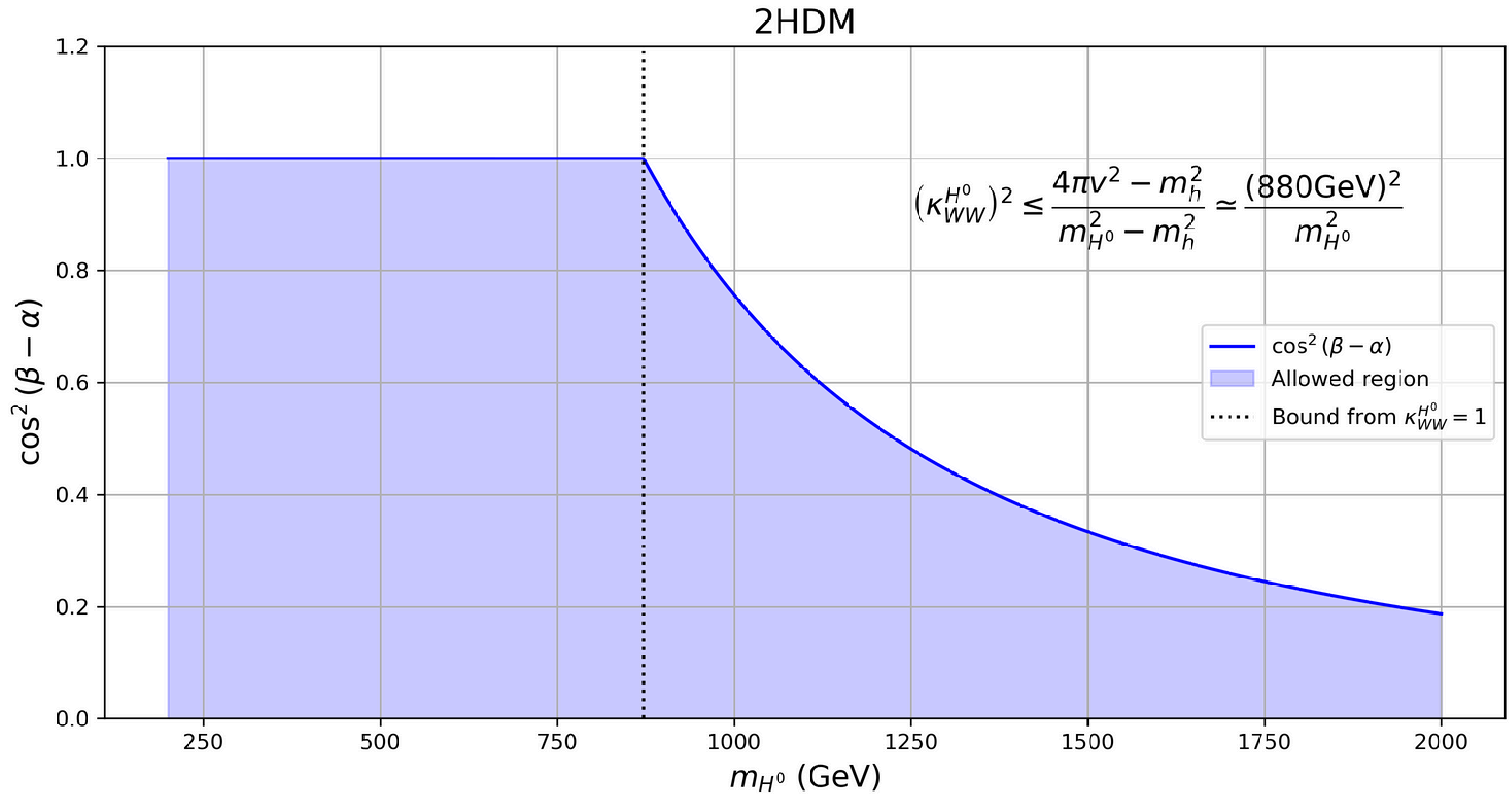


Figure: 2HDM: The relation sets a bound on H^0 coupling that falls with increasing m_{H0}

THEORETICAL AND ANALYTIC COMPUTATIONS

GEORGIE MACHACEK (GM)

GEORGIE MACHACEK (GM)

1 Higgs Doublet (ϕ), 1 Complex Triplet (Χ) & 1 Real Triplet (ξ) [4]

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$$

Hyper Charges:

$Y_\Phi = 1$

$Y_X = 2$

$Y_\xi = 0$

bi-doublet and bi-triplet under $SU(2)_L \times SU(2)_R$

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}.$$

neutral state decompsition

$$\phi^0 = \frac{v_\phi + \phi^{0,r} + \phi^{0,i}}{\sqrt{2}}$$

$$\chi^0 = v_\chi + \frac{\chi^{0,r} + \chi^{0,i}}{\sqrt{2}}$$

$$\xi^0 = v_\chi + \xi^{0,r}$$

attain vevs as

$$v^2 = v_\phi^2 + 8v_\chi^2 = \frac{4M_W^2}{g^2} = (246\ \mathrm{GeV})^2,$$

GAUGE FIELDS:

Doubly charged states:	$\chi^{\pm\pm}$
Singly charged states:	$\phi^\pm,\ \xi^\pm,\ \chi^\pm$
CP-even states:	$\phi^{0,r},\ \chi^{0,r},\ \xi^{0,r}$
CP-odd states:	$\phi^{0,i},\ \chi^{0,i}$

PHYSICAL FIELDS:

Custodial Singlet	$h^0,\ H^0$
Custodial Triplet	$H_3^\pm,\ H_3^0$
Custodial Fiveplet	$H_5^{\pm\pm},\ H_5^\pm,\ H_5^0$
Goldstone Bosons	$G^\pm,\ G^0$

GEORGIE MACHACEK (GM)

GM Coupling:

Interaction $(\kappa^{h^i}_{VV})$	Effective Coupling
$\kappa^{h^0}_{WW}$	$c_H c_\alpha - \frac{\sqrt{8}}{\sqrt{3}} s_H s_\alpha$
$\kappa^{h^0}_{ZZ}$	$c_H c_\alpha - \frac{\sqrt{8}}{\sqrt{3}} s_H s_\alpha$
$\kappa^{H^0_5}_{WW}$	$-\frac{2}{\sqrt{3}} s_H$
$\kappa^{H^0_5}_{ZZ}$	$-\frac{2}{\sqrt{3}} s_H$
$\kappa^{H^{++}_5}_{WW}$	$\sqrt{2} s_H$
$\kappa^{H^0}_{WW}$	$c_H s_\alpha + \frac{\sqrt{8}}{\sqrt{3}} s_H c_\alpha$
$\kappa^{H^0}_{ZZ}$	$c_H s_\alpha + \frac{\sqrt{8}}{\sqrt{3}} s_H c_\alpha$
$\kappa^{H^0_5}_{W^+W^-}$	$-\frac{1}{\sqrt{3}} s_H$
$\kappa^{H^+_5}_{W^\pm Z}$	s_H

$\rightarrow (\kappa^{h^0}_{VV})$

$\rightarrow (\kappa^{H^0}_{VV})$

Table: GM Model: Effective coupling

[1]

High Energy Vector Boson Scattering

1. $W^+W^- \xrightarrow{h^0/H^0/H^0_5/H_5^{++}} W^+W^-$
2. $W^\pm Z \xrightarrow{h^0/H^0/H^+_5} W^\pm Z$

GM CONTRIBUTIONS:

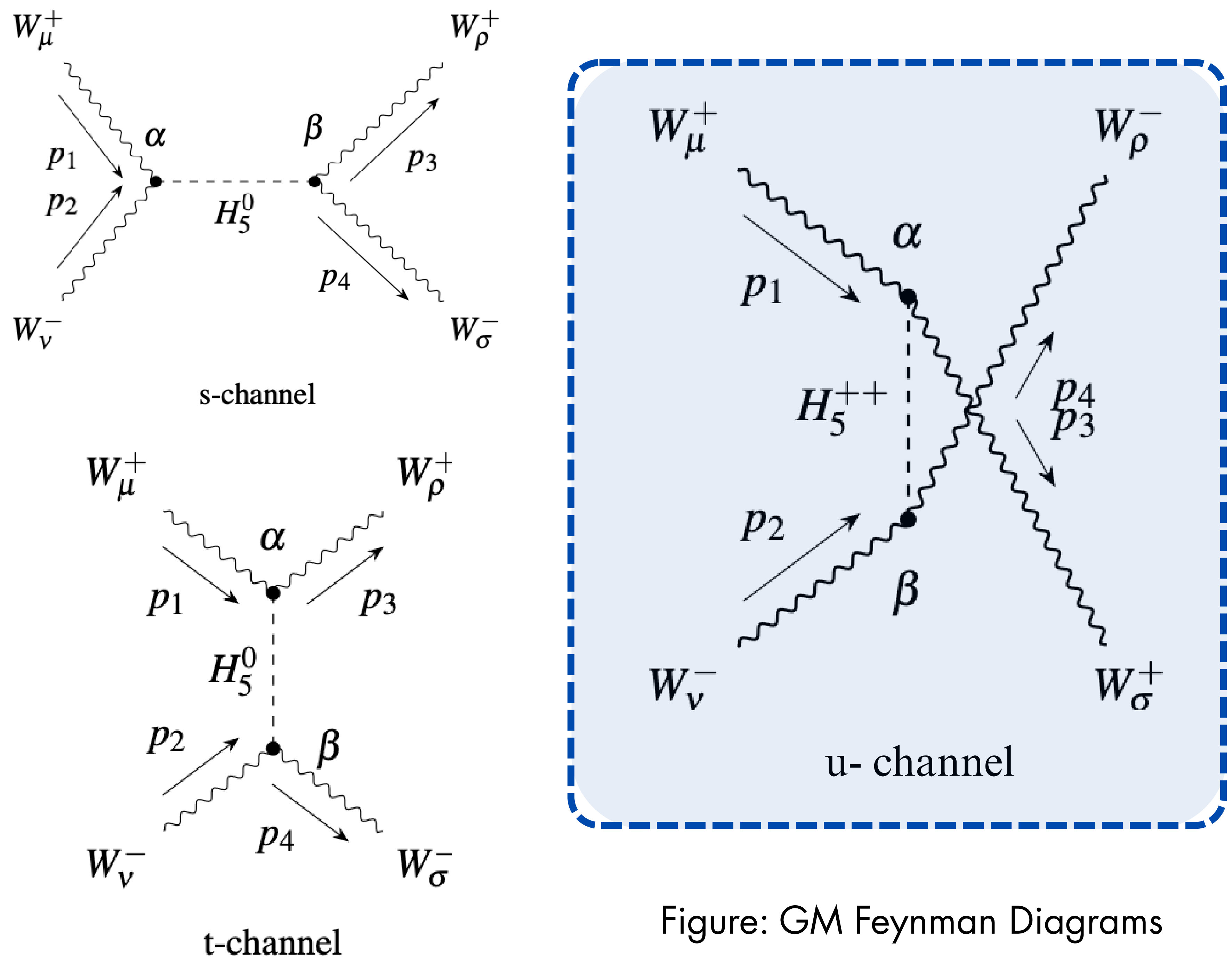


Figure: GM Feynman Diagrams

GEORGIE MACHACEK (GM)

Sum Rules in GM

1. $W^+W^- \xrightarrow{h^0/H^0/H^0_5/H_5^{++}} W^+W^-$

$$\boxed{\left(\kappa_{WW}^{h^0}\right)^2 + \left(\kappa_{WW}^{H^0}\right)^2 + \left(\kappa_{WW}^{H_5^0}\right)^2 - \left(\kappa_{WW}^{H_5^{++}}\right)^2 = 1}$$

Mass bound :

$$s_H^2 \leq \frac{3}{2} \cdot \frac{8\pi v^2 - 2m_h^2}{4m_5^2 + 5m_h^2} \simeq \left(\frac{734\ \mathrm{GeV}}{m_5}\right)^2$$

2. $W^\pm Z \xrightarrow{h^0/H^0/H^0_5/H^+_5} W^\pm Z$

$$\boxed{\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 + \left(\kappa_{WW}^{H_5^0}\kappa_{ZZ}^{H_5^0}\right) - \left(\kappa_{WZ}^{H_5^+}\right)^2 = 1}$$

Mass bound :

$$s_H^2 \leq \frac{3}{5\sqrt{2}} \frac{(16\pi v^2 - m_{h^0}^2)}{(m_{h^0}^2 - m_5^2)} \approx \left(\frac{(800\ \mathrm{GeV})^2}{m_5^2}\right).$$

