Universität Hamburg
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FAKULTÄT
FÜR MATHEMATIK,
INFORMATIK UND
NATURWISSENSCHAFTEN

Master Thesis Colloquium

Revisiting Gravitational Wave Detection with SCRF Cavities at DESY

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Outline

1. Concepts of heterodyne gravitational wave detection
2. The MAGO cavity
3. Theoretical description of the signal
4. Signal, noise and sensitivity
5. Outlook

General GW Detectors

LIGO Detector (Livingston)
https://www.ligo.caltech.edu/page/observatories-collaborations (access: 13/04/2023)

$\sim 10\ \mathrm{Hz}\ - 10\ \mathrm{kHz}$

Weber Bar Detector NAUTILUS (ROG Collaboration)
(arXiv:1009.1138 [Astro-ph.IM])

$\sim 1\ \mathrm{kHz}$

Wanted:

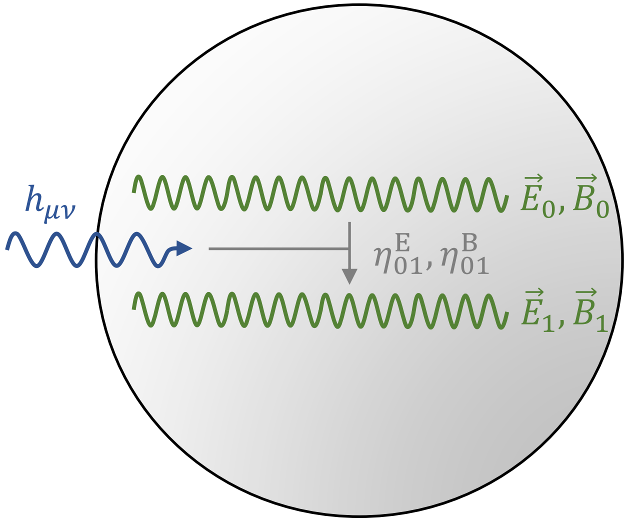
- Detector that is small <u>and</u> has a high sensitivity
- Detector that is sensitive across a large frequency range

Particularly interesting: $\gtrsim 10\ \mathrm{kHz}$: **<u>New Physics!</u>**

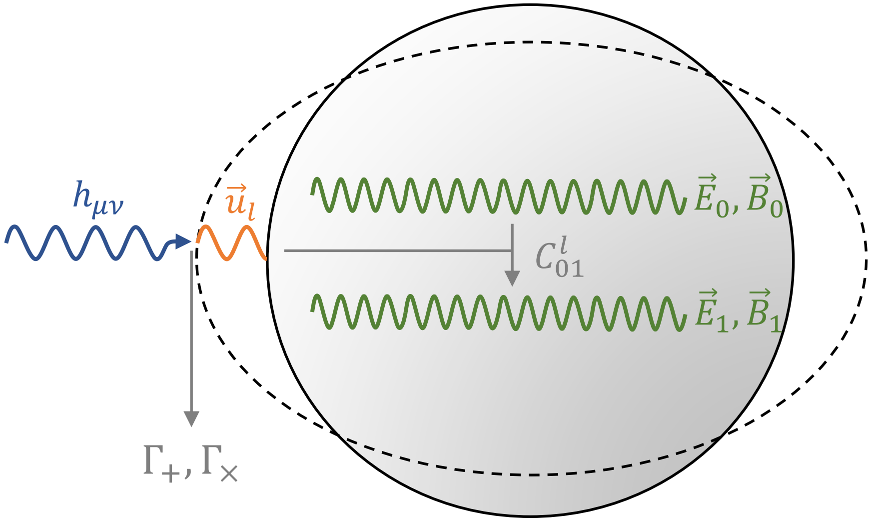
Heterodyne Cavity Experiments

- Idea: GW couples to the electromagnetic field in a cavity and is resonantly enhanced

Direct Coupling (Gertsenshtein Effect)



Mechanical Coupling

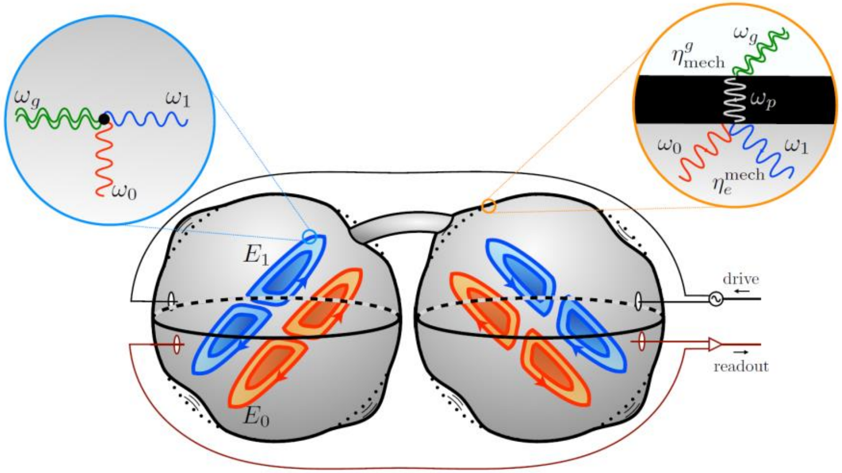


$$\boldsymbol{\omega_g \approx \omega_1 - \omega_0 \sim \mathcal{O}(1\ \mathrm{kHz})\ \mathrm{to}\ \mathcal{O}(1\ \mathrm{GHz})}$$

- Note: Modes should be distinguishable: High Quality factors desired!
  → Use Superconducting Radio Frequency (SCRF) Cavities ($Q \sim 10^{12}$)

Design

- Optimal Geometry (for mechanical coupling): **Sphere** *(Lobo, Phys. Rev. D, 52, 1995)*
- Problem: Spheres have no nearly degenerate modes and are not easily tunable
- Solution: Use two **coupled** spheres with central tuning cell
  (Ballantini et al, arXiv:gr-qc/0203024 (2005))



Berlin et al., arXiv:hep-ph/2303.01518v1 (2023)

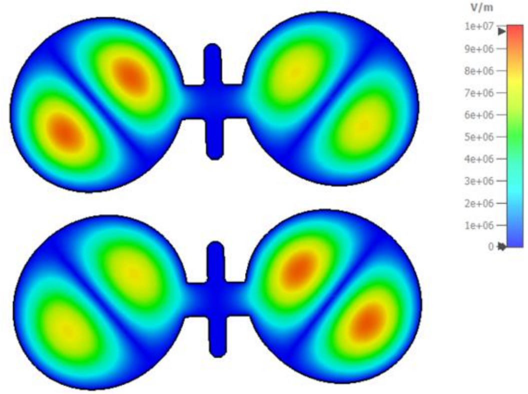
Electromagnetic Modes

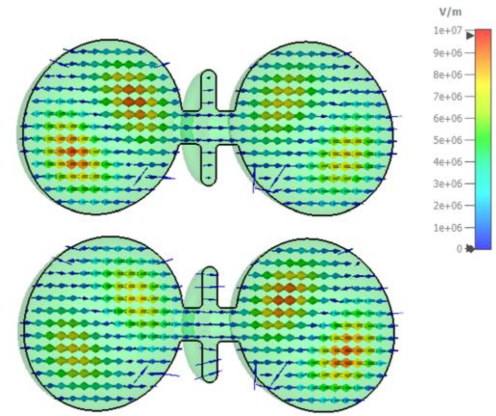
- Important Mode: TE_{011}, because it allows for a $l = 2$ transition between the modes

$\omega_0 = 1.773430\ \mathrm{GHz}$

$\Delta\omega \approx 16\ \mathrm{kHz}$

$\omega_1 = 1.773446\ \mathrm{GHz}$





The MAGO Collaboration

- The idea is **not** new! First proposals for heterodyne EM-detectors were made in the 1970s *(e.g.: V.B. Braginskii et al. Zh. Eksp. Teor. Fiz. 65 (1973))*
- They led to the MAGO collaboration at INFN, which has already built three prototypes

PACO Prototype (thanks to Gianluca Gemme)

MAGO constant coupling (thanks to Gianluca Gemme)

(Ballantini et al., arXiv:gr-qc/0203024 (2005))

The funding was stopped in 2005, MAGO never made a physics run

DESY and FNAL

DESY and FNAL are now interested in reactivating the research on heterodyne cavities

Main reasons:

- High frequency range (i.e. above $\sim 10\ \mathrm{kHz}$) is interesting for new physics
  Aggarwal et al., arXiv:gr-qc/2011.12414v2 (2021)
- Heterodyne cavity experiments could be also used for axion research
  Berlin et al., arXiv:hep-ph/2112.11465 (2019)
- DESY and FNAL host many experts on cavity technologies

The tunable prototype will come to DESY in a few weeks!

Main goals of my master thesis

- Understanding and developing the theoretical background
- Applying the results to the MAGO cavity

Theoretical Background

The starting point is the Einstein-Maxwell Action:

$$S_{\mathrm{EM}} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\right)$$

With the weak field approximation:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \qquad \sqrt{-g} = 1 + h^{\alpha}_{\ \alpha} + \mathcal{O}(h^2)$$

We get the Lagrangian:

$$\mathcal{L}_{\mathrm{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}j^{\mu}_{\mathrm{eff}}A_{\mu}$$

$$j^{\mu}_{\mathrm{eff}} = \partial_{\nu}\left(\frac{h^{\alpha}_{\ \alpha}}{2}F^{\mu\nu} + h^{\nu}_{\ \alpha}F^{\alpha\mu} - h^{\mu}_{\ \alpha}F^{\alpha\nu}\right)$$

(Inverse) Gertsenshtein-Effect!

