Probing the Higgs Low-Energy Theorem: $h \to \gamma\gamma$ and $h \to Z\gamma$ decays in the BSM Inert Doublet Model

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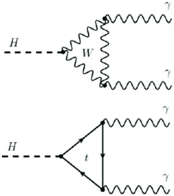
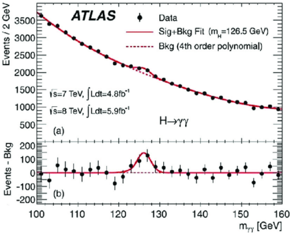
Outline

Higgs Boson in the Standard Model and Beyond

The 2012 discovery of the Higgs boson was a landmark event in Particle Physics.





Any deviation from the SM prediction of the Higgs boson decay channels would be a strong indication of new particles, pointing to **Physics Beyond the Standard Model**.

The Inert Doublet Model (IDM)

The IDM extends the Standard Model by adding a second scalar doublet. These two doublets are distinguished by an exact $\mathbb{Z}_2$ symmetry.

Standard Model Doublet (H_1):

- $H_1 \to H_1$.
- Acquires a VEV, breaking electroweak symmetry.
- Gives mass to SM particles.

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h^0 + iG^0) \end{pmatrix}$$

Inert Doublet (H_2):

- $H_2 \to -H_2$.
- Does not acquire a VEV.
- Contains new stable scalar particles.

$$H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H^0 + iA^0) \end{pmatrix}$$

The Tree-Level Higgs Potential

The tree-level potential $V(H_1, H_2)$ in the Inert Doublet Model is crucial for defining the masses and interactions of the scalar sector.

$$V(H_1, H_2) = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^\dagger H_2|^2 + \frac{1}{2}\lambda_5[(H_1^\dagger H_2)^2 + h.c.] \quad (1)$$

- **Mass Terms (μ_1^2, μ_2^2)**
- **Self-Interaction Terms (λ_1, λ_2)**
- **Mixed Interaction Terms ($\lambda_3, \lambda_4, \lambda_5$)**

The Scalar Masses at Tree Level

Following electroweak symmetry breaking, the scalar sector gives rise to five physical Higgs bosons [1]:

$$M_h^2 = \lambda_1 v^2 \tag{2}$$

$$M_H^2 = \mu_2^2 + \frac{1}{2}\lambda_{345} v^2 \tag{3}$$

$$M_A^2 = \mu_2^2 + \frac{1}{2}\bar{\lambda}_{345} v^2 \tag{4}$$

$$M_{H^\pm}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2 \tag{5}$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$ and $\bar{\lambda}_{345} \equiv \lambda_3 + \lambda_4 - \lambda_5$.

The lightest inert boson is a prime candidate for **dark matter** due to the $\mathbb{Z}_2$ symmetry preventing it from decaying into Standard Model fermions.

The Low-Energy Higgs Theorem

The Higgs low-energy theorem is a powerful tool that relates the amplitudes of two processes which differ by the insertion of a **Higgs-boson leg with zero external momentum** [2, 3].

$$\mathcal{A}_h = \sum_i \frac{\partial \mathcal{A}}{\partial \ln m_i} \frac{h}{v} \tag{6}$$

where m_i are the heavy particle masses in the loop.

- **First Assumption:** $p_h^\mu \to 0$
  - From the translational property, it follows that: $[P_\mu, h] = i\partial_\mu h = 0$.
  - The effect of the **constant field** h is equivalent to redefining all mass parameters of the theory: $m_i \to m_i \left(1 + \frac{h}{v}\right)$.

- **Second Assumption:** $m_h \ll m_{\text{loop}}$
  By integrating out heavy degrees of freedom, we get an **Effective Lagrangian** for a generic decay $h \to XX$:

$$\mathcal{L}^h_{\text{eff}} \supset -\frac{1}{4} C_{hXX}\, h\, X_{\mu\nu} X^{\mu\nu} \tag{7}$$

in which the coupling is defined as:

$$C_{hXX} = \frac{\partial}{\partial h} \Pi_{XX}(p^2 = 0)\Big|_{h=0} \tag{8}$$

and Π_{XX} is the gauge boson's vacuum polarization.

