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Florian Lika
Supervisors: Gudrid Moortgat-Pick, Sven Heinemeyer

SUSY-Parameter determination within Dark Matter Phenomenology at future e+e- colliders

Minimal Supersymmetric Standard Model

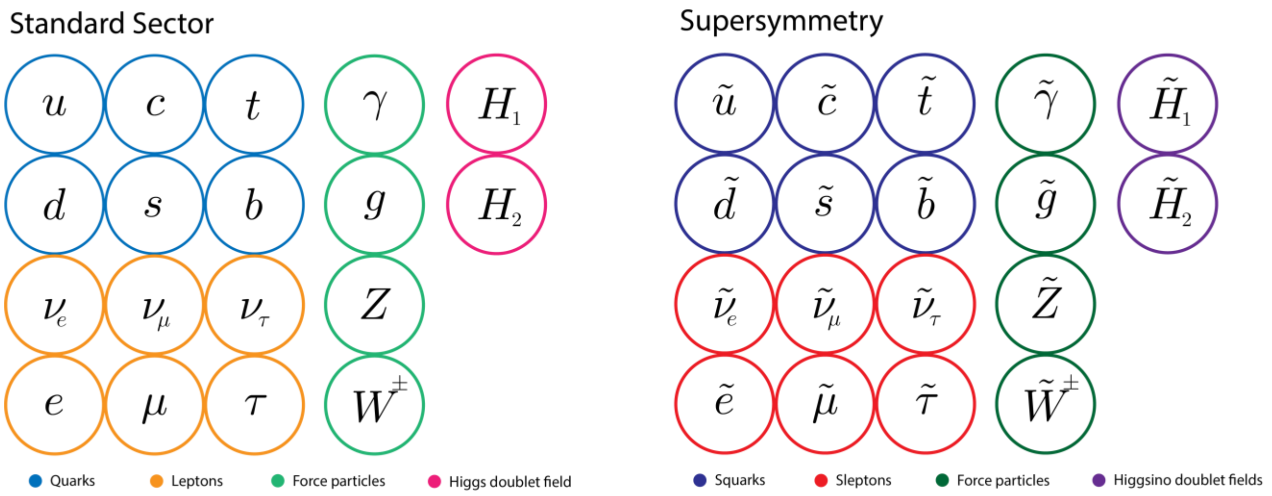
- Gaugino and higgsino fields mix into mass eigenstates

- Neutralinos

$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$

- Charginos

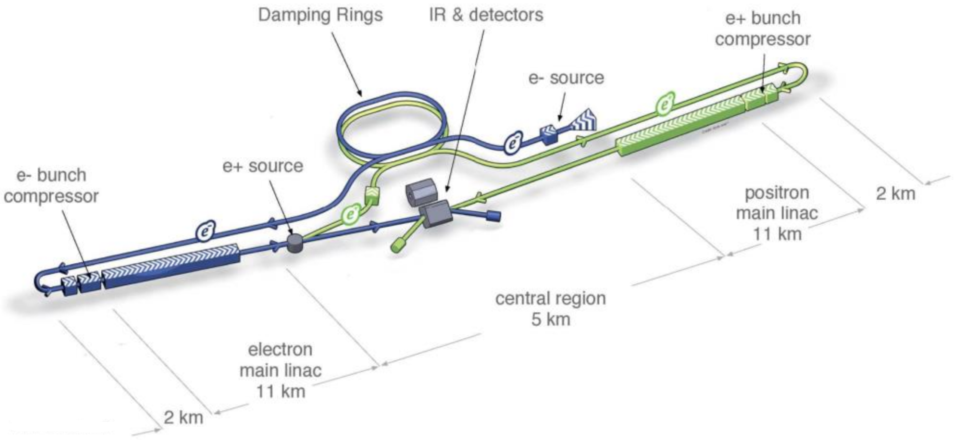
$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$



International Linear Collider

- Future e^+e^- linear collider
- Model-independent Higgs searches
- Spin polarised beams
- Possible future experiment:
  - $e^+e^- \to \tilde{\chi}_1{}^+\tilde{\chi}_1{}^-$
  - Luminosity $L = 500\ \mathrm{fb}^{-1}$
  - $\sqrt{s} = 500\ \mathrm{GeV}$

https://cds.cern.ch/record/2715199/files/CLICdp-Conf-2020-004.pdf

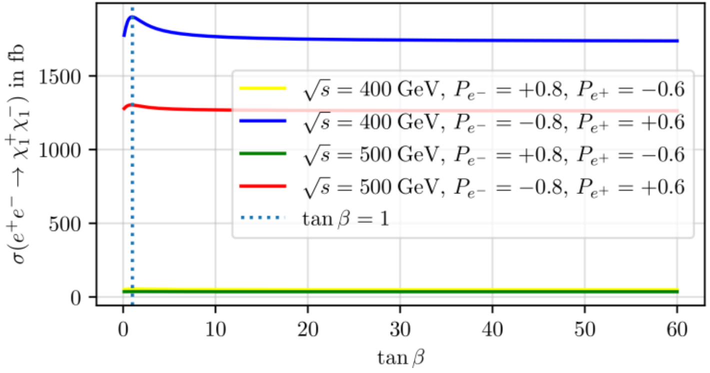


K. Desch, J. Kalinowski, G. Moortgat-Pick, M.M. Nojiri, G. Polesello
https://arxiv.org/abs/hep-ph/0312069

Neutralino Sector

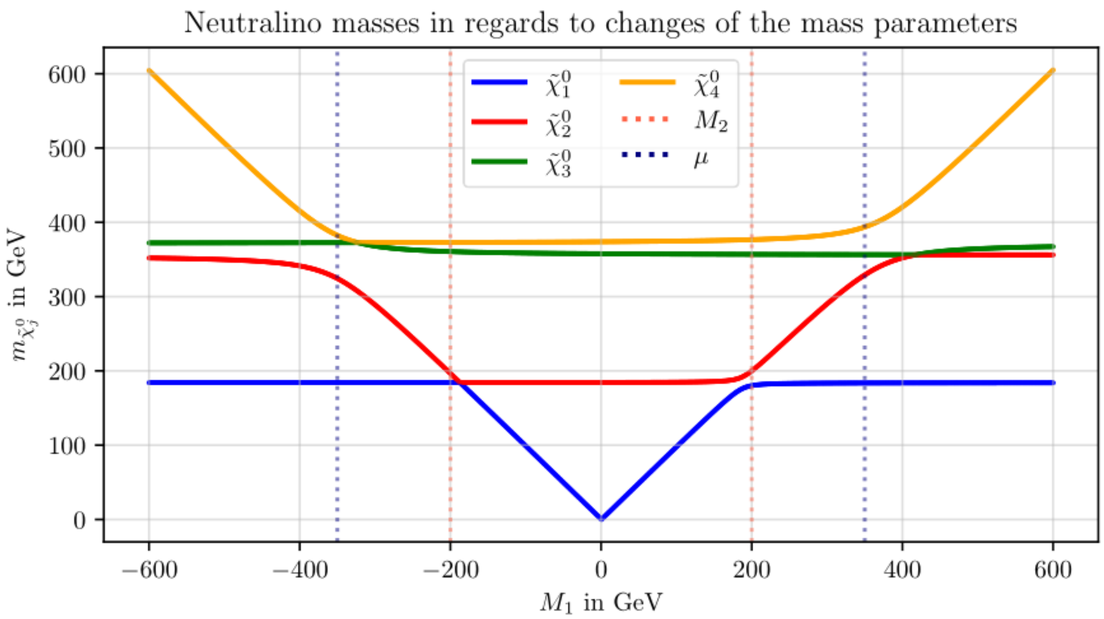
- Bino Parameter M_1
- Wino Paramter M_2
- Higgsino parameter $\mu, \tan\beta$
- $\tan\beta = \frac{v_1}{v_2}$
- M_1 can only be determined with Neutralinos
- $M_2, \mu, \tan\beta$ are determined using chargino data

$$\mathcal{M}_N = \begin{pmatrix} M_1\cos^2\theta_W + M_2\sin^2\theta_W & (M_2 - M_1)\sin\theta_W\cos\theta_W & 0 & 0 \\ (M_2 - M_1)\sin\theta_W\cos\theta_W & M_1\cos^2\theta_W + M_2\sin^2\theta_W & m_Z & 0 \\ 0 & m_Z & \mu\sin 2\beta & -\mu\cos 2\beta \\ 0 & 0 & -\mu\cos 2\beta & -\mu\sin 2\beta \end{pmatrix}$$



