Master Colloquium: Supersymmetric Four-Higgs Doublet Models

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WHY SUSY-4HDM?

- ATLAS und CMS found excesses in the experimental data potentially corresponding to a scalar at $\sim 95.4\,\text{GeV}$:
  (A) $pp \to \phi \to \gamma\gamma$ [13, 5]
  (B) $e^+e^- \to Z\phi \to Zb\bar{b}$
- excesses potetentially corresponding to a scalar of $\sim 400\,\text{GeV}$:
  (C) $A \to t\bar{t}$ [6, 15]
  (D) $\phi \to \tau^+\tau^-$ [6, 1]
  (E) $A \to ZH$ [6, 14, 2, 3]
- need Higgs sector analogous to 2HDM type IV to embed these in a theory

$\rightarrow$ supersymmetric four-Higgs doublet model

Scalar Potential

- 4 Higgs doublets with neutral and charged components [11]

$$H_{u1} = \begin{pmatrix} H_{u1}^{+} \\ H_{u1}^{0} \end{pmatrix} \qquad H_{u2} = \begin{pmatrix} H_{u2}^{+} \\ H_{u2}^{0} \end{pmatrix} \qquad H_{d} = \begin{pmatrix} H_{d}^{0} \\ H_{d}^{-} \end{pmatrix} \qquad H_{d2} = \begin{pmatrix} H_{d2}^{0} \\ H_{d2}^{-} \end{pmatrix} \tag{1}$$

- parametrization of neutral component $H_k^0 = \frac{1}{\sqrt{2}}(v_k + \eta_k + i\chi_k)$ [4]
- scalar potential for Higgs field [7]

$$\begin{aligned} V = &\sum_{i=1}^{2}\Big[\left(|\mu_{i1}|^2 + |\mu_{i2}|^2 + \tilde{m_{ui}}^2\right)\left(|H_{ui}^0|^2 + |H_{ui}^+|^2\right) + \left(|\mu_{1i}|^2 + |\mu_{2i}|^2 + \tilde{m_{di}}^2\right)\left(|H_{di}^0|^2 + |H_{di}^-|^2\right)\Big] \\ &+ \Big[\left(\mu_{11}^*\mu_{21} + \mu_{12}^*\mu_{22}\right)\left(H_{u1}^{0*}H_{u2}^0 + H_{u1}^{+*}H_{u2}^+\right) + \left(\mu_{11}^*\mu_{21} + \mu_{12}^*\mu_{22}\right)\left(H_{d1}^{0*}H_{d2}^0 + H_{d1}^{-*}H_{d2}^-\right) + \text{c.c.}\Big] \\ &+ \Big[\sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij}^2\left(H_{ui}^+H_{dj}^- - H_{ui}^0H_{di}^0\right) + \text{c.c.}\Big] + \frac{g^2 + g'^2}{8}\Big[\sum_{i=1}^{2}\left(|H_{ui}^0|^2 + |H_{ui}^+|^2 - |H_{di}^0|^2 + |H_{di}^-|^2\right)\Big]^2 \\ &+ \frac{g^2}{2}\Big[\sum_{i=1}^{2}|H_{ui}^{+*}H_{ui}^0 + H_{di}^{0*}H_{di}^-|^2 - \sum_{i=1}^{2}\sum_{j=1}^{2}\left(|H_{ui}^0|^2 - |H_{di}^0|^2\right)\left(|H_{uj}^+|^2 - |H_{dj}^-|^2\right)\Big] \end{aligned} \tag{2}$$

Derivation of the mass matrix

- vacuum expectation values [7]

$$v_{u1} = \frac{2M_Z}{\sqrt{(g^2 + g'^2) \cdot (1 + \tan\omega^2)}} \cdot \cos\beta \tag{3}$$

$$v_{u2} = \frac{2M_Z}{\sqrt{(g^2 + g'^2) \cdot (1 + \tan\omega^2)}} \cdot \tan\omega \sin\alpha \tag{4}$$

$$v_{d1} = \frac{2M_Z}{\sqrt{(g^2 + g'^2) \cdot (1 + \tan\omega^2)}} \cdot \sin\beta \tag{5}$$

$$v_{d2} = \frac{2M_Z}{\sqrt{(g^2 + g'^2) \cdot (1 + \tan\omega^2)}} \cdot \tan\omega \cos\alpha \tag{6}$$