Adding Mechanical Wall Deformation

Gravitational Waves induce a tidal force density

$$f_i(t,\vec{x}) = -\rho c^2 R_{0i0j}(t,\vec{x}) x_j$$

$$R_{\mu\alpha\beta\nu} = \frac{1}{2}\left(\partial_\alpha\partial_\beta h_{\mu\nu} + \partial_\mu\partial_\nu h_{\alpha\beta} - \partial_\mu\partial_\nu h_{\alpha\nu} - \partial_\alpha\partial_\nu h_{\mu\beta}\right)$$

leading to a displacement of the cavity boundaries governed by the equation

$$\rho(\vec{x})\frac{\partial^2\vec{u}}{\partial t^2} - \mu\Delta\vec{u} - (\lambda+\mu)\nabla(\nabla\cdot\vec{u}) = \vec{\mathrm{f}}(\vec{\mathrm{x}},\mathrm{t}).$$

$\vec{u}$: Displacement vector

We use a separation ansatz:

$$\vec{u}(t,\vec{x}) = \sum_{m=1}^{\infty}\vec{u}_m(\vec{x})q_m(t)$$

$$\int_{V_{\mathrm{cav}}}\vec{u}_n(\vec{x})\vec{u}_m(\vec{x})\rho(\vec{x})dV = M\delta_{nm}$$

$$f_m(t) = \int_{V_{\mathrm{cav}}}\vec{f}(\vec{x},t)\cdot\vec{u}_m(\vec{x})dV$$

The Full Lagrangian

We can now write down the full Lagrange function of a cavity disturbed by a gravitational wave:

$$L = \int d^3V \left[-\frac{1}{2} j^{\mu}_{\mathrm{eff}} A_{\mu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} \right] + \sum_l \left[\frac{1}{2} M \dot{q}_l^2 - \frac{1}{2} M \omega_l^2 q_l^2 + q_l f_l \right]$$

Mechanical Part

Electromagnetic Part

Note that: $\quad j^{\mu}_{\mathrm{eff}} = \mathcal{O}(h) \;\; \Rightarrow \;\; j^{\mu}_{\mathrm{eff}} A'_{\mu} = j^{\mu}_{\mathrm{eff}} A_{\mu} + \mathcal{O}\left(h^2\right)$

Using the Proper Detector Frame

- Gertsenshtein current is <u>not gauge independent</u>!

$$j^{\mu}_{\text{eff}} = \partial_{\nu}\left(\frac{h^{\alpha}{}_{\alpha}}{2}F^{\mu\nu} + h^{\nu}{}_{\alpha}F^{\alpha\mu} - h^{\mu}{}_{\alpha}F^{\alpha\nu}\right)$$

- We have to use a physical gauge, i.e. the proper detector frame:

  Marzlin K P Phys. Rev. D ***50*** *888 (1994)*

$$g_{00} = -(1+\vec{a}\cdot\vec{x})^2 + (\vec{\omega}\times\vec{x})^2 - \gamma_{00} - 2(\vec{\omega}\times\vec{x})^i\gamma_{0i} - (\vec{\omega}\times\vec{x})^i(\vec{\omega}\times\vec{x})^j\gamma_{ij}$$

$$g_{0i} = (\vec{\omega}\times\vec{x})_i - \gamma_{0i} - (\vec{\omega}\times\vec{x})^j\gamma_{ij}$$

$$g_{ij} = \delta_{ij} - \gamma_{ij}$$

$$\gamma_{00} = \sum_{n=0}^{\infty}\frac{2}{(n+3)!}x^k x^l x^{k_1}\dots x^{k_n}\left(\partial_{k_1}\dots\partial_{k_n}R_{0k0l}\right)(g)\cdot[(n+3)+2(n+2)\vec{a}\vec{x}+(n+1)(\vec{a}\vec{x})^2]$$

$$\gamma_{0i} = \sum_{n=0}^{\infty}\frac{2}{(n+3)!}x^k x^l x^{k_1}\dots x^{k_n}\left(\partial_{k_1}\dots\partial_{k_n}R_{0kil}\right)(g)\cdot[(n+2)+(n+1)\vec{a}\vec{x}]$$

$$\gamma_{ij} = \sum_{n=0}^{\infty}\frac{2}{(n+3)!}x^k x^l x^{k_1}\dots x^{k_n}\left(\partial_{k_1}\dots\partial_{k_n}R_{ikjl}\right)(g)\cdot[n+1]$$

COMPLICATED!

- Use long wavelength approximation $\lambda_g \gg L_{\text{cav}}$

  MAGO: $\omega_g \ll 1\ \text{GHz}$

$$\rightarrow \boxed{ds^2 = -dt^2\left(1-\frac{1}{2}\ddot{h}^{TT}_{kl}x^i x^j\right) + dx^i dx^j \delta_{ij}}$$

- Neglect rotation and acceleration $\vec{\omega} = 0$, $\vec{a} = 0$

The Equations of Motion

Dissipative Terms

External Oscillator

$$\ddot{b}_0(t) + \frac{\omega_0}{Q_0}\dot{b}_0(t) + \omega_0^2 b_0(t) = \omega_0^2 V_{\mathrm{cav}}^{-1/3} q_l(t)\left(C_{00}^l b_0(t) + \sqrt{\frac{U_0}{U_1}} C_{01}^l b_1(t) \right) + J_0 + \frac{\omega_0}{Q_0}\sqrt{\frac{U_d}{U_0}}\dot{b}_d$$

$$\ddot{b}_1(t) + \frac{\omega_1}{Q_1}\dot{b}_1(t) + \omega_1^2 b_1(t) = \omega_1^2 V_{\mathrm{cav}}^{-1/3} q_l(t)\left(C_{11}^l b_1(t) + \sqrt{\frac{U_1}{U_0}} C_{10}^l b_0(t) \right) + J_1 + \varepsilon\frac{\omega_1}{Q_1}\sqrt{\frac{U_d}{U_1}}\dot{b}_d$$

$$\ddot{q}_l(t) + \frac{\omega_l}{Q_l}\dot{q}_l(t) + \omega_l^2 q_l(t) = \frac{f_l}{M} + \frac{f_l^{ab}}{M}$$

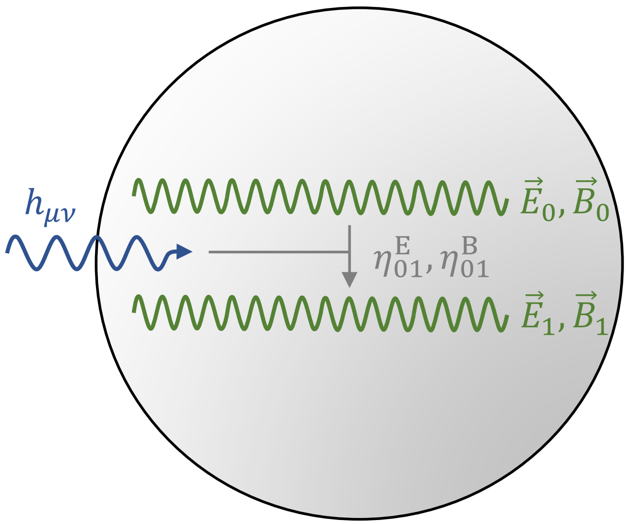
$$f_l^{ab} = U_0 C_{00}^l b_0^2 + U_1 C_{11}^l b_1^2 + 2\sqrt{U_0 U_1} C_{01}^l b_0 b_1$$

The EoMs can be solved assuming

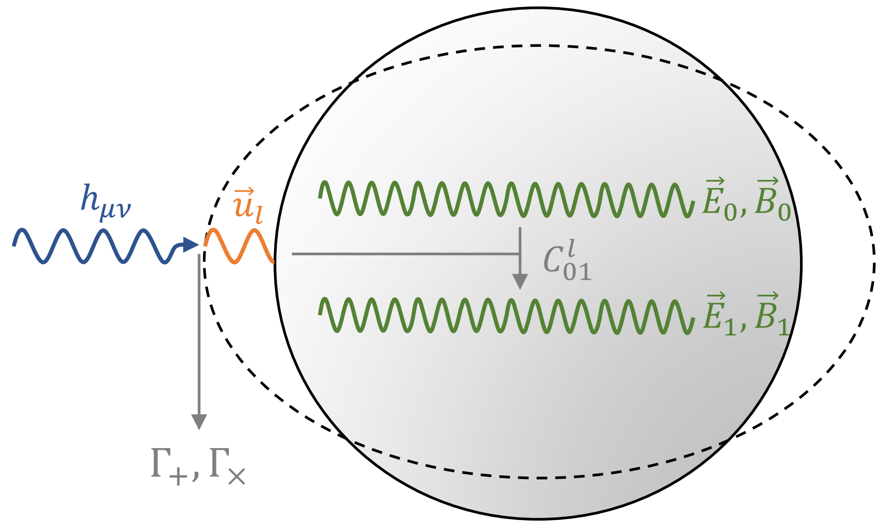
- The GW is monochromatic and propagates in z-direction
- $\omega_g \ll \omega_0, \omega_1$, so fast oscillating terms can be neglected ($Ae^{i\omega_1 t} + Be^{i\omega_g t} \approx Be^{i\omega_g t}$)
- $q_l(t)$ and $b_1(t)$ are small, we only need to consider leading order terms

Solution

Direct Coupling (Gertsenshtein Effect)



