Colloquium Bachelor Thesis

„Parameter determination for dark matter candidates in the MSSM“

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Universität Hamburg
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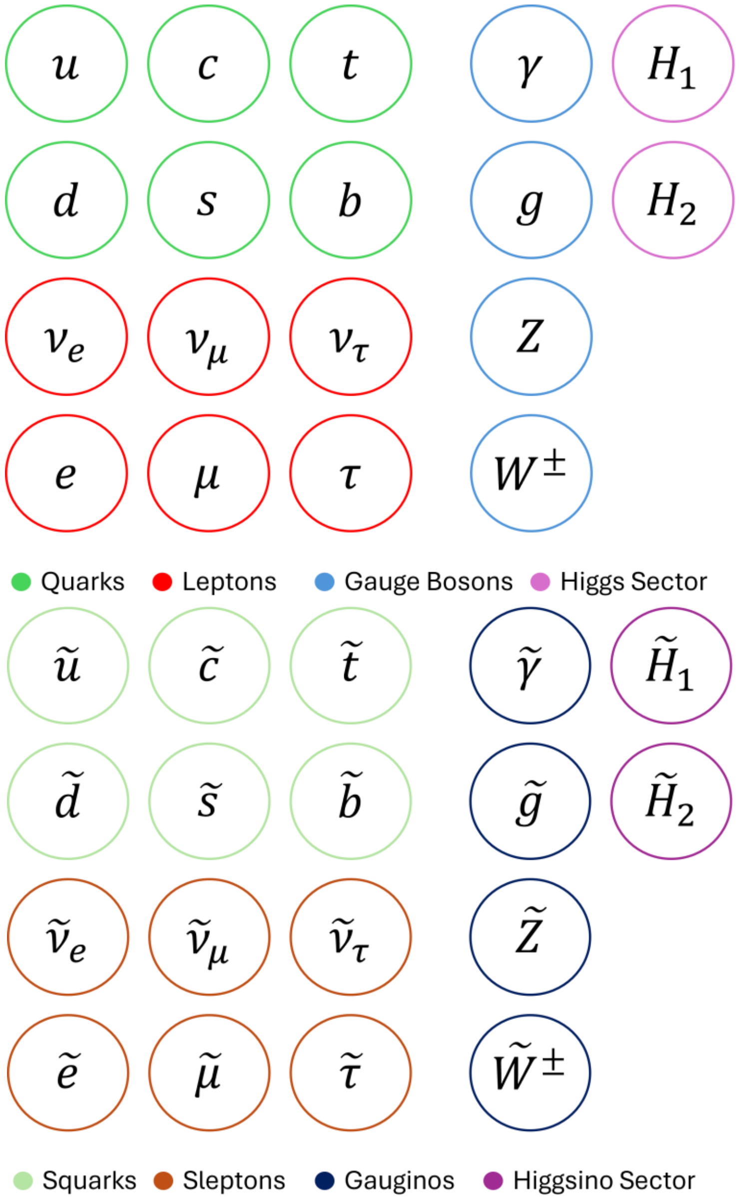
Goals

- Existing code for this endeavor from Florian Lika [1] is implemented
- Study of the reconstruction of supersymmetric parameters and their accuracy
- Employ measurements of the future possible Linear Collider ILC
- Calculation of dark matter relic density $\Omega\hbar^2$ and its accuracy
- Optimize the code in terms of accuracy, application on current excesses in CMS [2] and ATLAS [3]

The Minimal Supersymmetric Standard Model (MSSM)

- Minimal Supersymmetric Standard Model (MSSM) is assumed
- Fermions get bosonic partners (prefix „s“), Bosons get fermionic partners (suffix „ino“)
- MSSM adds 105 new parameters to existing SM parameters
- Of special interest are:
  1. M_1 (Bino mass parameter)
  2. M_2 (Wino mass parameter)
  3. μ (Higgsino mass parameter)
- $\tan\beta$ (ratio of vacuum expactation values of both Higgs-Doublets)



[4]

Mass matrices in the MSSM

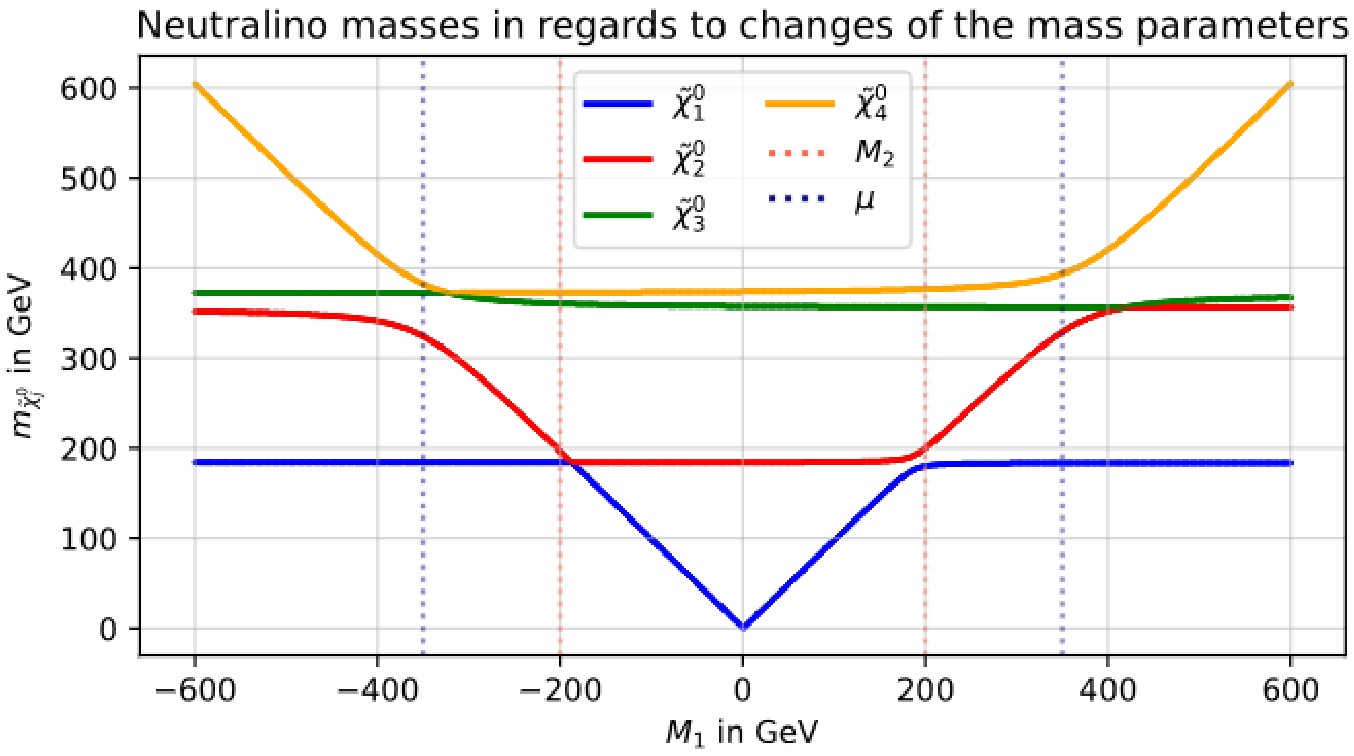
- MSSM can be characterized utilizing Bino-Wino-Basis
- Mass mixing matrix of Charginos $\tilde{\chi}_1^{\pm}, \tilde{\chi}_2^{\pm}$
- Mass mixing matrix Neutralinos $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$
- Find eigenvalues (diagonalizing)