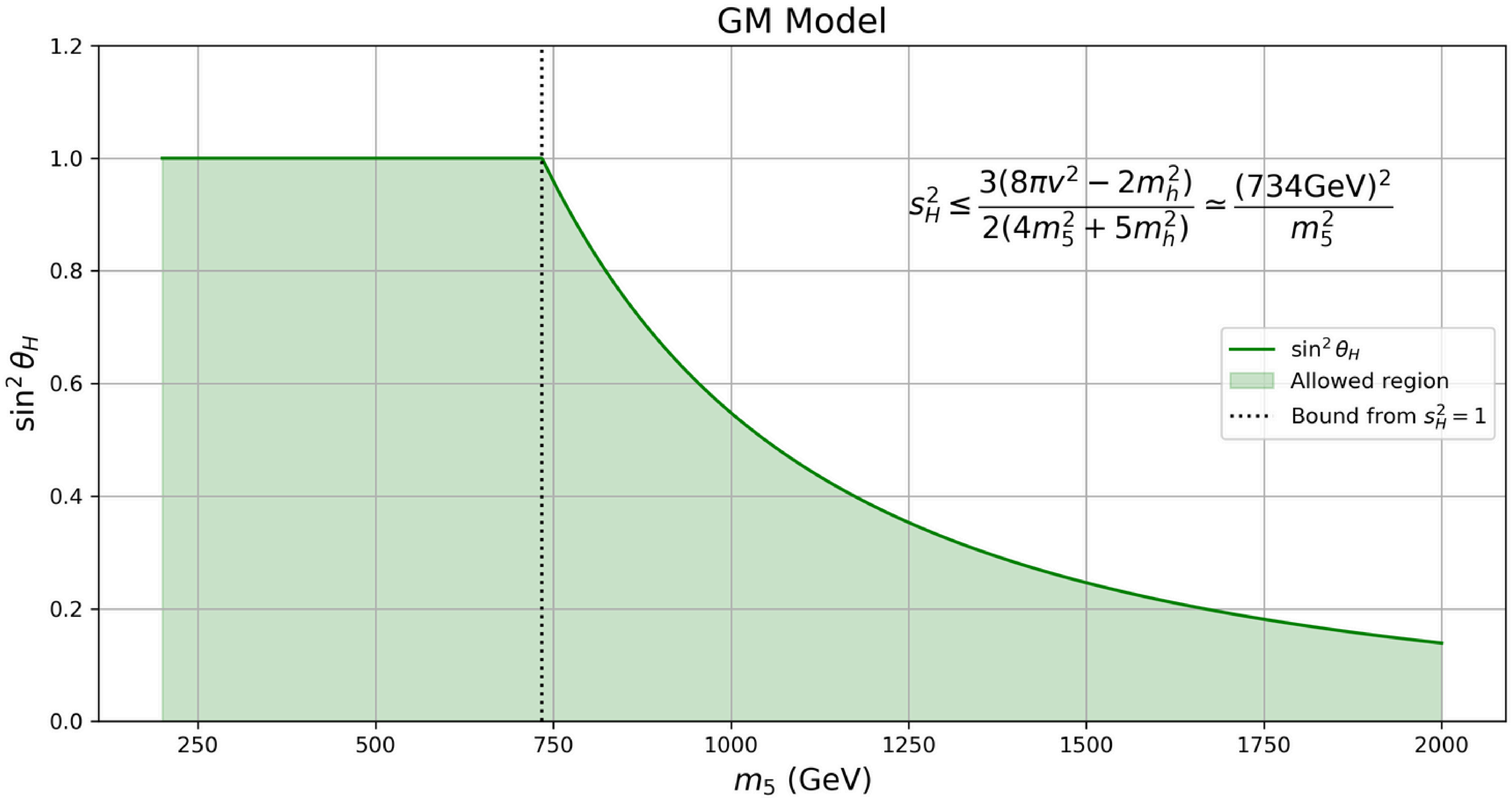


Figure: GM: mass bound sets upper bound on s_H that falls with increasing m_5

COMPARATIVE ANALYSIS

COMPARATIVE ANALYSIS

Feature	2HDM	GM Model	Septet Model
Field content and SU(2) representation	$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \ \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$	$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \ \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \ \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$	$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \ X = \begin{pmatrix} \chi^{+5} \\ \chi^{+4} \\ \chi^{+3} \\ \chi^{+2} \\ \chi_1^{+1} \\ \chi^0 \\ \chi_2^{-1} \end{pmatrix}$
Higgs couplings to Gauge Bosons	$\kappa_{VV}^{h^0} = \sin(\beta-\alpha)$ $\kappa_{VV}^{H^0} = \cos(\beta-\alpha)$	$\kappa_{VV}^{h^0} = c_{\theta_H} c_\alpha - \sqrt{\frac{8}{3}} c_{\theta_H} s_\alpha$ $\kappa_{VV}^{H^0} = c_{\theta_H} s_\alpha + \sqrt{\frac{8}{3}} c_{\theta_H} c_\alpha$ $\kappa_{VV}^{H_5^{++}} = \sqrt{2} s_{\theta_H}$	$\kappa_{VV}^{h^0} = c_7 c_\alpha - 4 s_7 s_\alpha$ $\kappa_{VV}^{H^0} = c_7 s_\alpha + 4 s_7 c_\alpha$ $\kappa_{VV}^{H^{++}} = \sqrt{15} s_7$
VEV Structure	$v^2 = v_1^2 + v_2^2$	$v^2 = v_\phi^2 + 8 v_\chi^2$	$v^2 = v_\phi^2 + 16 v_\chi^2$
$\tan(\beta, \theta_H, \theta_7)$ relation (VEV)	$\tan\beta = \frac{v_2}{v_1}$	$\tan\theta_H = \frac{2\sqrt{2} v_\chi}{v_\phi}$	$\tan\theta_7 = \frac{4 v_\chi}{v_\phi}$
Coupling Sum Rules	$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 = 1$	$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 + \left(\kappa_{VV}^{H_5^0}\right)^2 - \left(\kappa_{VV}^{H_5^{++}}\right)^2 = 1$	$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 - \left(\kappa_{VV}^{H^{++}}\right)^2 = 1$
Mass Bounds	$\cos^2(\beta-\alpha) \leq \left(\frac{880 \text{ GeV}}{m_{H^0}}\right)^2$	$s_H^2 \leq \left(\frac{734 \text{ GeV}}{m_5}\right)^2$	$s_7^2 \leq \left(\frac{260 \text{ GeV}}{m_{H^{++}}}\right)^2$

Table: Comparative analysis of 2HDM, GM and Septet Models

PHENOMENOLOGICAL APPLICATIONS FOR BSM

EXPERIMENTAL LIMITS ON BSM MODELS

Experimental limits: A portrait of the Higgs boson by the CMS experiment ten years after the discovery (arxiv:2207.00043)

Framework	Definition
Coupling Modifier (κ)	$\kappa_V^2 = \frac{\sigma^{BSM}}{\sigma^{SM}}$

Experimental Value
CMS Global Fit: $\kappa_V = 1.02 \pm 0.08$ $\Rightarrow \kappa_V \in [0.86, 1.18]$ (2σ)

[3]

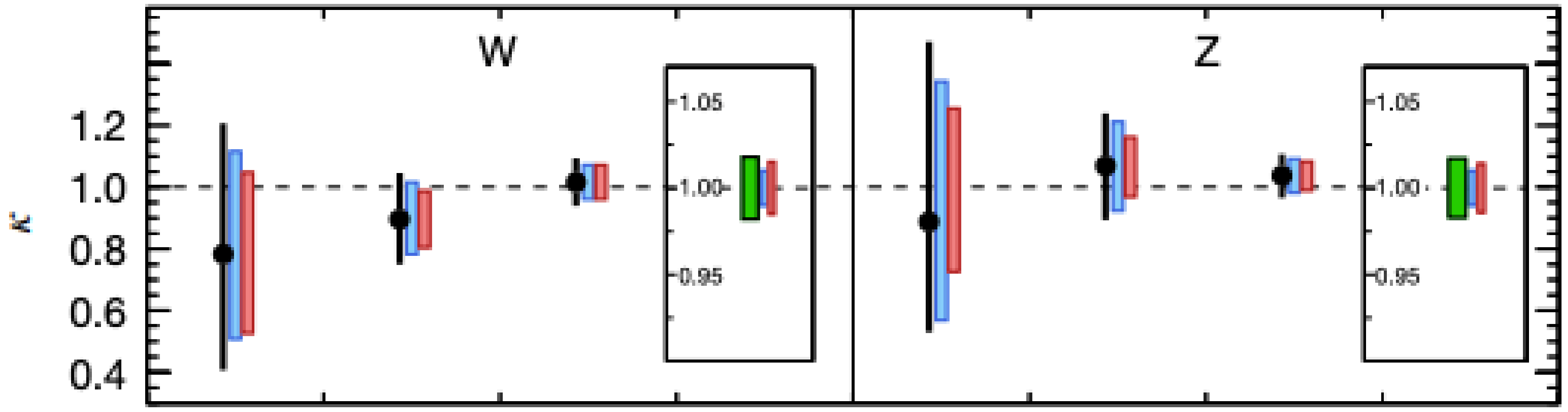
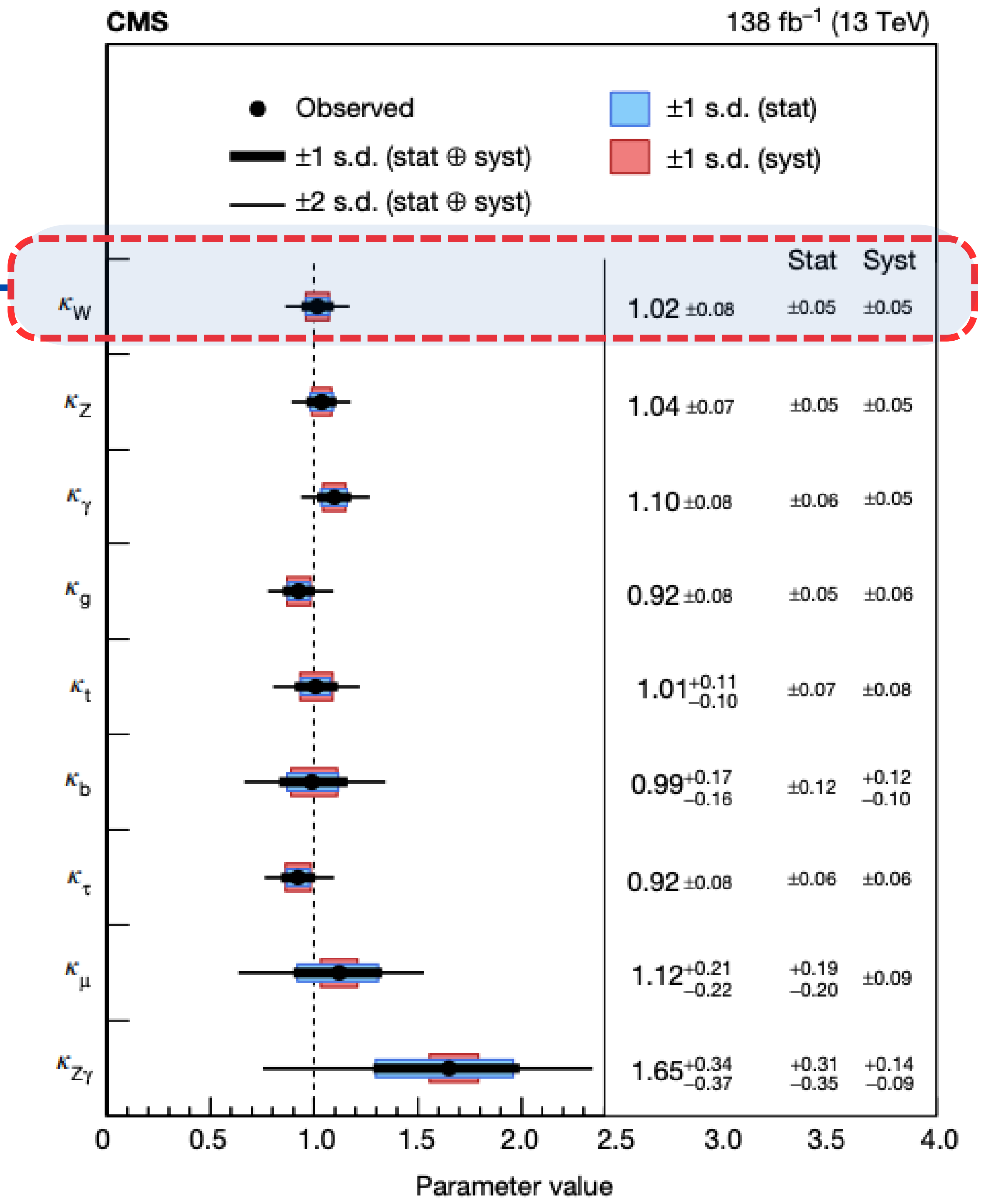


