Domain Walls in the N2HDM: Exploring Vacuum Trapping and Scalar Potential Evolution

Master Thesis

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Motivation

- The Standard Model is incomplete:
  - No explanation for dark matter, baryogenesis, or vacuum metastability
  - No solution to the hierarchy problem
- Extended Higgs sectors like the N2HDM offer
  - Richer vacuum structures and new phase transitions
  - Possibility of solving cosmological problems
- Research Questions
  - Can the scalar potential in the N2HDM lead to vacuum trapping?
  - Can domain walls rescue the universe from a false vacuum?
- Objectives
  - Study vacuum structure and trapping in the Z'_2-conserving N2HDM
  - Analyze loop effects and singlet composition on vacuum fate

Limitations of the Standard Model

- The Standard Model (SM) has several known shortcomings:
  - ➢ Hierarchy Problem:
    - Quadratic divergences in Higgs mass not naturally cancelled
  - ➢ No Dark Matter Candidate:
    - SM lacks a stable, neutral particle
  - ➢ Baryon Asymmetry:
    - SM CP-violation insufficient for observed matter-antimatter imbalance
  - ➢ Vacuum Metastability:
    - Higgs potential may not be absolutely stable at high energies
- Need for Extensions:
  - ➢ Provide strong 1st-order phase transitions
  - ➢ Introduce scalar dark matter candidates
  - ➢ Offer new symmetry-breaking patterns to stabilize the vacuum

The Two-Higgs Doublet Model (2HDM)

$$\text{2HDM} = \text{SM}(\phi_1) + \text{Second Higgs Doublet}(\phi_2)$$

$$V = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + h.c.) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2$$
$$+ \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + h.c.]$$

EW vacuum: $\Phi_1 = \begin{pmatrix} \phi_1^\dagger \\ \frac{1}{\sqrt{2}}(v_1+\rho_1 + i\,\eta_1) \end{pmatrix}$, $\Phi_2 = \begin{pmatrix} \phi_2^\dagger \\ \frac{1}{\sqrt{2}}(v_2+\rho_2 + i\,\eta_2) \end{pmatrix}$

Symmetries: Z_2: $\phi_1 \to \phi_1$, $\phi_2 \to -\phi_2$ only softly broken by m_{12}^2

Scalar Physical Spectrum: 2 CP-Even (h, H), 1 CP-Odd (A), and 2 Charged Scalars (H^+, H^-)

Parameter Space: $m_h, m_H, m_A, m_{H^\pm}, \tan\beta, \alpha, m_{12}^2$

Next-to 2 Higgs Doublet Model (N2HDM)

$$\begin{aligned}\text{N2HDM} &= \text{SM}(\phi_1) + \text{Second Higgs Doublet}(\phi_2) + \text{Real Scalar Singlet}(\phi_s)\\ &= \text{2HDM}(\phi_1, \phi_2) + \text{Real Scalar Singlet}(\phi_s)\end{aligned}$$

Scalar tree-level potential

$$\begin{aligned}V =\ & m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + h.c.) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2\\ & + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + h.c.]\\ & \left(+\frac{1}{2}m_S^2\Phi_S^2 + \frac{\lambda_6}{8}\Phi_S^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger\Phi_1)\Phi_S^2 + \frac{\lambda_8}{2}(\Phi_2^\dagger\Phi_2)\Phi_S^2\right)\end{aligned}$$

Symmetries: Z_2: $\phi_1 \to \phi_1$, $\phi_2 \to -\phi_2$ and $\phi_s \to \phi_s$, only softly broken by m_{12}^2
Z_2': $\phi_1 \to \phi_1$, $\phi_2 \to \phi_2$ and $\phi_s \to -\phi_s$, spontaneously broken by v_s

Next-to 2 Higgs Doublet Model (N2HDM)

EW vacuum: $\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \xi_1 + i\chi_1 \\ (v_1+\rho_1 + i\eta_1) \end{pmatrix}$, $\Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \xi_2 + i\chi_2 \\ (v_2+\rho_2 + i\eta_2) \end{pmatrix}$, $\Phi_S = v_S + \rho_S$

