Quartic Gauge-Higgs coupling: Constraints and Future Directions

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based on [2208.09334] with Anisha, O. Atkinson, A. Bhardwaj and C. Englert

Introduction

- Quartic Gauge-Higgs coupling κ_{2V}: Why is it of interest?
- Current direct constraints from experiments
- Going beyond the leading order: Higgs Effective Field Theory (HEFT)
- Constraints from single Higgs data, $H \to VV$
- Direct collider sensitivity with Graph Neural Networks (GNNs)
- Conclusion

Introduction

from 2207.00092

- Couplings of the discovered Higgs boson to SM matter follows SM expectations

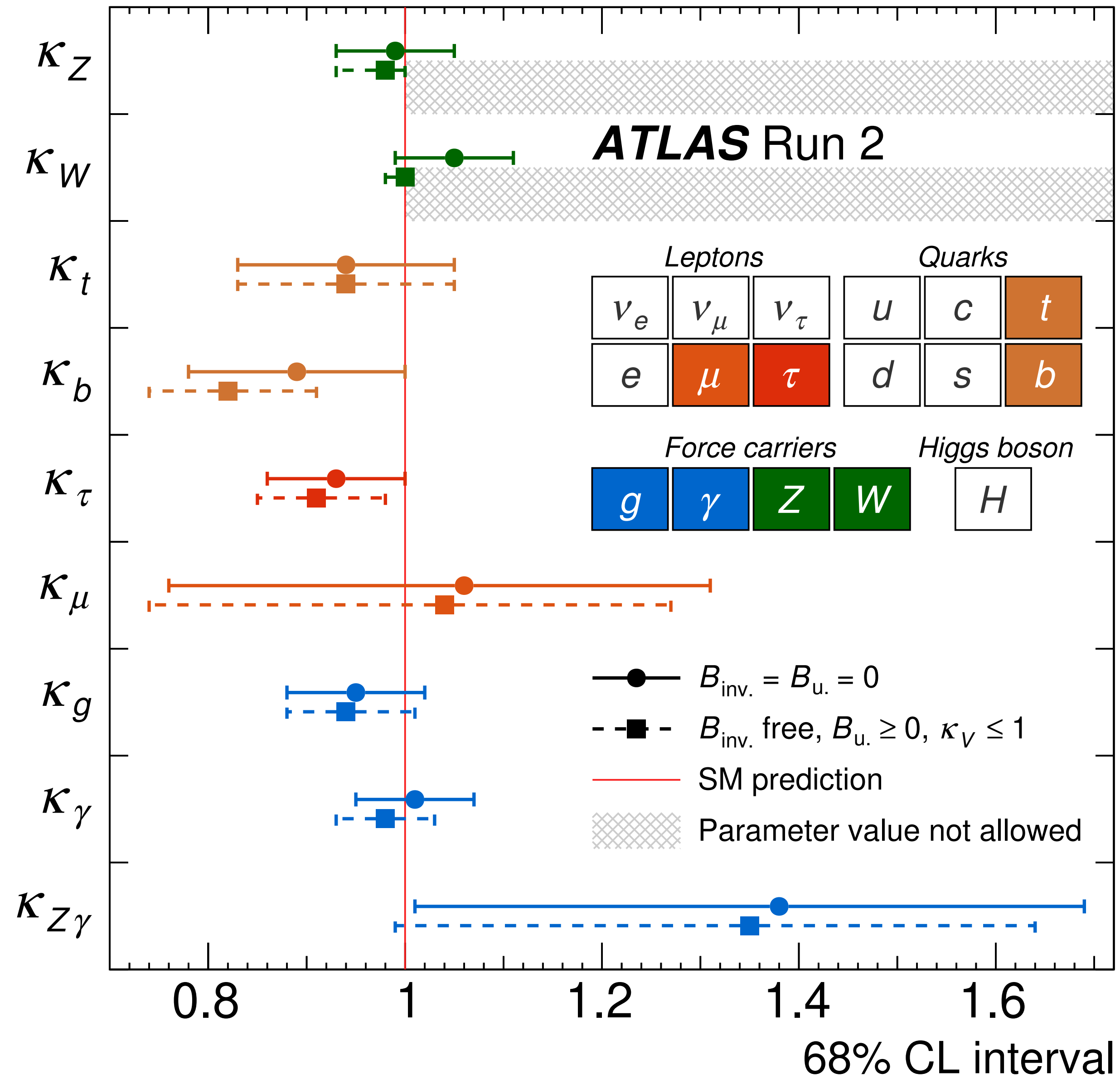
- κ-framework:

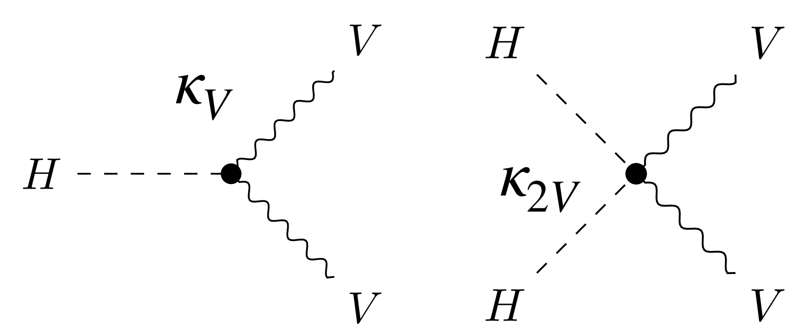
$$\kappa_i = \frac{g_i}{g_i^{\mathrm{SM}}}$$

- Any departure of gauge-Higgs couplings from SM expectation implies perturbative unitarity violation

- Less attention in quartic interactions between Higgs and gauge fields

Statistically limited





Introduction

- **In the SM**: gauge invariance and doublet character of Higgs imply full correlation of HVV and $HHVV$ interactions

- **BSM:**

  - In Multi-Higgs models with mixing, Higgs interactions modified by characteristic angle

$$\text{SM+singlet:} \quad \kappa_V = \cos(\chi) \qquad \kappa_{2V} = \cos^2(\chi)$$

$$\text{2HDM:} \quad \kappa_V = \sin(\beta - \alpha) \qquad \kappa_{2V} = 1$$

  - In composite Higgs, e.g. MCHM5 $\kappa_V = \sqrt{1 - \frac{v^2}{f^2}}$ but $\kappa_{2V} = 1 - 2\frac{v^2}{f^2}$

- For HVV both cases can be associated, e.g. $\sin(\beta - \alpha) = \sqrt{1 - \frac{v^2}{f^2}}$

- Could be distinguished from $HHVV$ measurement

Oblique corrections and κ_{2V}

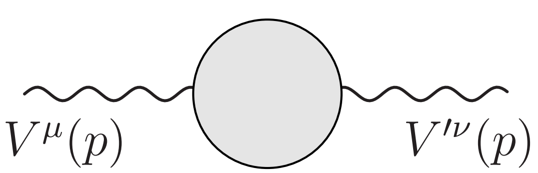
- Peskin-Takeuchi parameters

$$S = \frac{4 s_W^2 c_W^2}{\alpha}\left(\frac{\Pi_{ZZ}(M_Z)^2 - \Pi_{ZZ}(0)}{M_Z^2} - \frac{c_W^2 - s_W^2}{c_W s_W}\frac{\Pi_{AZ}(M_Z^2) - \Pi_{AZ}(0)}{M_Z^2} - \frac{\Pi_{AA}(M_Z^2)}{M_Z^2}\right)$$

$$T = \frac{1}{\alpha}\left(\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{2 s_W}{c_W}\frac{\Pi_{AZ}(0)}{M_Z^2}\right)$$

$$U = \frac{4 s_W^2}{\alpha}\left(\frac{\Pi_{WW}(M_W^2) - \Pi_{WW}(0)}{M_W^2} - c_W^2\frac{\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0)}{M_Z^2} - 2 s_W c_W \frac{\Pi_{AZ}(M_Z^2) - \Pi_{AZ}(0)}{M_Z^2} - s_W^2 \frac{\Pi_{AA}(M_Z^2)}{M_Z^2}\right)$$

constructed from vacuum polarisations



$$\sim \Pi^{\mu\nu}_{VV'}(p^2) = (p^2 - M_V^2)\delta_{VV'} + \Pi_{VV'}(p^2)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + B_{VV'}(p^2)\frac{p^\mu p^\nu}{p^2}$$