Figure: Coupling Modifier measurements



EXPERIMENTAL LIMITS ON BSM MODELS

Analytical limits:

Heavy Higgs mass estimates

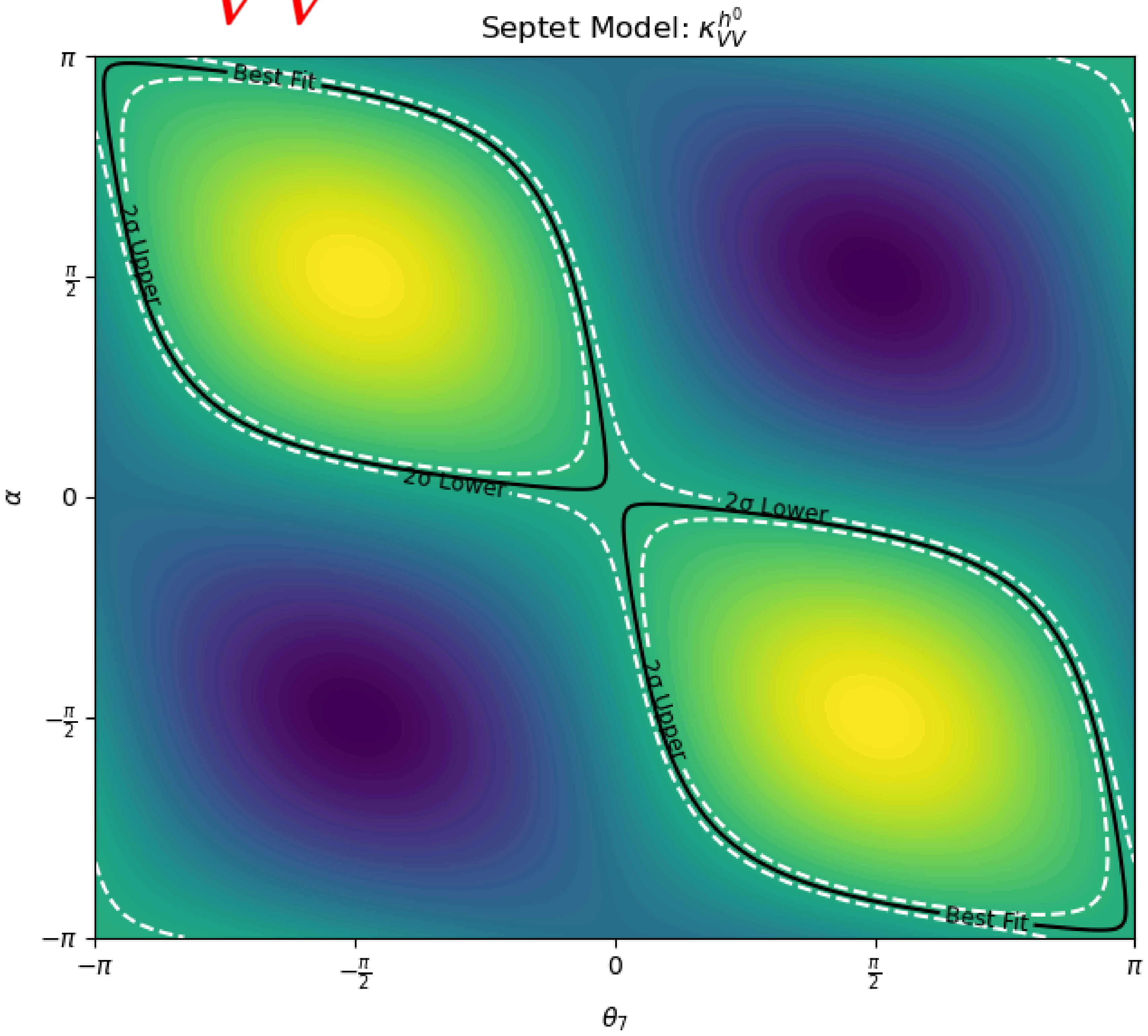
Model	Coupling Function	Mixing Values [Best, Low, High]	Mass Bound (TeV) [Low, High]	Mass Bound (TeV) [Best]
GM	$\sin^2\theta_H$	[0.02, 0.00, 0.23]	[1.53, 7474.16]	[4.80]
Septet	$\sin^2\theta_7$	[0.0027, 0.00, 0.0262]	[1.94, 3151.38]	[6.07]
2HDM	$\cos^2(\beta-\alpha)$	[0.00, 0.26, 0.00]	[1.70, 8630.42]	[1.70] (Low bound)

Table: Coupling and heavy Higgs mass estimates using experimental limits (‘0.00’ indicates unphysical values)

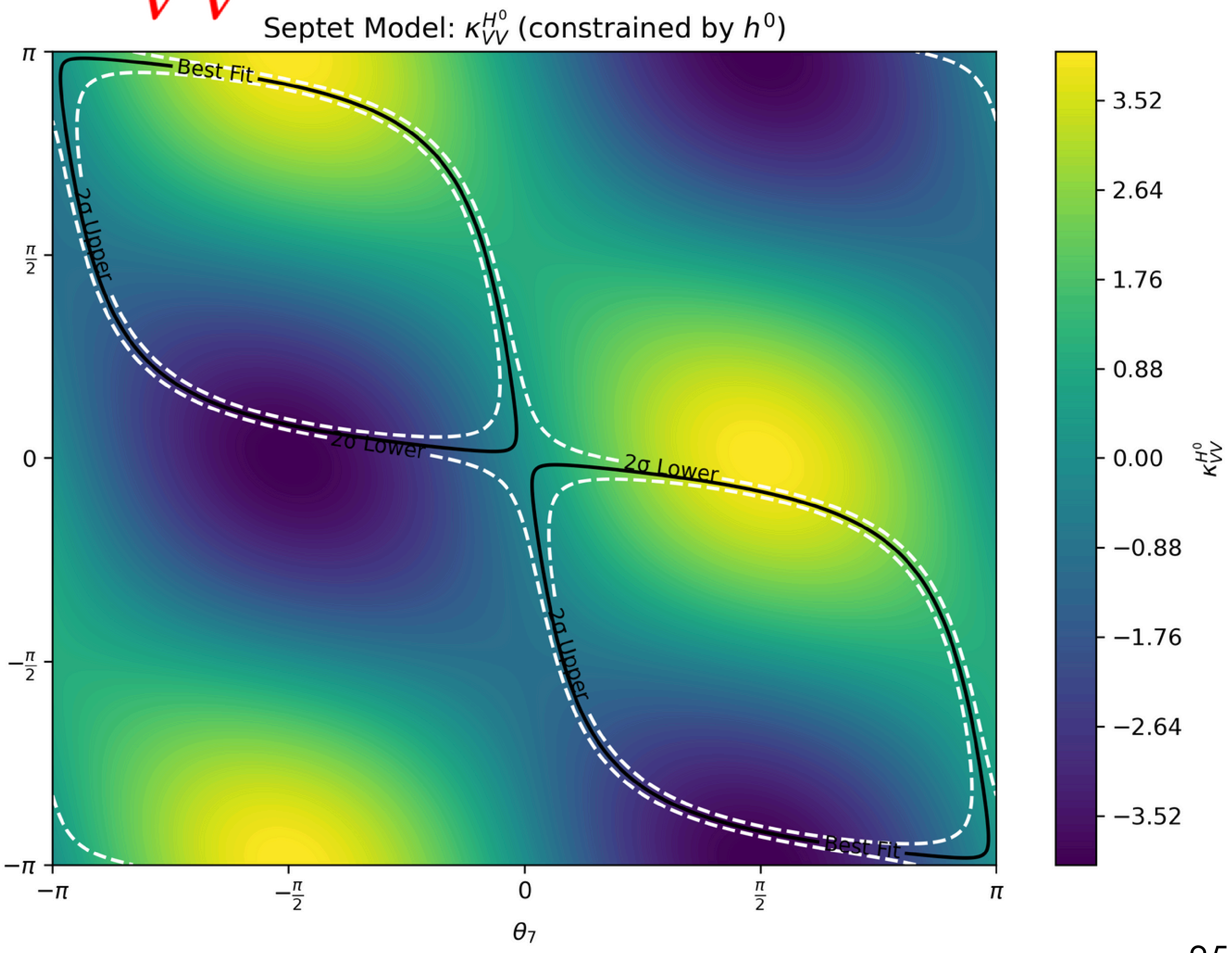
EXPERIMENTAL LIMITS ON BSM MODELS

CMS Global Fit:
$\kappa_V = 1.02 \pm 0.08$
$\Rightarrow \kappa_V \in [0.86, 1.18]$ (2σ)