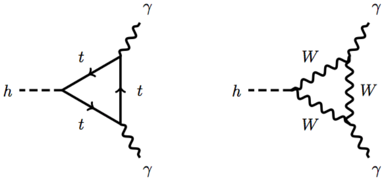
The effective coupling, C_{hXX}, is given by the HLET relation:

$$C_{hXX} = \frac{\partial \Pi_{XX}}{\partial \ln m_{\text{loop}}} \frac{1}{v} \tag{9}$$

This is a computational shortcut, as the derivative with respect to the loop mass (m_{loop}) is equivalent to the derivative with respect to the Higgs field (h) itself.

Methods and objectives

My work explores **one-loop-induced Higgs boson decays**.[4].



- Application of the **Background Field Method** [5, 6] for obtaining gauge-independent results for the photon self-energy in the HLET.
- Comparison between the SM results, both in the HLET approximation and the full loop calculation, and the IDM ones.
- Study of the **decoupling regime** as a function of the λ_3 coupling and the $H^{\pm}$ mass.

HLET Application to $h \to \gamma\gamma$

The **effective Lagrangian** [1] for this process is:

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{4} C_{h\gamma\gamma} h F_{\mu\nu} F^{\mu\nu} \tag{10}$$

The coupling $C_{h\gamma\gamma}$ is computed via the **HLET**:

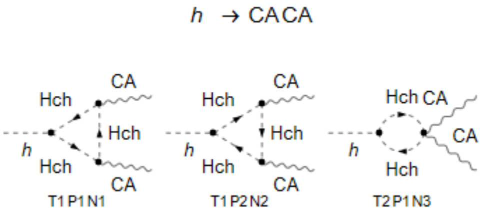
$$C_{h\gamma\gamma} = \frac{\partial}{\partial h} \Pi_{\gamma\gamma}(p^2 = 0)\Big|_{h=0} \tag{11}$$

where the vacuum polarization function $\Pi_{\gamma\gamma}(p^2)$ is defined from the **photon self-energy tensor**:

$$\Sigma^{\mu\nu}_{\gamma\gamma}(p^2) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\Pi_{\gamma\gamma}(p^2). \tag{12}$$

Comparison between SM and IDM decay

The IDM Feynman diagrams for $h \to \gamma\gamma$ include contributions from the charged Higgs boson (H^+), which are absent in the SM computation.



HLET One-Loop Corrections for $\Gamma(h \to \gamma\gamma)$

The one-loop corrections to the **Higgs to di-photon decay width** [7] are given by the following terms:

$$\Gamma(h \to \gamma\gamma) = \frac{\sqrt{2}\alpha_{\text{em}}^2 G_F m_h^3}{16\pi^3} \left| I_t^{(1)} + I_W^{(1)} + I_{H^\pm}^{(1)} \right|^2 \tag{13}$$

The expressions for the one-loop contributions read:

- $I_{H^\pm}^{(1)} = -\frac{1}{12}\left(1 - \frac{\mu_2^2}{m_{H^\pm}^2}\right) \quad = -\frac{1}{24}\left(\frac{\boldsymbol{\lambda_3} v^2}{m_{H^\pm}^2}\right)$
- $I_t^{(1)} = -\frac{4}{9}$
- $I_W^{(1)} = \frac{7}{4}$

Mathematica calculation steps

Calculation Procedure:

1. **Amplitude generation** using FeynArts for:
   - Top quark loop contribution
   - W-boson loop contribution
   - $H^{\pm}$ loop contribution (IDM)
2. **Algebraic manipulation** with FeynCalc:
   - Projection with projPi$^{\mu\nu}(p)$
   - Conversion to Passarino-Veltman integrals
   - Tensor reduction via TID
3. **Dimensional regularization** ($D = 4 - 2\epsilon$):