- Including modifiers κ_{2W} and κ_{2Z} gives

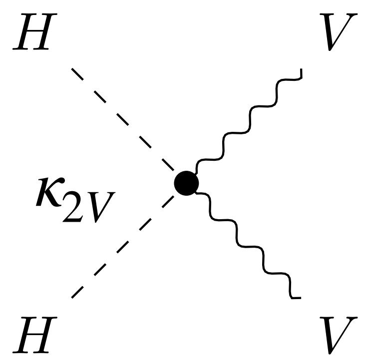
$$\Delta S = \Delta U = 0 \qquad\qquad \Delta T = \frac{\kappa_{2Z}^2 - \kappa_{2W}^2}{16\pi}\frac{M_H^2}{M_W^2} s_W^2 \log\frac{\Lambda}{M_H^2}$$

- T parameter is sensitive to custodial $SU(2)$ violation, need to keep $\kappa_{2W} \simeq \kappa_{2Z}$

- $\kappa_{2V} = \kappa_{2W} = \kappa_{2Z}$ removes oblique corrections

Perturbative unitarity and κ_{2V}

- Process relevant for κ_{2V} is $HV \to HV$ scattering
- Jacob-Wick expansion allows to extract partial waves



$\beta(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$

Wigner functions

$$a^J_{fi} = \frac{\beta^{1/4}(s, m^2_{f_1}, m^2_{f_1})\beta^{1/4}(s, m^2_{i_1}, m^2_{i_1})}{32\pi s} \int_{-1}^{1} \mathrm{d}\cos\theta\, \mathcal{D}^J_{\mu_i \mu_f}\, \mathcal{M}(s, \cos\theta)$$

- Wigner functions reduce to Legendre polynomials and $J = 0$ provides the dominant constraints
- Expressing the optical theorem in terms of partial waves gives us

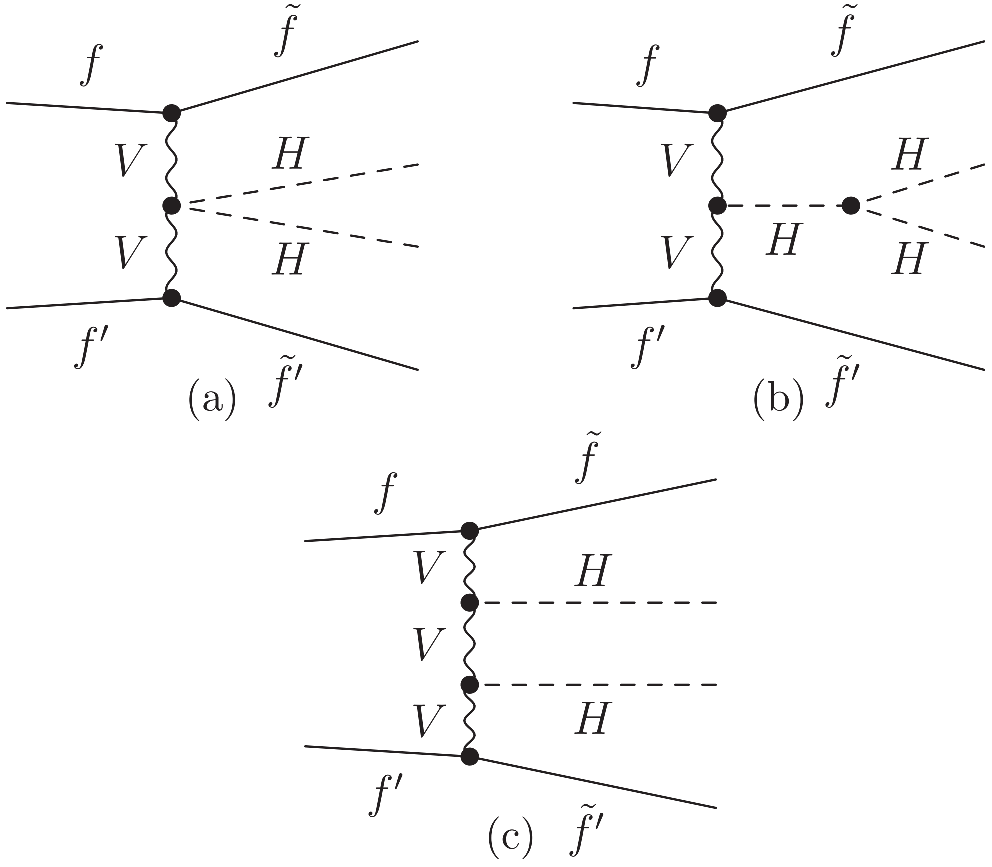
$$\mathrm{Im} a^0_{ii} \geq |a^0_{ii}|^2 \implies |\mathrm{Re} a^0_{ii}| \leq \frac{1}{2}$$

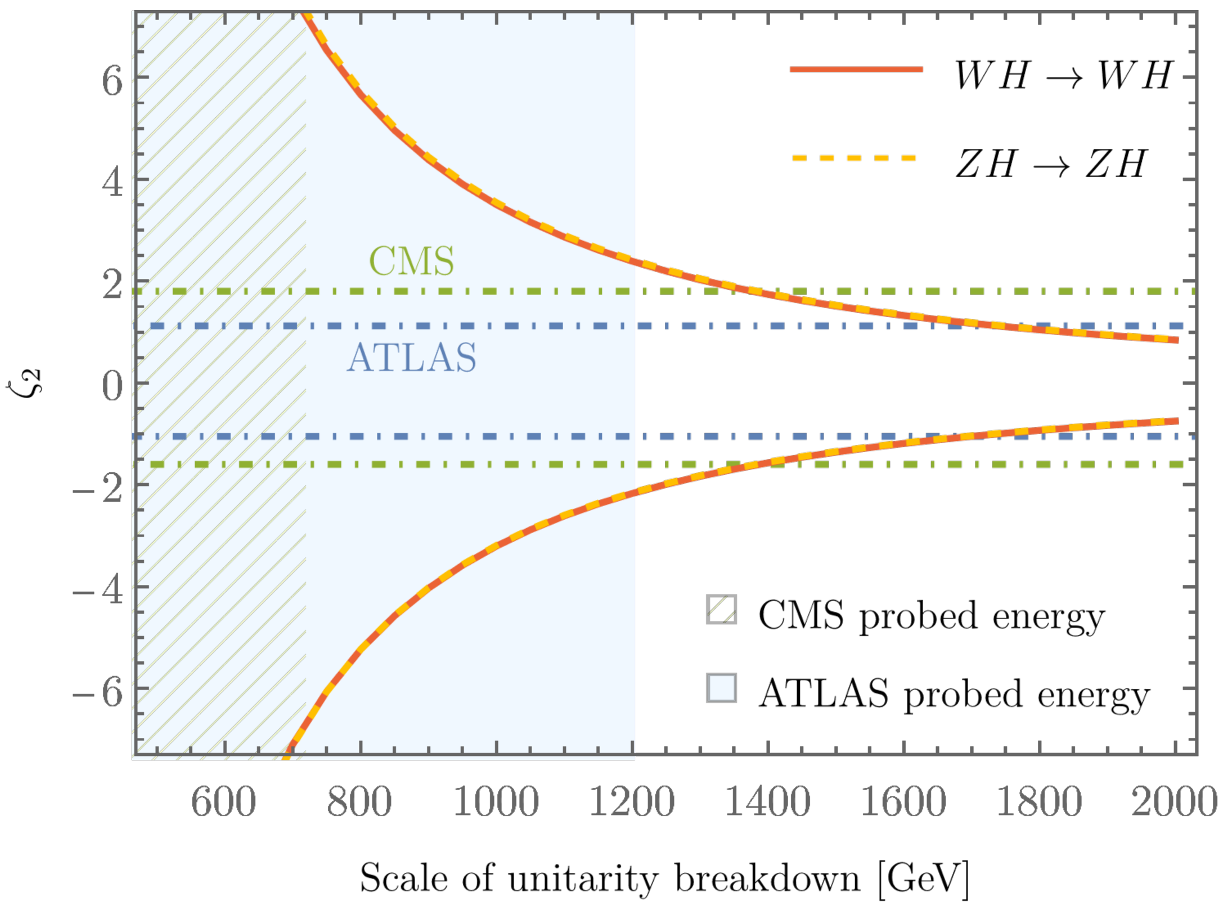
Perturbative unitarity and κ_{2V}

- Define $\zeta_2 = \kappa_{2V} - 1$
- Direct constraints from both ATLAS, CMS through VBF with $\kappa_V = 1$
- ATLAS uses the $HH \to b\bar{b}b\bar{b}$ final state (2001.05178 and CONF-HDBS-2022-35)
- CMS uses $HH \to b\bar{b}\tau^+\tau^-$ (2206.09401)

  - Measurements consistent with unitarity
  - κ_{2V} relatively unconstrained

- Can we do better by including single Higgs data $H \to VV$?