$\kappa_{VV}^{h^0}$



$\kappa_{VV}^{H^0}$



SEPTET MODEL

Figure: Heatmap for K^{h0}_{VV} and K^{H0}_{VV} with coupling strength limits

EXPERIMENTAL LIMITS ON BSM MODELS

$\kappa_{VV}^{H^{++}}$

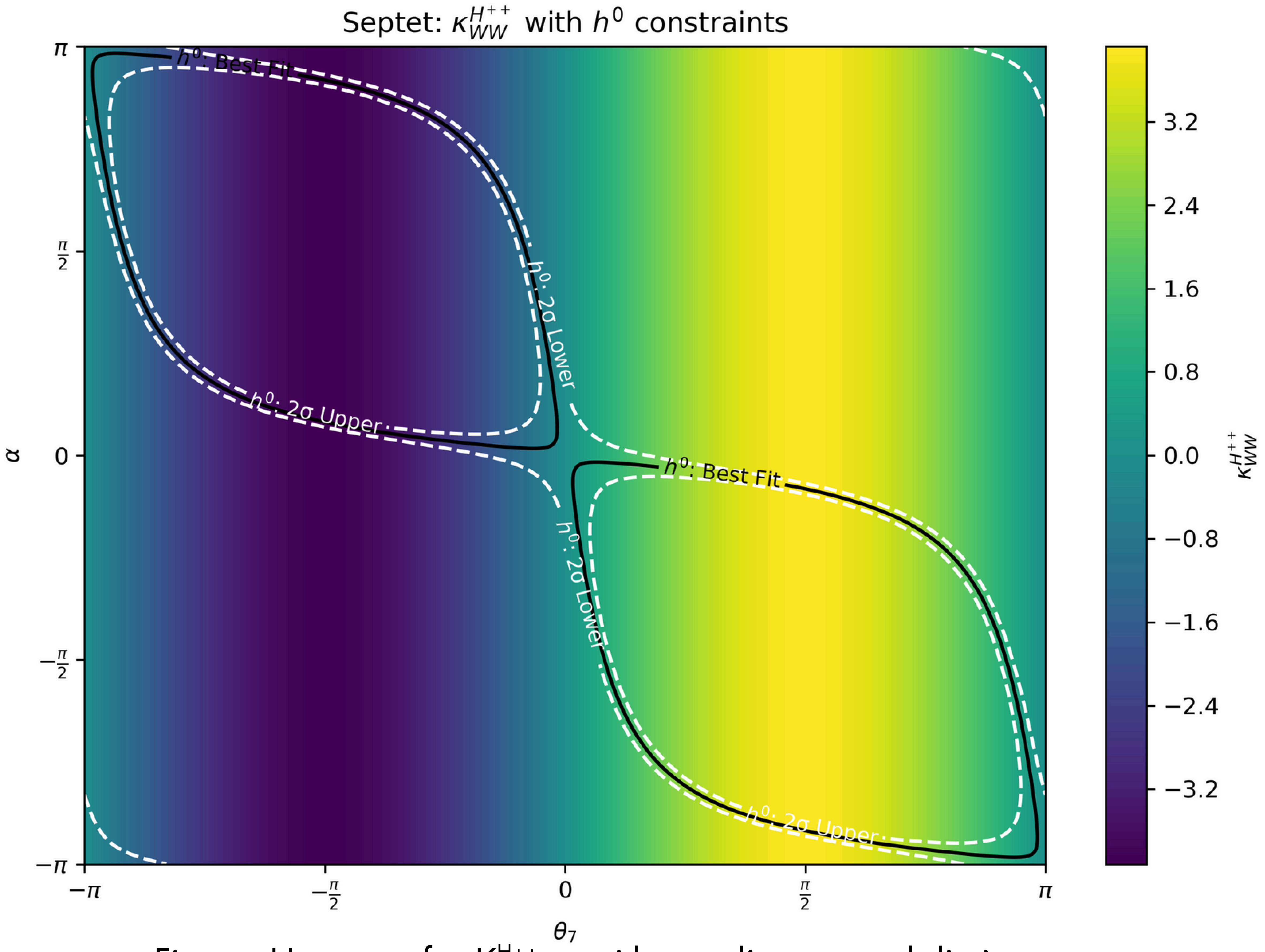
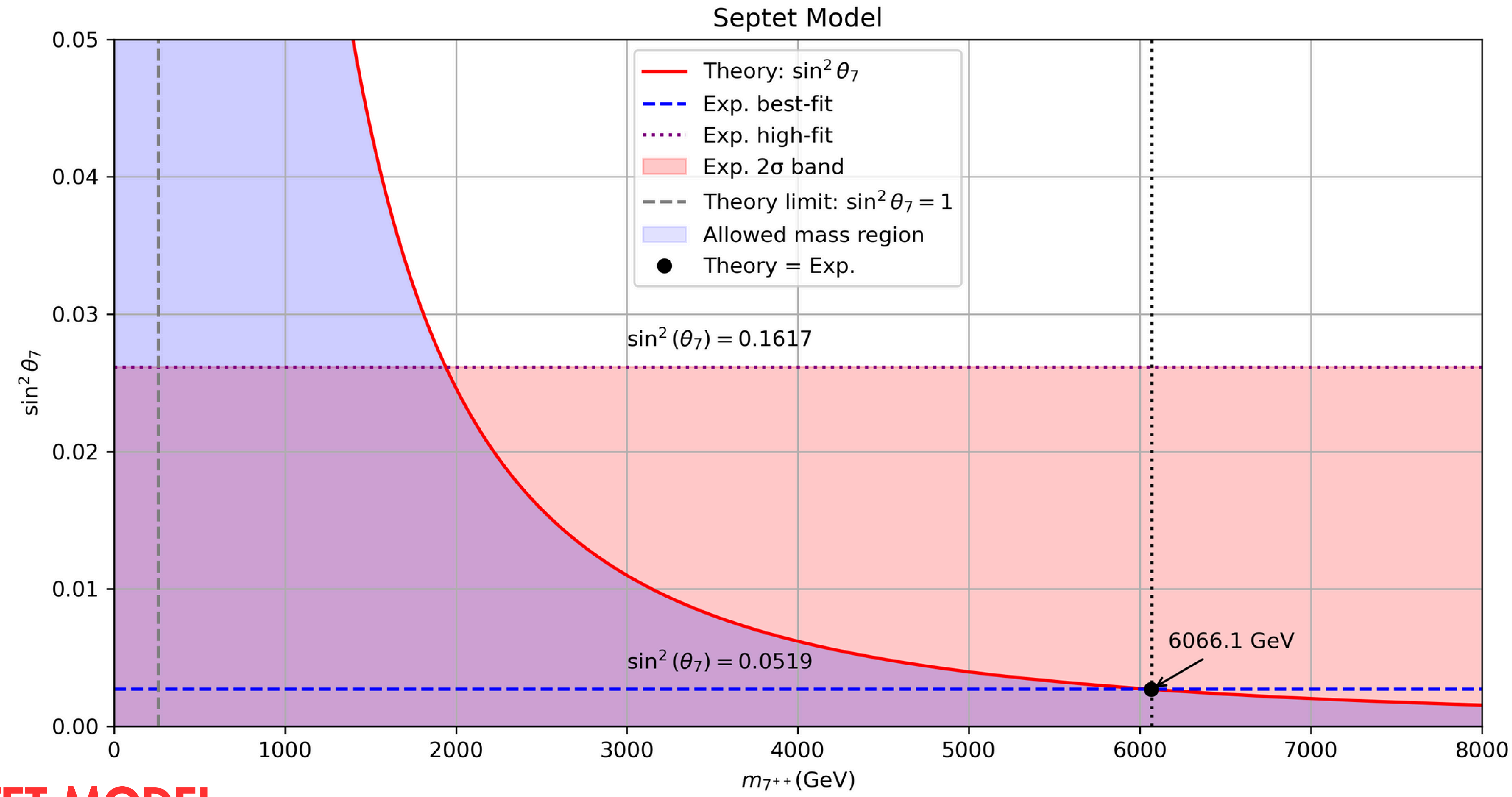


Figure: Heatmap for $K^{H^{++}}_{VV}$ with coupling strength limits

SEPTET MODEL

EXPERIMENTAL LIMITS ON BSM MODELS

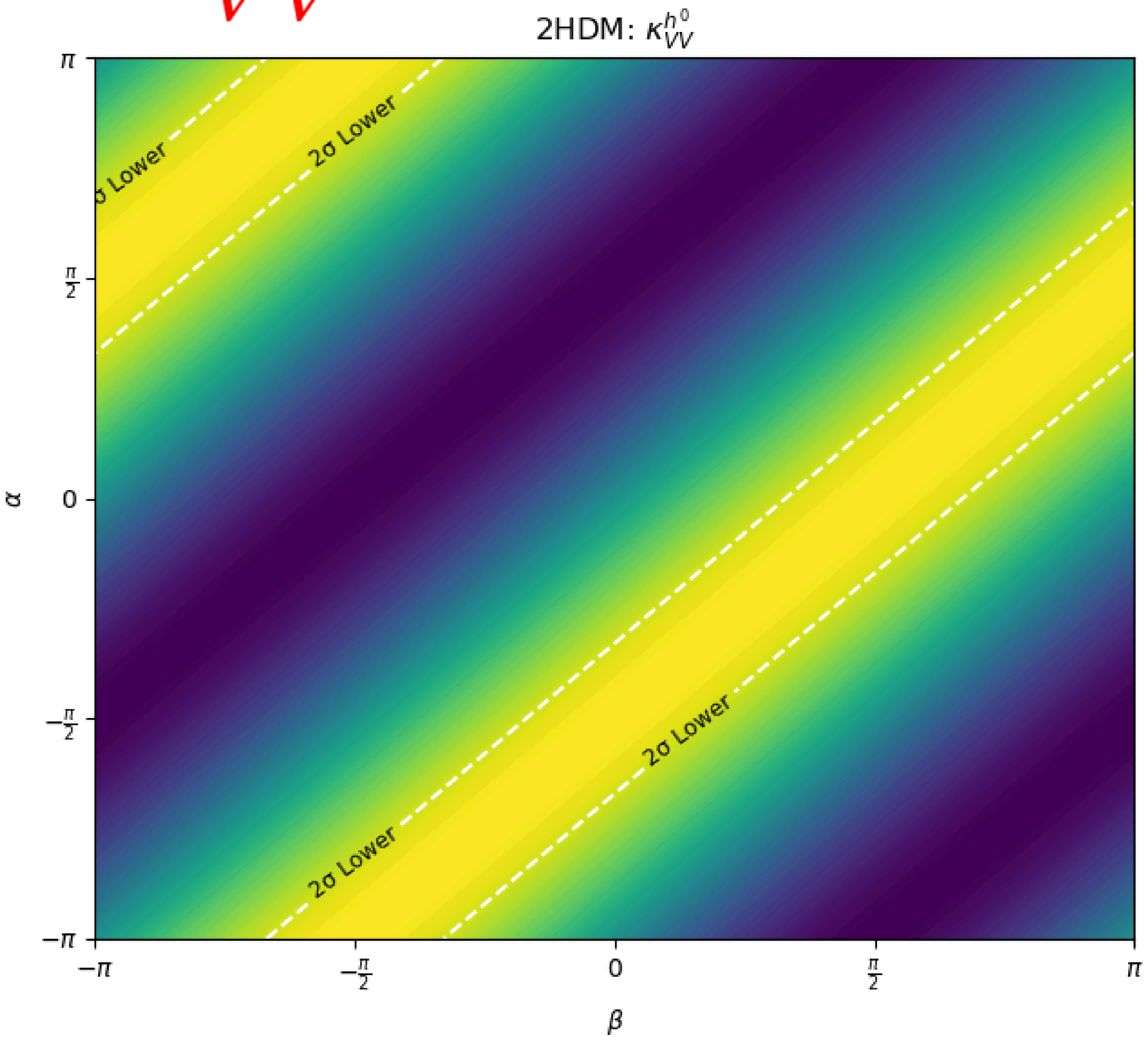


SEPTET MODEL

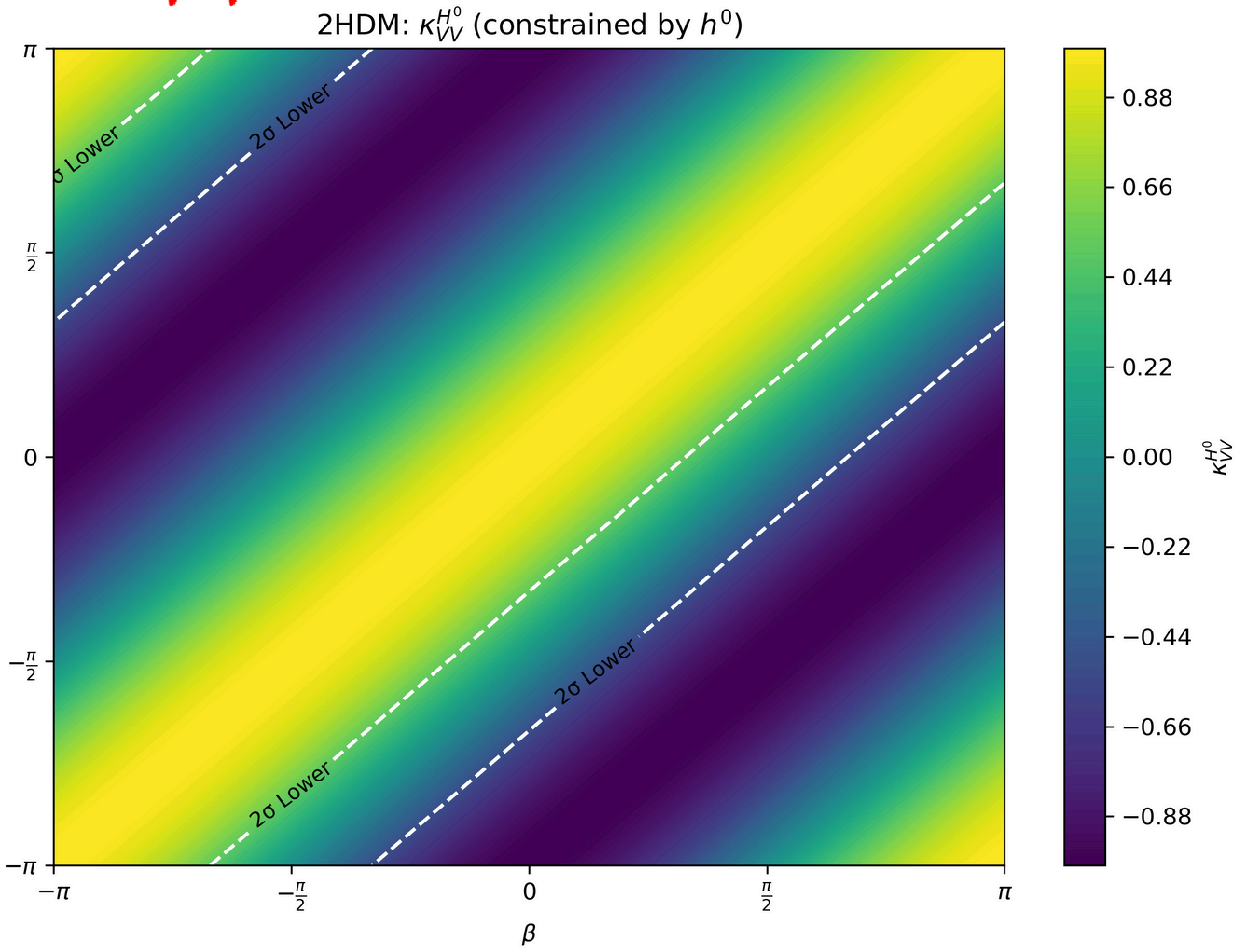
Figure: mass bound with mass estimates from K limit