$< \Phi_1 > |_{T=0} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$, $< \Phi_2 > |_{T=0} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$, $< \Phi_1 > |_{T=0} = v_S$

Scalar Spectrum: 3 CP-Even (H_1, H_2, H_3), 1 CP-Odd (A), and 2 Charged Scalars (H^+, H^-)

Extension of Z_2 to Yukawa sector ⇒ 4 types of the (N)2HDM

	u-type	**d-type**	**leptons**
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Type III (lepton-specific)	Φ_2	Φ_2	Φ_1
Type IV (flipped)	Φ_2	Φ_1	Φ_2

Next-to 2 Higgs Doublet Model (N2HDM)

Minimization Conditions:

$$\frac{\partial V}{\partial \phi_1} = v_1 \left(m_{11}^2 + \lambda_1 v_1^2 + \lambda_3 v_2^2 + \lambda_7 v_S^2 - m_{12}^2 \frac{v_2}{v_1} \right) = 0$$
$$\frac{\partial V}{\partial \phi_2} = v_2 \left(m_{22}^2 + \lambda_2 v_2^2 + \lambda_3 v_1^2 + \lambda_8 v_S^2 - m_{12}^2 \frac{v_1}{v_2} \right) = 0$$
$$\frac{\partial V}{\partial \phi_S} = v_S \left(m_S^2 + \lambda_6 v_S^2 + \lambda_7 v_1^2 + \lambda_8 v_2^2 \right) = 0.$$

Using the rotation matrices in terms of the mixing angles α1, α2, α3 and β, we switch to the mass basis

$$R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -(c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3}) & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ -(c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_3}) & -(c_{\alpha_1} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3} \end{pmatrix} \qquad R_\beta = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}$$

Loop Corrections and Coleman-Weinberg Potential

Why Loop Corrections?

- Tree-level potential may miss critical features of the vacuum structure
- Quantum corrections can shift the location and depth of minima
- Crucial for studying metastability and vacuum transitions

Coleman-Weinberg (CW) Potential

- One-loop correction to the effective potential:

$$V_{\text{eff}}(\phi) = V_{\text{tree}}(\phi) + V_{\text{CW}}(\phi)$$

- General form of the CW Potential:

$$V_{\text{CW}}(\phi) = \sum_j \frac{n_j}{64\pi^2}(-1)^{2s_j} m_j^4(\phi) \left[\ln\left(\frac{m_j^2(\phi)}{\mu^2}\right) - c_j \right]$$

  - Sums over all particle species: scalars, fermions and gauge bosons.

$m_j(\phi)$: Field-dependent mass of particle species j.
n_j: Number of degrees of freedom for particle j.
s_j: Spin of particle j.
μ: Renormalization scale

$$c_j = \begin{cases} \frac{3}{2}, & \text{for scalars and fermions,} \\ \frac{5}{6}, & \text{for gauge bosons.} \end{cases}$$

One-Loop Effects on Vacuum Nucleation

From Loop corrections to Tunneling Suppression

- The CW correction $V_{CW}(\phi)$ reshapes the potential:
  - Shifts minima positions and barrier height
  - Affects the difference $\Delta V = V_{false} - V_{true}$

Vacuum Nucleation

- Nucleation Rate:

$$\Gamma(T) \sim T^4 \exp\left(-\frac{S_3(T)}{T}\right)$$

Γ: probability per unit volume for bubble nucleation
S_3: bounce action — computed from the shape of the effective potential

  - Higher potential barriers ⇒ larger S_3 ⇒ suppressed tunneling
- One-loop effects can thus:
  - Prevent nucleation → **Vacuum Trapping**
  - Or enable transition by lowering the barrier

Vacuum Trapping

What is Vacuum Trapping?