Going beyond leading order

- Beyond tree-level, $\kappa_{2V} = 1 + \zeta_2 \neq 1$ is problematic within the SM, e.g. consider the $H \to ZZ$ renormalised amplitude in R_ξ gauge

$$\mathcal{M}^{\text{virt}} + \mathcal{M}_{\text{CT}} = -\frac{\alpha}{32\pi} \frac{e}{M_W s_W^5 c_W^2} \left(M_H^2 + 2M_Z^2 s_W^2 (3 + \xi_Z) \right) \times \Delta^{\text{UV}}(\mu^2, M_Z^2) \zeta_2 [\epsilon^\mu(Z_1)\epsilon_\mu(Z_2)]^* + \mathcal{O}(\varepsilon^0)$$

$\overline{\text{MS}}$ related divergence

- Gauge invariance is violated when only part of the kinetic Higgs term is modified

$$D_\mu \Phi^\dagger D^\mu \Phi \supset -g^{\text{SM}}_{HZZ} H Z_\mu Z^\mu - g^{\text{SM}}_{HHZZ} H^2 Z_\mu Z^\mu$$

- ζ_2 induces singularities not removed by bare HZZ SM interactions.

Model-independency beyond leading order

- We want to study correlations $\kappa_V - \kappa_{2V}$ which will help distinguishing BSM theories

- **How can we do a model-independent analysis as ATLAS and CMS and how do we include single Higgs data at NLO (e.g. $H \to VV*$)?**

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- **How can we do a model-independent analysis as ATLAS and CMS and how do we include single Higgs data at NLO (e.g. $H \to VV*$)?**

SM Effective Field Theory (SMEFT)

- Model independent up to some assumptions:
  - New physics decouple
  - Higgs is in an electroweak doublet

Not applicable, κ_V and κ_{2V} are not free parameters

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SM ⊂ SMEFT ⊂ HEFT

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Higgs Effective Field Theory (HEFT)

HEFT

- In HEFT the Higgs is a singlet and Goldstone bosons are parametrised non-linearly through matrix

$$U = \exp(i\pi^a \tau^a / v)$$

- Transformation under $L \in SU(2)_L$ and $R \in SU(2)_R$ as $U \to LUR^\dagger$

- Can be expanded as $U = \mathbb{1}_2 + i\frac{\pi^a}{v}\tau^a - \frac{2G^+G^- + G^0G^0}{2v^2}\mathbb{1}_2 + \ldots$

  where $G^\pm = \frac{1}{\sqrt{2}}(\pi^2 \pm i\pi^1)$ and $G^0 = -\pi^3$

- Covariant derivative is $D_\mu U = \partial_\mu U + ig_W(W_\mu^a \tau^a/2)U - igUB_\mu \tau^3/2$

- Gauge fields in physical basis defined through the rotation

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)\,, \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

HEFT

- Leading order Lagrangian given by

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \mathcal{L}_{\text{ferm}} + \mathcal{L}_{\text{Yuk}} + \frac{v^2}{4}\mathcal{F}_H \,\text{Tr}[D_\mu U^\dagger D^\mu U]$$
$$+\frac{1}{2}\partial_\mu H \partial^\mu H - V(H) + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}$$

- Higgs interactions with gauge and Goldstone bosons parametrised by

$$\mathcal{F}_H = \left(1 + 2\underbrace{(1+\zeta_1)}_{\kappa_V}\frac{H}{v} + \underbrace{(1+\zeta_2)}_{\kappa_{2V}}\left(\frac{H}{v}\right)^2 + \ldots\right)$$

- Potential: $V(H) = \frac{1}{2}M_H^2 H^2 + \kappa_3 \frac{M_H^2}{2v} H^3 + \kappa_4 \frac{M_H^2}{8v^2} H^4$

- Yukawa: $\mathcal{L}_{\text{Yuk}} = -\frac{v}{\sqrt{2}}\left(\bar{u}^i_L \quad \bar{d}^i_L\right) U \left(1 + c\frac{h}{v} + \ldots\right) \begin{pmatrix} y^u_{ij} u^j_R \\ y^d_{ij} d^j_R \end{pmatrix} + \text{h.c.}$

- Faddeev-Popov & Gauge-Fixing:

HEFT in R_ξ by Herrero and Morales
(2005.03537, 2107.07890, 2208.05900)

Additional operators

- HEFT is non-renormalisable in the 'classical' sense
- When considering radiative corrections the UV divergences will source new operator structures at one-loop
- Necessary to include all additional relevant operators at tree level for a consistent one-loop calculation
- Chiral Dimension 4:

$$\mathcal{L}_4 = \sum_i a_i \mathcal{O}_i$$

$\mathcal{O}_0$	$a_0(M_Z^2 - M_W^2)\mathrm{Tr}\left[U\tau^3U^\dagger \boldsymbol{\mathcal{V}}_\mu\right]\mathrm{Tr}\left[U\tau^3U^\dagger \boldsymbol{\mathcal{V}}_\mu\right]$
$\mathcal{O}_1$	$a_1\, g' g_W \mathrm{Tr}\left[UB_{\mu\nu}\frac{\tau^3}{2}U^\dagger W^a_{\mu\nu}\frac{\tau^a}{2}\right]$
$\mathcal{O}_{HBB}$	$-a_{HBB}\, g'^2 \frac{H}{v}\mathrm{Tr}\left[B_{\mu\nu}B^{\mu\nu}\right]$
$\mathcal{O}_{HWW}$	$-a_{HWW}\, g_W^2 \frac{H}{v}\mathrm{Tr}\left[W^a_{\mu\nu}W^{a\mu\nu}\right]$
$\mathcal{O}_{\Box\mathcal{V}\mathcal{V}}$	$a_{\Box\mathcal{V}\mathcal{V}}\frac{\Box H}{v}\mathrm{Tr}\left[\boldsymbol{\mathcal{V}}_\mu \boldsymbol{\mathcal{V}}^\mu\right]$
$\mathcal{O}_{H0}$	$a_{H0}(M_Z^2 - M_W^2)\frac{H}{v}\mathrm{Tr}\left[U\tau^3U^\dagger \boldsymbol{\mathcal{V}}_\mu\right]\mathrm{Tr}\left[U\tau^3U^\dagger \boldsymbol{\mathcal{V}}_\mu\right]$
$\mathcal{O}_{H1}$	$a_{H1}\, g' g_W \frac{H}{v}\mathrm{Tr}\left[UB_{\mu\nu}\frac{\tau^3}{2}U^\dagger W^a_{\mu\nu}\frac{\tau^a}{2}\right]$
$\mathcal{O}_{H11}$	$a_{H11}\frac{H}{v}\mathrm{Tr}\left[\mathcal{D}_\mu \boldsymbol{\mathcal{V}}^\mu \mathcal{D}_\nu \boldsymbol{\mathcal{V}}^\nu\right]$
$\mathcal{O}_{d1}$	$i a_{d1}\, g' \frac{\partial^\nu H}{v}\mathrm{Tr}\left[UB_{\mu\nu}\frac{\tau^3}{2}U^\dagger \boldsymbol{\mathcal{V}}^\mu\right]$
$\mathcal{O}_{d2}$	$i a_{d2}\, g_W \frac{\partial^\nu H}{v}\mathrm{Tr}\left[W^a_{\mu\nu}\frac{\tau^a}{2}\boldsymbol{\mathcal{V}}^\mu\right]$
$\mathcal{O}_{d3}$	$a_{d3}\frac{\partial^\nu H}{v}\mathrm{Tr}\left[\boldsymbol{\mathcal{V}}^\mu \mathcal{D}_\mu \boldsymbol{\mathcal{V}}^\mu\right]$
$\mathcal{O}_{\Box\Box}$	$a_{\Box\Box}\frac{\Box H \Box H}{v}$

$$\boldsymbol{\mathcal{V}}_\mu = (D_\mu U)U^\dagger$$

$$\mathcal{D}_\mu \boldsymbol{\mathcal{V}}^\mu = \partial_\mu \boldsymbol{\mathcal{V}}^\mu + i[g_W W^a_\mu \tfrac{\tau^a}{2}, \boldsymbol{\mathcal{V}}^\mu]$$