   UV divergence expansion:
   - $A_0(x) \to \epsilon A_{0e}(x) + A_0^{\text{fin}}(x) + \frac{x}{\epsilon} + \mathcal{O}(\epsilon^2)$
   - $B_0(x) \to \epsilon B_{0e}(x) + B_0^{\text{fin}}(x) + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^2)$
   - $C_0(x) \to \epsilon C_{0e}(x) + C_0^{\text{fin}}(x) + \mathcal{O}(\epsilon^2)$
4. **Finite part evaluation** at $p^2 \to 0$:

   Renormalized finite contributions:
   - $A_0^{\text{fin}}(x) \to x\left(1 - \log\left(\frac{x}{Q^2}\right)\right)$
   - $B_0^{\text{fin}}(p^2, x, x) \to \frac{(p^2)^2}{60x^2} + \frac{p^2}{6x} - \log\left(\frac{x}{Q^2}\right) + \mathcal{O}(p^4)$
   - $C_0^{\text{fin}}(0, 0, y, x, x, x) \to -\frac{y}{24x^2} - \frac{1}{2x} + \mathcal{O}(x)$ (valid for y « x)

HLET Implementation for $h \to \gamma\gamma$:

- $\mathcal{A}(h \to \gamma\gamma) = \mathcal{A}_W + \mathcal{A}_t + \mathcal{A}_{H^\pm}$

W-boson contribution:

- $\mathcal{A}_W \propto \frac{\partial}{\partial h}\Pi^W_{\gamma\gamma}(h)\Big|_{h=0}$
- $m_W(h) = m_W\left(1 + \frac{h}{v}\right)$
- `D[Pizero$G /. mW -> mW (v + h)/v, h] /. h -> 0`

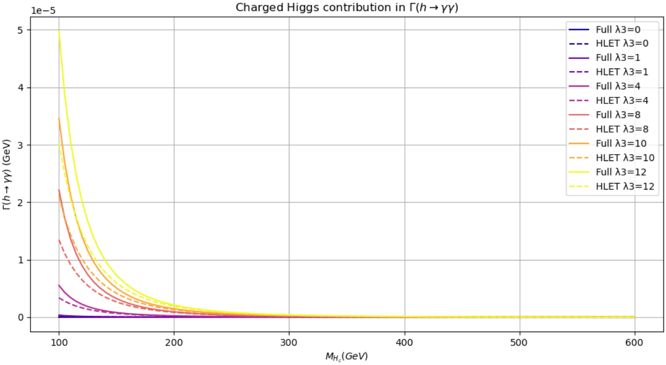
Top quark contribution:

- $\mathcal{A}_t \propto \frac{\partial}{\partial h}\Pi^t_{\gamma\gamma}(h)\Big|_{h=0}$
- $m_t(h) = m_t\left(1 + \frac{h}{v}\right)$
- `D[Pizero$t /. mt -> mt (v + h)/v, h] /. h -> 0`

Charged Higgs contribution (IDM):

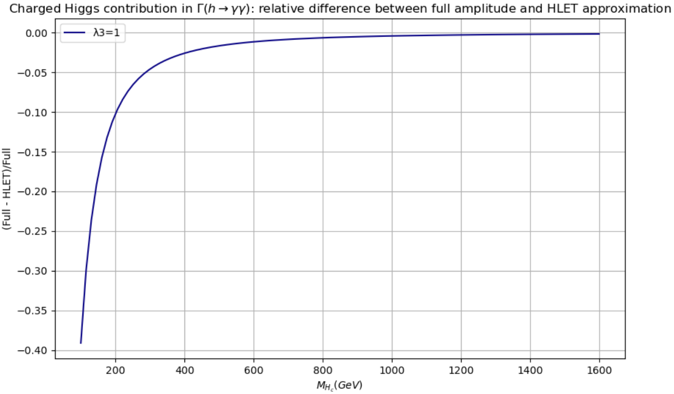
- $\mathcal{A}_{H^\pm} \propto \frac{\partial}{\partial h}\Pi^{H^\pm}_{\gamma\gamma}(h)\Big|_{h=0}$
- `D[Pizero$Hpm /. mHpm -> f[h], h] /. h -> 0`