$$M_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ 8\sqrt{2} m_W \sin\beta & \mu \end{pmatrix}$$

$$M_N = \begin{pmatrix} M_1 & 0 & -m_Z \cos\beta \sin\theta_W & m_Z \sin\beta \sin\theta_W \\ 0 & M_2 & m_Z \cos\beta \cos\theta_W & -m_Z \sin\beta \cos\theta_W \\ -m_Z \cos\beta \sin\theta_W & m_Z \cos\beta \cos\theta_W & 0 & -\mu \\ m_Z \sin\beta \sin\theta_W & -m_Z \sin\beta \cos\theta_W & -\mu & 0 \end{pmatrix}$$

[5]

Mass hierarchies of the $\tilde{\chi}_i^{\pm}$ and $\tilde{\chi}_j^0$



[5]

- Computed eigenvalues assigned as masses to $\tilde{\chi}_i^{\pm}$ and $\tilde{\chi}_j^0$ respectively, ascending hierarchy
- Determine, which parameter acts on which $m_{\tilde{\chi}_i^{\pm}}, m_{\tilde{\chi}_j^0}$, if mass parameter hierarchy known
- For example: $m_{\tilde{\chi}_1^0}$ mainly dependent on M_1, if $M_1 < M_2 < \mu$
- $\tilde{\chi}_1^0$ is then called „bino-like“

Data

- Input SLHA files contain points in SUSY parameter space
- The analyzed SLHA files are taken from *„The new “MUON G-2” result and supersymmetry”* [6] and conform to the following constraints:
  1. Experimental constraints, determined by the LHC
  2. Dark matter relic density needs to comply with constraints put up by *“Planck 2018 results”* [7]
  3. Direct dark matter searches restricting energy spectrum for WIMPS´s
  4. Points considered need to have stable vacuum state
  5. Muon $g-2$ contribution (does not play relevant role)
- Keep in mind, that $M_1 \approx M_2$, resulting in $m_{\tilde{\chi}_1^0} \approx m_{\tilde{\chi}_1^\pm}$

$100GeV \leq M_1 \leq 1TeV$	$M_1 \leq M_2 \leq 1.1M_1$
$1.1M_1 \leq \mu \leq 10M_1$	$5 \leq \tan\beta \leq 60$
$M_1 \leq m_{\tilde{l}_L} \leq 1.2M_1$	$M_1 \leq m_{\tilde{l}_R} \leq 10M_1$

Data Analysis

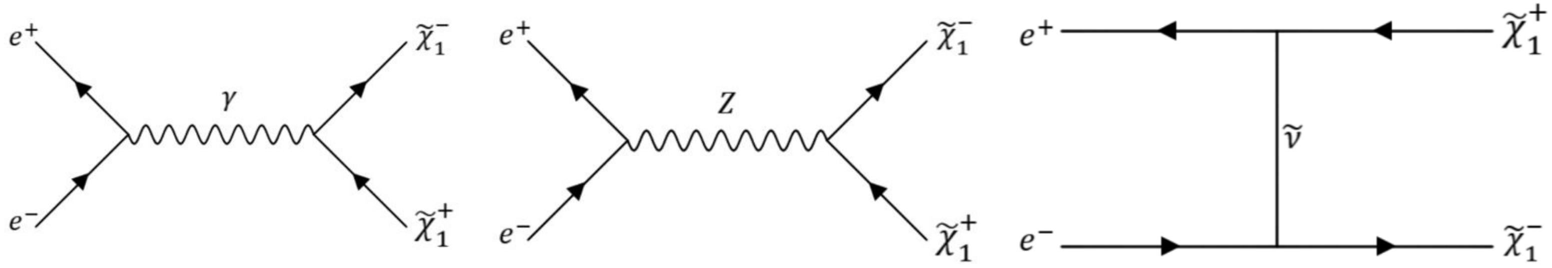
- Points are used to determine:
  1. chargino pair production cross-section $\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^+)$
  2. masses of lightest Neutralino and Chargino, $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_1^\pm}$
- Mixing angles $\Phi_{L,R}$ can be determined, utilizing cross-section
- Scanned over parameter space conforms to $M_1 < M_2 < \mu < 10M_2$, enforcing bino/wino-like dark matter
- Reconstructing parameters from $\Phi_{L,R}$, $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_1^\pm}$, this will show, within which uncertainty mass parameters could be determined in a real experiment
- With reconstructed SUSY parameter points (including their experimental uncertainties), dark matter relic density is constructed and compared to “real” dark matter relic density, obtained by original points

Chargino pair production

- Mixing angles and cross-section are related via

$$\sigma^{\pm}\{ij\} = c_1\, cos^2(2\Phi_L) + c_2 \cos(2\Phi_L) + c_3\, cos^2(2\Phi_R) + c_4 \cos(2\Phi_R) + c_5 \cos(2\Phi_L)\cos(2\Phi_R) + c_6$$