Mechanical Coupling



$$P_{\mathrm{sig}} = \frac{\omega_1}{Q_{\mathrm{cpl}}} \omega_g^4 U_0 \left| \underbrace{\frac{1}{2} \frac{\omega_1^2 C^l_{01} (h_+ \Gamma_+ + h_\times \Gamma_\times)}{\beta_1 \beta_l - \gamma_1 \gamma_l}}_{\text{Mechanical Coupling}} - \underbrace{\frac{\beta_l H \left(\kappa_1 \eta^E_{01} + \lambda_1 \eta^B_{01} \right)}{\beta_1 \beta_l - \gamma_1 \gamma_l}}_{\text{Gertsenshtein Effect}} \right|^2$$

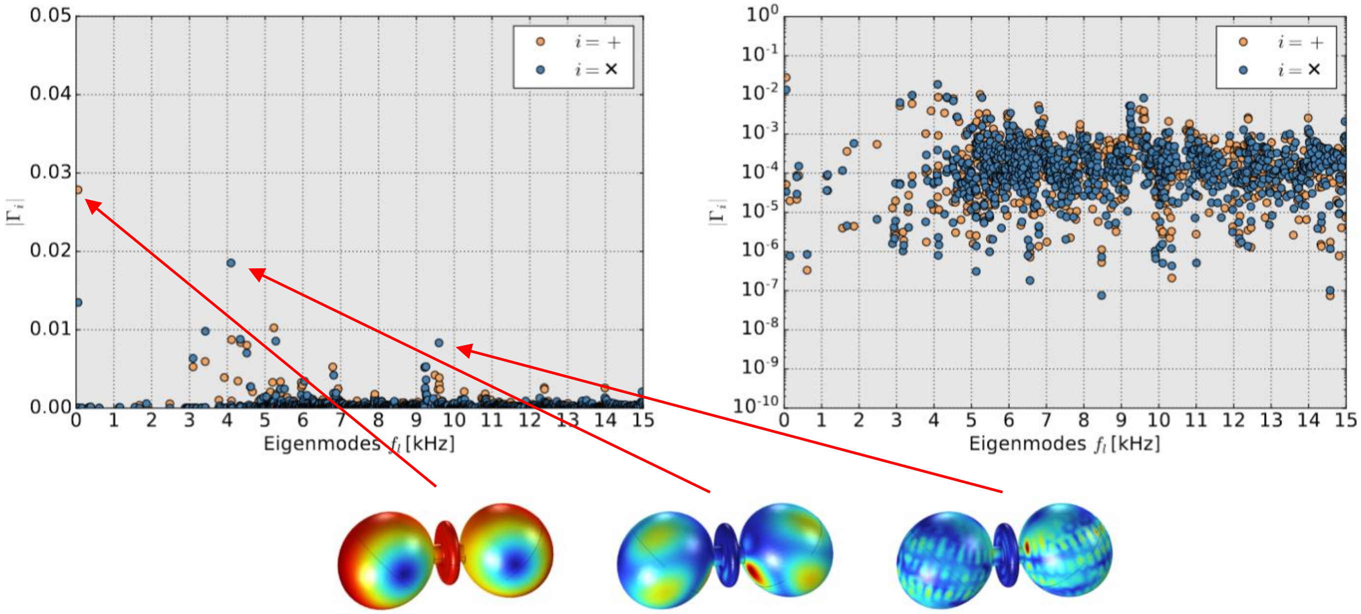
GW-Mechanical Coupling

- The Coupling Coefficients are given by:

$$\Gamma_+ := \frac{V_{\text{cav}}^{-1/3}}{M} \int_{V_{cav}} d^3x\, \rho(\vec{x})(x u_{l,x}(\vec{x}) - y u_{l,y}(\vec{x})) \qquad \Gamma_\times := \frac{V_{\text{cav}}^{-1/3}}{M} \int_{V_{cav}} d^3x\, \rho(\vec{x})(x u_{l,y}(\vec{x}) + y u_{l,x}(\vec{x}))$$

- Expected average scaling: $\Gamma_i(\omega_l) \sim \frac{1}{\omega_l^2}$

Linear Plot **Logarithmic Plot**



$\omega_l = 0.051$ kHz
$\Gamma_+ = -0.028$

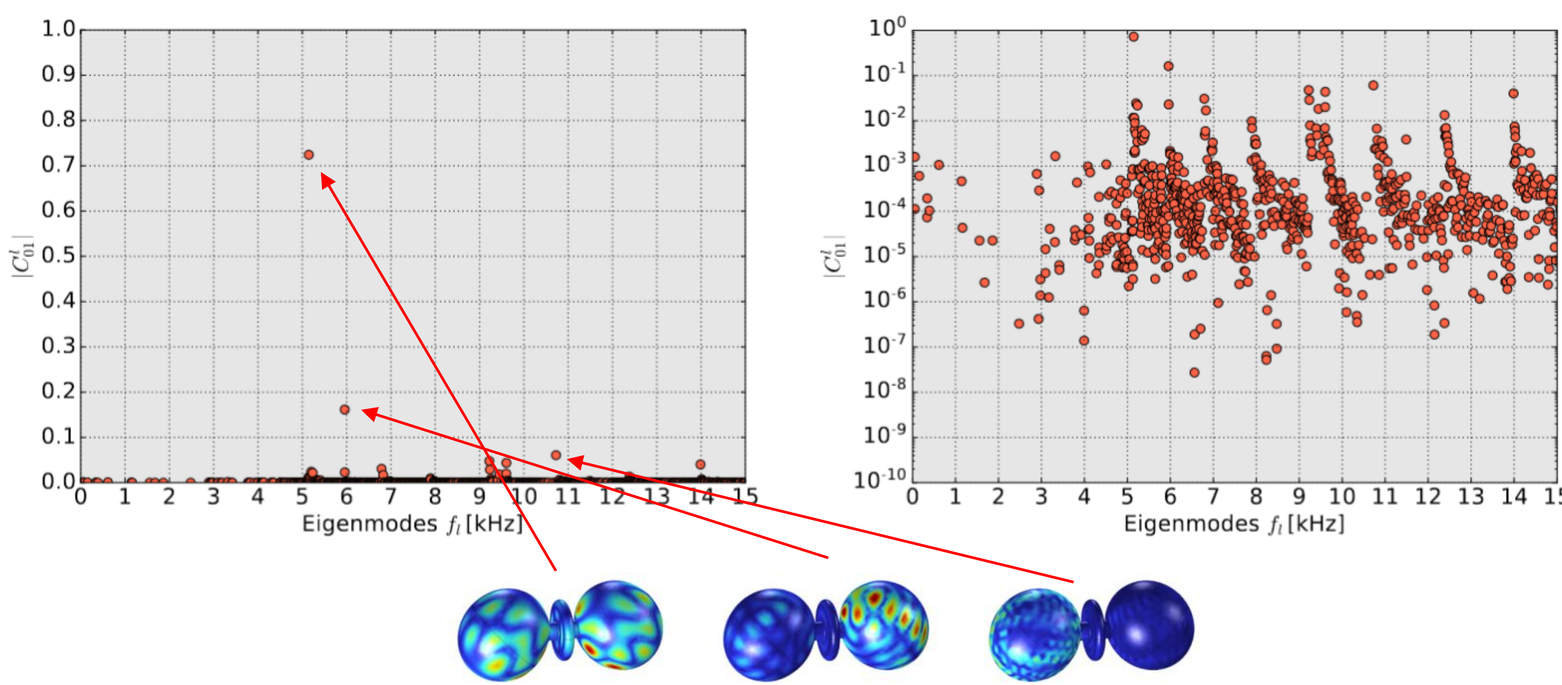
$\omega_l = 4.098$ kHz
$\Gamma_+ = -0.019$

$\omega_l = 9.597$ kHz
$\Gamma_+ = -0.008$

GW-Mechanical Coupling

- Dominating Coupling Coefficient: $$|C_{01}^{l}| = \frac{1}{C_{\text{norm}}}\left|\int_S d\vec{S}\,\vec{u}_l(\vec{x})\left[\frac{1}{\mu_0}\vec{B}_0\vec{B}_1 - \varepsilon_0\vec{E}_0\vec{E}_1\right]\right|^2$$
- Expected average scaling: $|C_{01}^{l}(\omega_l)| \sim 1$

Linear Plot **Logarithmic Plot**



$\omega_l = 5.147$ kHz $\quad C_{01}^{l} = 0.724$

$\omega_l = 5.958$ kHz $\quad C_{01}^{l} = 0.162$

$\omega_l = 10.726$ kHz $\quad C_{01}^{l} = 0.061$

GW-EM Coupling

Again, we consider a monochromatic GW propagating in z-direction

Symmetric TE_{011}-Mode

$$\eta_{01}^{\mathrm{E}} = \frac{1}{H\sqrt{U_0 U_1}} \int_{V_{\mathrm{cav}}} \mathrm{d}^3x H_0(\vec{x}) \varepsilon_0 \vec{E}_0(\vec{x}) \vec{E}_1(\vec{x}) \approx 0.2$$

$$\eta_{01}^{\mathrm{B}} = \frac{1}{H\sqrt{U_0 U_1}} \int_{V_{\mathrm{cav}}} \mathrm{d}^3x H_0(\vec{x}) \frac{1}{\mu_0} \vec{B}_0(\vec{x}) \vec{B}_1(\vec{x}) \approx 0.2$$

$\rightarrow \eta_{01}^{\mathrm{E}} \approx \eta_{01}^{\mathrm{B}} \approx 0.2$

Antisymmetric TE_{011}-Mode

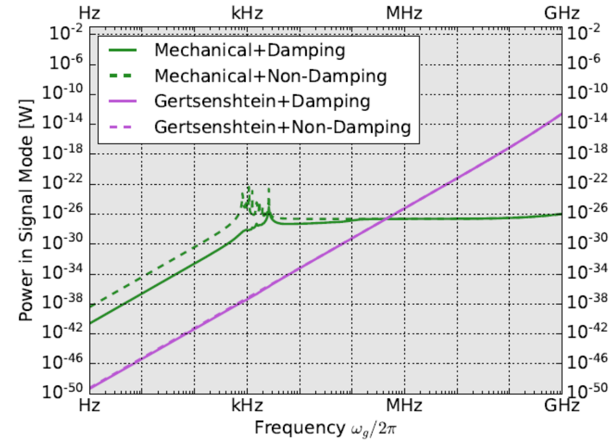
Signal Power

Signal Power for a GW with strain $h_0 = 10^{-20}$.