EXPERIMENTAL LIMITS ON BSM MODELS

$\kappa^{h^0}_{VV}$



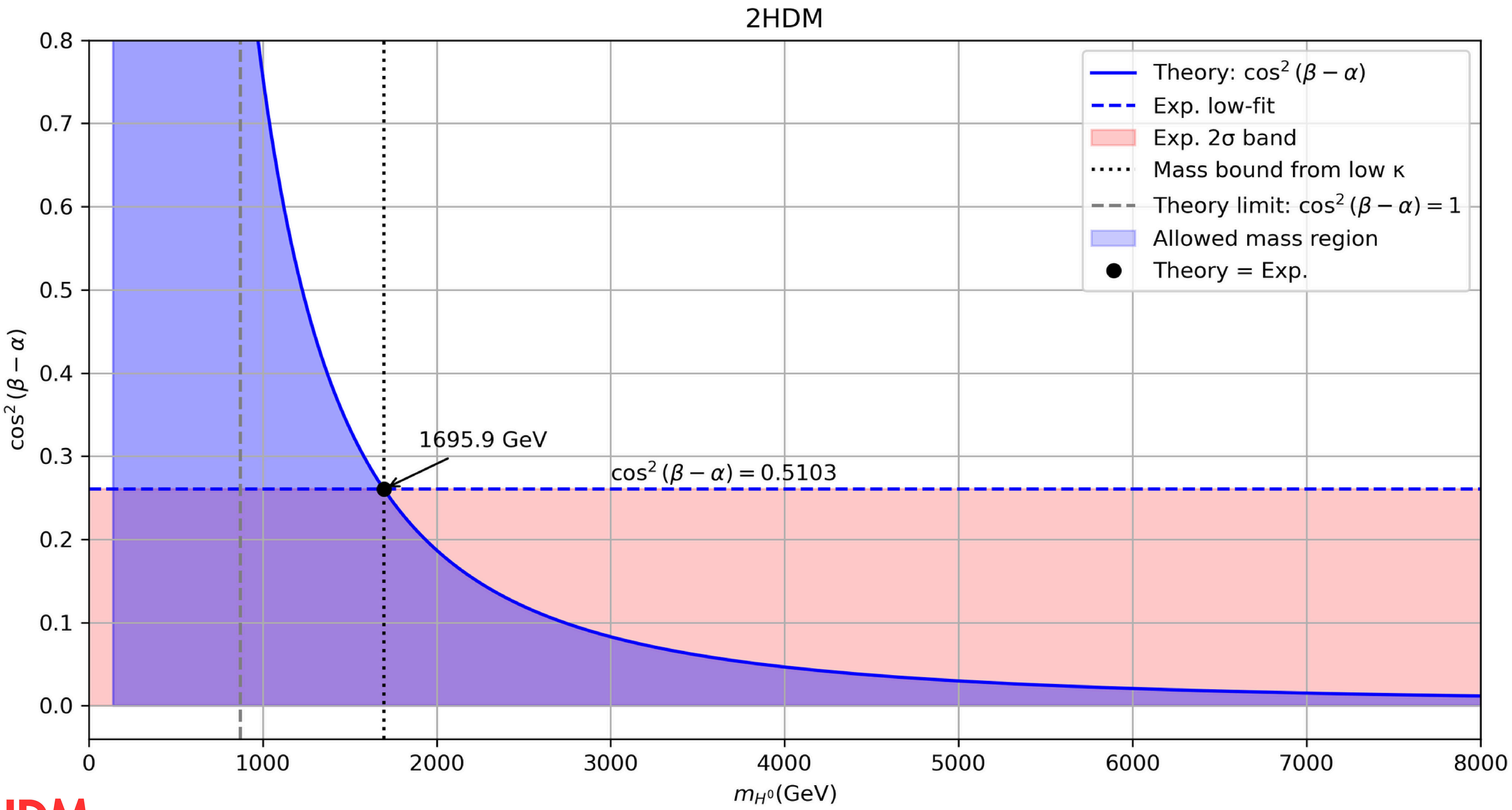
$\kappa^{H^0}_{VV}$



2HDM

Figure: Heatmap for K^{h0}_{VV} and K^{H0}_{VV} with coupling strength limits

EXPERIMENTAL LIMITS ON BSM MODELS

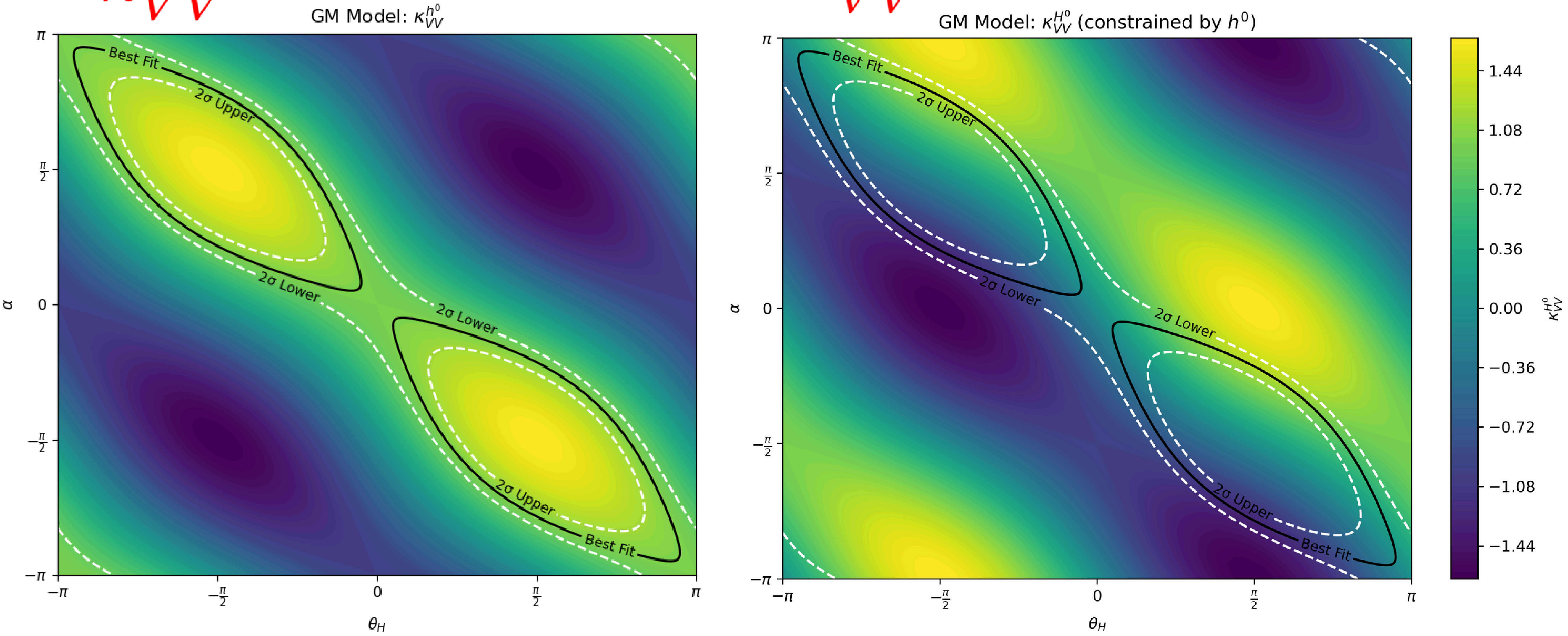


2HDM

Figure: mass bound with mass estimates from K limit

EXPERIMENTAL LIMITS ON BSM MODELS

$\kappa_{VV}^{h^0}$ $\kappa_{VV}^{H^0}$



GM MODEL

Figure: Heatmap for K^{h0}_{VV} and K^{H0}_{VV} with coupling strength limits

EXPERIMENTAL LIMITS ON BSM MODELS

$\kappa_{VV}^{H^{++}}$

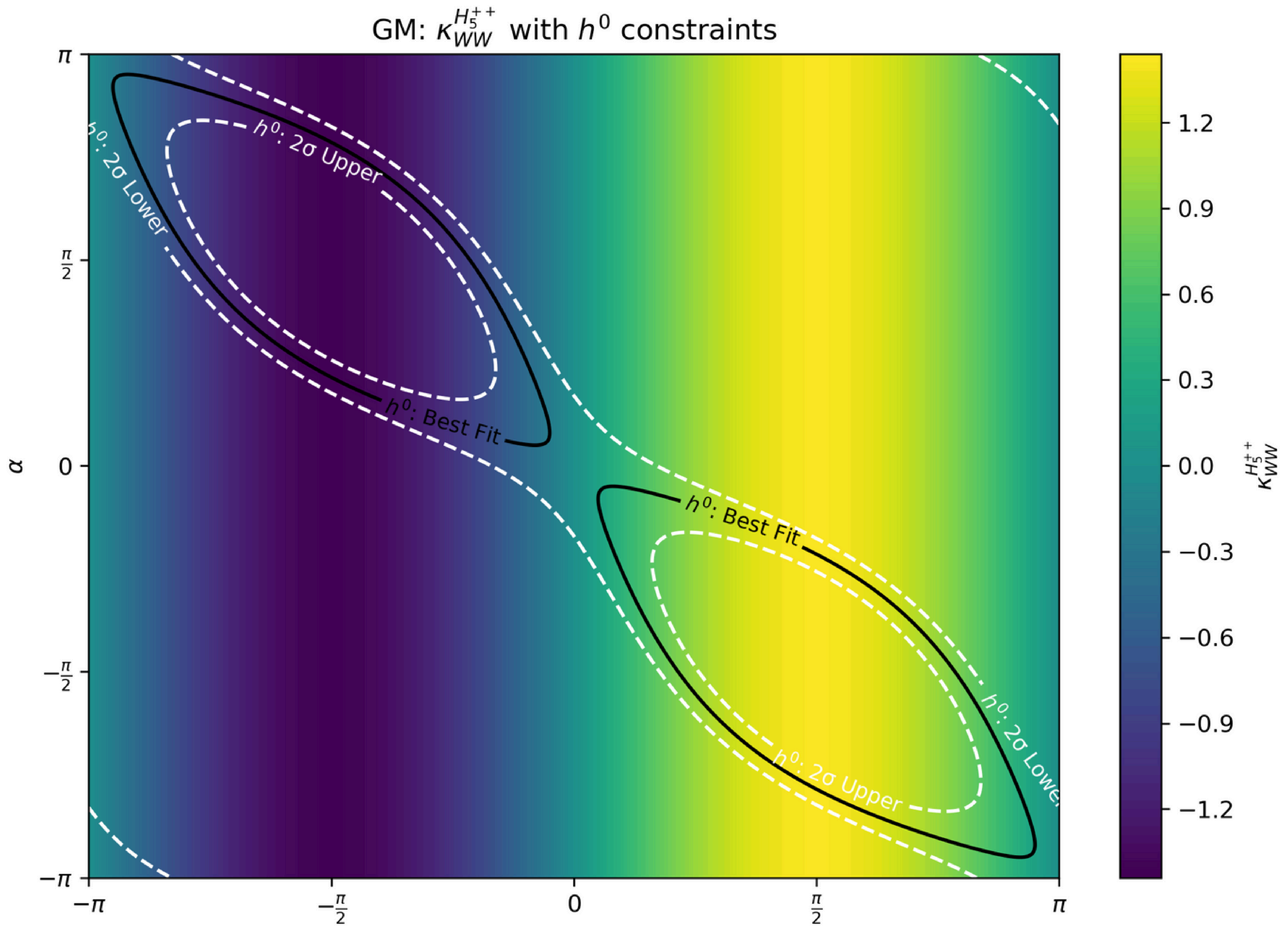
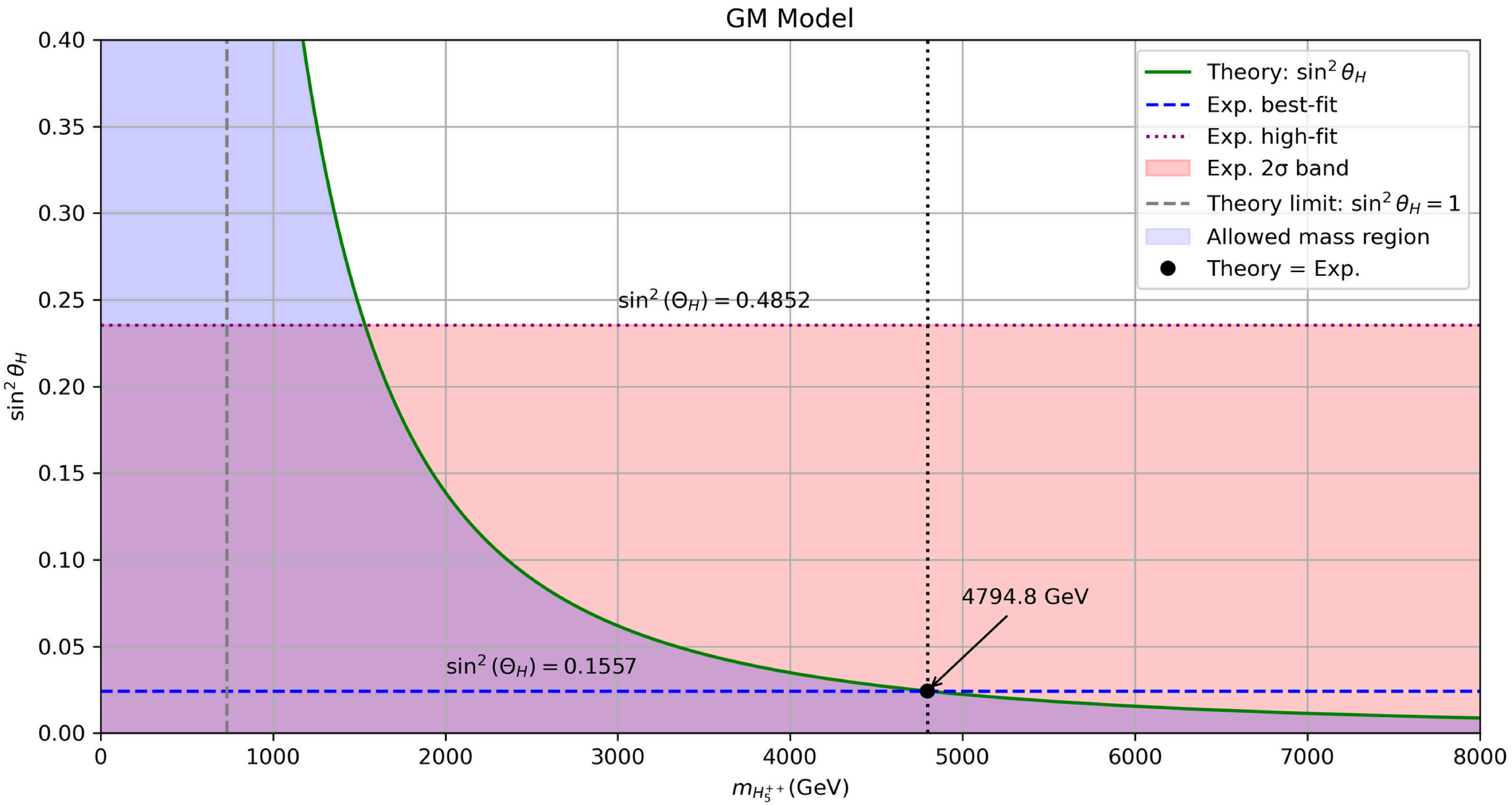