- Contributing first order Feynman diagrams for Chargino pair production



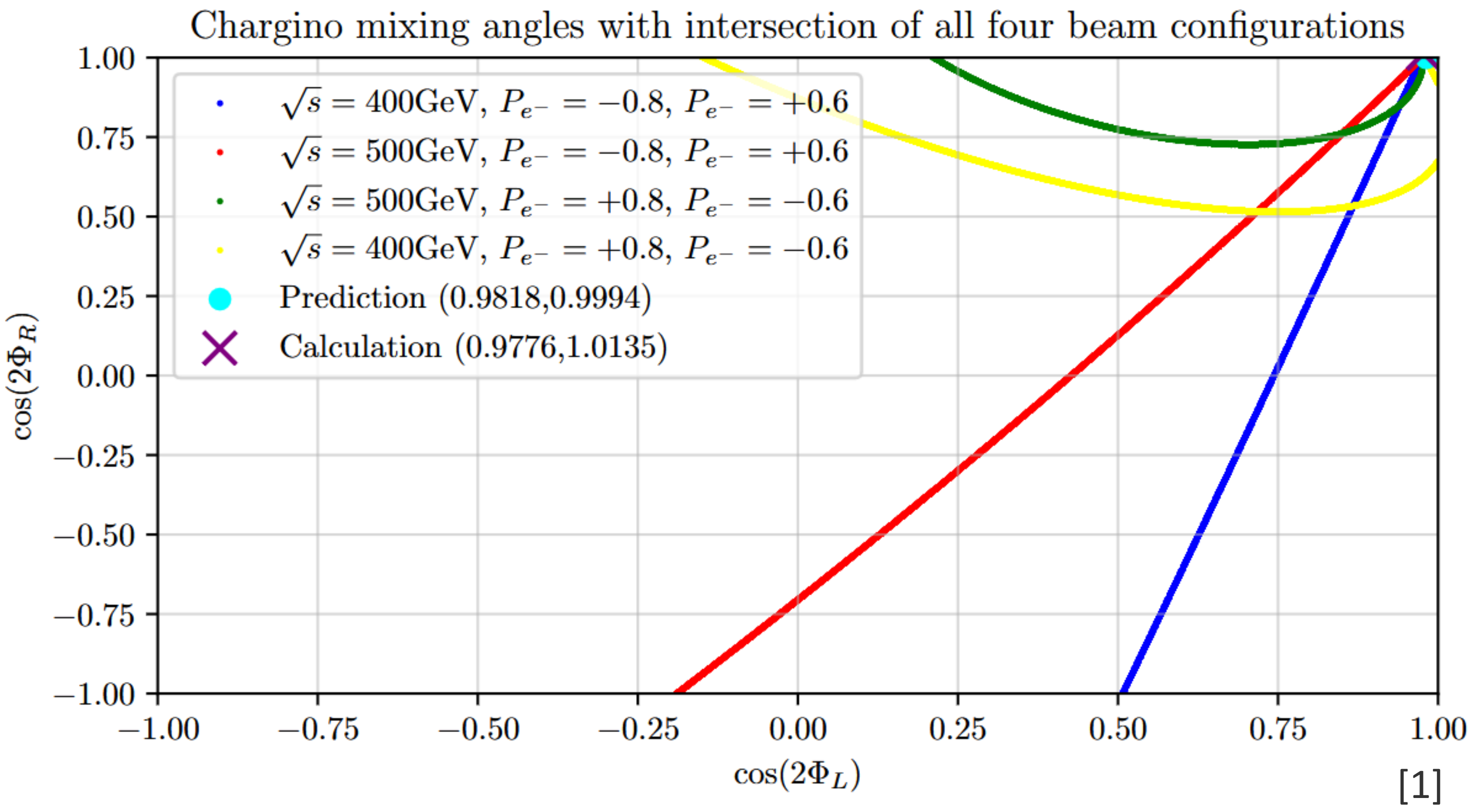
- In the following, reconstruction of parameters will be illustrated with an example point

- $M_{\tilde{\nu}}$ fixed, see table for example point

M_1	$175.09 GeV$	$\sigma^{400}_{-.8,+.6}$	$1744.2519 fb$
M_2	$178.25 GeV$	$\sigma^{400}_{+.8,-.6}$	$49.8956 fb$
μ	$1215.85 GeV$	$\sigma^{500}_{-.8,+.6}$	$1265.4737 fb$
$\tan\beta$	34.81	$\sigma^{500}_{+.8,-.6}$	$35.6168 fb$
$M_{\tilde{\nu}}$	$536.9 GeV$	$m_{\tilde{\chi}_1^{\pm}}$	$177.1484 GeV$

Mixing angles

- Results in ellipsis of valid angles
- 3 different beam configurations needed
- 1 solution in mixing-angle-space remains
- 4 differents beam configurations were used