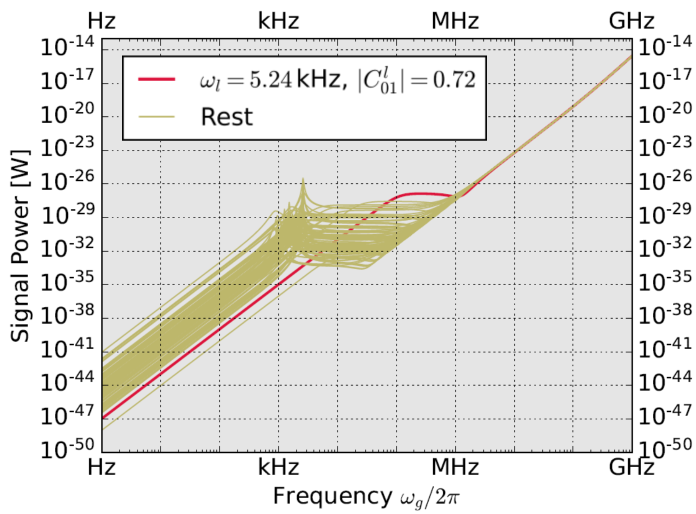
$$P_{\text{sig}} = \frac{\omega_1}{Q_{\text{cpl}}}\omega_g^4 U_0 \left| \frac{1}{2}\frac{\omega_1^2 C_{01}^l (h_+\Gamma_+ + h_\times\Gamma_\times)}{\beta_1\beta_l - \gamma_1\gamma_l} - \frac{\beta_l H(\kappa_1\eta_{01}^E + \lambda_1\eta_{01}^B)}{\beta_1\beta_l - \gamma_1\gamma_l} \right|^2$$

Damping Terms (Not considered in *Berlin et al. (2023)*)

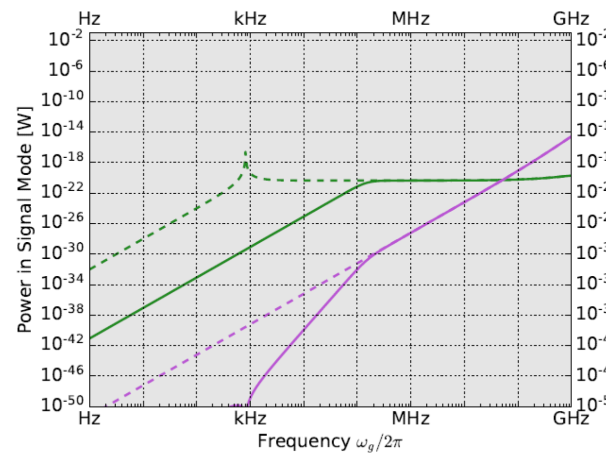
Note: $\gamma_1\gamma_l \sim \left|C_{01}^l\right|^2$



Simulated Couplings



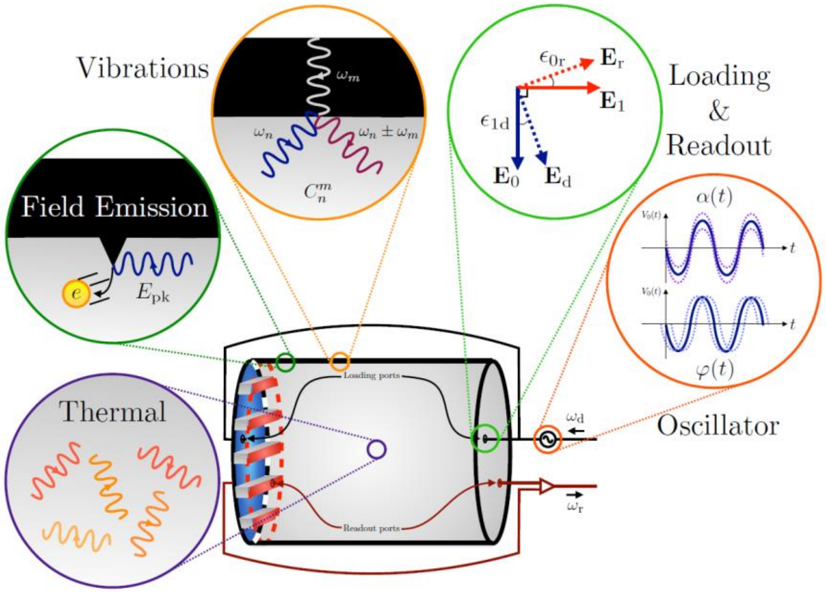
Power Signal for Different Mechanical Modes



Optimistic Values

$\left(\left|C_{01}^l\right| = 1, \omega_l = 5\ \text{kHz}, |\Gamma_+| = |\Gamma_\times| = 0.5\right)$

Noise Sources



Berlin et al, *arXiv:hep-ph/1912.11048v1 (2019)*

Mechanical Noise

$$S_{\text{mech}}(\omega) = \frac{\omega_1}{Q_1} U_0 V_{\text{cav}}^{-2/3} \left|C_{01}^{l}\right|^2 \frac{q_{\text{rms}}^2}{Q_l} \frac{4\pi\omega_{\text{min}}^3 \omega_1^4}{\left|\Lambda_1\left(\omega - (\omega_0 + \omega_g)\right)\right|^2} \frac{1}{\omega - \omega_0}$$

Thermal Mechanical Noise

$$S_{\text{th}}^{(\text{mech})}(\omega) = \frac{\omega_1}{Q_1} U_0 V_{\text{cav}}^{-2/3} \omega_1^4 \frac{\left|C_{01}^{l}\right|^2}{M} \frac{\omega_l}{Q_l} \frac{4\pi k_{\text{B}} T}{\left|\Lambda_1\left(\omega - (\omega_0 + \omega_g)\right)\right|^2} \frac{1}{\omega - \omega_0}$$

Thermal EM Noise

$$S_{\text{th}}^{(\text{EM})}(\omega) = \frac{\omega_1^2}{Q_1^2} \frac{4\pi k_{\text{B}} T \omega^2}{\left|\Lambda_2\left(\omega - (\omega_0 + \omega_g)\right)\right|^2}$$

Amplifier Noise

$$S_{\text{amp}}(\omega) = \pi\hbar\omega$$

Oscillator Phase Noise

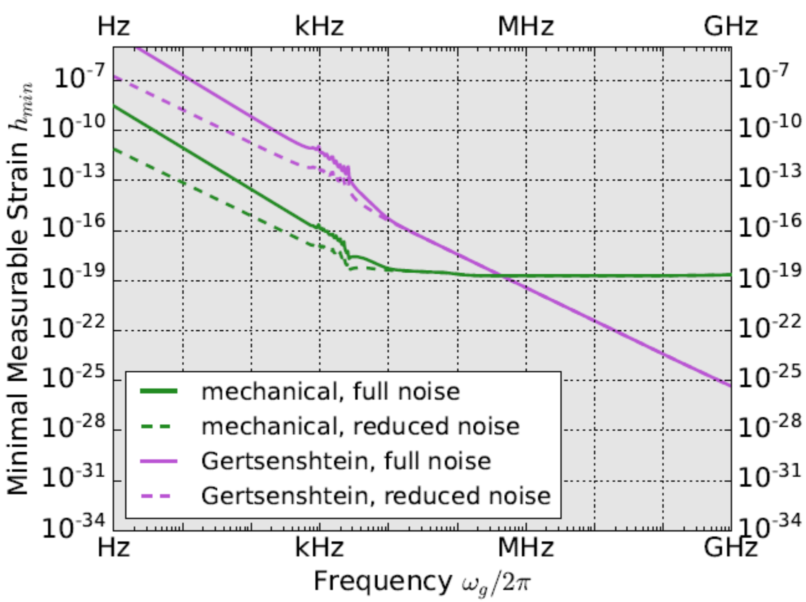
$$S_{\text{osc}}(\omega) = \epsilon^2 \frac{\omega_1^3}{Q_1^3} U_0 \omega_0^2 \frac{S_\varphi(\omega - \omega_0)}{\left|\Lambda_2\left(\omega - (\omega_0 + \omega_g)\right)\right|^2}$$

Total: $S_{tot}(\omega) = S_{\text{mech}}(\omega) + S_{\text{th}}^{(\text{mech})}(\omega) + S_{\text{th}}^{(\text{EM})}(\omega) + S_{\text{amp}}(\omega) + S_{\text{osc}}(\omega)$