$H^{\pm}$ contributions in the decay width



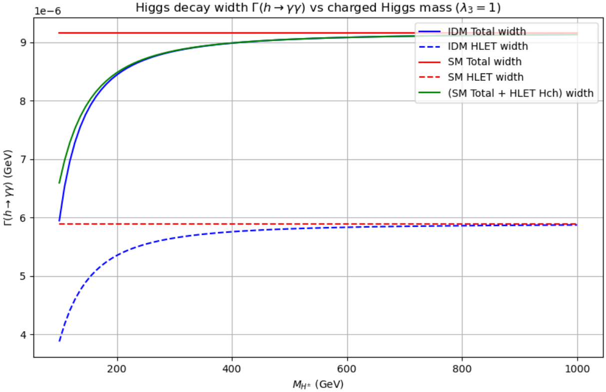
- Suppression of the $H^{\pm}$ loop contribution in the high mass limit;
- Enhancement of the contribution for higher values of the λ_3 coupling.

$H^{\pm}$ contributions: relative difference between full amplitude and HLET approximation



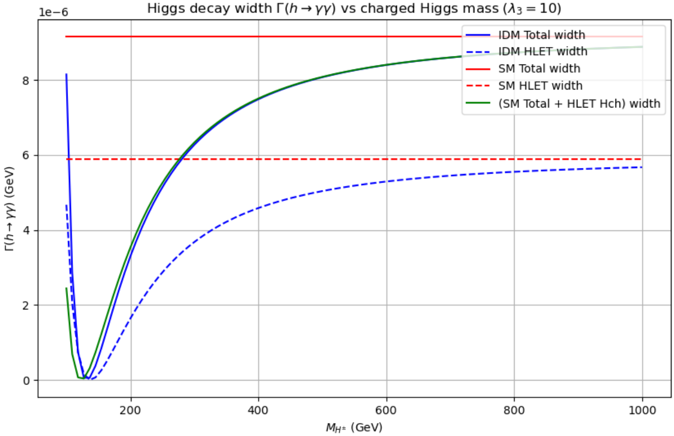
- The HLET approximation becomes accurate for $M_{H^{\pm}} > 600 GeV$.

$\lambda_3 = 1$



- The HLET **underestimates** the decay widths both for SM and IDM;
- The IDM width converges to the SM prediction for $M_{H^\pm} > 500 - 600\,GeV$;
- The mixed approach (green line SM + HLET for $H^\pm$) is accurate in the decoupling limit of $M_{H^\pm} \to \infty$.

$\lambda_3 = 10$


Higgs decay width Γ(h → γγ) vs charged Higgs mass (λ₃ = 10)
1e−6
IDM Total width
IDM HLET width
SM Total width
SM HLET width
(SM Total + HLET Hch) width
Γ(h → γγ) (GeV)
0
2
4
6
8
200
400
600
800
1000
M_{H±} (GeV)

HLET Application to $h \to Z\gamma$

- **Effective Lagrangian for $Z\gamma$ [2]:**

$$\mathcal{L}_{Z\gamma} = -\frac{1}{4} F^{\rho\mu\nu} Z^{0}_{\rho\mu\nu} \Pi^{0}_{Z\gamma}(0), \tag{14}$$

where $\Pi^{0}_{Z\gamma}(0)$ is the top-quark contribution to the dimensionless $Z\gamma$ mixing amplitude at zero momentum transfer.