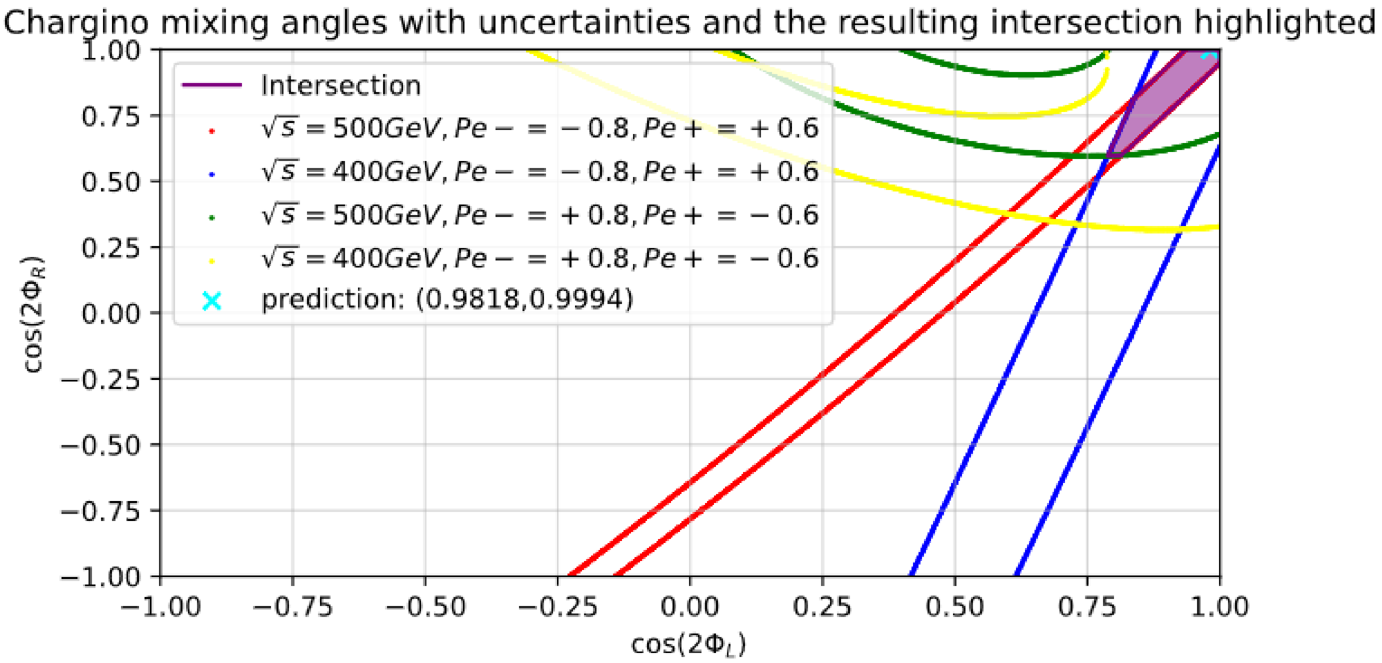


[1]

$\sqrt{s}$	P_{e^-}	P_{e^+}
400 GeV	-0.8	+0.6
400 GeV	+0.8	-0.6
500 GeV	-0.8	+0.6
500 GeV	+0.8	-0.6

Mixing angles and masses

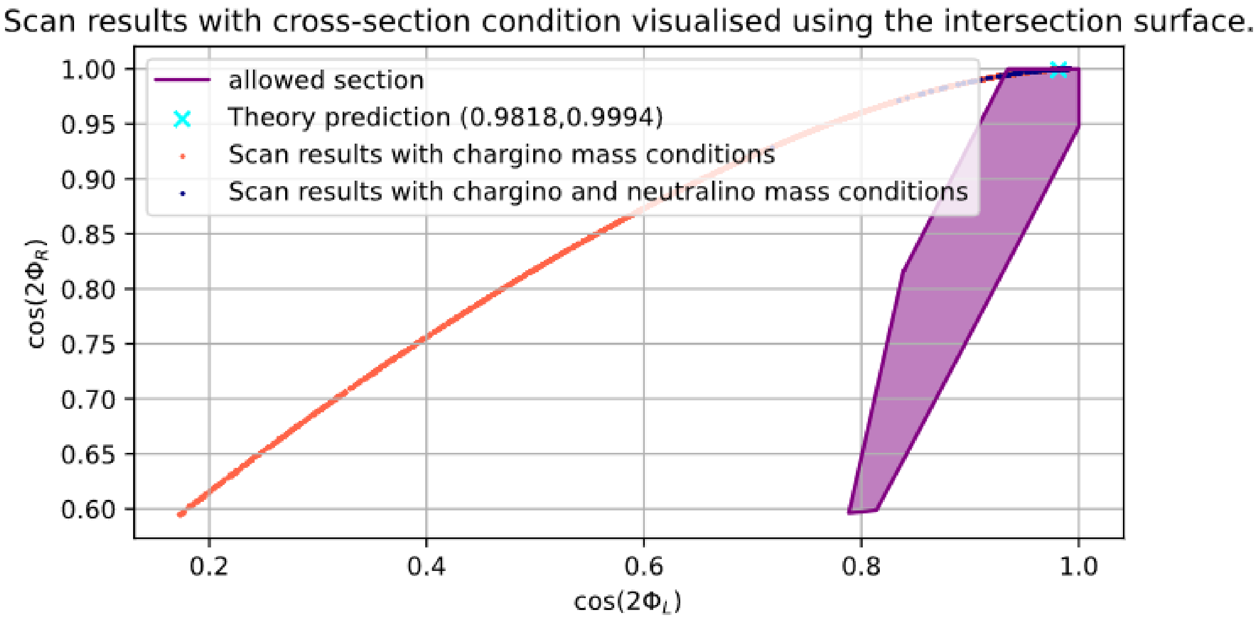
- Uncertainties in polarization, cross-section must be considered
- Ellipses expand into elliptical bands
- Intersection point (single solution) becomes crossing surface (many solutions)
- Data reconstruction can now begin