Neutralino Sector

- Particles assigned by mass hierarchy
- Parameter hierarchy determines sensibility



M_2 = 200 GeV

μ = 350 GeV

$\tan\beta$ = 20

K. Desch, J. Kalinowski, G. Moortgat-Pick, M.M. Nojiri, G. Polesello
https://arxiv.org/abs/hep-ph/0312069

Chargino Sector

- Two unitary transformations for chargino mixing $U_{L,R}$
- Two mixing angles $\Phi_{L,R}$

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W \cos\beta \\ \sqrt{2}m_W \sin\beta & \mu \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}^- \end{pmatrix}_{L,R}$$

$$U_{L,R} = \begin{pmatrix} \cos\Phi_{L,R} & \sin\Phi_{L,R} \\ -\sin\Phi_{L,R} & \cos\Phi_{L,R} \end{pmatrix}$$

Goal:

- Measure $\Phi_{L,R}$, $m_{\tilde{\chi}_1^\pm}$, $m_{\tilde{\chi}_1^0}$ to reconstruct SUSY-parameters

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}(M_2^2 + \mu^2 + 2m_W^2 \mp \Delta_C)$$

$$\cos 2\phi_{L,R} = -(M_2^2 - \mu^2 \mp 2m_W^2 \cos 2\beta)/\Delta_C$$

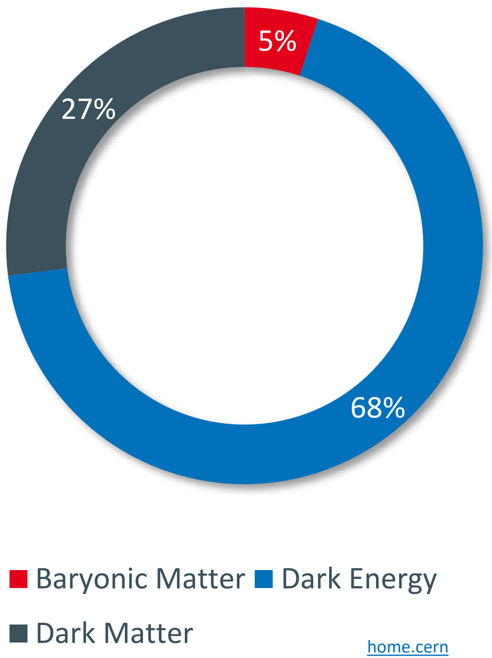
$$\Delta_C = [(M_2^2 - \mu^2)^2 + 4m_W^4 \cos^2 2\beta + 4m_W^2(M_2^2 + \mu^2) + 8m_W^2 M_2 \mu \sin 2\beta]^{1/2}$$

Planck 2018
https://arxiv.org/abs/1807.06209

Dark Matter

- Dark matter relic density $\Omega h^2 = 0.12$ determined by Planck Collaboration
- $\tilde{\chi}_1^0$ is dark matter candidate in MSSM
- Dependent on $M_1, M_2, \mu, \tan\beta$
- Relic density determined with micrOMEGAs
- Chargino coannihilation scenario $M_1 {\sim} M_2$

→ DM relic density is reduced by $\tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm$ decay into SM particles
→ Bino/Wino-like DM resulting from $M_1 {\sim} M_2$



home.cern

Strategy

- Take a parameter point with correct dark matter relic density
- Assume measurement of $\sigma(e^+e^- \rightarrow \tilde{\chi}_1{}^+\tilde{\chi}_1{}^-)$ and lightest chargino mass
    - at $\sqrt{s} = 400\text{ GeV},\ 500\text{ GeV}$
    - With $P_{e^-} = \mp 0.8$ and $P_{e^+} = \pm 0.6$
    - Considering necessary uncertainties especially from $M_{\tilde{\nu}}$
- Calculate chargino mixing angles
- Redetermine chargino SUSY parameters – $M_2, \mu, \tan\beta$,
- Assume measurement of lightest neutralino mass
- Redetermine neutralino SUSY parameters – M_1
- Use experimentally determined parameters to calculate DM relic density
- Compare indirectly derived DM relic density with correct DM relic density

Manimala Chakraborti, Sven Heinemeyer, Ipsita Saha
https://arxiv.org/abs/2104.03287

Dataset

Constraints:

- Muon $(g-2)$ – *BNL* and *Fermilab*
- Vacuum stability – stable and correct EW vacuum
- LHC constraints – all relevant SUSY searches
- Dark matter relic density constraints – *Planck 2018*
- Direct dark matter detection – *XENON1T*

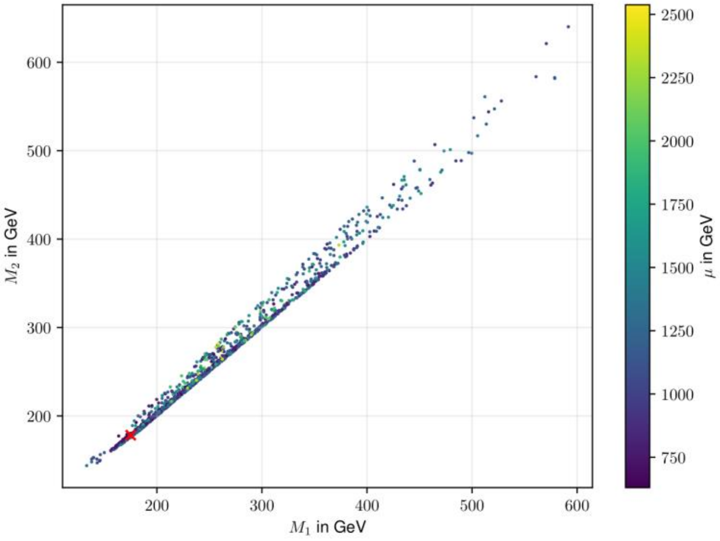
$$
\begin{aligned}
100\ \mathrm{GeV} &\leq M_1 \leq 1\ \mathrm{TeV} \\
M_1 &\leq M_2 \leq 1.1 M_1 \\
1.1 M_1 &\leq \mu \leq 10 M_1 \\
5 &\leq \tan\beta \leq 60 \\
100\ \mathrm{GeV} &\leq m_{\tilde{l}_{L,R}} \leq 1\ \mathrm{TeV}
\end{aligned}
$$

M_1	175.09 GeV	$\sigma^{400}_{-0.8,+0.6}$	1744.2519 fb
M_2	178.25 GeV	$\sigma^{400}_{+0.8,-0.6}$	49.8956 fb
μ	1215.85 GeV	$\sigma^{500}_{-0.8,+0.6}$	1265.4737 fb
$\tan\beta$	34.81	$\sigma^{500}_{+0.8,-0.6}$	35.6168 fb
$M_{\tilde{\nu}}$	536.9 GeV	$m_{\chi_1^\pm}$	177.1484 GeV