MAGO Sensitivity

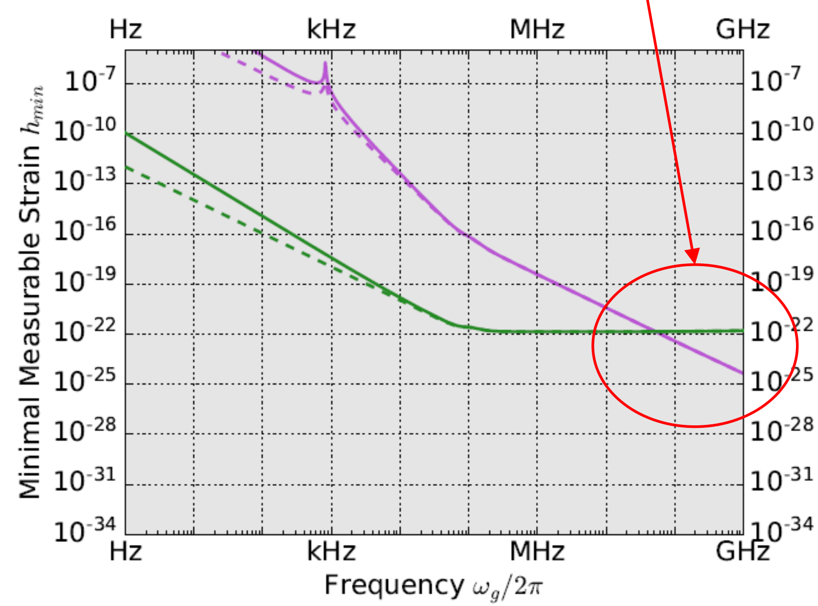
We can now estimate the minimal measurable strain by setting:

$$\mathrm{SNR} = \frac{P_{\mathrm{sig}}}{P_{\mathrm{noise}}} \overset{!}{=} 1$$



Simulated Couplings

The long wavelength approach may break down here

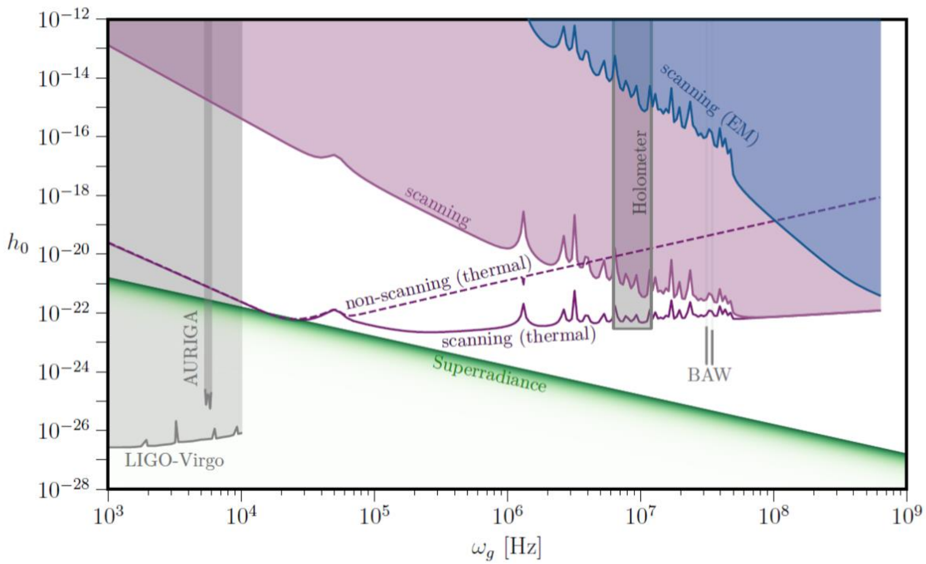


Optimistic Values

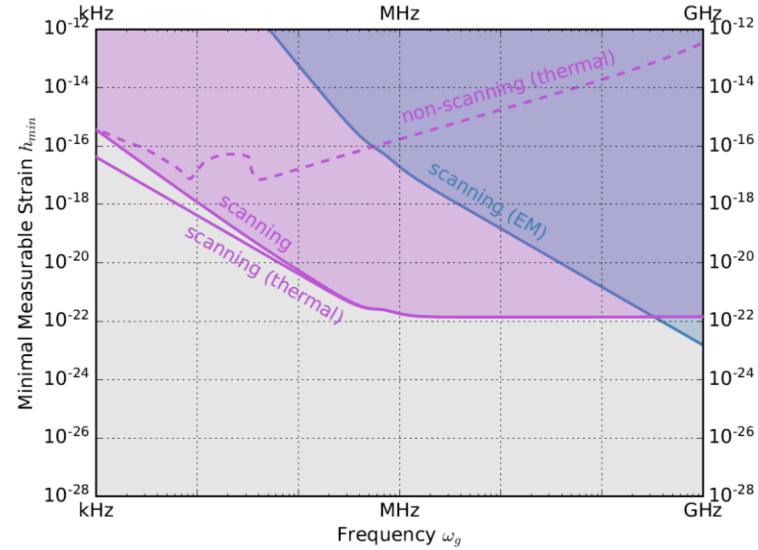
$$\boldsymbol{(|C^l_{01}| = 1,\ \omega_l = 5\ \mathrm{kHz},\ |\Gamma_+| = |\Gamma_\times| = 0.5)}$$

Comparison to Previous Results

We compare the results to *Berlin et al. (2023)*, which made the calculations for idealized spherical cavities and unspecified coupling.



Berlin et al., arXiv:hep-ph/2303.01518v1 (2023)



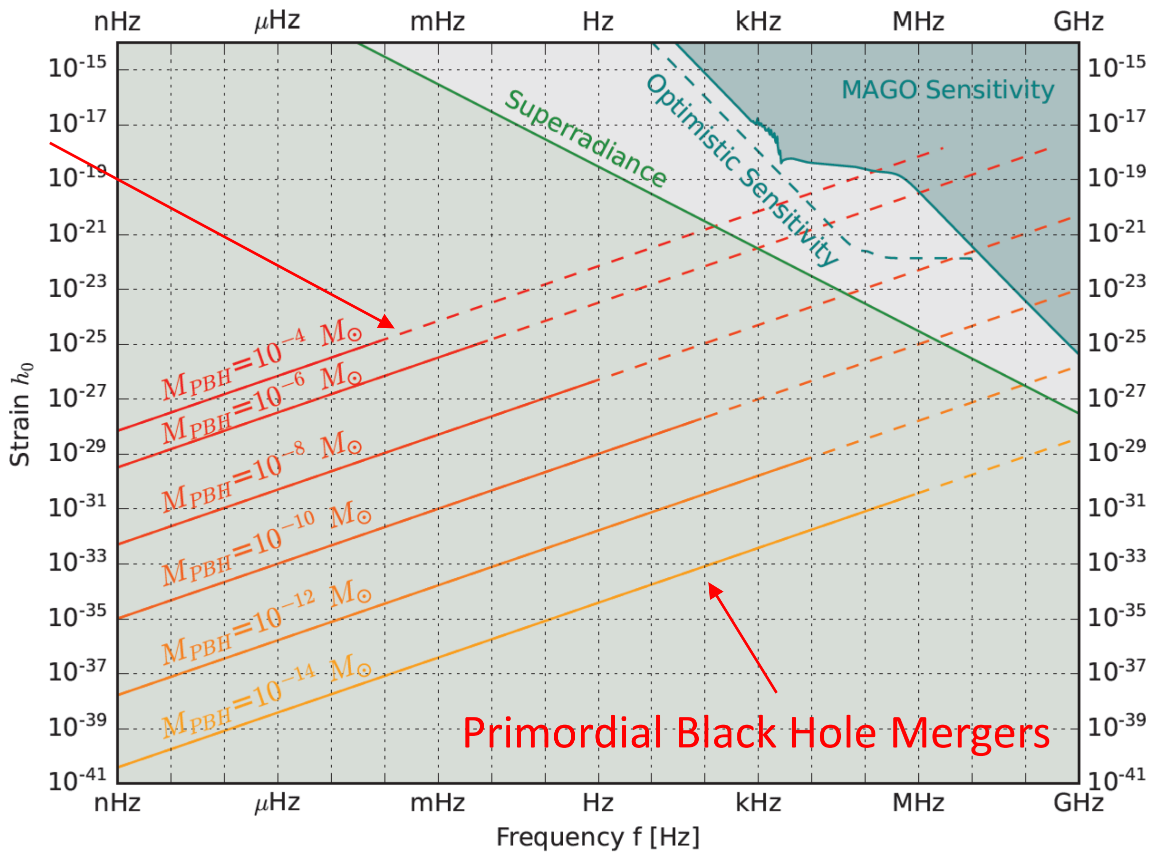
Our Result

- Main Difference: *Berlin et al. (2023)* assumed lowest lying mode at ~ 10 kHz
- We found the mode at ~ 0.05 kHz, which significantly reduces the mechanical noise

Sensitivity and Sources

MAGO Sensitivity compared to possible exotic sources

Restriction due to cavity ring-up time



- Superradiance may be detectable, but PBHs seem to be out of reach

Take Home Messages

- Heterodyne Cavity experiments can probe a very broad GW spectrum across $1\ \mathrm{kHz}\ - 1\ \mathrm{GHz}$, where new physics could show up
- The GW-detector interaction is described by the mechanical and the Gertsenshtein coupling
- The sensitivity of MAGO-like detectors already approaches the promising region for new physics
- The MAGO prototype will soon come to DESY, where first measurements will be carried out which are then compared with our simulations

Outlook

There are still a lot of things to do on the experimental <u>and</u> theoretical part of the project!