- mass matrix elements $\mathcal{M}^2_{i,j} = \frac{1}{2}\frac{\partial^2 V}{\partial H_{\nu,i}\partial H_{\sigma,j}}$ [8]
- minimalization conditions $\frac{\partial V}{\partial H_{\nu i}} = 0$ [7]
- deviations from results in *A Private SUSY 4HDM with FCNC in the Up-sector*

Mass spectrum for CP-even neutral scalars

$$m^2_{u1u1} = \frac{1}{2}\Delta_{u1} + \frac{1}{2}M_Z^2\left[\frac{1}{2}s_\alpha^2 s_\omega^2 - s_\beta^2 c_\omega^2 - \frac{1}{4}c_\omega^2 t_\omega^2 + \frac{3}{4}c_\omega^2\right], \quad (7)$$

$$m^2_{d1d1} = \frac{1}{2}\Delta_{d1} - \frac{1}{2}M_Z^2\left[\frac{1}{2}s_\alpha^2 s_\omega^2 + s_\beta^2 c_\omega^2 - \frac{1}{4}c_{2\omega}\right], \quad (8)$$

$$m^2_{d2d2} = \frac{1}{2}\Delta_{d2} - \frac{1}{2}M_Z^2\left[s_\alpha^2 c_\omega^2 t_\omega^2 + \frac{1}{2}s_\beta^2 c_\omega^2 + s_\omega^2 - \frac{1}{4}\right], \quad (9)$$

$$m^2_{u2u2} = \frac{1}{2}\Delta_{u2} + \frac{1}{2}M_Z^2\left[s_\alpha^2 s_\omega^2 - \frac{1}{2}s_\beta^2 c_\omega^2 + \frac{1}{4}c_{2\omega}\right], \quad (10)$$

$$m^2_{u1d1} = -\frac{1}{8}M_Z^2 s_{2\beta} c_\omega^2 - \frac{1}{2}b_{11}^2, \quad (11)$$

$$m^2_{u1d2} = -\frac{1}{4}M_Z^2 s_\omega c_\alpha c_\beta c_\omega - \frac{1}{2}b_{12}^2, \quad (12)$$

$$m^2_{d1u2} = -\frac{1}{4}M_Z^2 s_\alpha s_\beta s_\omega c_\omega - \frac{1}{2}b_{21}^2, \quad (13)$$

$$m^2_{d2u2} = -\frac{1}{4}\ M_Z^2 s_\alpha s_\omega^2 c_\alpha - \frac{1}{2}b_{22}^2, \quad (14)$$

$$m^2_{u1u2} = \frac{1}{4}M_Z^2 s_\alpha s_\omega c_\beta c_\omega + \frac{1}{2}\left(\mu_{11}^*\mu_{21} + \mu_{12}^*\mu_{22}\right), \quad (15)$$

$$m^2_{d1d2} = \frac{1}{4}M_Z^2 s_\beta s_\omega c_\alpha c_\omega + \frac{1}{2}\left(\mu_{11}^*\mu_{12} + \mu_{21}^*\mu_{22}\right). \quad (16)$$

Mass spectrum for CP-odd neutral scalars

$$m_{11}^2 = b_{11}^2 \cot\beta + \csc\beta \tan\omega \left(b_{21}^2 \cos\alpha - (\mu_{11}\mu_{12} + \mu_{21}\mu_{22}) \sin\alpha\right) , \tag{17}$$
$$m_{22}^2 = b_{11}^2 \tan\beta + \sec\beta \tan\omega \left(b_{12}^2 \sin\alpha - (\mu_{11}\mu_{21} + \mu_{12}\mu_{22}) \sin\alpha\right) , \tag{18}$$
$$m_{33}^2 = b_{22}^2 \cot\alpha + \csc\beta \cot\omega \left(b_{12}^2 \cos\beta - (\mu_{11}\mu_{12} + \mu_{21}\mu_{22}) \sin\beta\right) , \tag{19}$$
$$m_{44}^2 = b_{22}^2 \tan\alpha + \cot\omega \sec\alpha \left(b_{21}^2 \sin\beta - (\mu_{11}\mu_{21} + \mu_{12}\mu_{22}) \cos\beta\right) , \tag{20}$$
$$m_{12}^2 = b_{11}^2, \tag{21}$$
$$m_{13}^2 = \mu_{11}^*\mu_{12} + \mu_{21}^*\mu_{22}, \tag{22}$$
$$m_{14}^2 = b_{21}^2, \tag{23}$$
$$m_{23}^2 = b_{12}^2, \tag{24}$$
$$m_{24}^2 = \mu_{11}^*\mu_{12} + \mu_{21}^*\mu_{22}, \tag{25}$$
$$m_{34}^2 = b_{22}^2. \tag{26}$$