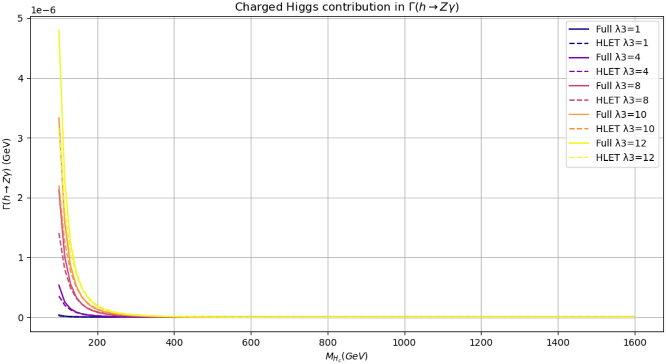
- Differentiating the expression with respect to m_t leads to the **effective $hZ\gamma$ Lagrangian**:

$$\mathcal{L}_{HZ\gamma} = \frac{N_c e_t v_t}{6\pi} \sqrt{\frac{\alpha_{em} G_F M_Z^2}{8\sqrt{2}\pi}} F^{\mu\nu} Z_{\mu\nu} \frac{H}{v}, \tag{15}$$

- **Decay width:**

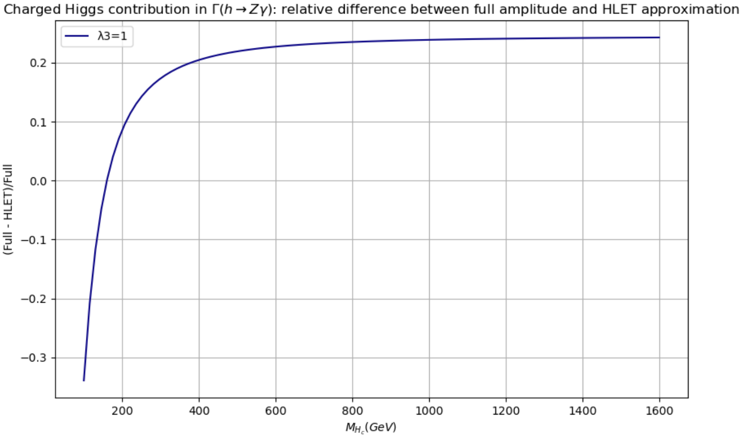
$$\Gamma(h \to Z\gamma) = \frac{\sqrt{2}\alpha_{\text{em}}^2 G_F m_h^3}{128\pi^3} \left(1 - \frac{m_z^2}{m_h^2}\right)^3 \left| J_t^{(1)} + J_W^{(1)} + J_{H^\pm}^{(1)} \right|^2 \tag{16}$$

$H^{\pm}$ contributions in the decay width



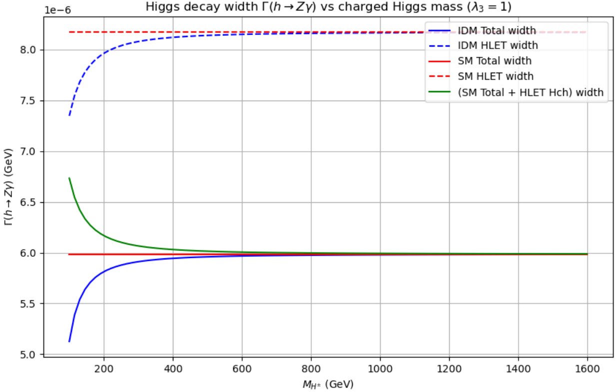
- Suppression of the $H^{\pm}$ loop contribution in the high mass limit;
- Enhancement of the contribution for higher values of the λ_3 coupling.