Some ideas for the theoretical part:

- Making simulations with a <u>precise</u> model of the MAGO cavity
  → *Compare them with measurements*

- Finding geometries allowing for scans in the MHz-GHz regime

- Analyse the GWs that propagate along arbitrary directions

- Considering arrays of cavities in order to improve the sensitivity and spatial resolution

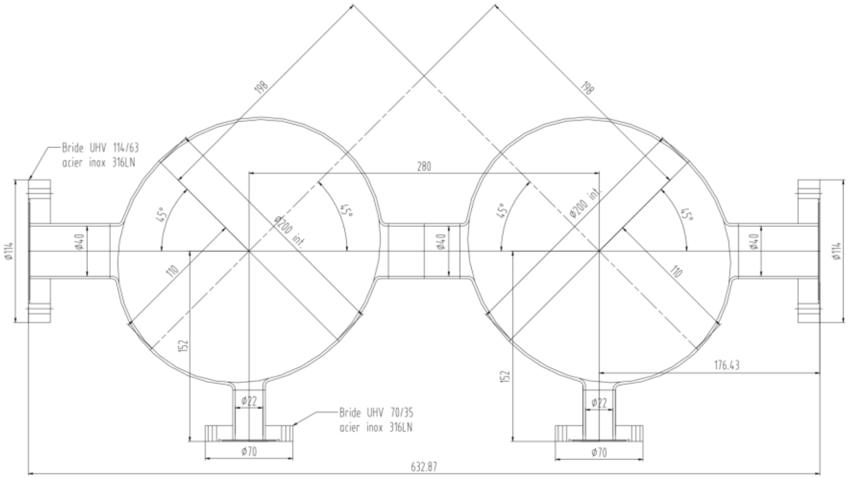
- How do the couplings change when the cavity is deformed?

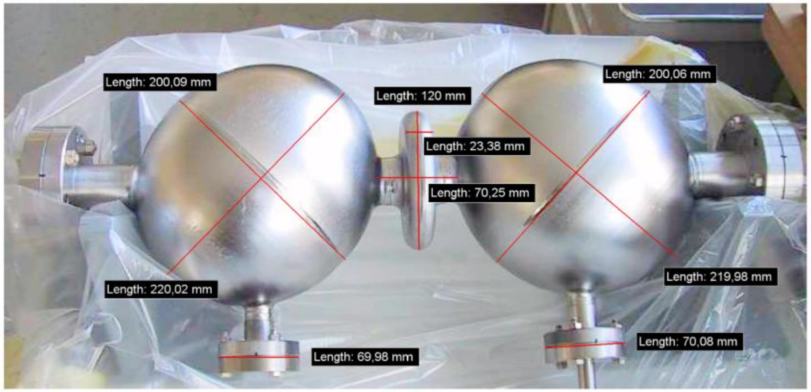
⋮

Thank You!

Backup 1: MAGO-Size and Parameters

We only had technical drawings for the cavity with constant coupling





Material	Temperature [K] [K]	Density [kg/m^3]	Young's Modulus [GPa]	Poisson's Ratio
Cu OFE Annealed	293	8930	115	0.344
SS 316LN		7950	196	0.27
Nb RRR 300		**8570**	**106**	**0.40**
Cu OFE Annealed	2	8930	138	0.377
SS 316LN		7950	208	N/A
Nb RRR 300		**8570**	**104.8**	**N/A**

Backup 2: Closer Look at the Signal Power

We found the total signal power

$$P_{\mathrm{sig}} = \frac{\omega_1}{Q_{\mathrm{cpl}}} \omega_g^4 U_0 \left| \frac{1}{2} \frac{\omega_1^2 C_{01}^l (h_+ \Gamma_+ + h_\times \Gamma_\times)}{\beta_1 \beta_l - \gamma_1 \gamma_l} - \frac{\beta_l H\left(\kappa_1 \eta_{01}^E + \lambda_1 \eta_{01}^B\right)}{\beta_1 \beta_l - \gamma_1 \gamma_l} \right|^2$$

The remaining constant that we did not mentioned in the talk are

$$\alpha_1 = \frac{\omega_1}{Q_1} + 2i(\omega_0 + \omega_g) \qquad \beta_1 = \omega_1^2 - \left(\omega_0 + \omega_g\right)^2 + i \frac{\omega_1}{Q_1}(\omega_0 + \omega_g) \qquad \gamma_1 = V_{\mathrm{cav}}^{-1/3} \omega_1^2 C_{01}^l$$

$$\alpha_l = 2i\omega_g + \frac{\omega_l}{Q_l} \qquad \beta_l = \omega_l^2 - \omega_g^2 + i\omega_g \frac{\omega_l}{Q_l} \qquad \gamma_l = \frac{1}{M} V_{\mathrm{cav}}^{-1/3} U_0 C_{01}^l$$

$$\kappa_1 = i \frac{\omega_1}{8c^2}(\omega_0 + \omega_g)$$

$$\lambda_1 = \frac{\omega_1^2}{8c^2}$$

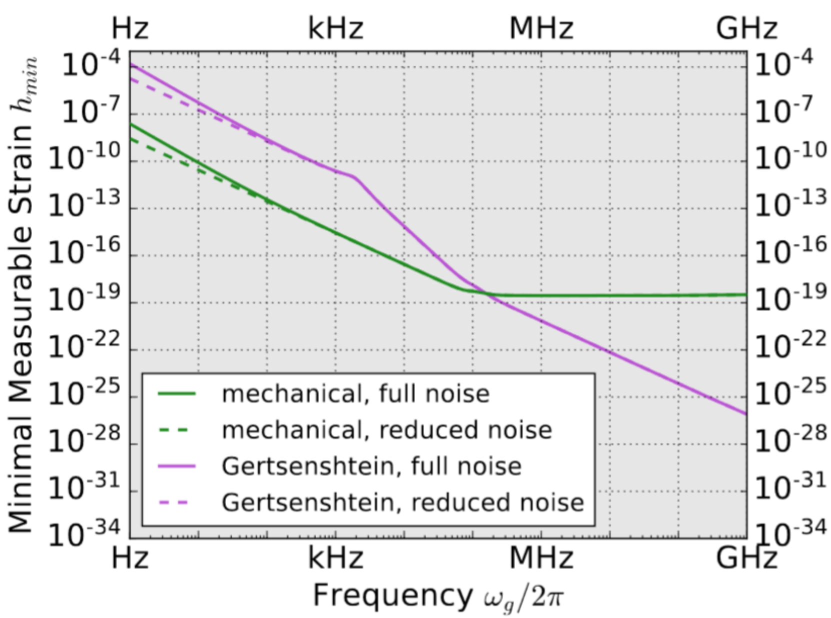
Resonance Functions:

$$\Lambda_1(\omega) \coloneqq \left(\beta_1 - \omega^2 + i\omega\alpha_1\right)\left(\beta_2 - \omega^2 + i\omega\alpha_2\right) - \gamma_1\gamma_2$$

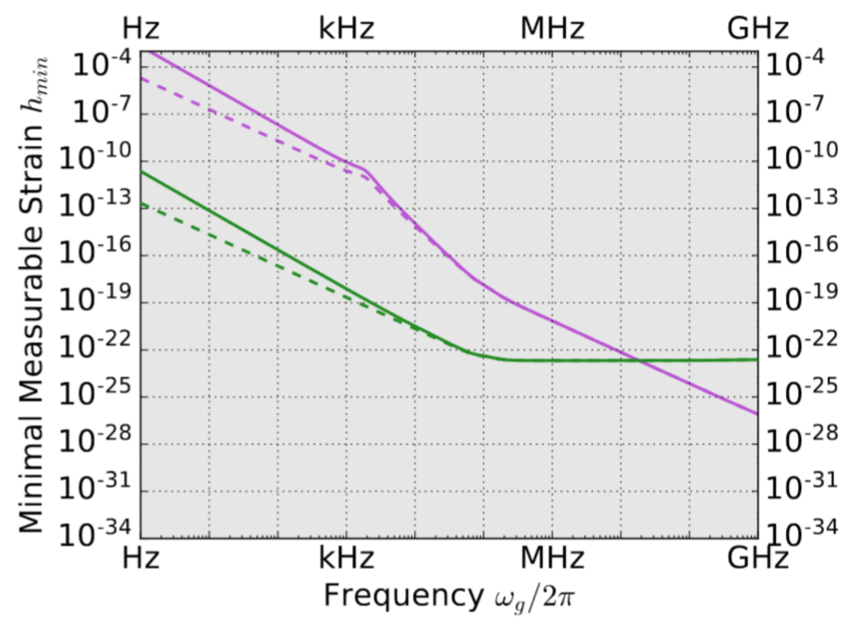
$$\Lambda_2(\omega) \coloneqq \Lambda_1(\omega)\left(\beta_l - \omega^2 + i\omega\alpha_l\right)^{-1}$$

Backup 3: Mechanical and EM Separated

These plots compare the sensitivity due to the mechanical and Gertsenshtein signal



Simulated Couplings



Optimistic Values

$(|C_{01}^{l}| = 1, |\Gamma_{+}| = |\Gamma_{\times}| = 0.5)$

Backup 4: Axions

MAGO may be able to detect Axions as well

PSD: $$P_{sig} = \frac{\omega_1}{Q_{\mathrm{cpl}}} U_0 \left(\eta_{01}^{\mathrm{A}} \omega_1 g_{a\gamma\gamma}\right)^2 \frac{2\rho_{DM}}{\left|\Lambda_2 \left(m_a - \omega_g\right)\right|^2}$$