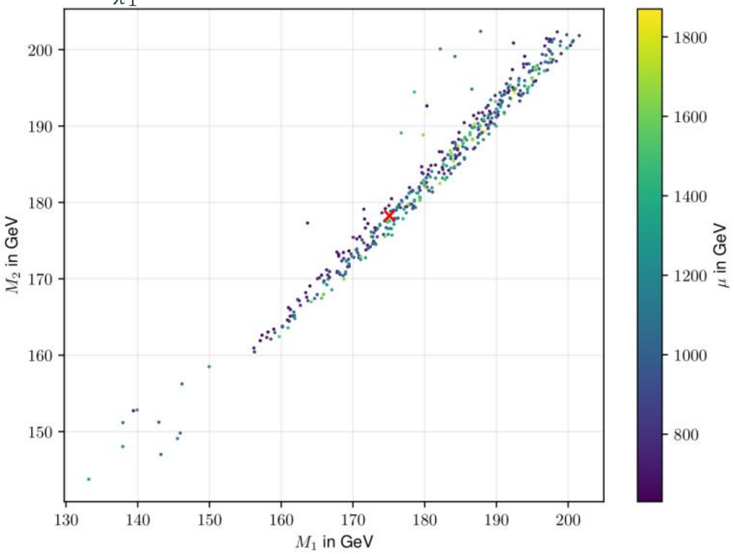
100 GeV	$\leq$	M_1	$\leq$	1 TeV
M_1	$\leq$	M_2	$\leq$	$1.1M_1$
$1.1M_1$	$\leq$	μ	$\leq$	$10M_1$
5	$\leq$	$\tan\beta$	$\leq$	60
100 GeV	$\leq$	$m_{\tilde{l}_{L,R}}$	$\leq$	1 TeV

Dataset

Full data set



$m_{\chi_1^\pm} < 200$ GeV



- Red dot corresponds to example point

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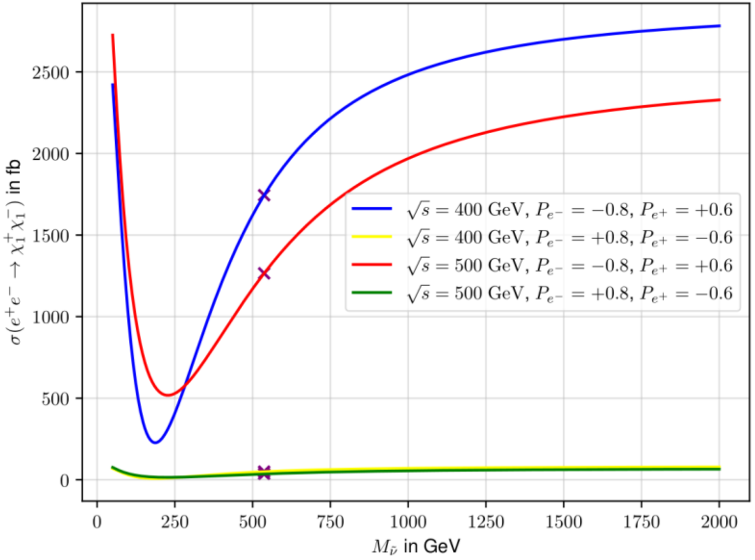
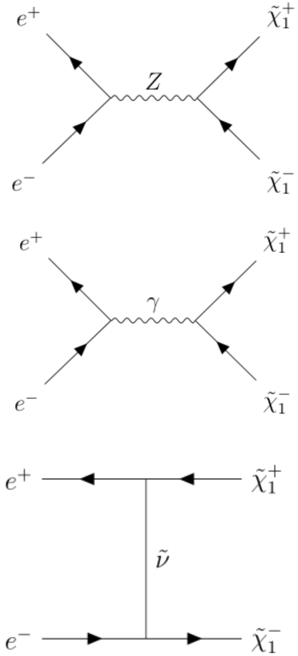
Cross sections

- Sneutrino mass $M_{\tilde{\nu}}$ relevant in t-channel propagator

→ Finding sensible limit becomes important objective





$$\sigma^\pm\{ij\} = \sigma(e^+e^- \to \tilde{\chi}_i^\pm \tilde{\chi}_j^\mp)$$

$$\sigma^\pm\{ij\} = c_1 \cos^2 2\Phi_L + c_2 \cos 2\Phi_L + c_3 \cos^2 2\Phi_R + c_4 \cos 2\Phi_R + c_5 \cos 2\Phi_L \cos 2\Phi_R + c_6$$

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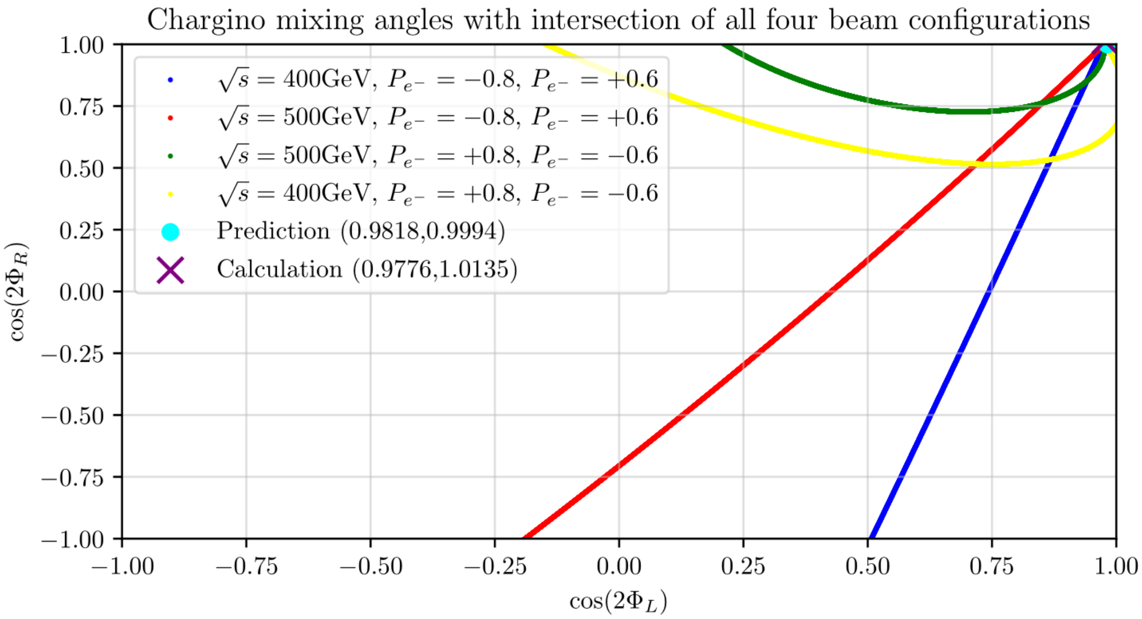
Chargino mixing angles

- Single beam energy causes ambiguities

→ At least three beam configurations remove ambiguity

- Ellipses are not continuous

→ Intersection difficult to obtain



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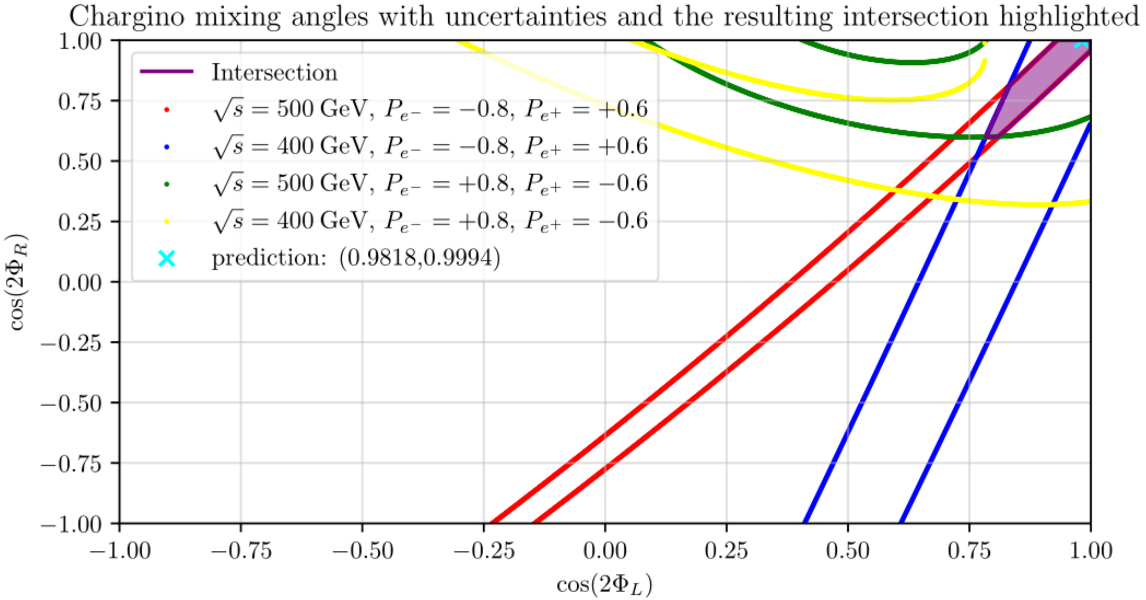
Chargino mixing angles