$H^{\pm}$ contributions: relative difference between full amplitude and HLET approximation



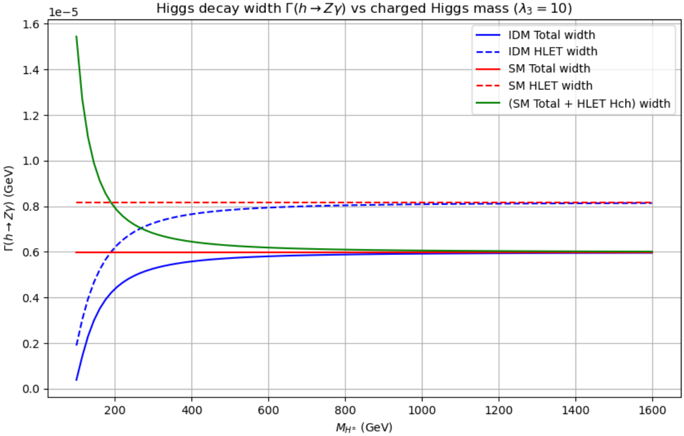
- The HLET approximation becomes accurate for approximately $M_{H^{\pm}} > 800 GeV$.

$\lambda_3 = 1$



- The HLET here **overestimates** the decay widths both for SM and IDM;
- The IDM width converges to the SM prediction for about $M_{H\pm} > 600 GeV$ as well;
- The mixed approach (green line SM + HLET for $H^{\pm}$) remains accurate in the decoupling limit of $M_{H\pm} \to \infty$.

$\lambda_3 = 10$


Higgs decay width Γ(h → Zγ) vs charged Higgs mass (λ3 = 10)
1e−5
Γ(h → Zγ) (GeV)
M_{H±} (GeV)
IDM Total width
IDM HLET width
SM Total width
SM HLET width
(SM Total + HLET Hch) width
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
200
400
600
800
1000
1200
1400
1600

Conclusions

- **Comparison between full SM and HLET results**
  The Low-Energy Theorem fails to accurately describe the W loop contribution because the assumption $m_h \ll m_W$ is invalid, resulting in a ∼30% error. As a consequence, it underestimates the $\gamma\gamma$ decay width and overestimates $Z\gamma$.

- **IDM contributions**
  The IDM introduces contributions from the charged Higgs boson which, in the decoupling limit ($M_{H^\pm} \to \infty$), approach zero, as expected.

- **HLET high-mass validity in the IDM**
  The HLET approximation (SM Total + HLET $H^\pm$) is shown to be reliable at high values of $M_{H^\pm}$.

My experience as a Summer Student at DESY

Thank you!

Appendix: Theoretical Constraints on the Higgs Potential

- **Vacuum Stability**
  The scalar potential must be bounded from below.
  - $\lambda_1 > 0$, $\lambda_2 > 0$
  - $\lambda_3 + \lambda_4 + \lambda_5 + \sqrt{\lambda_1 \lambda_2} > 0$

- **Perturbative Unitarity**
  Scattering probabilities of scalar particles must not violate unitarity.

- **Inert Vacuum Condition**
  The model requires that the Standard Model-like vacuum $(\langle H_1 \rangle \neq 0, \langle H_2 \rangle = 0)$ is the global minimum of the potential.
  - $m_{H_2}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2 > 0$

Appendix:
Full Mathematica results for $h \to \gamma\gamma$

$$\text{Amplitude level prefactor} = \frac{E_L^2}{2\pi^2 v}$$

$$\text{HLET conversion factor} = \frac{2}{m_h^2}$$

$$\text{Decay width prefactor} = \frac{\sqrt{2}\alpha_{em}}{16\pi^3} \cdot G_F \cdot m_h^3$$

SM Amplitude contributions:

$$\mathcal{A}_{\text{top}} = \frac{E_L^3 M_T^2}{12\pi^2 M_W S_W}\left[2(m_h^2 - 4M_T^2)C_0(0, 0, m_h^2, M_T^2, M_T^2, M_T^2) - 4\right]$$

$$\mathcal{A}_{\text{gauge}} = \frac{E_L^3}{16\pi^2 M_W S_W}\left[-6M_W^2(m_h^2 - 2M_W^2)C_0(0, 0, m_h^2, M_W^2, M_W^2, M_W^2) + m_h^2 + 6M_W^2\right]$$