Overlap:

$$\eta_{01}^{\mathrm{A}} = \frac{1}{2\sqrt{U_0 U_1}} \int_{V_{\mathrm{cav}}} \mathrm{d}^3 x \sqrt{\frac{\varepsilon_0}{\mu_0}} \vec{E}_1(\vec{x}) \vec{B}_0(\vec{x}) \approx 4.06 \times 10^{-7}$$

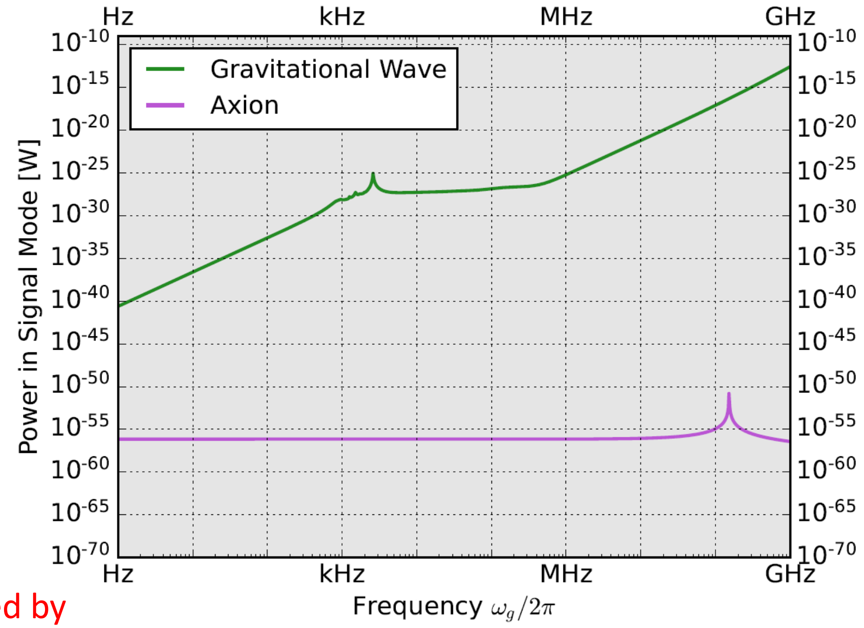
Parameters:

$$\rho_{DM} = 3.07 \times 10^{-42}\ \mathrm{GeV}^4$$

$$g_{a\gamma\gamma} = 5 \times 10^{-17} \mathrm{GeV}^{-1}$$

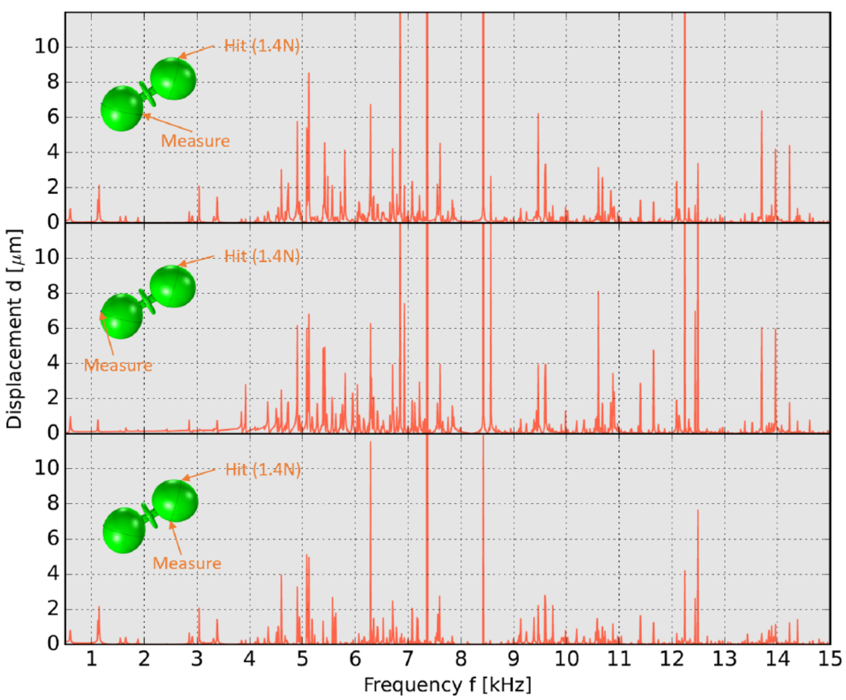
$$m_a = 10^{-7} \mathrm{eV} \approx 152\ \mathrm{MHz}$$

Very restrictive parameters motivated by dark matter searches of ALP DM

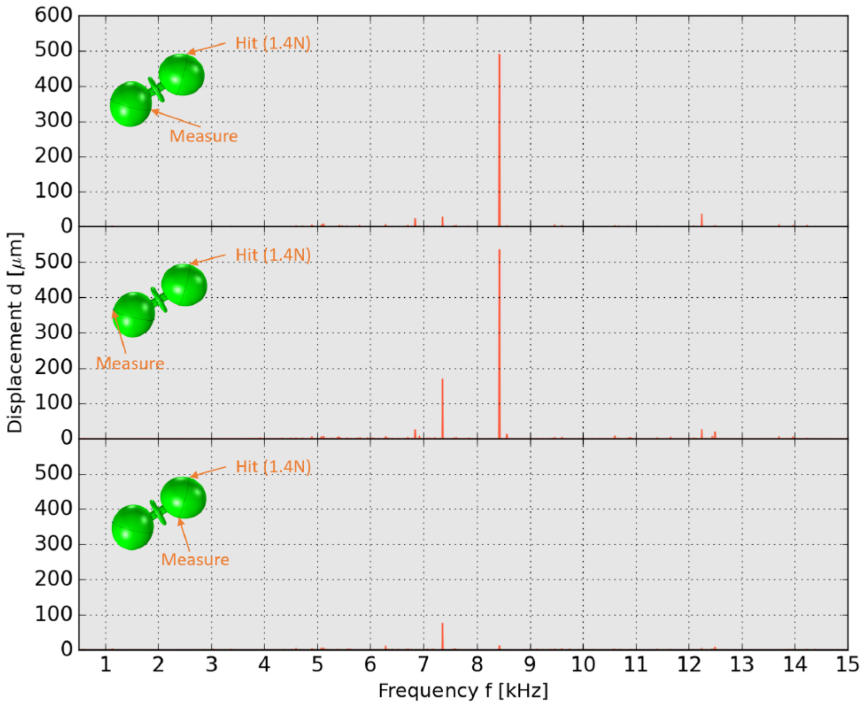


Backup 5: MAGO Vibrational Spectrum

We also simulated the mechanical spectrum of MAGO as a preparation for the experiment



Low-Scale Spectrum

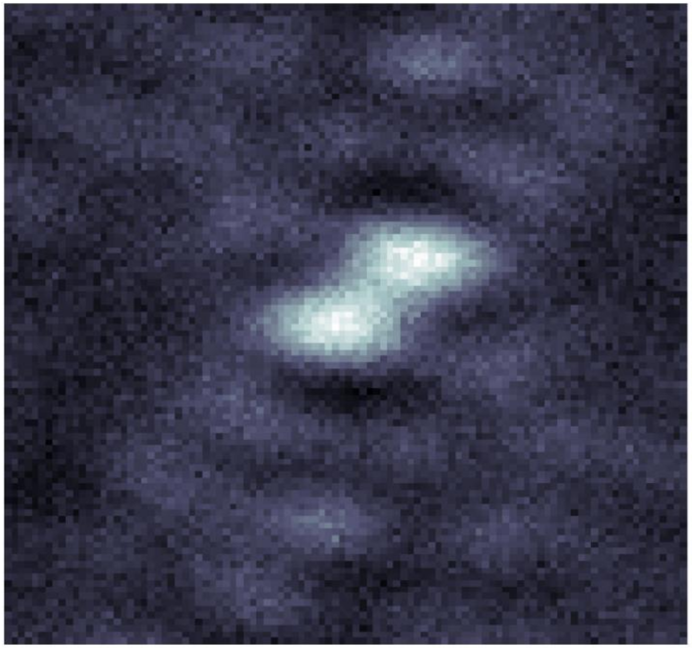


Large-Scale Spectrum

Backup 5: Multiple Cavities

An array of N MAGO-like cavities would be allow for an improved sensitivity and a localization of the source on the sky

- The picture below shows a simulation of the MAGO collaboration for an array of $N = 10$ cavities. They could reach a spatial resolution of $\Delta\theta = \Delta\phi = 0.1°$



(Ballantini et al., arXiv:gr-qc/0203024 (2005))