- Adding uncertainties
  - 0.5% on chargino mass
  - Gaussian error on cross section
  - 0.5% on polarisation
  - → Sneutrino mass error not included
- 1D Curves now become 2D bands
- Intersection becomes 2D area



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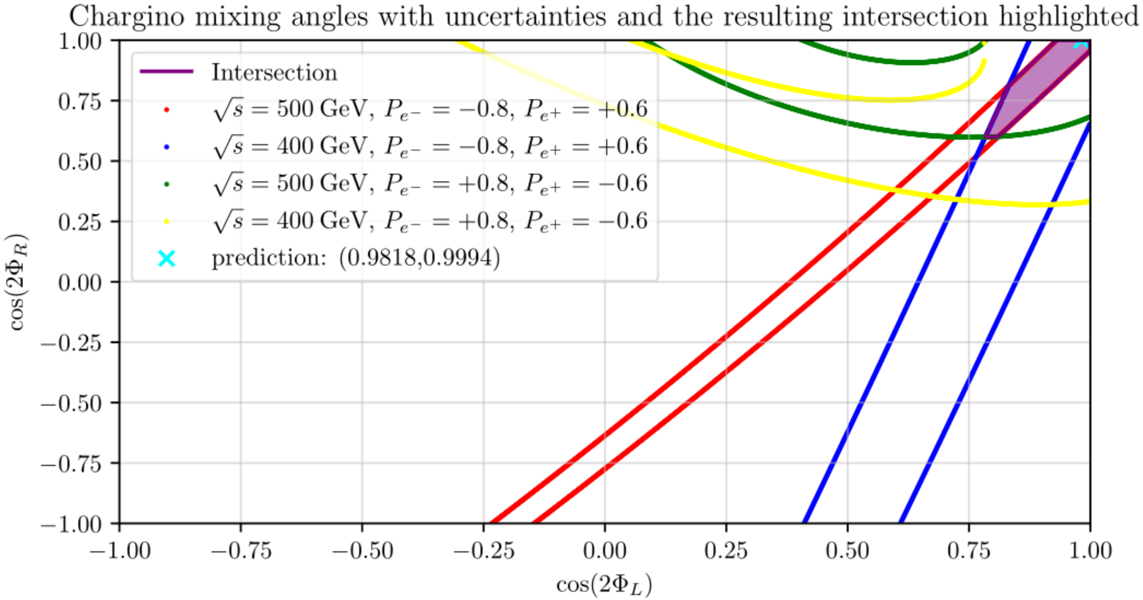
Chargino mixing angles

- Set theory approach
  - Mixing angle bands are defined as polygons using the *shapely* Python libary
  - Intersection of polygons is calculated
- Accurately describes all the points within and on the boundary
- Calculation is easier, more efficient and less ambiguous compared to 1D case



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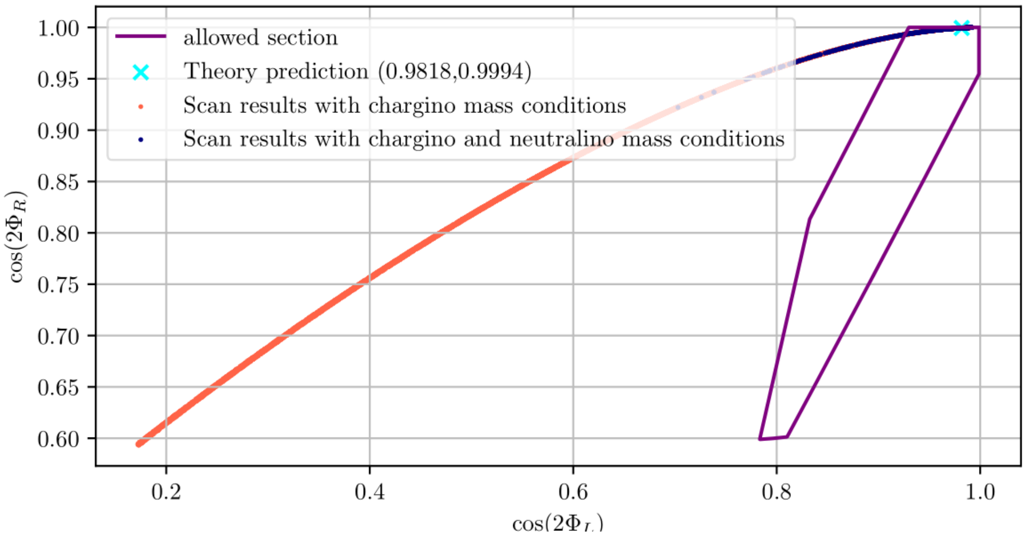
Parameter Reconstruction – Scan

- Reconstruction done by parameter scan
- Random points from M_1, M_2, μ space are filtered by constraints
- Constraints:
  - Chargino masses within 0.5%
  - Cross-section condition
  - Neutralino masses within 0.5%

Scan results with cross-section condition visualised using the intersection surface.



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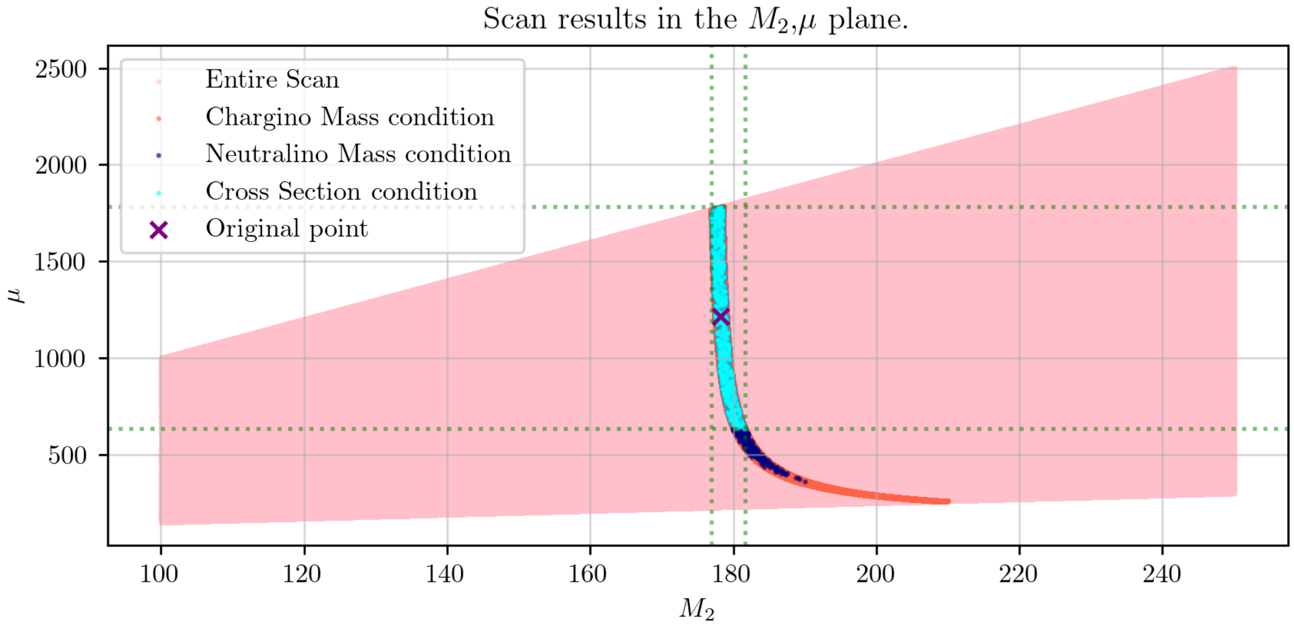
100 GeV	$\leq$	M_1	$\leq$	1 TeV
M_1	$\leq$	M_2	$\leq$	$1.1M_1$
$1.1M_1$	$\leq$	μ	$\leq$	$10M_1$
5	$\leq$	$\tan\beta$	$\leq$	60
100 GeV	$\leq$	$m_{\tilde{l}_{L,R}}$	$\leq$	1 TeV