Figure: Heatmap for K^{H++}_{VV} with coupling strength limits

GM MODEL

EXPERIMENTAL LIMITS ON BSM MODELS



GM MODEL

Figure: mass bound with mass estimates from K limit

SUMMARY AND FUTURE PLAN FOR THESIS

SUMMARY AND FUTURE PROSPECTS

Summary: Comparative Analysis of 2HDM, GM, Septet Model

Model	Coupling Expressions	Sum Rule	Mass Bound Constraint
Septet	$\kappa_{VV}^{h^0} = \cos\theta_7 \cos\alpha - 4\sin\theta_7 \sin\alpha$ $\kappa_{VV}^{H^0} = \cos\theta_7 \sin\alpha + 4\sin\theta_7 \cos\alpha$ $\kappa_{VV}^{H^{++}} = \sqrt{15}\sin\theta_7$	$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 - \left(\kappa_{VV}^{H^{++}}\right)^2 = 1$	$\sin^2\theta_7 \lesssim \frac{(260\,\mathrm{GeV})^2}{m_{H^{++}}^2}$
GM	$\kappa_{VV}^{h^0} = \cos\theta_H \cos\alpha - \sqrt{\frac{8}{3}}\sin\theta_H \sin\alpha$ $\kappa_{VV}^{H^0} = \cos\theta_H \sin\alpha + \sqrt{\frac{8}{3}}\sin\theta_H \cos\alpha$ $\kappa_{VV}^{H_5^{++}} = \sqrt{2}\sin\theta_H$	$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 + \left(\kappa_{VV}^{H_5^0}\right)^2 - \left(\kappa_{VV}^{H_5^{++}}\right) = 1$	$\sin^2\theta_H \lesssim \frac{(734\,\mathrm{GeV})^2}{m_5^2}$
2HDM	$\kappa_{VV}^{h^0} = \sin(\beta - \alpha)$ $\kappa_{VV}^{H^0} = \cos(\beta - \alpha)$	$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 = 1$	$\cos^2(\beta - \alpha) \lesssim \frac{(880\,\mathrm{GeV})^2}{m_{H^0}^2}$

SUMMARY AND FUTURE PROSPECTS

Mixing angle and Mass estimates from K (Coupling Modifier) limits

Model	Mixing Values [Best, Low, High]	Mass Bound (TeV) [Low, High]	Mass Bound (TeV) [Best]
GM	[0.02, 0.00, 0.23]	[1.53, 7474.16]	[4.80]
Septet	[0.0027, 0.00, 0.0262]	[1.94, 3151.38]	[6.07]
2HDM	[0.00, 0.26, 0.00]	[1.70, 8630.42]	[1.70] (Low bound)

Heavy Higgs masses

Future Prospects:

- Deriving the Cross Section(σ_{BSM}) using Higgs Tools [6] from K experimental limits
- Differentiating the different model parameters

THANK YOU

APPENDIX: SEPTET MODEL

The Electroweak Lagrangian for Septet :

$$\mathcal{L} \supset (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) + (\mathcal{D}_\mu X)^\dagger (\mathcal{D}^\mu X),$$

Replacement Rules and physical fields

$$h^0 = \cos\alpha\, \phi^{0,r} - \sin\alpha\, \chi^{0,r},$$
$$H^0 = \sin\alpha\, \phi^{0,r} + \cos\alpha\, \chi^{0,r}.$$

$$G^0 = \cos\theta_7\, \phi^{0,i} + \sin\theta_7\, \chi^{0,i},$$
$$A^0 = -\sin\theta_7\, \phi^{0,i} + \cos\theta_7\, \chi^{0,i},$$

$$H^{++} \equiv \chi^{+2}, \quad \chi^{+3}, \quad \chi^{+4}, \quad \chi^{+5},$$

Covariant Derivative

$$\mathcal{D}_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - \frac{ie}{s_W c_W} Z_\mu (T^3 - s_W^2 Q) - ieA_\mu Q.$$

$$G^+ = \cos\theta_7\, \phi^+ + \sin\theta_7 \left(\sqrt{\frac{5}{8}}\chi^{+1} - \sqrt{\frac{3}{8}}(\chi^{-1})^*\right),$$

$$H_f^+ = -\sin\theta_7\, \phi^+ + \cos\theta_7 \left(\sqrt{\frac{5}{8}}\chi^{+1} - \sqrt{\frac{3}{8}}(\chi^{-1})^*\right),$$

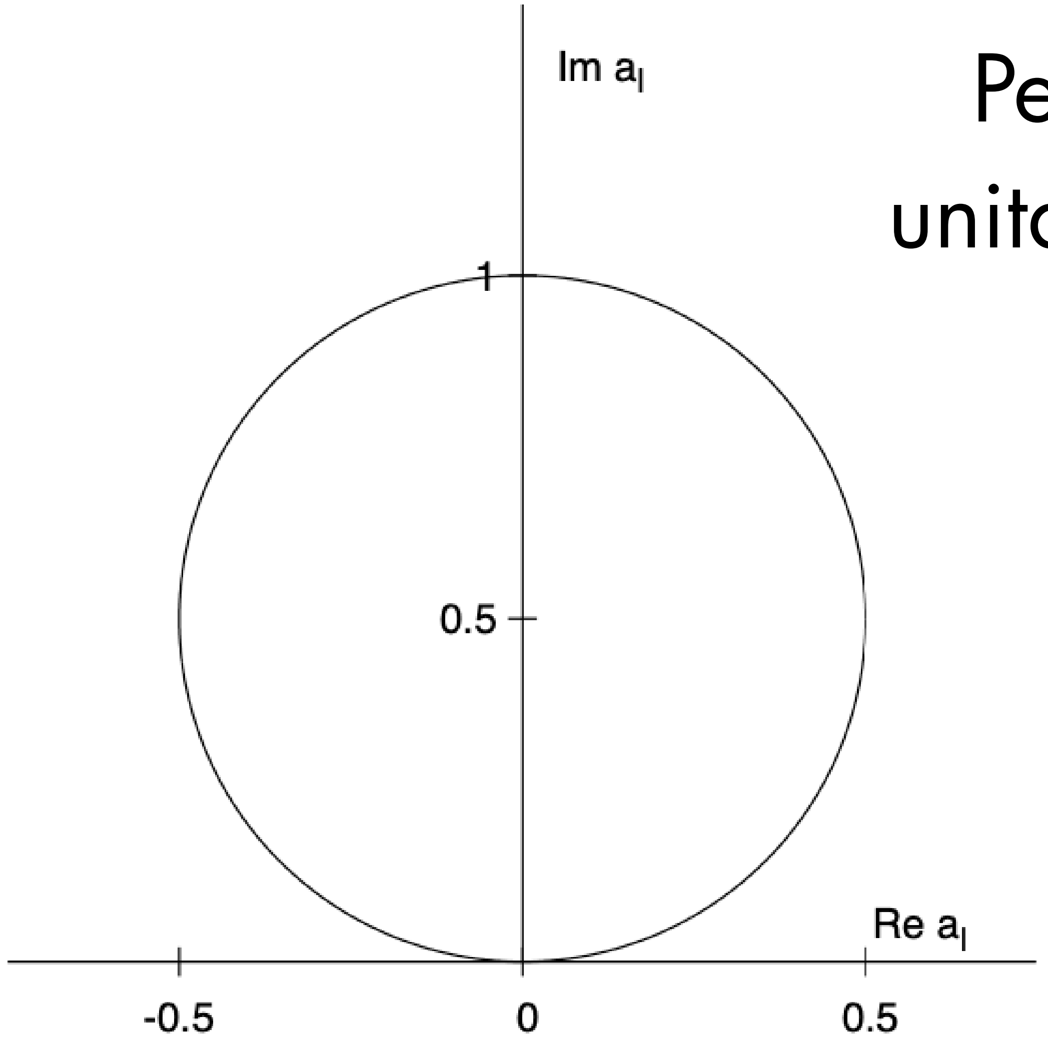
$$H_V^+ = \sqrt{\frac{3}{8}}\chi^{+1} + \sqrt{\frac{5}{8}}(\chi^{-1})^*.$$

$$H_1^+ = \cos\gamma\, H_f^+ - \sin\gamma\, H_V^+,$$
$$H_2^+ = \sin\gamma\, H_f^+ + \cos\gamma\, H_V^+;$$

APPENDIX: PERTURBATIVE UNITARITY

Unitarity Conservation with Higgs

$$\mathcal{M} = 16\pi \sum_{l=0}^{\infty} (2l+1)\, a_l(|\vec{p}|)\, P_l \cos\theta,$$



Perturbative unitarity bounds

$$|a_l| \le 1; \quad |Re(a_l)| \le \frac{1}{2}; \quad 0 \le |Im(a_l)| \le 1.$$

Mass Bound calculation:

$$(\kappa_V^h)^2 + (\kappa_V^H)^2 - (\kappa_{WW}^{H^{++}})^2 = 1,$$

$$\kappa_{WW}^{H} = 0$$

$$(\kappa_V^h)^2\big|_{\max} = 1 + (\kappa_{WW}^{H^{++}})^2.$$

$$2(\kappa_V^h)^2 m_h^2 + 2(\kappa_V^H)^2 m_H^2 + (\kappa_{WW}^{H^{++}})^2 m_{H^{++}}^2 \le 8\pi v^2.$$

$$(\kappa_{WW}^{H^{++}})^2 \le \frac{8\pi v^2 - 2m_h^2}{m_{H^{++}}^2 + 2m_h^2}.$$