Mass spectrum for charged scalars

$$m_{11}^2 = \tan\omega \csc\beta \left(b_{21}\cos\alpha - (\mu_{11}\mu_{12} + \mu_{21}\mu_{22})\sin\alpha\right) \\ + b_{11}\cot\beta + \frac{g'^2 M_W^2}{4g^2}\left(\cos^2\beta\cos^2\omega + \cos(2\alpha)\sin^2\beta\right), \tag{27}$$

$$m_{22}^2 = -\cos\alpha\sec\beta\tan\omega(\mu_{11}\mu_{21} + \mu_{12}\mu_{22}) + b_{12}\sin\alpha\sec\beta\tan\omega \\ + b_{11}\tan(\beta) + \frac{g'^2 M_W^2}{4g^2}\left(\sin^2\beta\cos^2\omega - \cos(2\alpha)\sin^2\omega\right), \tag{28}$$

$$m_{33}^2 = \csc\alpha\cot\omega(b_{12}\cos\beta - \sin\beta(\mu_{11}\mu_{12} + \mu_{21}\mu_{22})) \\ + b_{22}\cot\alpha + \frac{g'^2 M_W^2}{4g^2}\left(\cos^2\alpha\sin^2\omega + \cos(2\beta)\cos^2\omega\right), \tag{29}$$

$$m_{44}^2 = b_{21}\sec\alpha\sin\beta\cot\omega - \sec\alpha\cos\beta\cot\omega(\mu_{11}\mu_{21} + \mu_{12}\mu_{22}) \\ + b_{22}\tan\alpha + \frac{g'^2 M_W^2}{4g^2}\left(\sin^2\alpha\sin^2\omega - \cos(2\beta)\cos^2\omega\right), \tag{30}$$

(31)

Mass spectrum for the charged Higgs

$$m_{12}^2 = b_{11} + \frac{1}{8g^2} g^{'2} M_W^2 \sin(2\beta)\cos^2\omega, \tag{32}$$

$$m_{13}^2 = \mu_{11}\mu_{12} + \mu_{21}\mu_{22} + \frac{g^{'2} M_W^2}{8g^2} \sin\alpha \sin\beta \sin(2\omega), \tag{33}$$

$$m_{14}^2 = b_{21} + \frac{g^{'2} M_W^2}{8g^2} \cos\alpha \sin\beta \sin(2\omega), \tag{34}$$

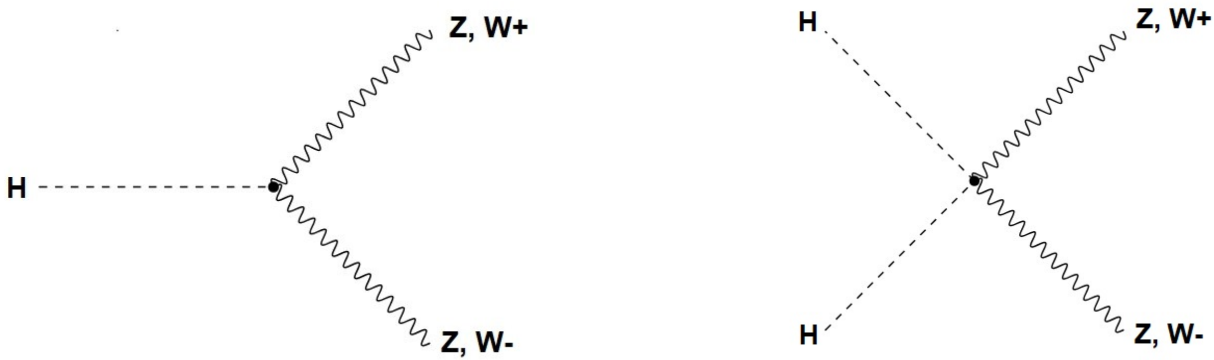
$$m_{23}^2 = b_{12} + \frac{g^{'2} M_W^2}{8g^2} \sin\alpha \cos\beta \sin(2\omega), \tag{35}$$

$$m_{24}^2 = \mu_{11}\mu_{21} + \mu_{12}\mu_{22} + \frac{g^{'2} M_W^2}{8g^2} \cos\alpha \cos\beta \sin(2\omega), \tag{36}$$

$$m_{34} = b_{22} + \frac{g^{'} 2 M_W^2}{8g^2} \sin(2\alpha)\sin^2\omega. \tag{37}$$