Parameter Reconstruction – Scan

- Pink area: full scanning range
- Cyan points fulfill all conditions
- Green dotted lines describe allowed interval



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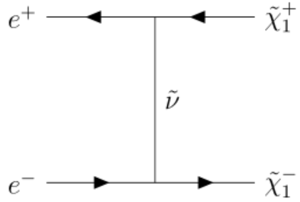
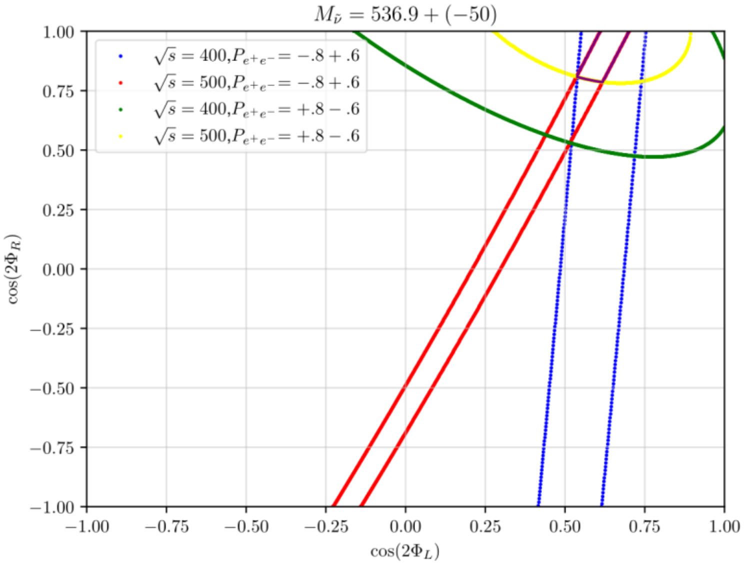
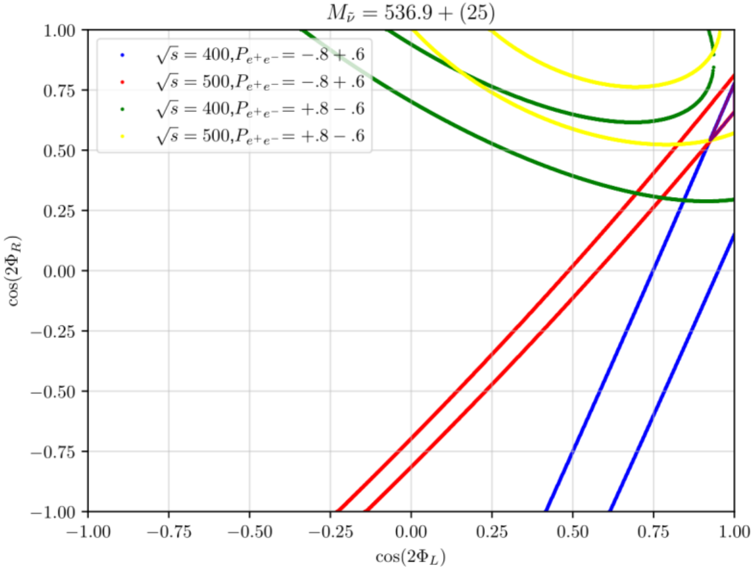


Mixing angles - $M_{\tilde{\nu}}$





- Intersection shifts to border of physically allowed range

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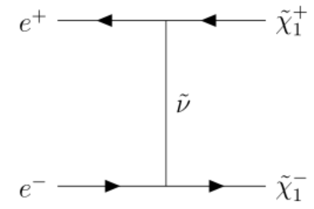
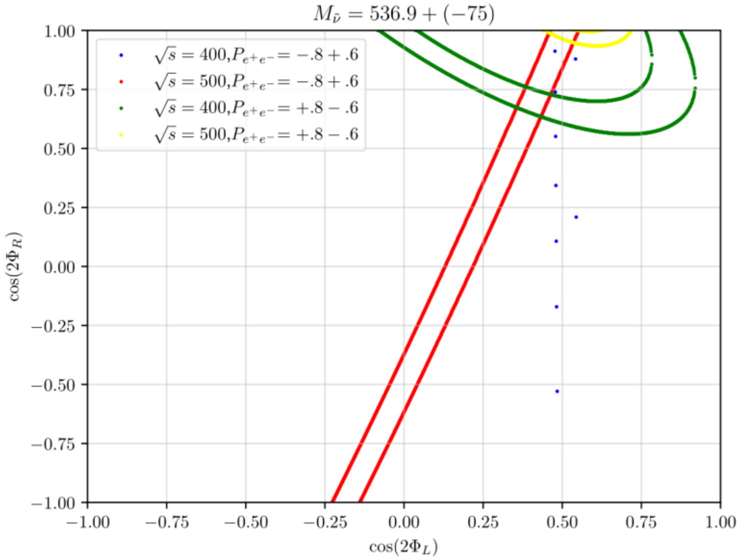
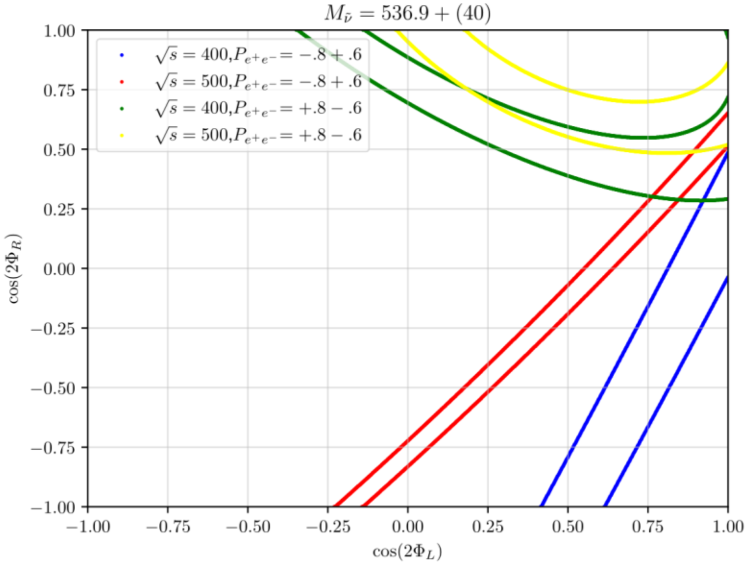