Renormalisation

- Treat VEV and s_W as derived quantities $v = \frac{2M_W s_W}{e}$ $\quad s_W^2 = 1 - c_W^2 = 1 - \frac{M_W^2}{M_Z^2}$
- Usual renormalisation of SM quantities:

$$
\begin{aligned}
e_0 &= (1+\delta Z_e)e \\
M_{0,W}^2 &= M_W^2 + \delta M_W^2 \\
M_{0,Z}^2 &= M_Z^2 + \delta M_Z^2 \\
s_{W,0} &= s_W + \delta s_W \\
c_{W,0} &= c_W + \delta c_W
\end{aligned}
$$

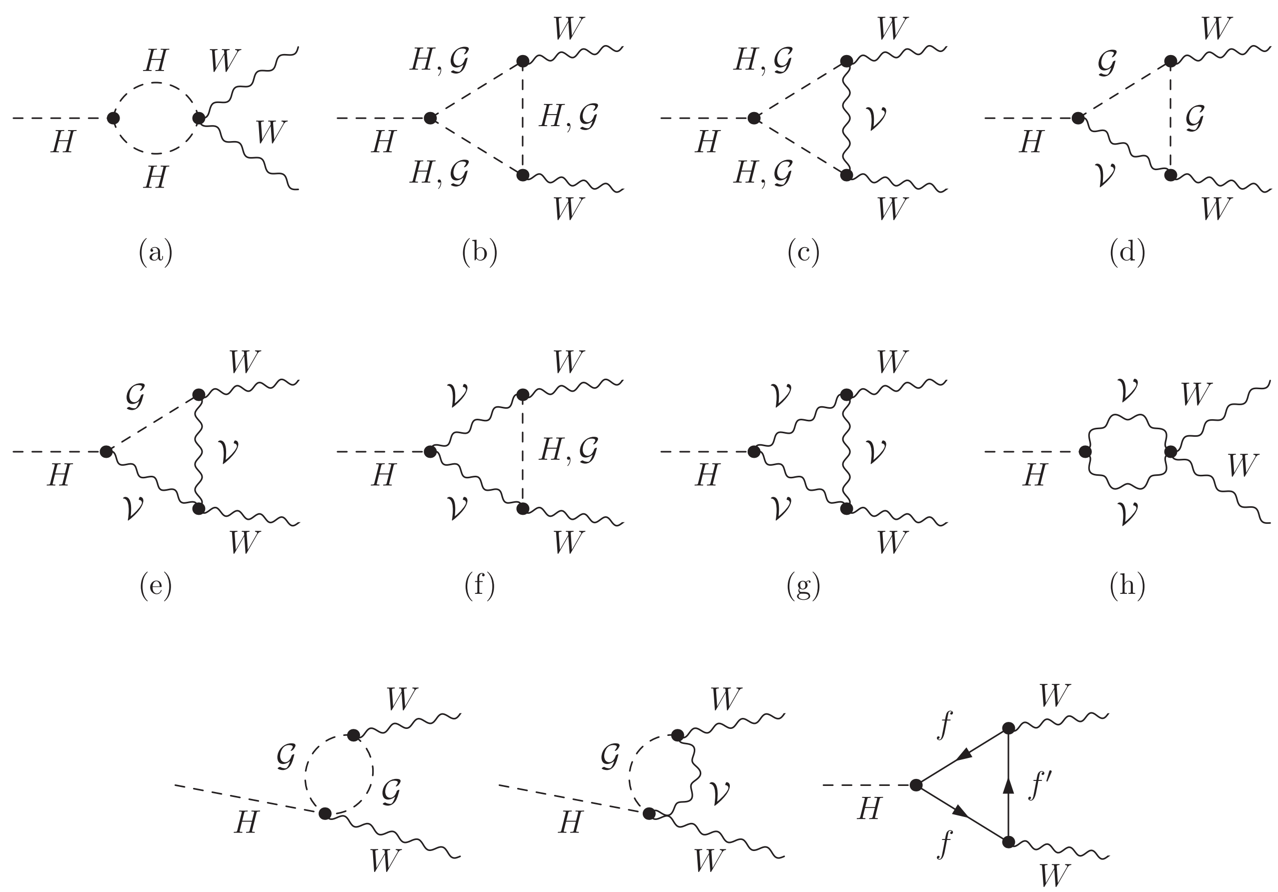
$$
\begin{aligned}
W_{0,\mu}^{\pm} &= (1+\delta Z_W/2)W_\mu^{\pm} \\
H_0 &= (1+\delta Z_H/2)H \\
\begin{pmatrix} A_0 \\ Z_0 \end{pmatrix}_\mu &= \begin{pmatrix} 1+\delta Z_{AA} & \delta Z_{AZ} \\ \delta Z_{ZA} & 1+\delta Z_{ZZ} \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix}_\mu
\end{aligned}
$$

- On-shell renormalisation conditions for gauge and Higgs sectors
- Electric charge renormalised in the Thomson limit
- HEFT coefficients: $\zeta_{0,i} = \zeta_i + \delta\zeta_i \quad a_{0,i} = a_i + \delta a_i$
- HEFT coefficients renormalised in $\overline{\text{MS}}$ scheme

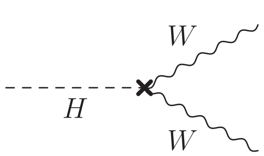
HVV at one-loop

- Can calculate one-loop processes and fix $\delta a_i, \delta\zeta_i$ RCs by removing singularities
- Can calculate $H \to VV$

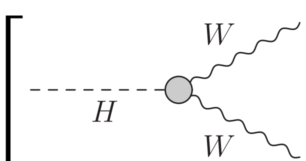
Triangle diagrams:


(a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k)

Counterterm:



$$= \frac{eM_W(1+\zeta_1)}{s_W}\left(\delta Z_e + \frac{\delta M_W^2}{2M_W^2} - \frac{\delta s_W}{s_W} + \frac{\delta\zeta_1}{1+\zeta_1} + \frac{\delta Z_H}{2} + \delta Z_W\right) g^{\mu\nu}$$

$+\delta\Big[$  $\Big]$

Terms with HEFT δa_i

Decay widths

- Only include 1-loop interference with LO: $\mathcal{M}^2 = |\mathcal{M}_{\rm LO}|^2 + 2\,{\rm Re}\left\{\mathcal{M}_{\rm LO}\mathcal{M}^*_{\text{1-loop}}\right\}$
- We parameterise both LO and 1-loop amplitudes as

Vector propagator

polarisation vector

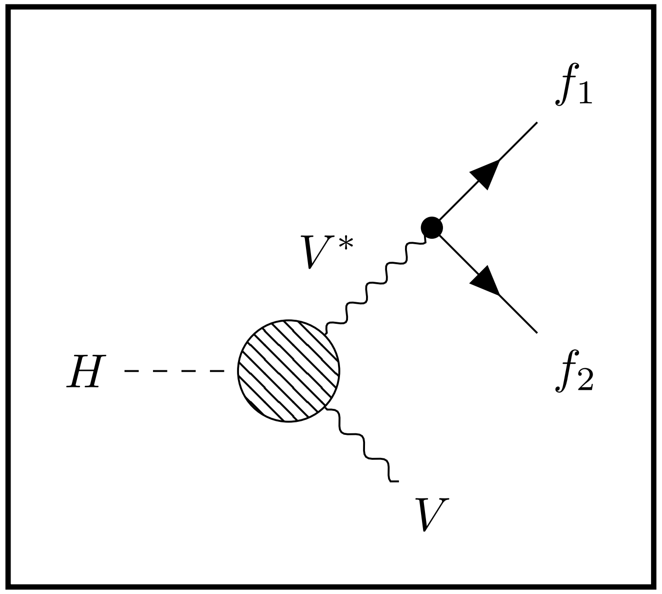
$$\mathcal{M} = \mathcal{M}^\mu \epsilon^*_\mu = \mathcal{M}^{\mu\nu}_{HVV}\Delta^{\nu\rho}_{VV}\mathcal{M}^\rho_{ffV}\epsilon^*_\mu$$

Levi-Civita tensor

$$\mathcal{M}^{\mu\nu}_{HVV} = (F_1^{\rm LO} + F_1^{\text{1-loop}})g^{\mu\nu} + F_2^{\text{1-loop}}(p_1+p_2)^\mu p_3^\nu + F_3^{\text{1-loop}}\varepsilon^{\mu\nu}_{\rho\sigma}p^\rho_H(p_1+p_2)^\sigma$$