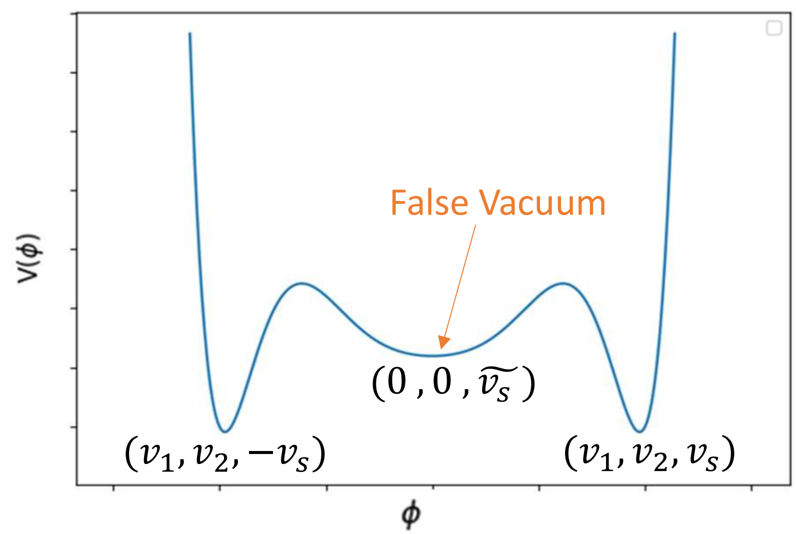
- During the early universe, phase transitions occur as the temperature drops.
- Universe could find itself in a local vacuum state rather than the global one.
- If the barrier separating this from the true vacuum is too high, the field becomes *trapped*.

Consequences

- The universe remains in a metastable vacuum:
  - Prevents Electroweak Symmetry Breaking (EWSB)
  - Unphysical particle masses or couplings

Conditions for Vacuum Trapping

- Coexisting minima along with:
  - Large potential barrier
  - Weak tunneling (large bounce action)

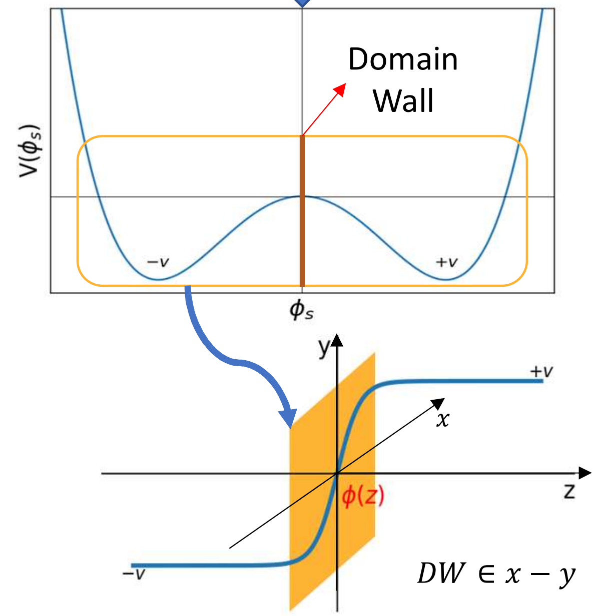


Domain Walls

Discrete Symmetry —SSB→ Degenerate vacuum

What are Domain Walls?

- Topological defects arising from a spontaneously broken discrete symmetry.
- Act as boundaries/transition region for distinct degenerate vacuum states.
- Inside the Domain Wall,
  Energy of Metastable state > True EW Vacuum
- Problematic ⟶ Dominate the energy of the universe at some point in time.
- Permitted ⟶ For approx. discrete symmetries
  - If annihilation occurs before energy domination



Domain Walls

Thin-Wall Approximation

- Bubble of true vacuum forms inside false vacuum, separated by a thin-wall
- Energy needed to nucleate a bubble of true vacuum