Mixing angles - $M_{\tilde{\nu}}$





- No intersection at sufficient deviation of $M_{\tilde{\nu}}$

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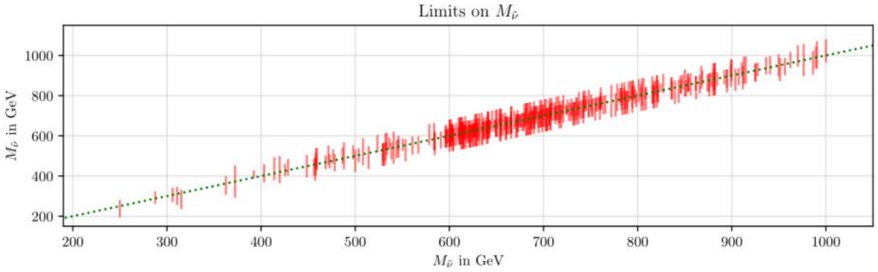
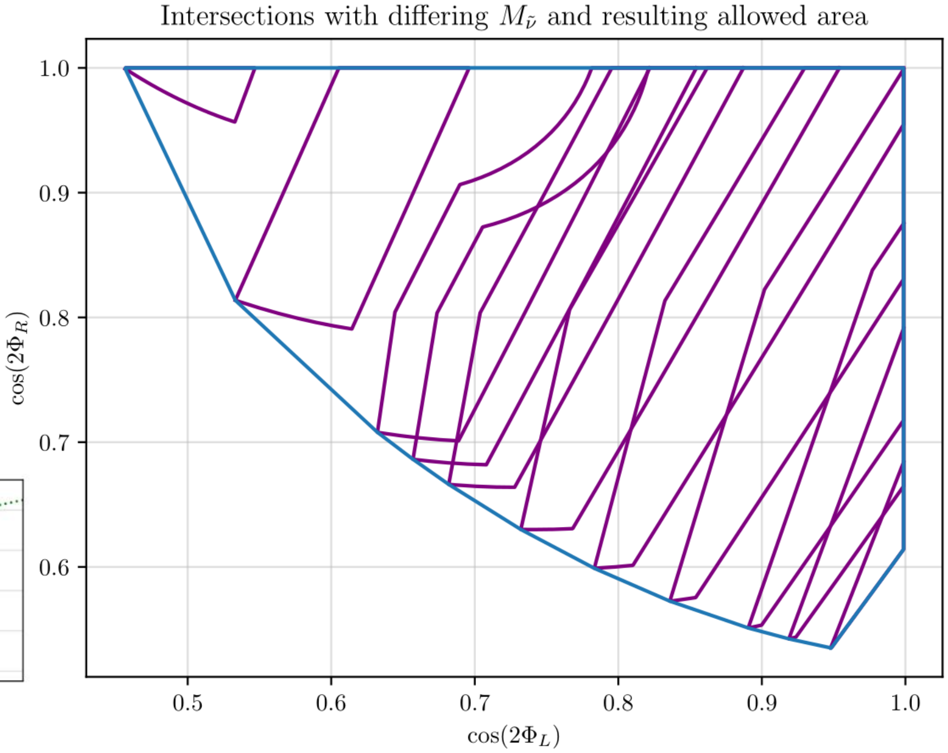
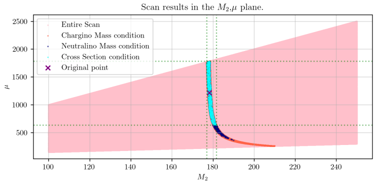
Mixing angles - $M_{\tilde{\nu}}$

- Cross-section area increases
- Blue border is created by taking the convex hull of all shapes
- Important:
  → Do not change cross-section

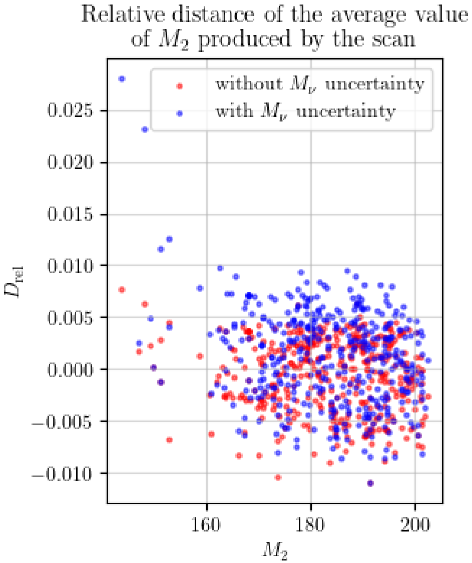


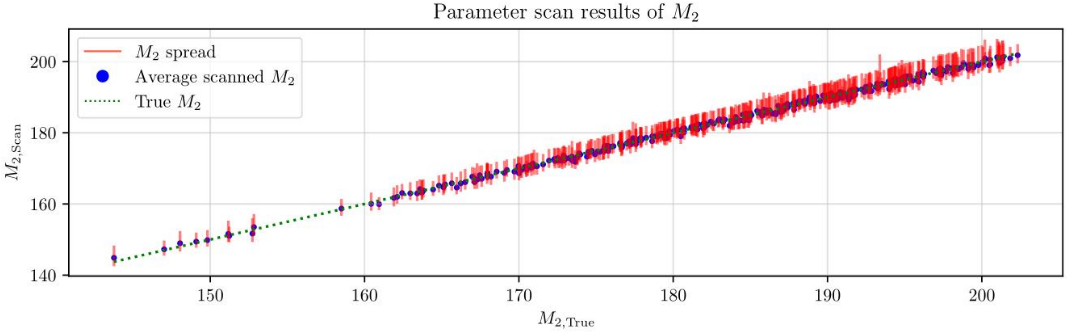


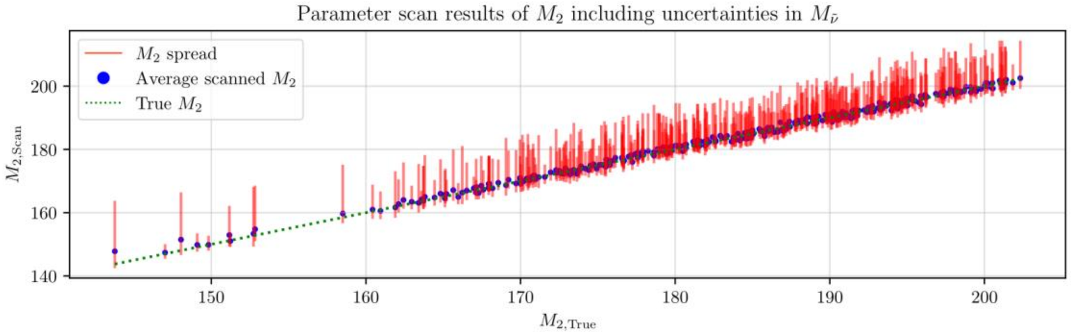


$$D_{\mathrm{rel}} = \frac{\bar{Q}_{\mathrm{Scan}} - Q_{\mathrm{True}}}{Q_{\mathrm{True}}} \qquad Q \in \{M_1, M_2, \mu, \Omega h^2\}$$

Results – M_2



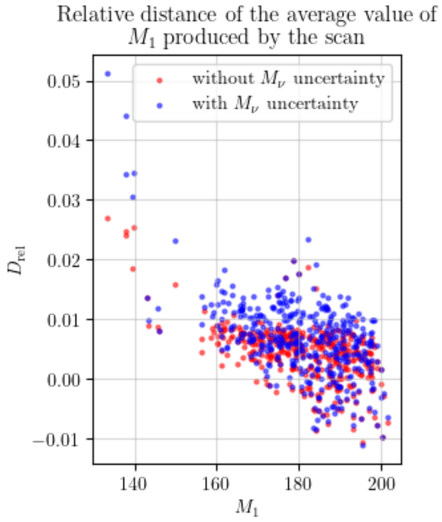


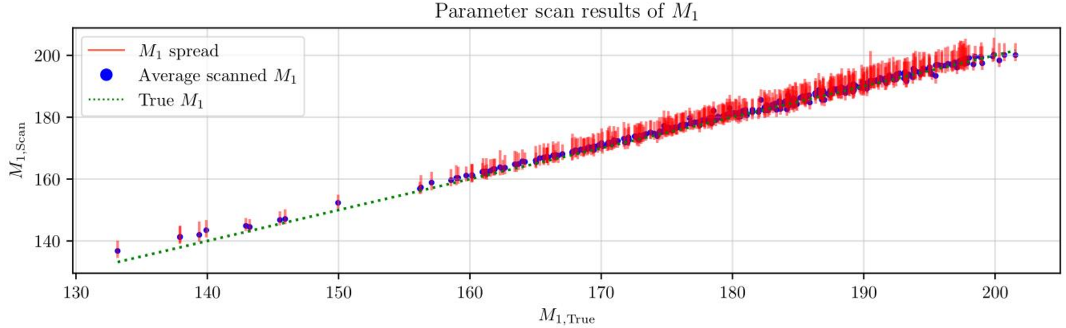


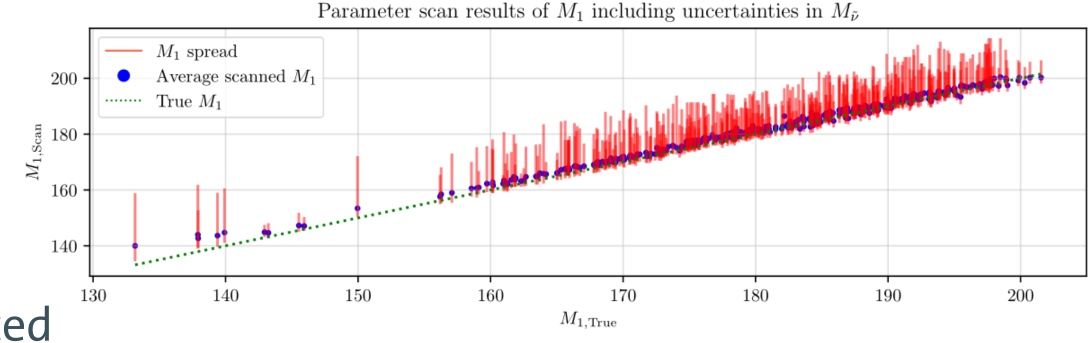
- Upper limit increasing with $M_{\tilde{\nu}}$ uncertainty

$$D_{\text{rel}} = \frac{\bar{Q}_{\text{Scan}} - Q_{\text{True}}}{Q_{\text{True}}} \qquad Q \in \{M_1, M_2, \mu, \Omega h^2\}$$