- Decay width: (from S. Dawson and P. Giardino, 1807.11504 and 1801.01136)

$$m_{23^{\rm up}_{\rm low}} = \frac{1}{2}\left(M_H^2 + M_V^2 - m_{12}^2 \mp \sqrt{\lambda(m_{12}, M_H, M_V)}\right)$$



$$\Gamma[H(p_H)\to f(p_1)f(p_2)V(p_3)] = \int_0^{(M_H-M_V)^2} dm^2_{12}\int_{m^2_{23,\rm low}}^{m^2_{23,\rm up}} dm^2_{23}\,\frac{\mathcal{M}^2}{32(2\pi)^3M_H^3}$$

$$m_{ij} = (p_i+p_j)^2$$

- Integration performed numerically after identifying form factors F_i

$\kappa_V - \kappa_{2V}$ bounds from Higgs data

- Renormalisation non-trivial and requires additional HEFT Lorentz structures and coefficients a_i

- Aim here is to obtain a consistent $\kappa_{2V} - \kappa_V$ correlation, not perform a comprehensive HEFT fit → set $a_i = 0$ and perform χ^2 fit for bounds on $\zeta_1 - \zeta_2$

Naive rescaling by luminosity

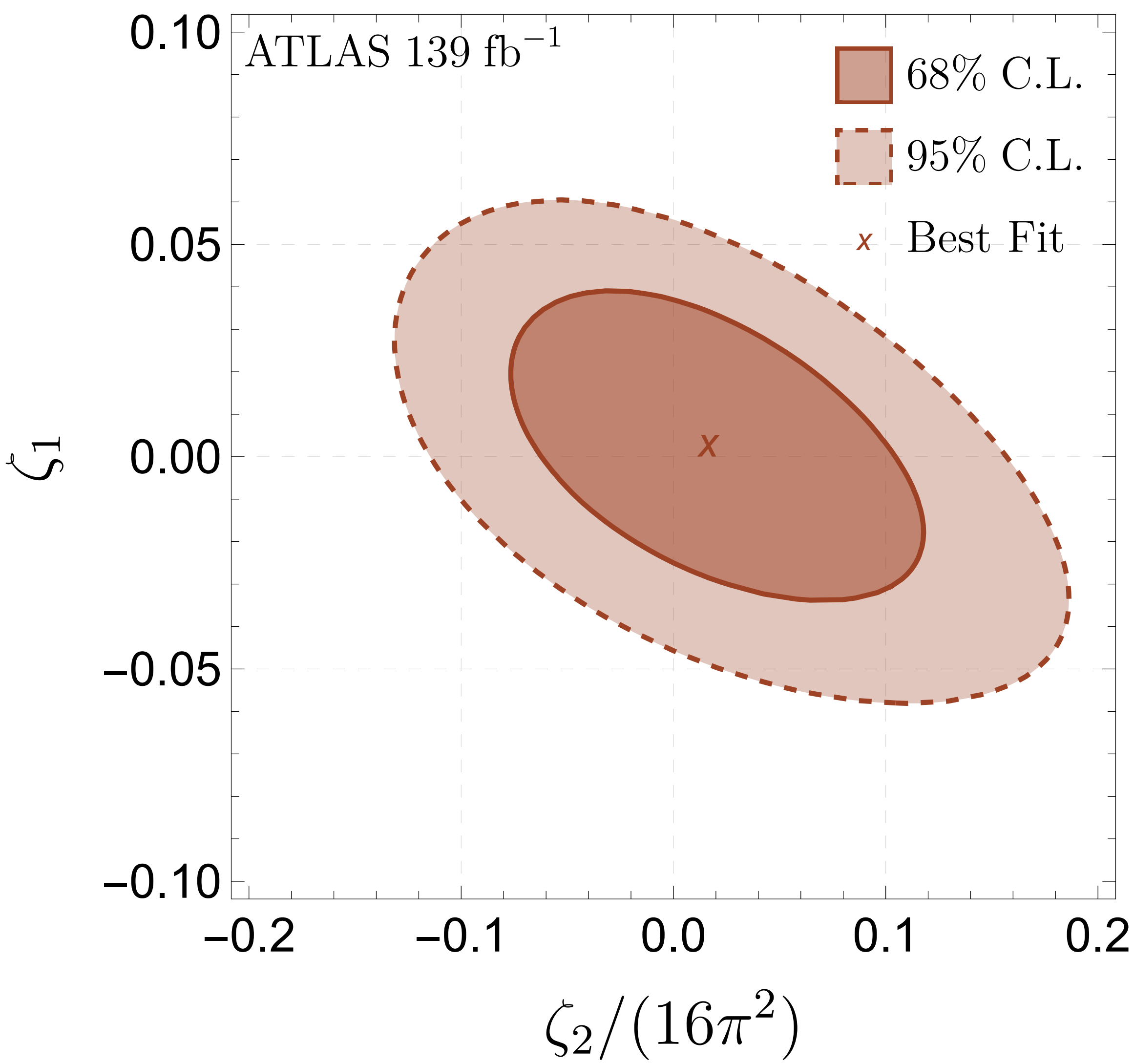
Parameters	ATLAS Run 2 data [*]	HL-LHC uncertainties	Correlation Matrix [*]			
κ_Z	$0.99^{+0.06}_{-0.06}$	± 0.012	1	0.40	0.44	0.09
κ_W	$1.05^{+0.06}_{-0.06}$	± 0.013		1	0.47	0.08
κ_γ	$1.01^{+0.06}_{-0.06}$	± 0.013			1	0.12
$\kappa_{Z\gamma}$	$1.38^{+0.31}_{-0.37}$	± 0.073				1

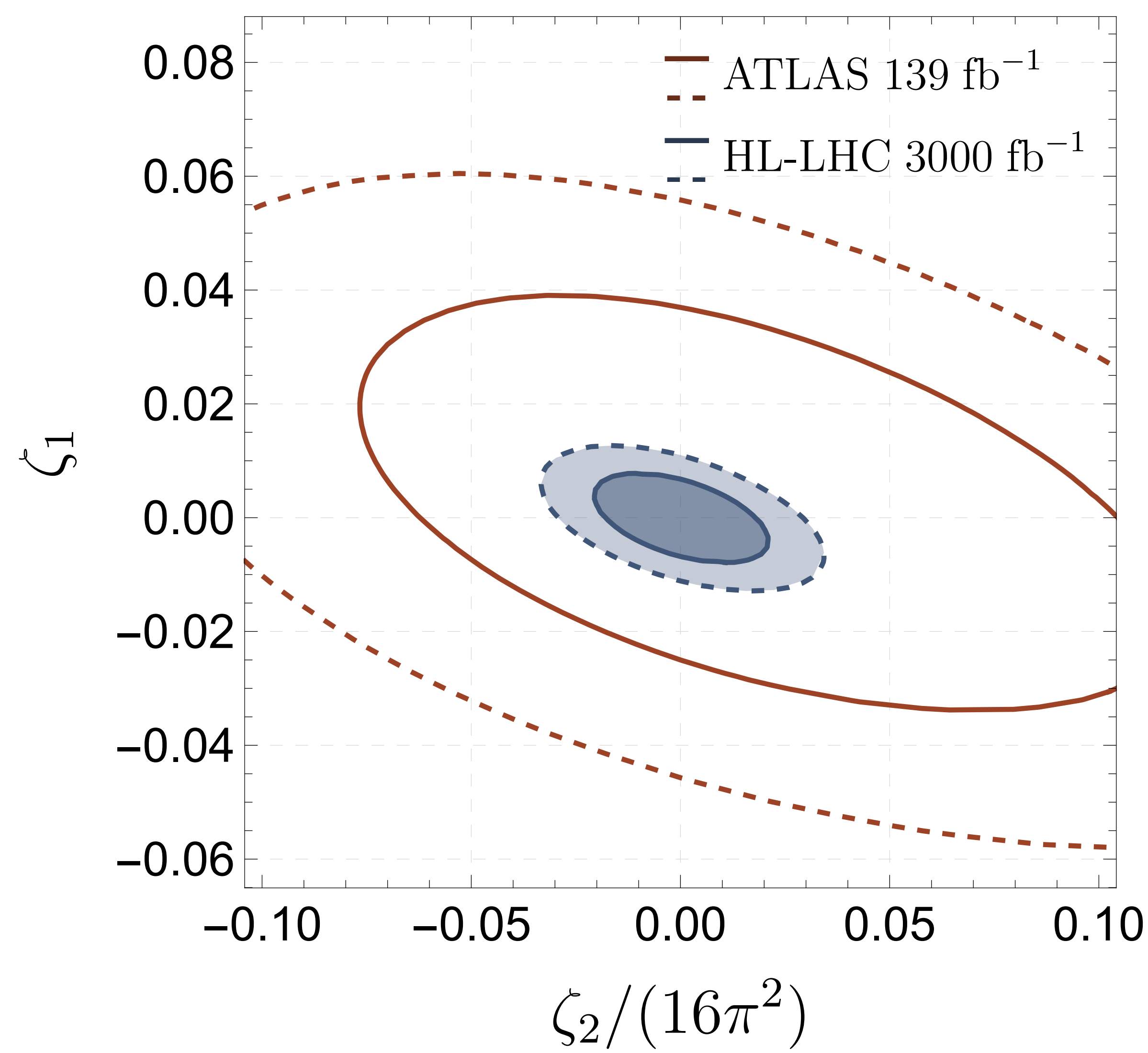
*ATLAS (2207.00092)

$$\chi^2(\zeta_1, \zeta_2) = \sum_{i,j=1}^{\text{data}} \left(\kappa_{i,\text{exp}} - \kappa_{i,\text{th}}(\zeta_1, \zeta_2)\right) (V_{ij})^{-1} \left(\kappa_{j,\text{exp}} - \kappa_{j,\text{th}}(\zeta_1, \zeta_2)\right)$$