APPENDIX: 2HDM (1)

Two Complex SU(2)L Doublets : Φ_i (i = 1,2)

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}.$$

parametrization

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix},$$

$$\Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix},$$

$$\tan\beta = \frac{v_2}{v_1},$$

$$\tan 2\alpha = \frac{2\left(m_{12}^2 - \lambda_{345} v^2 s_\beta c_\beta\right)}{m_{12}^2\left(c_\beta - s_\beta\right) - v^2\left(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2\right)},$$

Replacement Rules and physical fields

CP-even mixing angle

$$R_\alpha = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$$

CP-odd & Charged mixing angle

$$R_\beta = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}$$

$$\rho_1 = H^0\cos\alpha - h^0\sin\alpha,$$
$$\rho_2 = H^0\sin\alpha + h^0\cos\alpha,$$
$$\eta_1 = G^0\cos\beta - A^0\sin\beta,$$
$$\eta_2 = G^0\sin\beta + A^0\cos\beta,$$
$$\omega_1^\pm = G^\pm\cos\beta - H^\pm\sin\beta,$$
$$\omega_2^\pm = G^\pm\sin\beta + H^\pm\cos\beta.$$

where $c_\alpha = \cos\alpha, s_\alpha = \sin\alpha$

$c_\beta = \cos\beta, s_\beta = \sin\beta.$

The Electroweak Lagrangian for 2HDM :

$$\mathscr{L} = \left(D_\mu\Phi_1\right)^\dagger\left(D_\mu\Phi_1\right) + \left(D_\mu\Phi_2\right)^\dagger\left(D_\mu\Phi_2\right).$$

Covariant Derivative $D_\mu = \partial_\mu - ig\frac{\sigma_a}{2}W_\mu^a - ig'\frac{Y}{2}B_\mu$

$$M_W^2 = \frac{1}{4}g^2v^2,$$
$$M_Z^2 = \frac{1}{4}(g^2+g'^2)v^2,$$
$$M_\gamma^2 = 0.$$

2HDM Potential

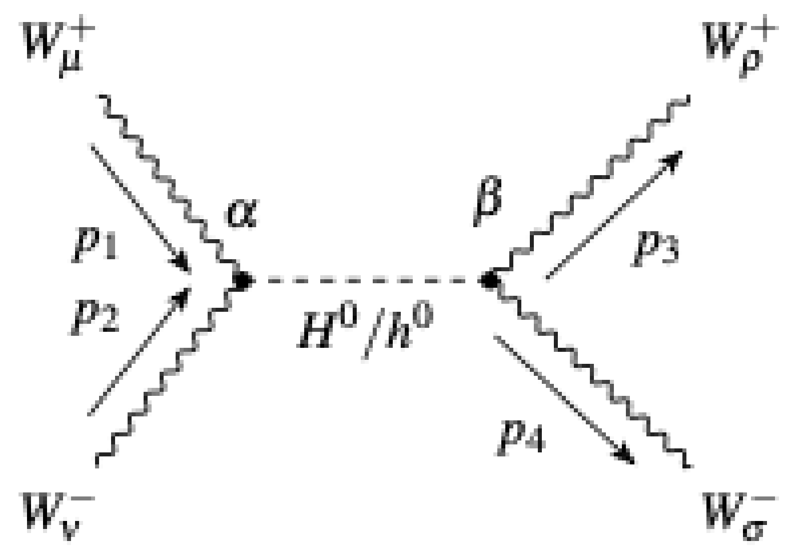
$$\begin{aligned} V_{2HDM}(\Phi_1,\Phi_2) = & m_{11}^2\left(\Phi_1^\dagger\Phi_1\right) + m_{22}^2\left(\Phi_2^\dagger\Phi_2\right) - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) \\ & + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) \\ & + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{\lambda_5}{2}\left(\left(\Phi_1^\dagger\Phi_2\right)^2 + \left(\Phi_2^\dagger\Phi_1\right)^2\right) \end{aligned}$$

APPENDIX: 2HDM (2)

High Energy Vector Boson Scattering

1. $W^+W^- \xrightarrow{h^0/H^0} W^+W^-$
2. $Z\,Z \xrightarrow{h^0/H^0} Z\,Z$
3. $W^\pm Z \xrightarrow{h^0/H^0/H^+_5} W^\pm Z$

Eg. $W^+W^- \xrightarrow{h^0/H^0} W^+W^-$



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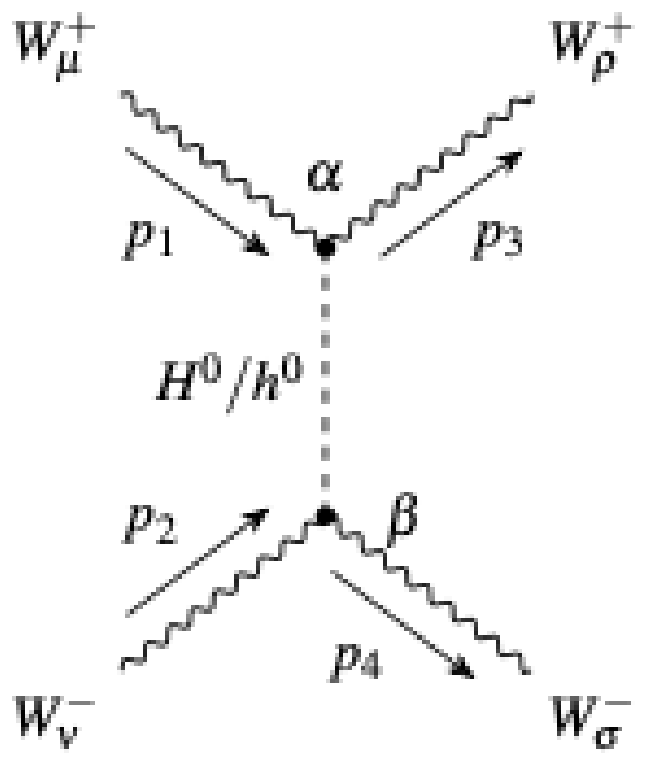

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SM: h_{SM} = 125 Gev

Matrix Element and Sum Rule

$$i\mathcal{M} = -i\frac{g^2}{4m_W^2}\left[\left(\kappa_{WW}^{h^0}\right)^2\left(\frac{s^2}{s-m_{h^0}^2}+\frac{t^2}{t-m_{h^0}^2}\right)+\left(\kappa_{WW}^{H^0}\right)^2\left(\frac{s^2}{s-m_{h^0}^2}+\frac{t^2}{t-m_{h^0}^2}\right)\right],$$

$$i\mathcal{M} \simeq -\frac{i}{v^2}\left[\left(\underline{\left(\kappa_{WW}^{h^0}\right)^2+\left(\kappa_{WW}^{H^0}\right)^2}\right)(s+t)+2\left(\kappa_{WW}^{h^0}\right)^2 m_{h^0}^2+2\left(\kappa_{WW}^{H^0}\right)^2 m_{H^0}^2+\ldots\right].$$

1st Sum Rule:

$$\boxed{\begin{gathered}\left(\kappa_{WW}^{h^0}\right)^2+\left(\kappa_{WW}^{H^0}\right)^2=1, \quad \text{or} \\ \sum_i \left(g_{h_iWW}\right)^2=\left(g_{H_{SM}WW}\right)^2, \quad h_i=h^0,H^0\end{gathered}}$$

Theoretical Check:
SUCCESS!!

$$\sin^2(\beta-\alpha)+\cos^2(\beta-\alpha)=1$$

APPENDIX: 2HDM

Mass Matrices from Potential

$$\frac{\partial V_{2HDM}}{\partial \Phi_1}|_{\langle\Phi_1\rangle,\langle\Phi_2\rangle} = 0, \quad \frac{\partial V_{2HDM}}{\partial \Phi_2}|_{\langle\Phi_1\rangle,\langle\Phi_2\rangle} = 0,$$

$$V|_{bilinear} = \frac{1}{2}\begin{pmatrix}\rho_1 & \rho_2\end{pmatrix} M_\rho^2 \begin{pmatrix}\rho_1 \\ \rho_2\end{pmatrix} + \frac{1}{2}\begin{pmatrix}\eta_1 & \eta_2\end{pmatrix} M_\eta^2 \begin{pmatrix}\eta_1 \\ \eta_2\end{pmatrix} + \frac{1}{2}\begin{pmatrix}\omega_1^+ & \omega_2^+\end{pmatrix} M_\omega^2 \begin{pmatrix}\omega_1^- \\ \omega_2^-\end{pmatrix}.$$

CP-even mixing angle $R_\alpha = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$ **CP-odd & Charged mixing angle** $R_\beta = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}$

Mass matrix (non-diagonal) for CP-Even Higgs(Scalar) [2]:

$$\begin{aligned} M^2_{\rho_{11}} &= \frac{\lambda_1 v_1^3 + m_{12}^2 v_2}{v_1} \\ M^2_{\rho_{22}} &= \frac{\lambda_2 v_2^3 + m_{12}^2 v_1}{v_2} \\ M^2_{\rho_{12}} &= (\lambda_3+\lambda_4+\lambda_5)v_1 v_2 - m_{12}^2 \end{aligned} \quad \longleftrightarrow \quad M_\rho^2 = \begin{pmatrix} M^2_{\rho_{11}} & M^2_{\rho_{12}} \\ M^2_{\rho_{12}} & M^2_{\rho_{22}} \end{pmatrix},$$

where $c_\alpha = \cos\alpha, s_\alpha = \sin\alpha, c_\beta = \cos\beta, s_\beta = \sin\beta$.

Mass matrix (non-diagonal) for CP-Odd Higgs (Pseudoscalar):

$$\begin{aligned} M^2_{\eta_{11}} &= (-\lambda_5 v_1 v_2 + m_{12}^2)\frac{v_2}{v_1} \\ M^2_{\eta_{22}} &= (-\lambda_5 v_1 v_2 + m_{12}^2)\frac{v_1}{v_2} \\ M^2_{\eta_{12}} &= \lambda_5 v_1 v_2 - m_{12}^2 \end{aligned} \quad \longleftrightarrow \quad M_\eta^2 = \begin{pmatrix} M^2_{\eta_{11}} & M^2_{\eta_{12}} \\ M^2_{\rho_{12}} & M^2_{\rho_{22}} \end{pmatrix},$$

Mass matrix (non-diagonal) for Charged Higgs:

$$\begin{aligned} M^2_{\omega_{11}} &= (2m_{12}^2 - (\lambda_4+\lambda_5)v_1 v_2)\frac{v_2}{2v_1} \\ M^2_{\omega_{22}} &= (2m_{12}^2 - (\lambda_4+\lambda_5)v_1 v_2)\frac{v_1}{2v_2} \\ M^2_{\omega_{12}} &= \frac{1}{2}(\lambda_4+\lambda_5)v_1 v_2 - m_{12}^2 \end{aligned} \quad \longleftrightarrow \quad M_\omega^2 = \begin{pmatrix} M^2_{\omega_{11}} & M^2_{\omega_{12}} \\ M^2_{\omega_{12}} & M^2_{\omega_{22}} \end{pmatrix}.$$

APPENDIX: GM(1)

REPLACEMENT RULES:

$\sin\Theta_H = \sqrt{8}v_\chi/v$ and $\cos\Theta_H = v_\phi/v$,

CP - even neutral states:

$$h^0 = \cos\alpha\phi^r - \sin\alpha\left(\frac{1}{\sqrt{3}}\xi^r + \sqrt{\frac{2}{3}}\chi^r\right)$$

$$H^0 = \sin\alpha\phi^r + \cos\alpha\left(\frac{1}{\sqrt{3}}\xi^r + \sqrt{\frac{2}{3}}\chi^r\right)$$

$$H_5^0 = \sqrt{\frac{2}{3}}\xi^r - \frac{1}{\sqrt{3}}\chi^r.$$

CP -odd neutral states

$$G^0 = \cos\Theta_H\phi^i - \sin\Theta_H\chi^i,$$

$$H_3^0 = -\sin\Theta_H\phi^i + \cos\Theta_H\chi^i.$$

Singly charged states:

$$G^+ = \cos\Theta_H\phi^+ + \sin\Theta_H\frac{(\chi^+ + \xi^+)}{\sqrt{2}},$$

$$H_3^+ = -\sin\Theta_H\phi^+ + \cos\Theta_H\frac{(\chi^+ + \xi^+)}{\sqrt{2}},$$

$$H_5^+ = \frac{(\chi^+ - \xi^+)}{\sqrt{2}}.$$

Doubly charged states:

$$H_5^{++} = \chi^{++}$$

The Electroweak Lagrangian for GM :

$$\mathscr{L} = (D^\mu\phi)^\dagger(D_\mu\phi) + (D^\mu\chi)^\dagger(D_\mu\chi) + \frac{1}{2}(D^\mu\xi)^\dagger(D_\mu\xi) - V_{GM}(\Phi,\Delta),$$