Results – M_1







- Lower values of M_1 not reproduced
- Upper limit increasing with $M_{\tilde{\nu}}$ uncertainty

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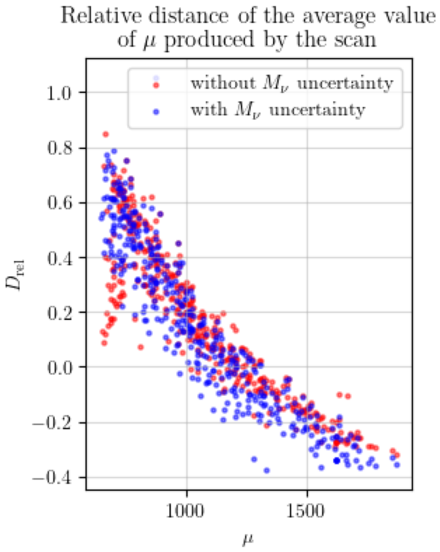
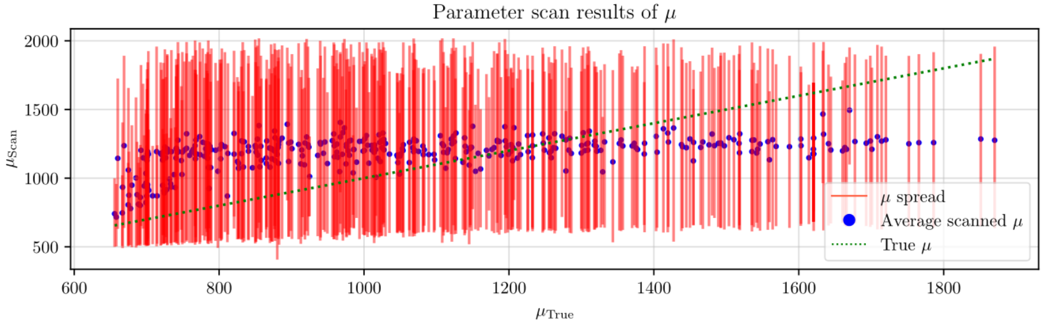
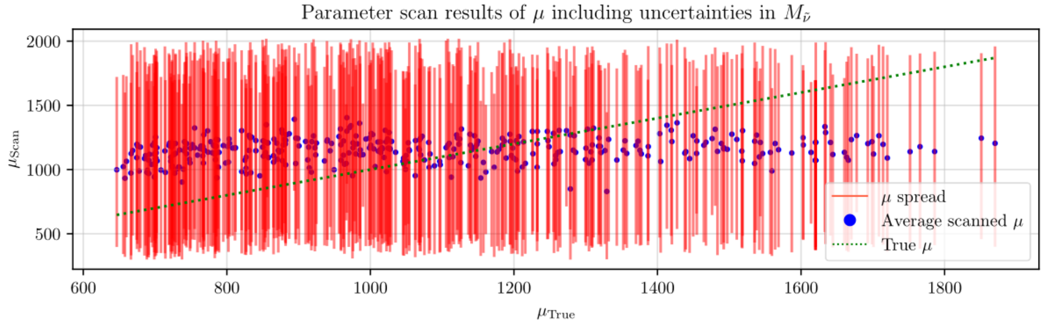
$$D_{\text{rel}} = \frac{\bar{Q}_{\text{Scan}} - Q_{\text{True}}}{Q_{\text{True}}} \qquad Q \in \{M_1, M_2, \mu, \Omega h^2\}$$

Results – μ







- Different scenario or heavier particles necessary for accurate reproduction

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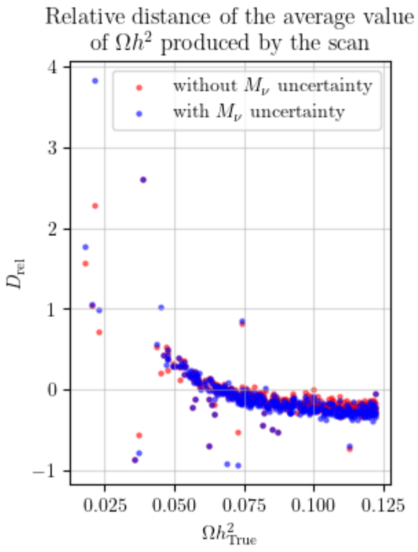
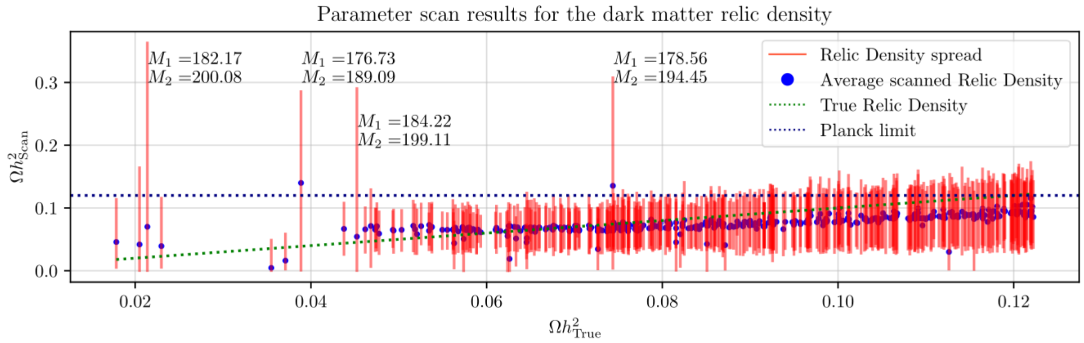
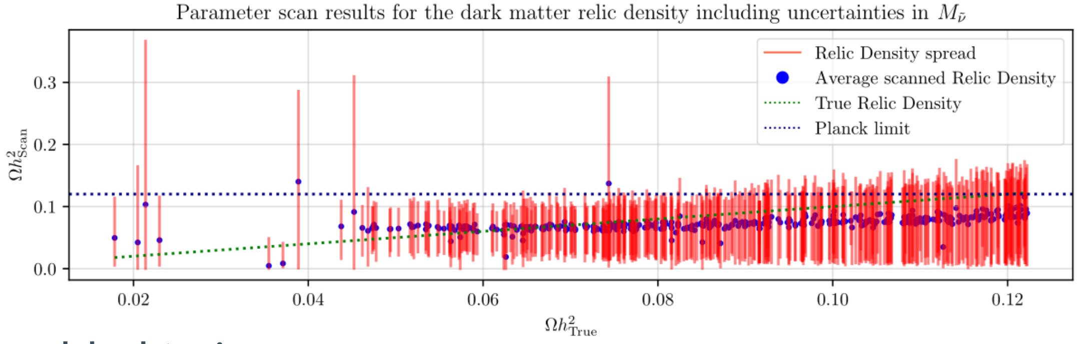
$$D_{\text{rel}} = \frac{\bar{Q}_{\text{Scan}} - Q_{\text{True}}}{Q_{\text{True}}} \qquad Q \in \{M_1, M_2, \mu, \Omega h^2\}$$