$\kappa_V - \kappa_{2V}$ bounds from Higgs data

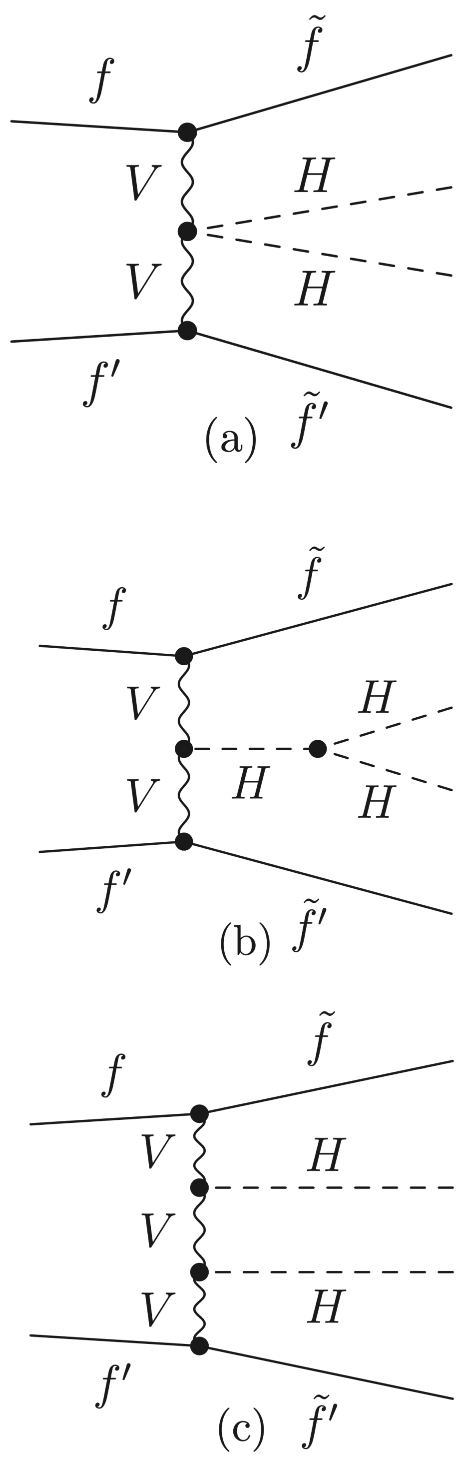
- ζ_2 is loop suppressed
- Loose bounds on κ_{2V} and even at HL-LHC $|\zeta_2| \sim 4$
- Indicates the need to increase the sensitivity of direct searches





Enhancing direct sensitivity

- Usual selection for VBF topology $pp \to (H \to b\bar{b})(H \to b\bar{b})jj$
  - 2 forward jets in opposite hemispheres, $\eta_{j_1}\eta_{j_2} < 1$
  - large invariant mass of jets, $m_{jj} > 500$ GeV
- Main background from multijet QCD processes



- Sensitivity enhancement:
  - represent events as fully-connected bidirectional graphs
  - use a Graph Neural Network (GNN) for signal-background discrimination
  - Supervised learning: background → 0, signal → 1

Graph Neural Network

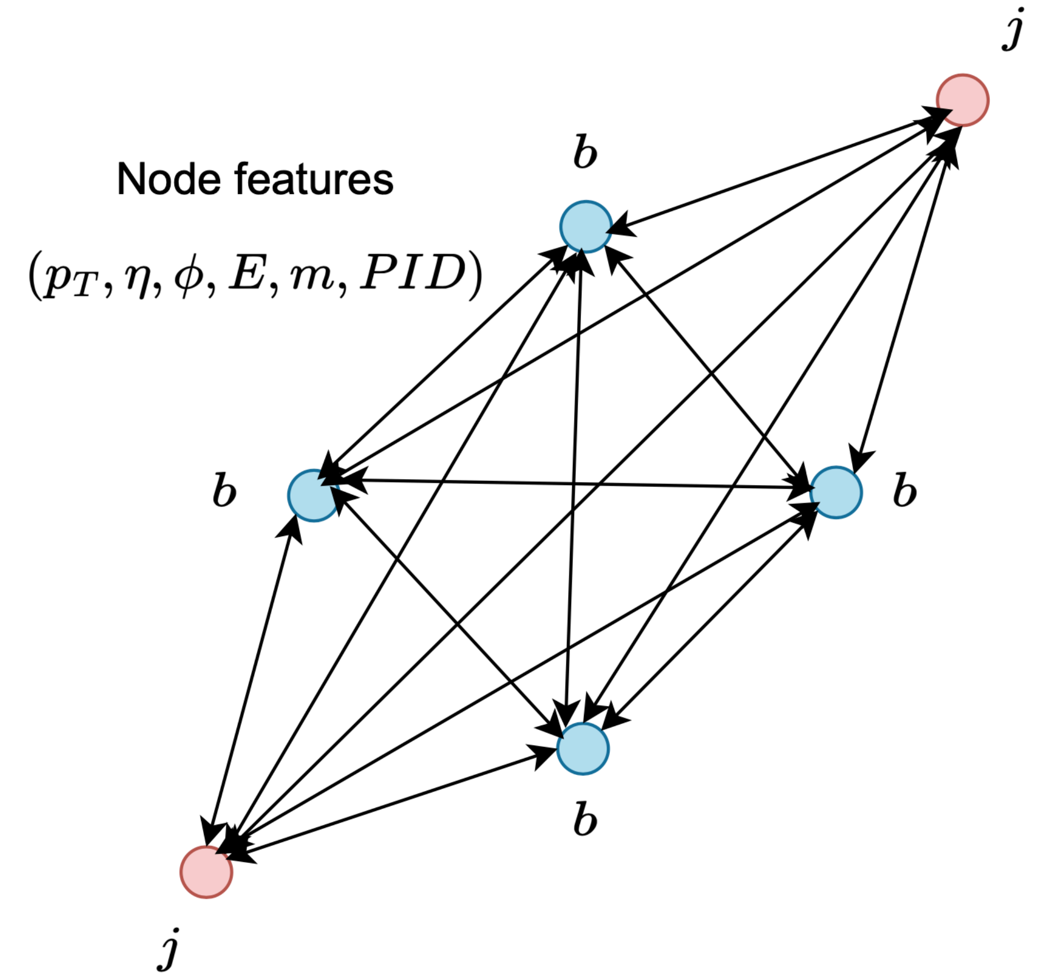
- Each node is assigned node features $\vec{x}_i^{(0)}$ as input

- Node features updated for each 'message passing layer' with Edge Convolution

$$\vec{x}_i^{(l+1)} = \frac{1}{|\mathcal{N}(i)|} \sum_{j \in \mathcal{N}(i)} \textrm{ReLU}\left(\Theta \cdot (\vec{x}_j^{(l)} - \vec{x}_i^{(l)}) + \Phi \cdot (\vec{x}_i^{(l)})\right)$$

Linear Layers

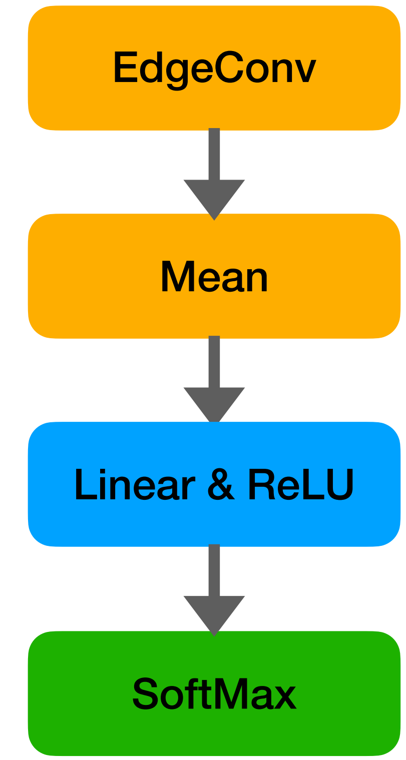
Nodes in 'neighbourhood' of i (connected)