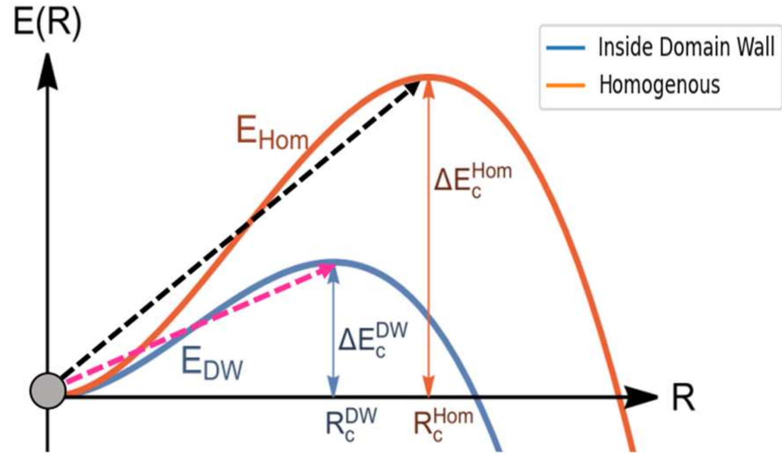
$$E(R) = 4\pi R^2 \sigma_B - \frac{4}{3}\pi R^3 \Delta V - \pi R^2 \sigma_{\rm DW}$$

- Critical radius R_c at which the nucleated bubble becomes energetically favorable

$$R_c = \frac{2}{\Delta V}\left(\sigma_B - \frac{1}{4}\sigma_{\rm DW}\right)$$

- Modifies the Bounce Action and hence the tunneling probability

$$\frac{S_3}{T} \approx \frac{16\pi}{3T}\frac{(\sigma_B - \sigma_{\rm DW}/4)^3}{\Delta V^2}$$



S. Blasi and A. Mariotti, *Domain Walls Seeding the Electroweak Phase Transition*

Analysis of Vacuum Trapping in N2HDM

Tools used

- *ScannerS* for parameter space scanning
- *BSMPT* for effective potential and analysing phase transitions

Physical Input Parameters

- Type 2 N2HDM
- Parameters chosen in a way to support a First-Order Electroweak Phase Transition (FOEWPT)
- Consistent with electroweak precision constraints

- *CP*-even Higgs bosons $m_{h_1},\ m_{h_2}, m_{h_3}$
- *CP*-odd Higgs bosons m_A
- *Charged* Higgs bosons $m_{H^\pm}$
- Soft breaking Z_2 term m_{12}^2

- Singlet VEV v_s
- Ratio of the VEVs $\tan\beta = \frac{v_2}{v_1}$
- EW scale $v = \sqrt{v_1^2 + v_2^2} \approx 246$ GeV
- Mixing angles $\alpha_1,\ \alpha_2,\ \alpha_3$

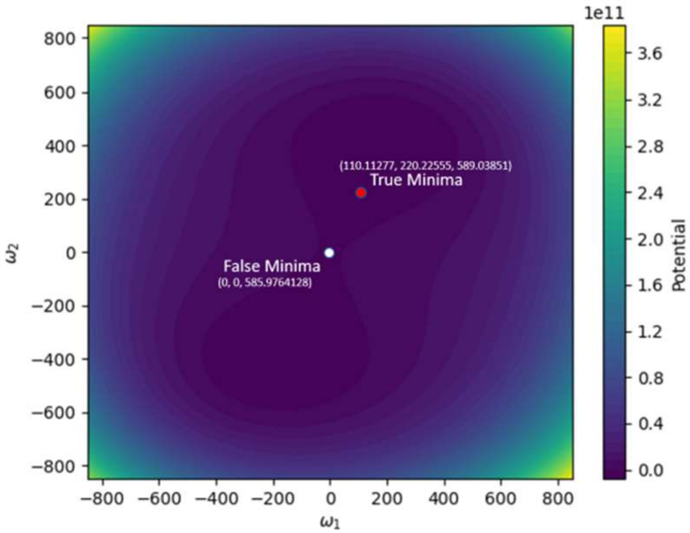
Vacuum Trapping in N2HDM

Parameter space scanned in the N2HDM

m_{h_1}(GeV)	m_{h_2}(GeV)	m_{h_3}(GeV)	$m_{A,H^\pm}$ (GeV)	$\tan\beta$	α_1	α_2	α_3	m_{12}^2(GeV)	v_s(GeV)
125.09	400	[300, 1000]	650	2	1.10714	1.82 x 10^{-8}	[-1.5, 1.5]	255^2	[1, 1000]