Results – Dark Matter

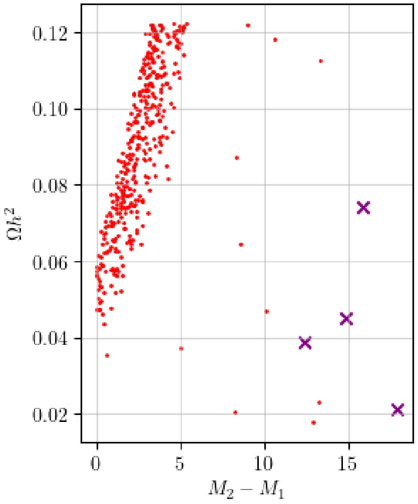


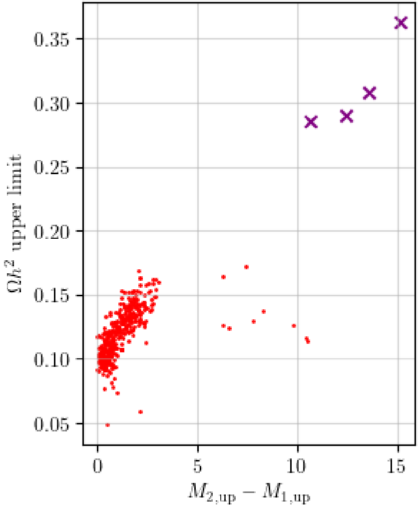


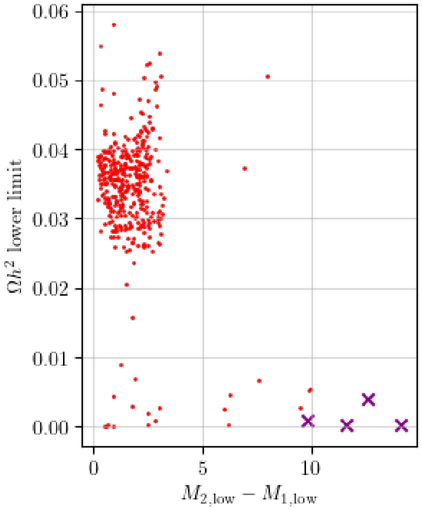


- Additional constraints could be added to increase accuracy

Results – ΔM







- Relic density dependent on $M_2 - M_1$ explaining the outliers

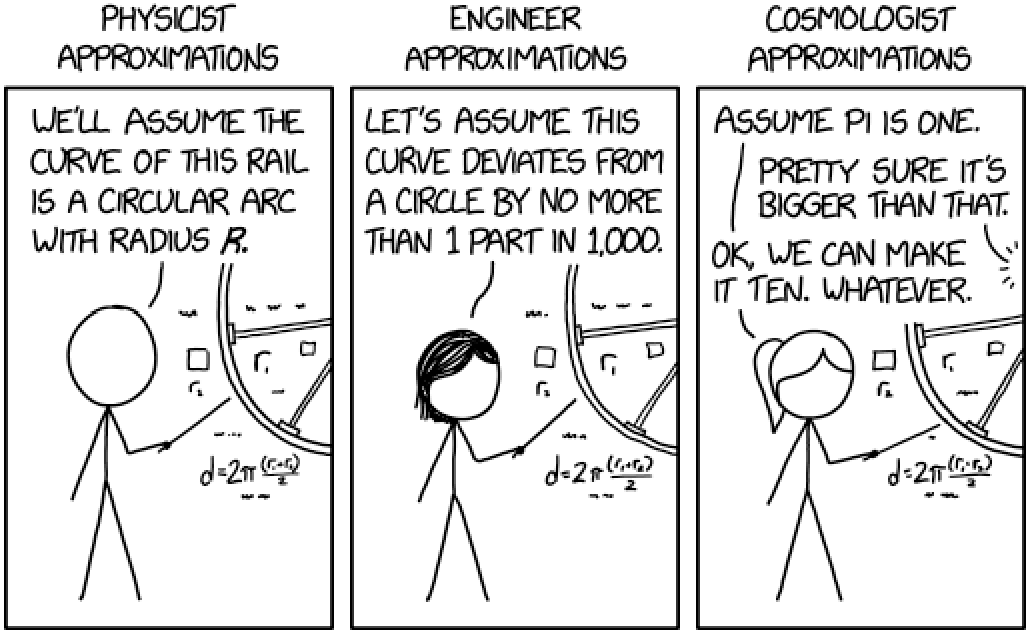
Conclusion

- M_2 reproduced most accuratly
- M_1 reproduced slightly less accuratly compared to M_2
  - Smaller values could not be reproduced
- More particles need to be analysed to reproduce μ conclusively
- Ωh^2 reproduced largely within allowed range
  - Accuracy can be improved by adding constraints

Next Steps

- Analysis for different scenarios
  - $\tilde{l}$ coannihilation, Wino DM, higgsino DM
- Add analysis of heavier particles into the established analysis
- Different Beam configurations
  - Increase Energy
  - Different collider setups
- Increase performance and streamline evaluation

Thank you



https://xkcd.com/2205/

Backup

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

$$R = \sin^2\theta_W$$
$$L = -\frac{1}{2} + \sin^2\theta_W$$
$$\int_C = \frac{q_{\tilde{\chi}}}{2E_b^3\pi}\int \mathrm{d}\cos\theta_W$$
$$G = \frac{e^2}{s}$$
$$Z = \frac{g^2}{\cos^2\theta_W(s - m_Z^2 + im_Z\Gamma_Z)}$$
$$\tilde{N} = \frac{g^2}{(t - m_{\tilde{\nu}}^2)}$$
$$c_{LR} = (1 - P(e^-))(1 + P(e^+))$$
$$c_{RL} = (1 + P(e^-))(1 - P(e^+))$$
$$f_1 = (p_1p_2)(p_2p_3)$$
$$f_2 = (p_1p_3)(p_2p_4)$$
$$f_3 = \frac{s m_{\tilde{\chi}_i^\pm}}{2}$$

$$c_1 = \int_C |Z|^2\{c_{LR}L^2 f_2 + c_{RL}R^2 f_1\}$$

$$c_2 = \int_C |Z|^2\{c_{LR}L^2(1-4L)(2f_2+f_3) + c_{RL}R^2(1-4R)(2f_1+f_3)\} - \int_C G\tilde{N}4\{c_{LR}L(2f_2+f_3) + c_{RL}R(2f_1+f_3)\} - \int_C Re(Z)\tilde{N}c_{LR}Lf_3$$

$$c_3 = \int_C |Z|^2(c_{LR}L^2 f_1 + c_{RL}R^2 f_2) - \int_C Z\tilde{N}2c_{LR}Lf_1 + \int_C \tilde{N}^2 c_{LR} f_1$$

$$c_4 = \int_C |Z|^2(1-4L)\{c_{LR}L^2(2f_1+f_3) + c_{RL}R^2(2f_2+f_3)\} + \int_C \tilde{N}^2 2c_{LR}f_1 + \int_C Re(Z)\tilde{N}c_{LR}L\{-4f_1 - f_3 + 4L(2f_1+f_3)\} + \int_C G\tilde{N}4c_{LR}(2f_1+f_3) - \int_C GRe(Z)4\{c_{LR}L(2f_1+f_3) + c_{RL}R(2f_2+f_3)\}$$

$$c_5 = \int_C |Z|^2(c_{LR}L^2 + c_{RL}R^2)f_3 - \int_C Re(Z)\tilde{N}c_{LR}Lf_3$$

$$c_6 = \int_C |Z|^2\{c_{LR}L^2(1-8L) + c_{RL}R^2(1-8L) + 16L^2(c_{LR}L^2 + c_{RL}R^2)\}(f_1+f_2+f_3) - \int_C Re(Z)\tilde{N}c_{LR}L(1-4L)(2f_1+f_3) + \int_C G^2(c_{LR}+c_{RL})(f_1+f_2+f_3) - \int_C Re(Z)G8\{c_{RL}R + c_{LR}L(1-4L)\}(f_1+f_2+f_3) + \int_C \tilde{N}^2 c_{LR}f_1 + \int_C G\tilde{N}4c_{LR}(2f_1+f_3)$$