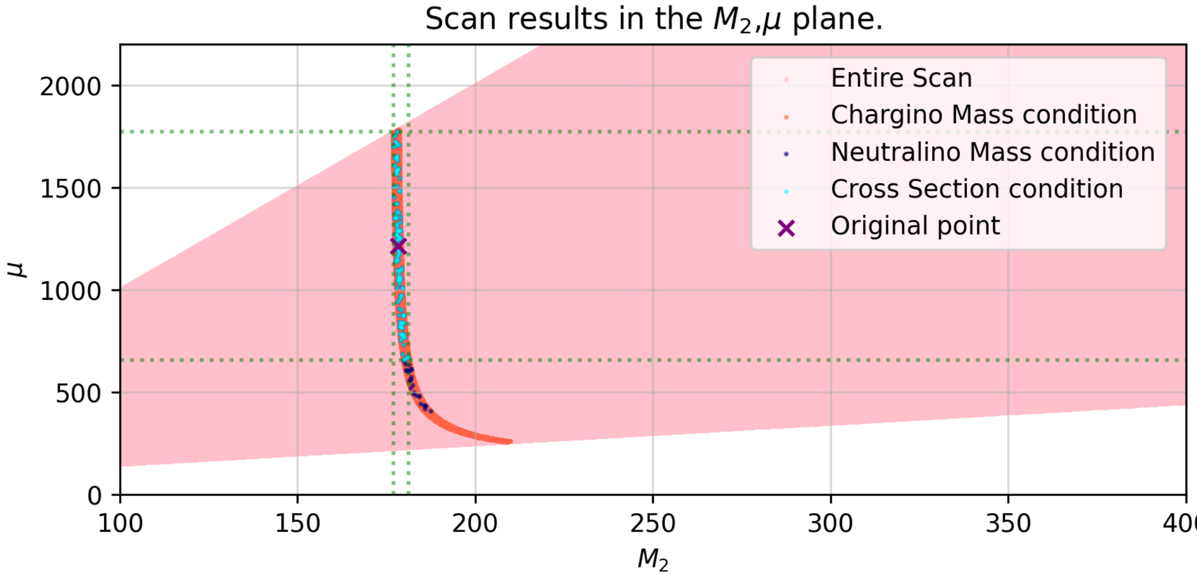


statistical cross-section uncertainties	$\sigma_{stat} = \frac{\sqrt{\sigma L}}{L}$
polarization uncertainties	$\delta P_{e^-} = \pm 0.004, \ \ \delta P_{e^-} = \pm 0.003$
used polarization including uncertainties	$P_{e^-} = \pm 0.804, \ \ P_{e^+} = \mp 0.603$ $P_{e^-} = \pm 0.796, \ \ P_{e^+} = \mp 0.597$
Chargino mass uncertainty	$\delta m_{\tilde{\chi}_1^\pm} = 0.005 m_{\tilde{\chi}_1^\pm}$

Scanning for valid points

- 3 criteria need to be met for a point to be considered valid:
  1. Charginomass $m_{\tilde{\chi}_1^\pm}$ of point is checked against measured mass (0.5% error)
  2. Mixing angles are determined, if point within crossing surface, point is valid
  3. Neutralinomass $m_{\tilde{\chi}_1^0}$ is calculated, if it is within error of 0.5% of measured mass, point is valid
- Spread of valid points showcases uncertainties of reconstruction





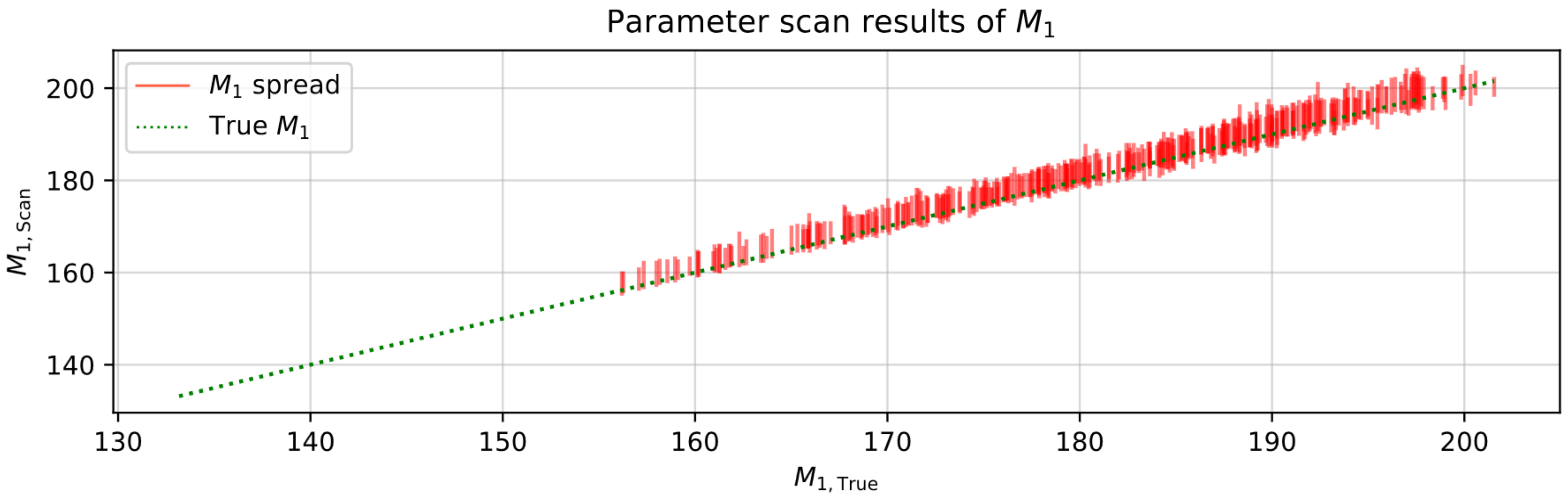
Results

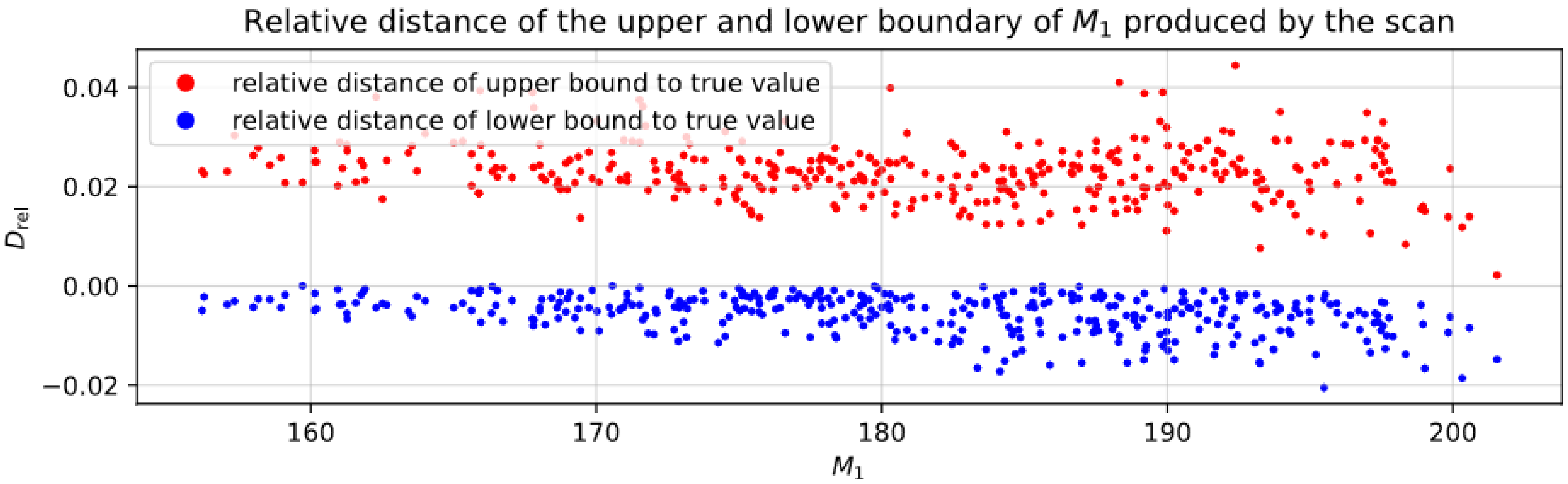
- **From this point, shown data encompasses all data points, not just the example point!**
- Goal was, to determine accuracy of reconstruction for mass parameters and dark matter relic density
- Relative distance of upper and lower boundary of a scan spread to true value of parameter will be used, i.e. largest and smallest value of a scan spread studied