SM HLET Amplitude contributions:

$$\mathcal{A}_{\text{top}} = \frac{2E_L^2}{9\pi^2 v}$$

$$\mathcal{A}_{\text{gauge}} = -\frac{7E_L^2}{8\pi^2 v}$$

IDM Amplitude contributions:

$$\mathcal{A}_{\rm top} = \frac{E_L^3 M_T^2}{12\pi^2 M_W S_W}\Big[2(m_h^2 - 4M_T^2)C_0(0,0,m_h^2,M_T^2,M_T^2,M_T^2) - 4\Big]$$

$$\mathcal{A}_{H\pm} = \frac{E_L^2\lambda_3 v}{8\pi^2}\Big[2M_{H\pm}^2 C_0(0,0,m_h^2,M_{H\pm}^2,M_{H\pm}^2,M_{H\pm}^2) + 1\Big]$$

$$\mathcal{A}_{\rm gauge} = \frac{1}{16\pi^2 S_W^2 v}\Big[m_h^2(4M_W^2 S_W^2 - E_L^2 v^2)C_0(0,0,m_h^2,M_W^2,M_W^2,M_W^2)$$
$$-3E_L^2 v^2(m_h^2 - 2M_W^2)C_0(0,0,m_h^2,M_W^2,M_W^2,M_W^2) + 3E_L^2 v^2 + 2m_h^2 S_W^2\Big]$$

HLET IDM Amplitude contributions:

$$\mathcal{A}_{\rm top} = \frac{2E_L^2}{9\pi^2 v}$$

$$\mathcal{A}_{H\pm} = \frac{E_L^2\lambda_3 v}{48\pi^2 M_{H\pm}}$$

$$\mathcal{A}_{\rm gauge} = -\frac{7E_L^2}{8\pi^2 v}$$

Appendix: Full Mathematica results for $h \to Z\gamma$

$$\text{Amplitude level prefactor} = \frac{16 M_W \pi^2 S_W}{E_L^3(-m_h^2 + M_Z^2)}$$

$$\text{Top amplitude level prefactor} = 3 \times \frac{2}{3}$$

$$\text{Gauge amplitude level prefactor} = 2$$

$$H^{\pm}\text{ amplitude level prefactor} = \frac{2vE_L}{M_W}$$

$$\text{HLET conversion factor} = \frac{vE_L m_h^2}{4M_W S_W}$$

$$\text{Decay width prefactor} = \frac{\sqrt{2}\alpha^2 G_F m_h^3}{128\pi^3}\left(1 - \frac{M_Z^2}{m_h^2}\right)^3$$

SM Amplitude contributions:

$$\mathcal{A}_{\text{top}} = \frac{E_L^3 M_T^2 (4C_W^2 - 4S_W^2 - 1)}{48\pi^2 C_W M_W S_W^2} \left[\frac{2(M_Z^2 B_0(m_h^2, M_T^2, M_T^2) - M_Z^2 B_0(M_Z^2, M_T^2, M_T^2) + m_h^2 - M_Z^2)}{m_h^2 - M_Z^2} \right.$$
$$\left. +(-m_h 2 + 4M_T 2 + M_Z 2)C_0(0, m_h 2, M_Z 2, M_T 2, M_T 2, M_T 2) \right]$$
$$\mathcal{A}_{\text{gauge}} = \frac{E_L^3}{32\pi^2 C_W M_W S_W^2} \left[2M_W 2 C_0(0, m_h 2, M_Z 2, M_W 2, M_W 2, M_W 2) \right.$$
$$\times \left(m_h^2 (7C_W^2 + S_W^2 - 2) + 2(-C_W^2 (5M_W^2 + 4M_Z^2) + M_W^2 S_W^2 + M_Z^2) \right)$$
$$- \frac{(C_W^2 (m_h^2 + 10M_W^2) - S_W^2 (m_h^2 + 2M_W^2))}{(m_h - M_Z)(m_h + M_Z)}$$
$$\left. \times \left(M_Z^2 (B_0(m_h^2, M_W^2, M_W^2) - B_0(M_Z^2, M_W^2, M_W^2)) + (m_h - M_Z)(m_h + M_Z) \right) \right]$$