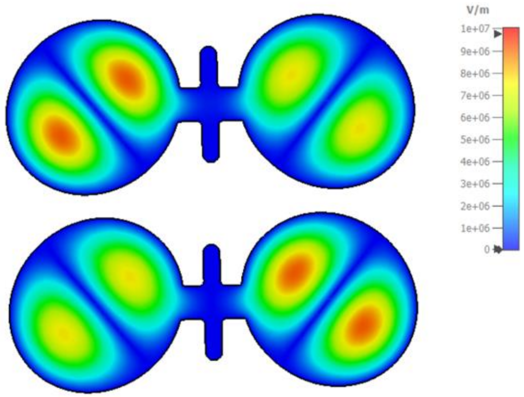
Backup 6: MAGO EM-Fields

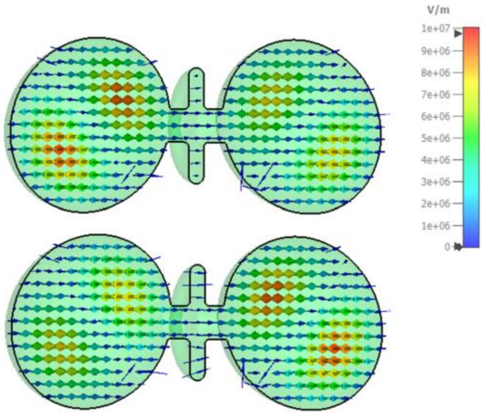
- Important Mode: TE_{011}, because it allows for a $l = 2$ transition between the modes

$$\omega_0 = 1.773430 \text{ GHz}$$

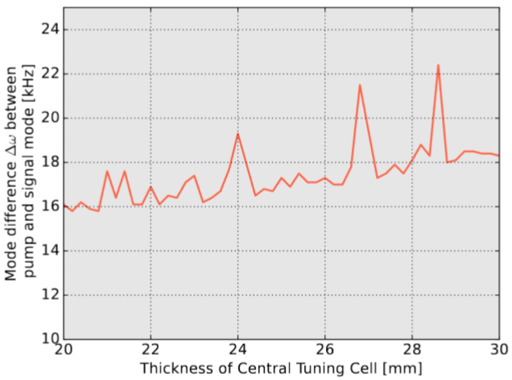
$$\Delta\omega \approx 16 \text{ kHz}$$

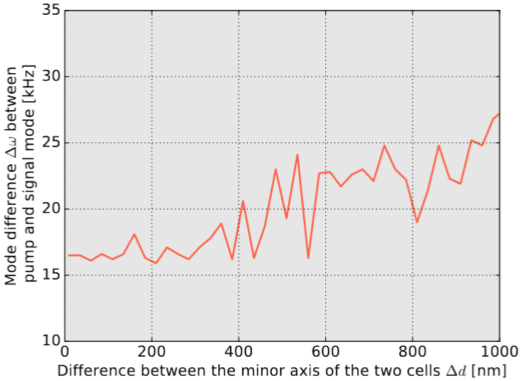
$$\omega_1 = 1.773446 \text{ GHz}$$





- Tuning Mechanisms:





Backup 7: Cavity Perturbation Theory

We consider the Lagrange function of the mechanical coupling:

$$U_n = \frac{1}{2\mu_0}\int_{V_{cav}} d^3x\, \vec{E}_n^2(\vec{x})$$

$$L_{\text{mech}} = \sum_n U_n'(e_n' - b_n') + \sum_l \left[\frac{1}{2}M\dot{q}_l^2 - \frac{1}{2}M\omega_l^2 q_l^2 + q_l f_l\right]$$

$$F_{\mu\nu}F^{\mu\nu} = \frac{2}{\mu_0}\vec{B}^2 - 2\varepsilon_0\vec{E}^2$$

$$\vec{E}(t,\vec{x}) = \sum_n e_n(t)\vec{E}_n(\vec{x})$$

The primed fields of the deformed cavity can be expressed in terms of the eigenmodes of the unperturbed cavity → **Cavity Perturbation Theory**

$$e_n' = e_n + \sum_{m\neq n}\frac{U_m}{U_n}\alpha_{nm}e_m$$

$$b_n' = b_n - \frac{1}{2}C_{nn}b_n + \sum_{m\neq n}\frac{U_m}{U_n}\beta_{nm}e_m$$

$$\alpha_{nm} = \frac{\omega_n\omega_m}{\omega_m^2 - \omega_n^2}C_{nm}$$

$$\beta_{nm} = \frac{\omega_n^2}{\omega_m^2 - \omega_n^2}C_{nm}$$

$$\omega_n' = \omega_n - \frac{1}{2}C_{nn}\omega_n$$

Backup 8: GW-EM Coupling

Again, we consider a monochromatic GW propagating in z-direction

$$\eta_{01}^{\mathrm{E}} = \frac{1}{H\sqrt{U_0 U_1}} \int_{V_{\mathrm{cav}}} \mathrm{d}^3x H_0(\vec{x}) \varepsilon_0 \vec{E}_0(\vec{x}) \vec{E}_1(\vec{x}) \approx 0.2$$

$$\eta_{01}^{\mathrm{B}} = \frac{1}{H\sqrt{U_0 U_1}} \int_{V_{\mathrm{cav}}} \mathrm{d}^3x H_0(\vec{x}) \frac{1}{\mu_0} \vec{B}_0(\vec{x}) \vec{B}_1(\vec{x}) \approx 0.2$$

$$\rightarrow \eta_{01}^{\mathrm{E}} \approx \eta_{01}^{\mathrm{B}} \approx 0.2$$

Other parameters:

$$H^{+} = h_{+} \times \sqrt{\frac{1}{V_{\mathrm{cav}}} \int_{V_{\mathrm{cav}}} \mathrm{d}^3x (x^2 - y^2)^2} \approx h_{+} \times 12.08\ \mathrm{m}^2$$

$$H^{\times} = h_{\times} \times \sqrt{\frac{1}{V_{\mathrm{cav}}} \int_{V_{\mathrm{cav}}} \mathrm{d}^3x (2xy)^2} \approx h_{\times} \times 12.26\ \mathrm{m}^2$$

$$H \approx H^{+} \approx H^{\times} \approx h_0 \times 12\ \mathrm{m}^2$$

$V_{\mathrm{cav}} \approx 9.56\ \mathrm{L}$ $A_{\mathrm{cav}} \approx 0.31\ \mathrm{m}^2$ $U_0 \approx 40\ \mathrm{J}$

Backup 9: Primordial Black Holes

- We focus on Primordial Black Hole (PBH) mergers with <u>equal</u> masses

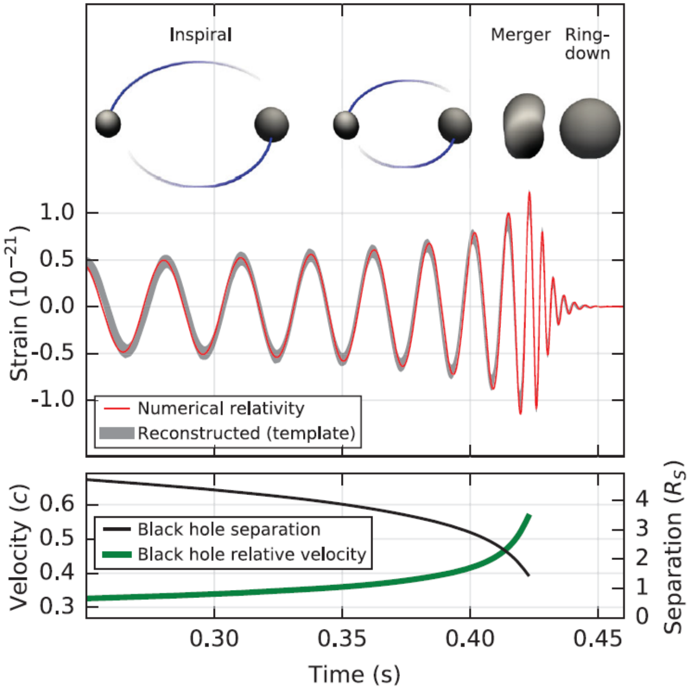
- The frequency increases until reaching the ISCO frequency (= Innermost Stable Circular Orbit).

$$f_{\mathrm{ISCO}} = 2200\ \mathrm{Hz} * \left(\frac{m_{\mathrm{PBH}}}{M_{\odot}}\right)^{-1}$$

- Outcoming Strain

$$h_0 \approx 9.77 \cdot 10^{-34} \times \left(\frac{f}{\mathrm{GHz}}\right)^{\frac{2}{3}} \left(\frac{m_{\mathrm{PBH}}}{10^{12} M_{\odot}}\right)^{\frac{5}{3}} \left(\frac{d_{\mathrm{L}}}{\mathrm{kpc}}\right)^{-1}$$

We took distances depending on the mass from *Franciolini et al. arXiv:2205.02153v1*



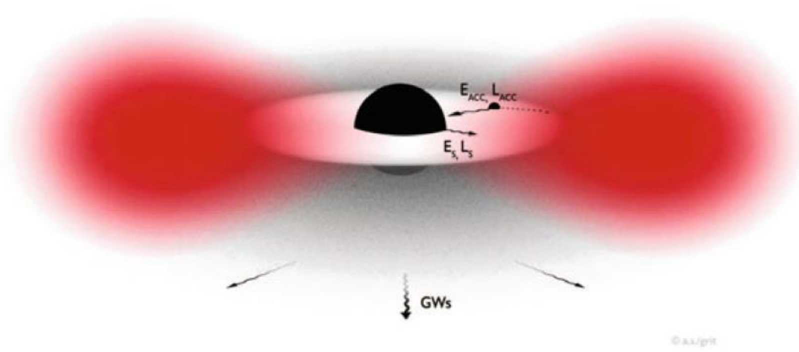
Abbott et al. PRL 116, 061102 (2016)

Backup 10: Superradiance

- Needed: Light (axion-like) scalar field and spinning black hole
- Superradiance instability in ergoregion of the black hole leads to a growing, bounded scalar field.
- Black hole and scalar field form atom-like structure
- GW production through annihilation of the scalar particles
- Strain:

$$h_0 \approx 10^{-23} \times \left(\frac{\Delta a_*}{0.1}\right)\left(\frac{1\ \mathrm{kpc}}{D}\right)\left(\frac{M_{BH}}{1 M_\odot}\right)\left(\frac{\alpha}{0.2}\right)^7$$

Typically very coherent!



Brito et al. *Superradiance – New Frontiers in Black Hole Physics*, page 200*,* Springer (2020)