$$D_{rel} = \frac{Q_{scan} - Q_{true}}{Q_{true}}$$

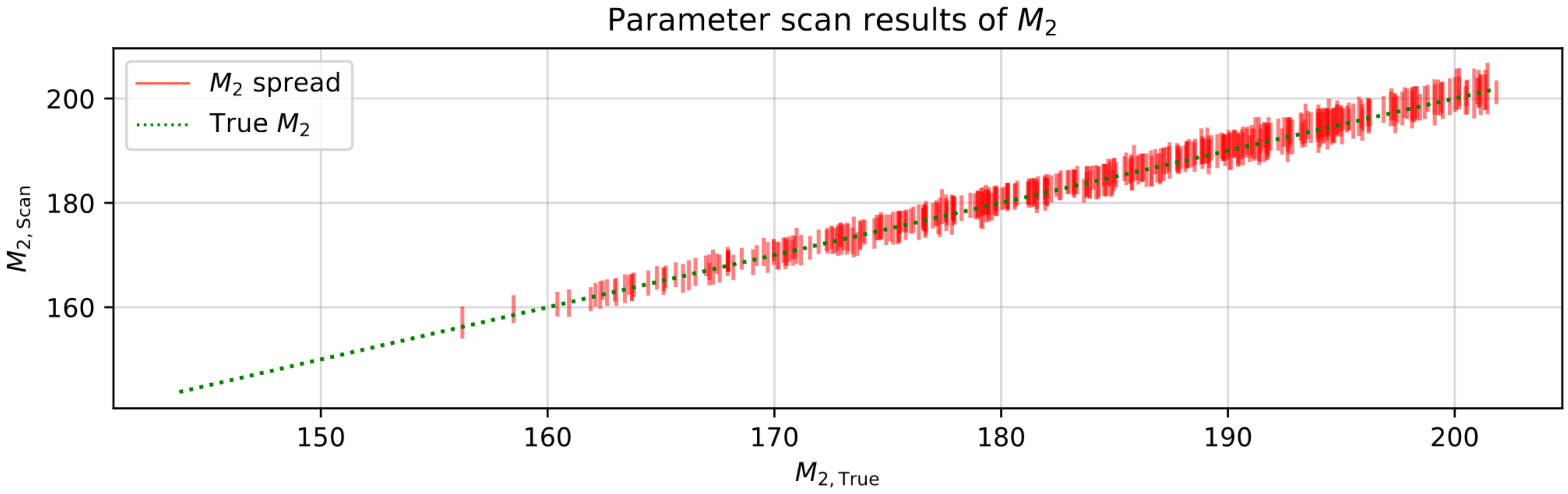
- Q_{scan} is upper or lower boundary of scan spread of that parameter point, Q_{true} is true value of a MSSM parameter
- Positive D_{rel} are interpreted as a boundary being larger than the true value, negative D_{rel} mean a boundary being smaller