Structure of the Effective Potential in Field Space

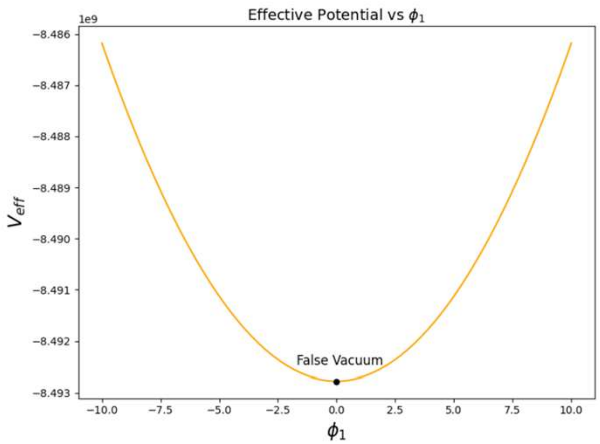
- Contour Plot suggests the presence of both a local (false) and global (true) minimum
- Variation in the effective potential values across the field space is numerically small



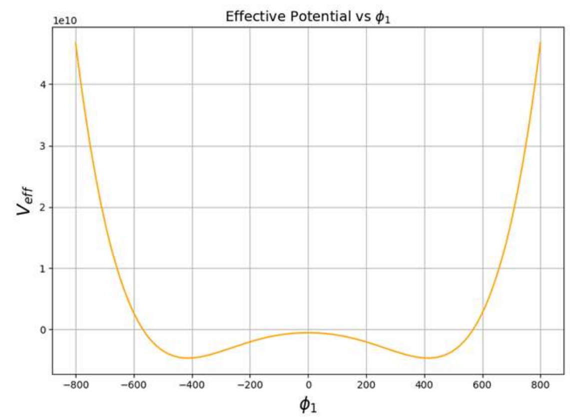
Vacuum Trapping in N2HDM

Isolating the minima

- For the false minimum, we compute $V_{eff}(\Phi 1)$ while fixing Φ2 = 0 and ΦS = vS, and scanning over Φ1
- For the global minimum, we perform 1D scans along Φ1, fixing the remaining fields at values corresponding to the known minimum configurations



- These 1D slice highlights the fine structure of the false minimum
- Such configurations are prime examples of vacuum trapping