This custodial symmetry prevents mixing between states that transform in different representations of SU(2)c.

$$\langle\Phi\rangle = \begin{pmatrix} v_\phi/\sqrt{2} & 0 \\ 0 & v_\phi/\sqrt{2} \end{pmatrix}$$

H_3^+ does not mix with H_5^+

Couples to fermions but not vector boson
Degenerate mass: m_3

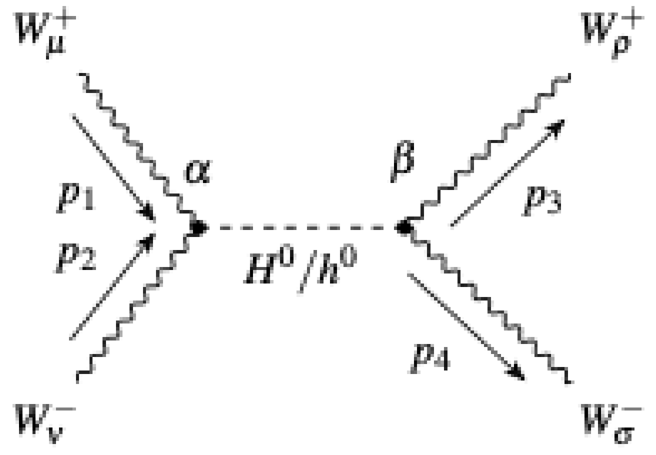
Couples to vector boson but not fermions
Degenerate mass: m_5

$$\langle X\rangle = \begin{pmatrix} v_\chi & 0 & 0 \\ 0 & v_\chi & 0 \\ 0 & 0 & v_\chi \end{pmatrix}$$

APPENDIX: GM(2)

SM: h_{SM} = 125 Gev

Eg. $W^+W^- \xrightarrow{h^0/H^0/H^0_5/H_5^{++}} W^+W^-$



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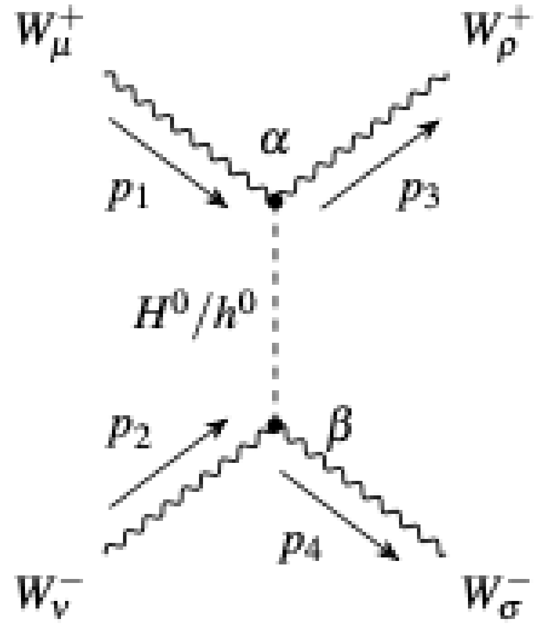

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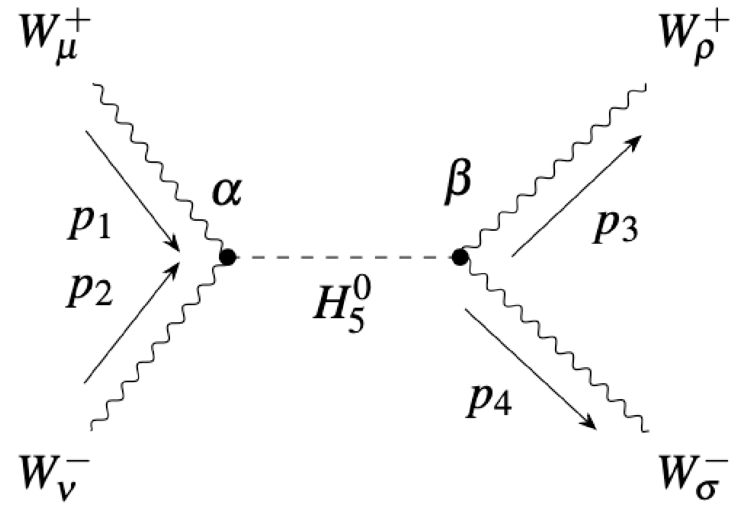

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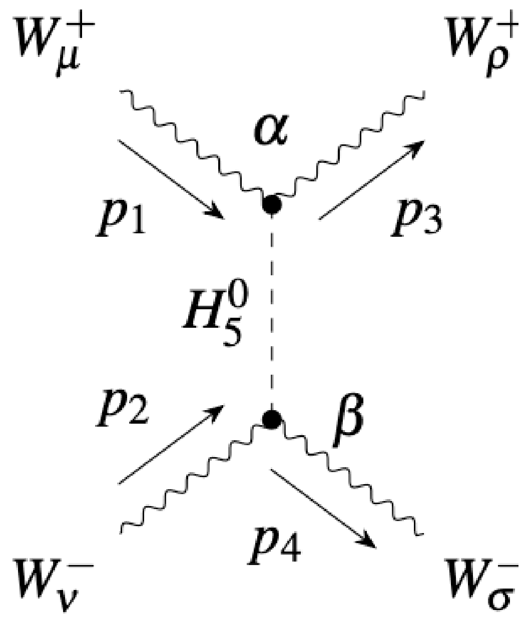

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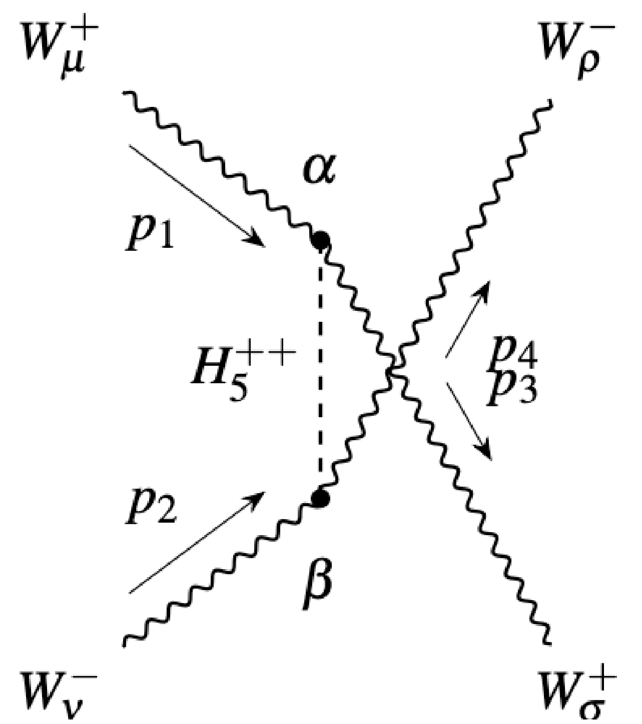

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Matrix Element and Sum Rule:

$$\mathscr{M} = -\frac{1}{v^2}\left[\left(\left(\kappa_{WW}^{h^0}\right)^2 + \left(\kappa_{WW}^{H^0}\right)^2 + \left(\kappa_{WW}^{H_5^0}\right)^2 - \left(\kappa_{WW}^{H_5^{++}}\right)^2\right)(s+t) + 2\left(\kappa_{WW}^{h^0}\right)^2 m_{h^0}^2 + 2\left(\kappa_{WW}^{H^0}\right)^2 m_{H^0}^2 + 2\left(\kappa_{WW}^{H_5^0}\right)^2 m_5^2 + \left(\kappa_{WW}^{H_5^{++}}\right)^2 m_5^2\right],$$

1[st] Sum Rule:

$$\left(\kappa_{WW}^{h^0}\right)^2 + \left(\kappa_{WW}^{H^0}\right)^2 + \left(\kappa_{WW}^{H_5^0}\right)^2 - \left(\kappa_{WW}^{H_5^{++}}\right)^2 = 1, \quad \text{or}$$

$$\sum_i g^2_{h_i W^+W^-} - g_{h^{--}W^+W^+} g_{h^{++}W^-W^-} = g^2_{H_{SM}WW},$$

$$c_H^2 + \frac{8}{3}s_H^2 + \frac{1}{3}s_H^2 - 2s_H^2 = c_H^2 + s_H^2 = 1,$$

Theoretical Check: SUCCESS!!

APPENDIX: Mass estimates using Experimental limits

Mass Bound Calculations

Septet Model

From the sum rule:

$$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 + \left(\kappa_{WW}^{H^{++}}\right)^2 = 1 \Rightarrow \left(\kappa_{WW}^{H^{++}}\right)^2 = \kappa^2 - 1$$

Using: $\kappa_{\text{best}} = 1.02$

$$\left(\kappa_{WW}^{H^{++}}\right)^2 = 0.0404 \Rightarrow \sin^2\theta_7 = \frac{0.0404}{15} \approx 0.0027$$

Perturbative bound:

$$\sin^2\theta_7 \leq \frac{8\pi v^2 - 2m_h^2}{15(m_7^2 + 2m_h^2)} \Rightarrow m_7 \leq \sqrt{\frac{8\pi v^2 - 2m_h^2 - 30m_h^2\sin^2\theta_7}{15\sin^2\theta_7}}$$

Evaluating:

$$m_7 \approx 6088 \text{ GeV}$$

APPENDIX: Mass estimates using Experimental limits

GM Model

From the sum rule:

$$\left(\kappa_{VV}^{h^0}\right)^2 + \left(\kappa_{VV}^{H^0}\right)^2 + \left(\kappa_{WW}^{H^{++}}\right)^2 = 1 \Rightarrow \left(\kappa_{WW}^{H^{++}}\right)^2 = \frac{6}{5}(\kappa^2 - 1)$$

Using: $\kappa_{\text{best}} = 1.02$

$$\left(\kappa_{WW}^{H^{++}}\right)^2 = \frac{6}{5}(1.02^2 - 1) = 0.048 \Rightarrow \sin^2\theta_H = \frac{1}{2} \cdot 0.048 = 0.024$$

From perturbative unitarity:

$$\sin^2\theta_H \leq \frac{3(8\pi v^2 - 2m_h^2)}{2(4m_5^2 + 5m_h^2)} \Rightarrow m_5 \leq \sqrt{\frac{3(8\pi v^2 - 2m_h^2) - 10m_h^2\sin^2\theta_H}{8\sin^2\theta_H}}$$

Evaluating:

$$m_5 \approx 4800\ \text{GeV for}\ \sin^2\theta_H = 0.024$$

APPENDIX: Mass estimates using Experimental limits

2HDM

From:

$$\kappa^2 = \sin^2(\beta - \alpha) \Rightarrow \cos^2(\beta - \alpha) = 1 - \kappa^2$$

Using: $\kappa_{\text{low}} = 0.86 \Rightarrow \cos^2(\beta - \alpha) = 1 - 0.86^2 = 0.26$
Then the theoretical mass bound:

$$\cos^2(\beta - \alpha) \leq \frac{4\pi v^2 - m_h^2}{m_H^2 - m_h^2} \Rightarrow m_H \leq \sqrt{\frac{(4\pi v^2 - m_h^2) + m_h^2 \cos^2(\beta - \alpha)}{\cos^2(\beta - \alpha)}}$$

Evaluating:

$$m_H \approx 1695 \text{ GeV}$$

Unphysical Regions and Their Exclusion

- **GM Model:** The low bound on $\kappa_{WW}^{H^{++}}$ gives $(\kappa^2 - 1) < 0$, which is unphysical as $\sin^2\theta_H$ becomes negative. Hence, we take $\sin^2\theta_H = 0$ as the conservative lower limit.
- **Septet Model:** Similarly, the 2σ lower bound leads to negative $\sin^2\theta_7$, which is unphysical, and we set it to zero.
- **2HDM:** The best-fit and upper values of κ exceed 1, leading to negative values of $\cos^2(\beta - \alpha)$ and hence undefined behavior. We only retain the 2σ lower bound.

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