Coupling of the CP-even neutral Higgs to gauge bosons

- derivation of couplings from the covariant term in the Lagrangian



- lightest Higgs boson seems to have highest value for coupling to the gauge bosons $\rightarrow$ potentially necessary to add a scalar for inclusion of the 95 GeV-Higgs
- coupling of two different Higgs to gauge boson(s) vanishes

Sum rule for gauge boson - Higgs coupling

- Sum Rule [9]

$$\sum_{i=1}^{4}\left(\frac{g_i}{g}\right)^2 = 1 \tag{38}$$

- for HWW-processes $\mathcal{L} \supset \frac{g^2}{2}\sum_{i=1}^{4}\sum_{j=1}^{4}\xi_{ij}v_jH_iW^+W^-$
- to prove sum rule, show that

$$\sum_{i=1}^{4}\left(\sum_{j=1}^{4}\xi_{ij}v_j\right)^2 = v^2 \tag{39}$$

- take the square and use the unitarity of the rotation matrix with entries ξ_{ij}
- result:

$$\sum_{i=1}^{4}\left(\sum_{j=1}^{4}\xi_{ij}v_j\right)^2 = \sum_{j=1}^{4}v_j^2 = v^2\Box \tag{40}$$

- proof analogous for *HZZ*-coupling

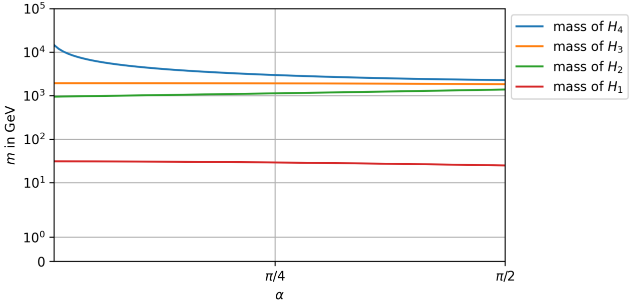
Z_2-SYMMETRY

- Z_2-symmetry to prevent FCNC at tree-level
- implementation [10]: $H_{u2} \to -H_{u2}$
  - $\mu_{11}^*\mu_{21} + \mu_{12}^*\mu_{22} = 0$ and $b_{2,j} = 0$ for $j = 1, 2$
- exact Z_2 symmetry lead to an additional $U(1)$-symmetry for H_{u2} in the potential $\to$ additional (unphysical) Goldstone bosons

$$
\begin{aligned}
V = \sum_{i=1}^{2} \Big[& \left(|\mu_{i1}|^2 + |\mu_{i2}|^2 + \tilde{m}_{ui}{}^2\right)\left(|H_{ui}^0|^2 + |H_{ui}^+|^2\right) + \left(|\mu_{1i}|^2 + |\mu_{2i}|^2 + \tilde{m}_{di}{}^2\right)\left(|H_{di}^0|^2 + |H_{di}^-|^2\right)\Big] \\
& + \left(\mu_{11}^*\mu_{12} + \mu_{21}^*\mu_{22}\right)\left(H_{d1}^{0*}H_{d2}^0 + H_{d1}^{-*}H_{d2}^-\right) + \text{c.c.}\Big] + \Big[\sum_{j=1}^{2} b_{1j}^2\left(H_{u1}^+H_{dj}^- - H_{u1}^0H_{di}^0\right) + \text{c.c.}\Big] \\
& + \frac{g^2 + g'^2}{8}\Big[\sum_{i=1}^{2}\left(|H_{ui}^0|^2 + |H_{ui}^+|^2 - |H_{di}^0|^2 + |H_{di}^-|^2\right)\Big]^2 + \frac{g^2}{2}\Big[\sum_{i=1}^{2}|H_{ui}^{+*}H_{ui}^0 + H_{di}^{0*}H_{di}^-|^2 \\
& - \sum_{i=1}^{2}\sum_{j=1}^{2}\left(|H_{ui}^0|^2 - |H_{di}^0|^2\right)\left(|H_{uj}^+|^2 - |H_{dj}^-|^2\right)\Big],
\end{aligned}
\tag{41}
$$

- soft-breaking by setting any of the parameters $\mu_{11}^*\mu_{21} + \mu_{12}^*\mu_{22} \neq 0$ or $b_{2,j} \neq 0$
  - best results for setting both bilinear terms $b \sim m_{\text{soft}}$, therefore $b \sim 1000\,\text{GeV}$ [12]