- 'Graph readout operation' → mean

- We then have a vector for the 'graph properties'

- A last linear layer produces a two-dimensional vector →signal/background scores

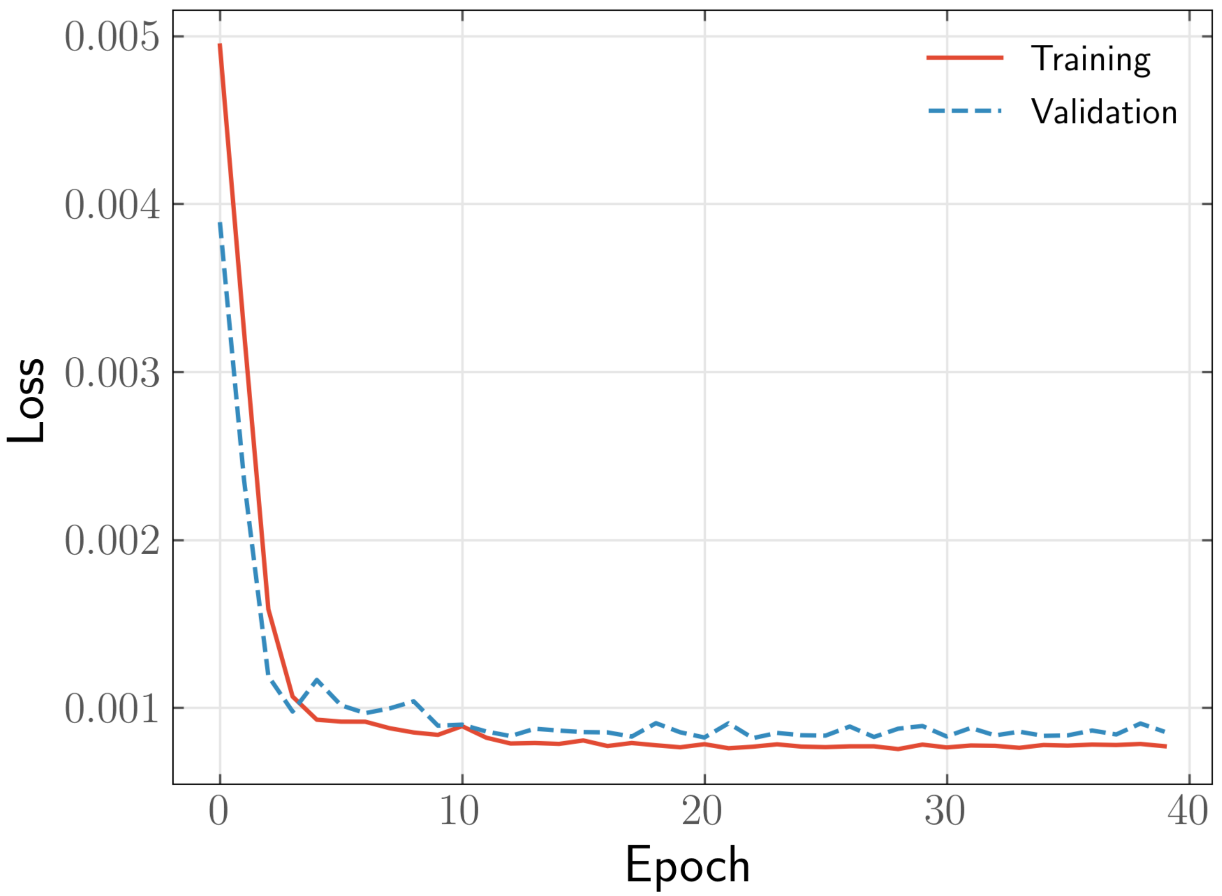


Performance

- Train by minimising cross-entropy loss function with ADAM

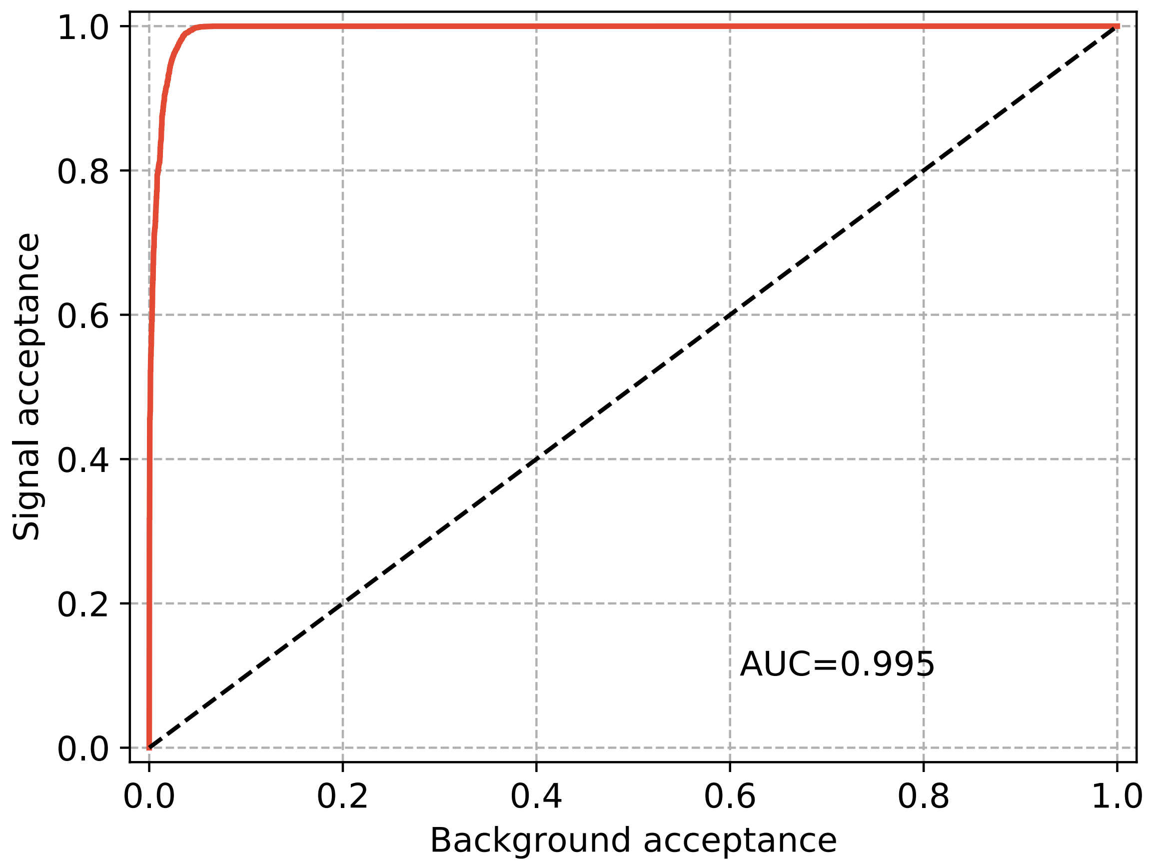
$$\mathcal{L} = -\sum_{i=1}^{2} y_i \log \hat{y}_i$$