SM HLET Amplitude contributions:

$$\mathcal{A}_{\text{top}} = -\frac{E_L^2 (3C_W^2 - 5S_W^2)}{36\pi^2 C_W S_W v}$$
$$\mathcal{A}_{\text{gauge}} = \frac{E_L^2 (43C_W^2 + S_W^2)}{48\pi^2 C_W S_W v}$$

IDM Amplitude contributions:

$$\begin{aligned}
\mathcal{A}_{\rm top} &= \frac{E_L^3 M_T^2(4C_W^2 - 4S_W^2 - 1)}{48\pi^2 C_W M_W S_W^2}\Bigg[\frac{2(M_Z^2 B_0(m_h^2, M_T^2, M_T^2) - M_Z^2 B_0(M_Z^2, M_T^2, M_T^2) + m_h^2 - M_Z^2)}{m_h^2 - M_Z^2} \\
&\quad + (-m_h^2 + 4M_T^2 + M_Z^2) C_0(0, m_h^2, M_Z^2, M_T^2, M_T^2, M_T^2)\Bigg] \\
\mathcal{A}_{H\pm} &= -\frac{E_L^2 \lambda_3 v (C_W^2 - S_W^2)}{16\pi^2 C_W S_W (m_h^2 - M_Z^2)}\Big[M_Z^2 B_0(m_h^2, M_{H\pm}^2, M_{H\pm}^2) - M_Z^2 B_0(M_Z^2, M_{H\pm}^2, M_{H\pm}^2) \\
&\quad + (m_h^2 - M_Z^2)(2M_{H\pm}^2 C_0(0, m_h^2, M_Z^2, M_{H\pm}^2, M_{H\pm}^2, M_{H\pm}^2) + 1)\Big] \\
\mathcal{A}_{\rm gauge} &= \frac{E_L^3}{32\pi^2 C_W M_W S_W^2}\Big[2M_W^2 C_0(0, m_h^2, M_Z^2, M_W^2, M_W^2, M_W^2) \\
&\quad \times \left(m_h^2(7C_W^2 + S_W^2 - 2) + 2(-C_W^2(5M_W^2 + 4M_Z^2) + M_W^2 S_W^2 + M_Z^2)\right) \\
&\quad - \frac{(C_W^2(m_h^2 + 10M_W^2) - S_W^2(m_h^2 + 2M_W^2))}{(m_h - M_Z)(m_h + M_Z)} \\
&\quad \times \left(M_Z^2(B_0(m_h^2, M_W^2, M_W^2) - B_0(M_Z^2, M_W^2, M_W^2)) + (m_h - M_Z)(m_h + M_Z)\right)\Big]
\end{aligned}$$

HLET IDM Amplitude contributions:

$$\begin{aligned}
\mathcal{A}_{\rm top} &= -\frac{E_L^2(3C_W^2 - 5S_W^2)}{36\pi^2 C_W S_W v} \\
\mathcal{A}_{H\pm} &= -\frac{E_L^2 \lambda_3 v (C_W^2 - S_W^2)}{96\pi^2 C_W S_W M_{H\pm}^2} \\
\mathcal{A}_{\rm gauge} &= \frac{E_L^2(43C_W^2 + S_W^2)}{48\pi^2 C_W S_W v}
\end{aligned}$$

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[1] M.Aiko, J.Braathen, and S.Kanemura. “Leading two-loop corrections to the Higgs di-photon decay in the Inert Doublet Model”. In: (2025).

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