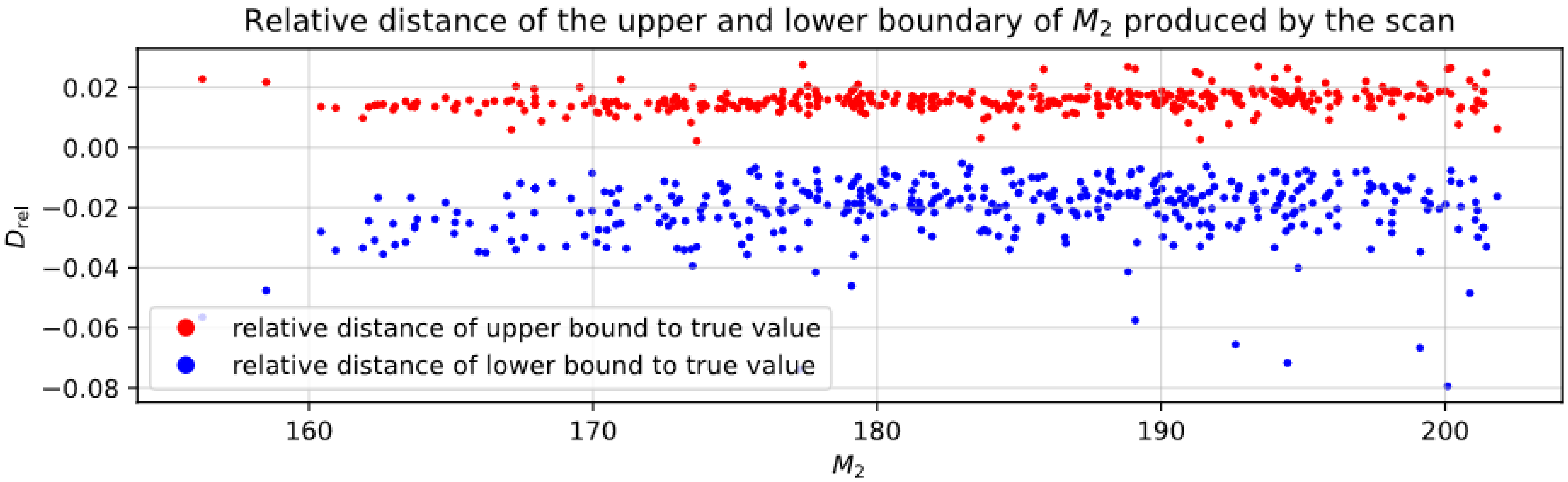
M_1 scan results



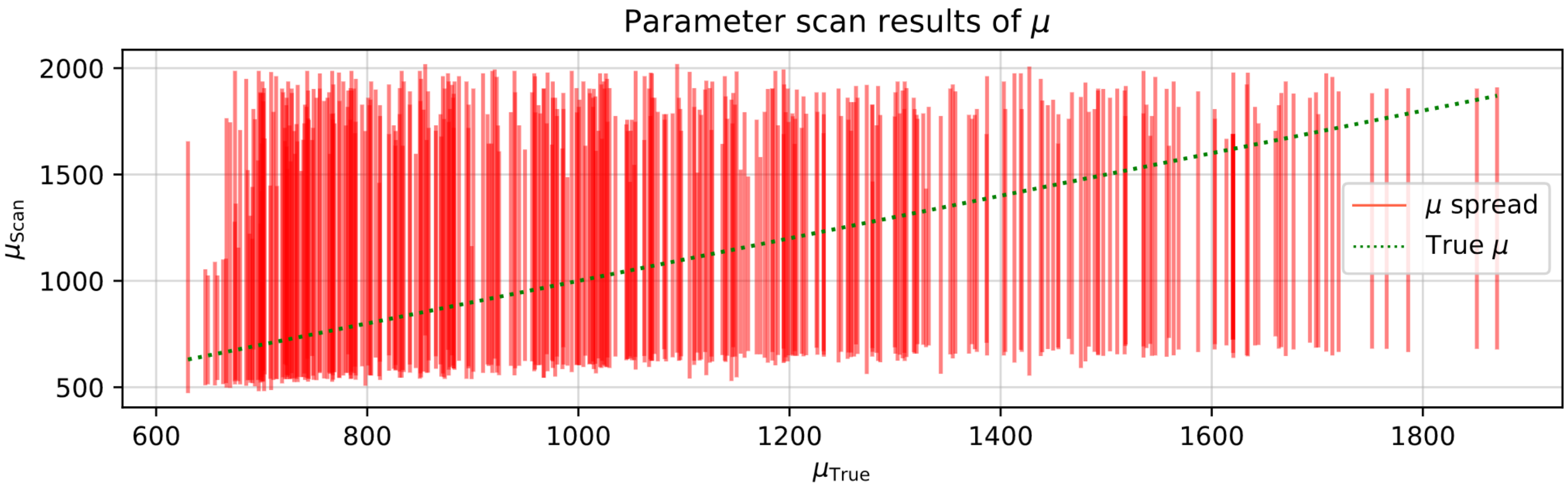


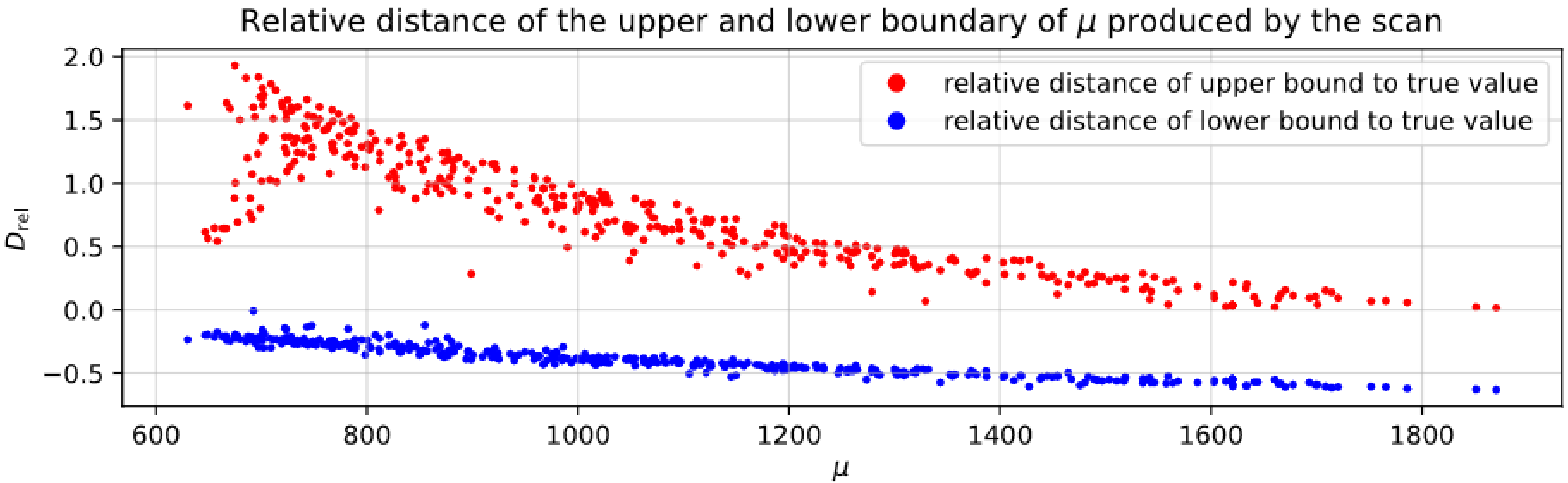
M_2 scan results





μ scan results

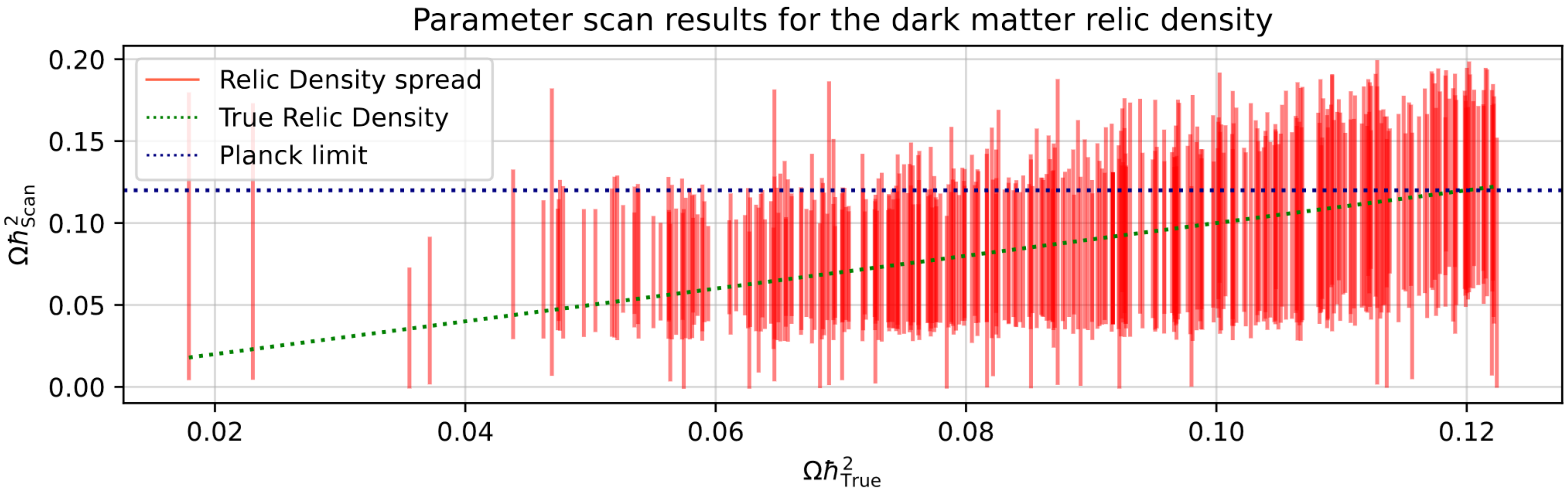


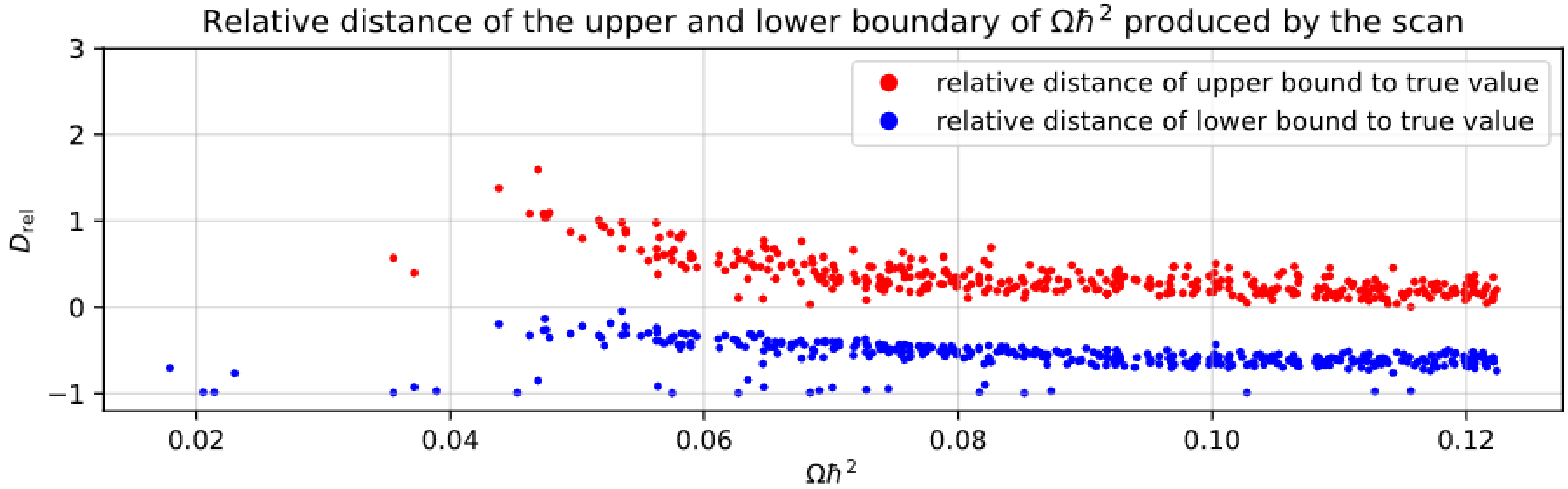


Assessment of the reconstruction

- M_1 and M_2 were reconstructed with rather high accuracy, while μ was reconstructed highly inaccurate
- Remember the dependency of the mass hierarchy ($m_{\tilde{\chi}_i^\pm}, m_{\tilde{\chi}_j^0}$) on the mass parameter hierarchy ($M_1 < M_2 \ll \mu$)
- $m_{\tilde{\chi}_1^\pm}$ and $m_{\tilde{\chi}_1^0}$ are highly dependent on M_1 and M_2, while being insensitive to μ
- Thus, the high accuracy in reconstruction of M_1 and M_2 and low accuracy of μ is expected
- DM relic density in Chargino co-annihilation scenario mainly dependent on M_1 and M_2 as well, low accuracy of μ is no problem

$\Omega\hbar^2$ results





Conclusion

- Code was successfully implemented
- D_{rel} of scan boundaries, instead of D_{rel} of average scan value [1]
- Fixing of Neutralino mass matrix, every scan spread now encompasses true value
- M_1 and M_2 could be reconstructed with accuracy of $1\% - 4\%$
- μ reconstruction highly inaccurate
- $\Omega\hbar^2$ calculation accuracy low, even though M_1, M_2 reconstructed with high accuracy

Outlook

- Code could be adapted to include different scenarios

  $\longrightarrow$ Better μ reconstruction accuracy

- Further studies on low calculation accuracy of relic density $\Omega\hbar^2$
- Usage of existing code to study data points from the excesses found at CMS [2] and ATLAS [3]

  $\longrightarrow$ Scanning over SLHA files studied in *“Consistent Excesses in the Search for $\tilde{\chi}_2^0\tilde{\chi}_1^\pm$: Wino/bino vs. Higgsino Dark Matter”* [8]

References

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