Mass spectrum plots



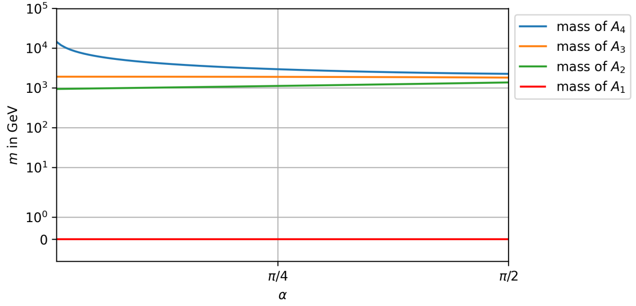


Figure. Dependence of the four mass eigenvalues of the CP-even (l) and CP-odd (r) neutral Higgs with a softly-broken Z_2-symmetry on the mixing angle α. The parameter-space is fixed at $\mu_{12} = 100\,\text{GeV}$, $\mu_{21} = 100\,\text{GeV}$, $\mu_{22} = 100\,\text{Gev}$, $b_{11}^2 = 1000^2\,\text{GeV}^2$, $b_{12}^2 = 1000^2\,\text{GeV}^2$, $\alpha = 0.38$, $\beta = 0.45$. The soft-breaking is achieved by setting $b_{22}^2 = 1000^2\,\text{GeV}^2$ and $b_{21}^2 = 1000^2\,\text{GeV}^2$. The colors indicate the different Higgs particles $H_{1,2,3,4}$.

Mass spectrum plots

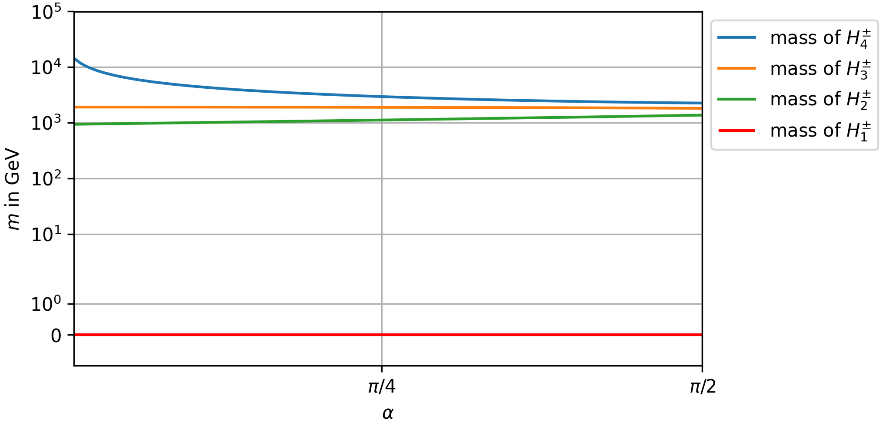


Figure. Dependence of the four mass eigenvalues of the charged Higgs with a softly-broken Z_2-symmetry on the mixing angle α. The parameter-space is fixed at $\mu_{12} = 100\,\text{GeV}$, $\mu_{21} = 100\,\text{GeV}$, $\mu_{22} = 100\,\text{Gev}$, $b_{11}^2 = 1000^2\,\text{GeV}^2$, $b_{12}^2 = 1000^2\,\text{GeV}^2$, $\alpha = 0.38$, $\beta = 0.45$. The soft-breaking is achieved by setting $b_{22}^2 = 1000^2\,\text{GeV}^2$ and $b_{21}^2 = 1000^2\,\text{GeV}^2$. The colors indicate the different Higgs particles $H_{1,2,3,4}$.

Summary and Outlook

Summary

- mass spectrum for the Higgs sector Higgs bosons
- Coupling of CP-even neutral Higgs to gauge bosons
- sum rule is fulfilled due to the unitarity of eigenvector-matrix
- lightest Higgs boson H_1 has mass $m_{H1} < m_Z$
- implementation of a Z_2-symmetry to restrict tree-level FCNC

Outlook

- special cases to derive analytic solutions for the mass spectrum
- higher-order loop corrections for numerically stable results
- study SUSY particle sector
- Yukawa sector $\rightarrow$ hopefully: explanation of excess at 95 GeV (and 400 GeV)
  $\rightarrow$ addition of a singlet?!
- study FCNC

Danke für ihre Aufmerksamkeit!

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