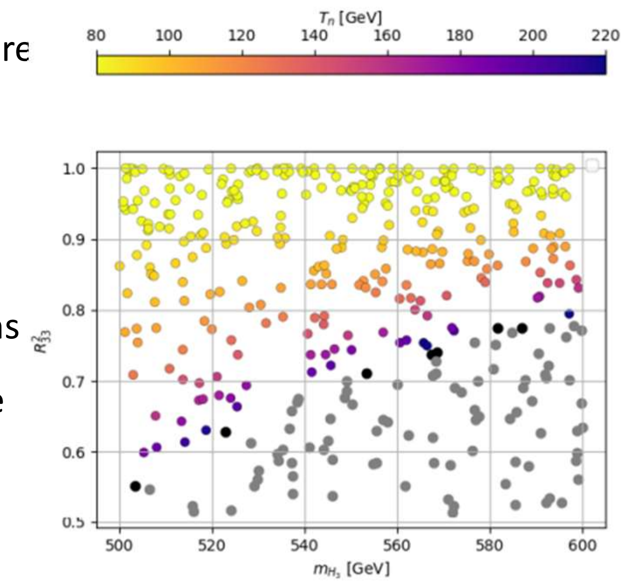


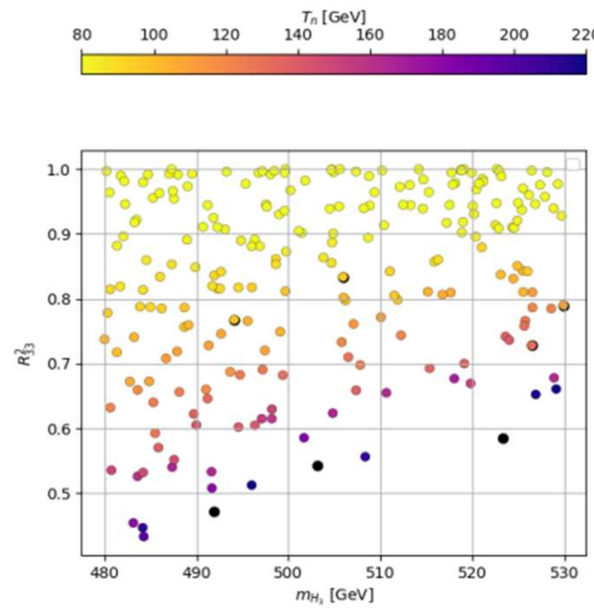
Thermal Evolution and BSMPT Analysis

- As the temperature decreases, during the universe's expansion, the structure of V_{eff} evolves dynamically
- Using BSMPT, we determine the Critical Temperature T_{crit} and the Nucleation Temperature T_{nuc}
- Scan was performed at finite temperatures ranging from T = 0 GeV up to T = 1000 GeV
- Classification of the parameter points:
  - Complete transitions: Points where both T_{crit} and T_{nuc} are defined
  - Trapped vacua candidates: Points where T_{crit} is defined, but T_{nuc} is undefined (suggesting vacuum trapping)
  - Undefined transitions: Points where T_{crit} could not be determined

Thermal Evolution and BSMPT Analysis

- Two different plots between R_{33}^2 vs m_{H_3} are shown for different ranges of the value of m_{H_3}
- The different points are represented as
  - Black points: Trapped vacuum candidates
  - Colored points: Completed phase transitions
  - Gray points: Undefined critical temperature





Interpretation

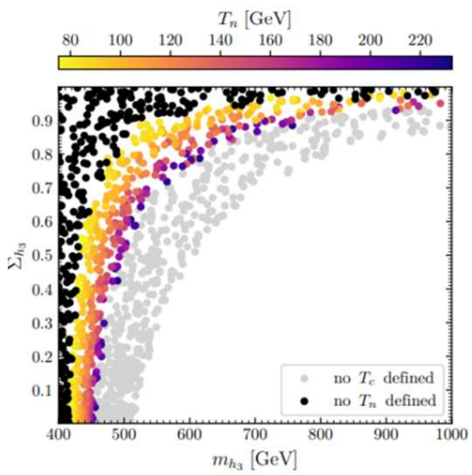
- Vacuum trapping candidates (black points) tend to appear for intermediate singlet compositions
- For the gray points, the Universe remains in the electroweak (EW) minimum throughout the entire temperature range upto T= 600 GeV
- Thus, for the gray points, electroweak symmetry is not restored at high temperatures.

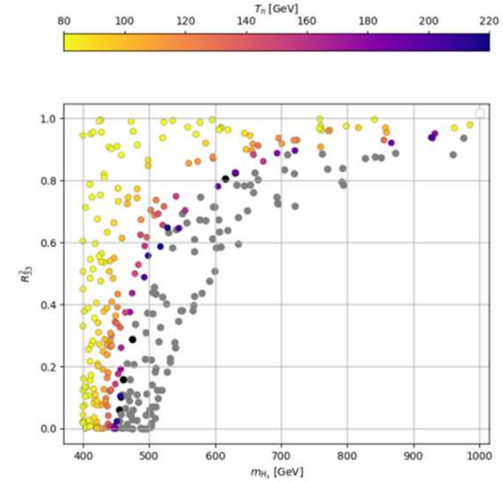
Thermal Evolution and BSMPT Analysis

Cross-Check with Reference points

- Exact Nucleation Condition $$\frac{\Gamma(T_n)}{H^4(T_n)} = 1$$

- Approximate Nucleation Condition $$\frac{S_3(T_n)}{T_n} \sim 140,$$





- Only a small subset of the points indicated vacuum trapping whereas most points successfully completed their phase transitions when analyzed using the exact nucleation condition

Future Prospects

- Improved Phenomenology
  - Study full **bubble wall profiles** beyond thin-wall approximation
- Incorporate quantum corrections to the scalar potential of the N2HDM
- Extending the study to incorporate full one-loop quantum corrections
- Reevaluate vacuum trapping points under radiative corrections and determine how loop effects modify the scalar landscape
- Investigate domain wall stability and its interplay with thermal tunneling using loop-corrected potentials beyond existing tools

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