True label (1 or 0) → y_i; Predicted score → $\hat{y}_i$



- Validation indicates avoidance of overfitting

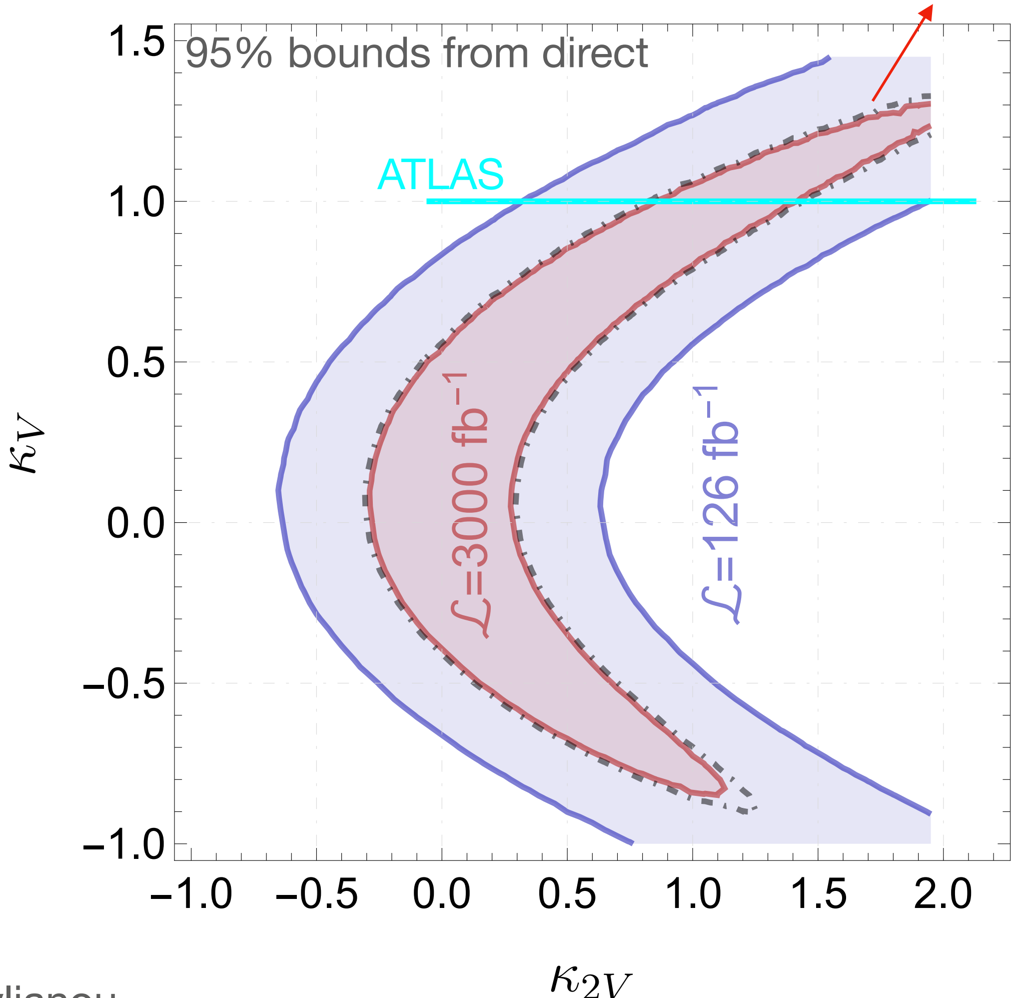
- Optimal working point identified from Receiver Operating Characteristic (ROC) curve to significantly reduce background

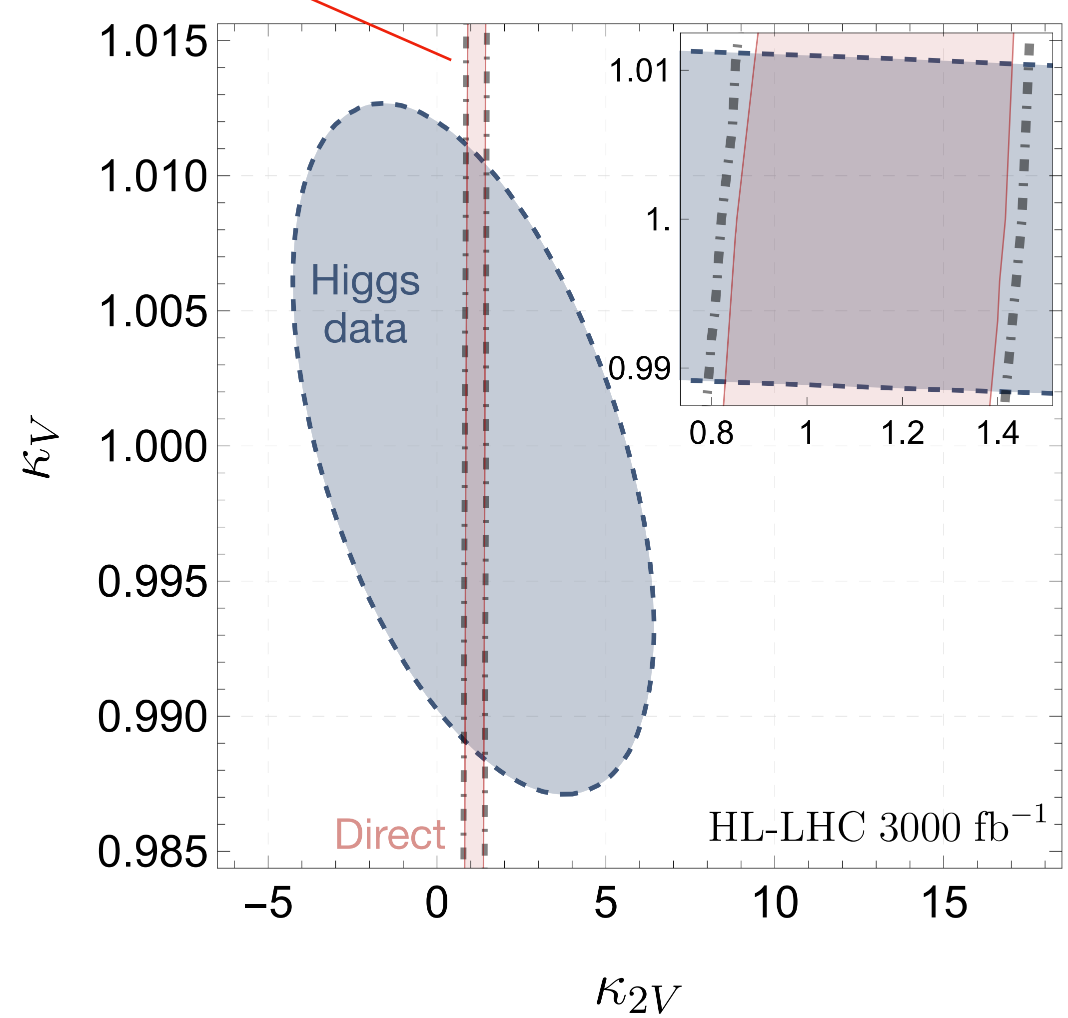


Bounds on $\kappa_V - \kappa_{2V}$

- Relative agreement gives confidence for HL-LHC extrapolation
- Improvement indicate that we might be optimistic:
  - No full detector simulation
  - No subdominant backgrounds (e.g. multi-boson and top)
- Also consider additional 25% background for HL-LHC

Dot-dashed: 25% background increase





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↓

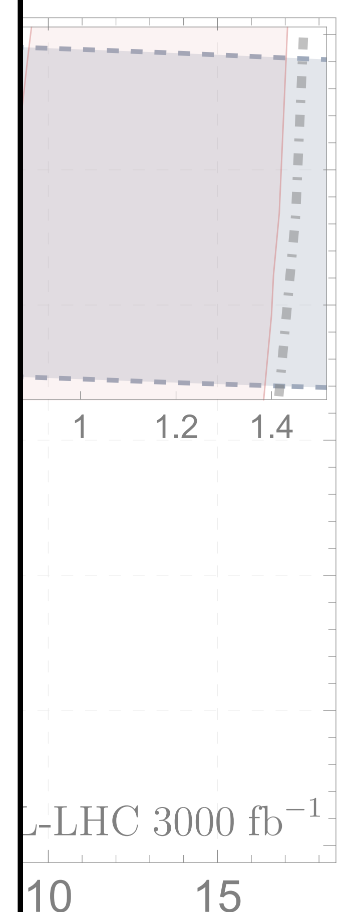
This can affect the GNN performance

↓

<u>But:</u>

- Such backgrounds have different kinematics
- Multi-class classification separating different backgrounds can enhance results in that case
- More sophisticated graph-embedding (e.g. physics-inspired) can further improve results

Explored this in [2111.01838] for semi-leptonic top decays in SMEFT



κ_{2V} κ_{2V}

Conclusion

- The quartic gauge-Higgs coupling κ_{2V} is currently relatively unconstrained
- The correlation between $\kappa_V - \kappa_{2V}$ is an essential ingredient in distinguishing BSM scenarios (e.g. models with Higgs mixing vs compositeness or SMEFT vs HEFT)
- Including single Higgs data is non-trivial at NLO but HEFT provides a theoretically consistent framework to achieve this
- However, constraints from Higgs data on κ_{2V} are relatively loose
- Despite our idealistic analysis for direct detection, competitiveness of GNNs demonstrates that such techniques deserve consideration as part of realistic experimental analyses
- Improvements of $\kappa_{2V} = 1 \pm 40\,\%$ could be within reach of the HL-LHC