PROBING INFLATIONARY MODELS WITH GRAVITATIONAL WAVES

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CONTENTS

INFLATION

Horizon Problem:

- Points of the CMB are correlated, even if they are outside each others particle horizons

Solution: **Rapid Inflation**

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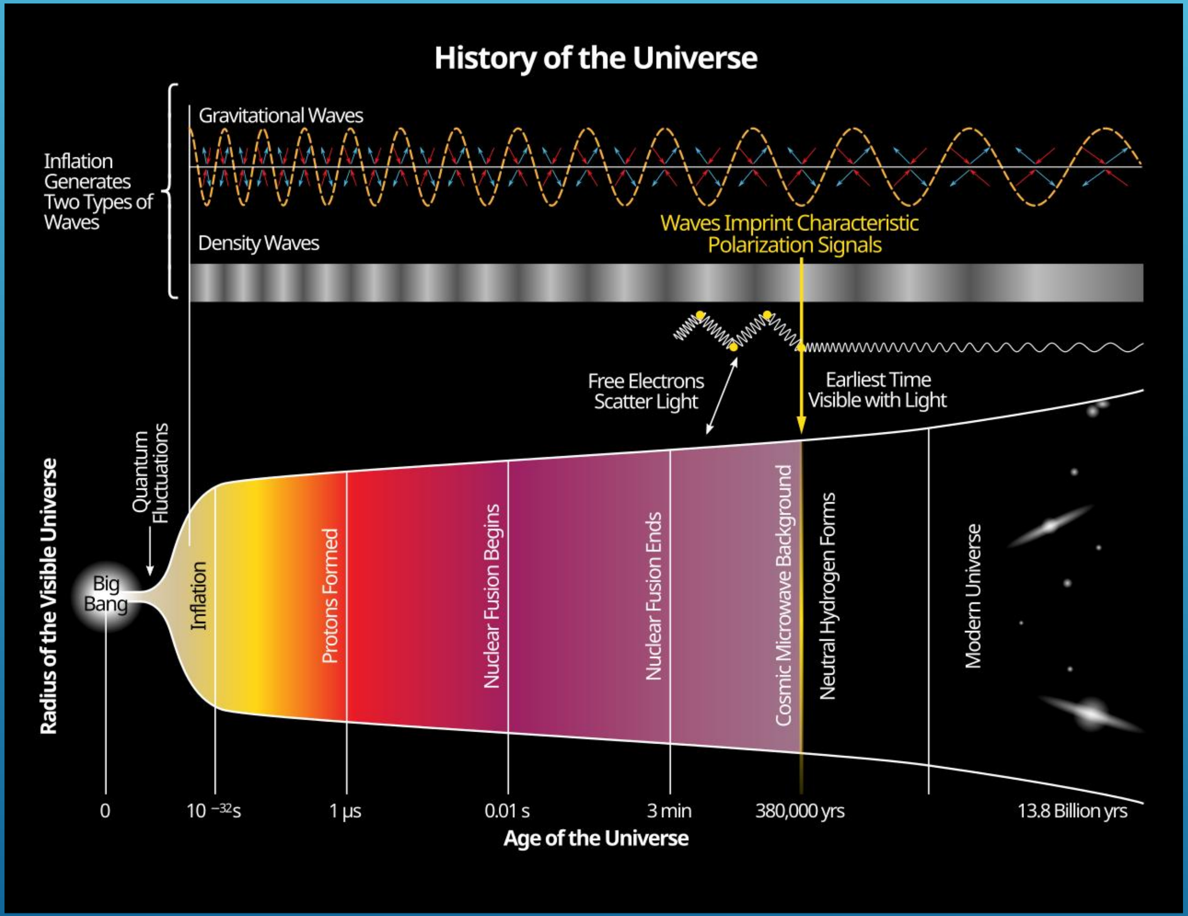
INFLATION



[1]

The universe rapidly grew 26 order of Magnitude

Grain of Rice (1,5mm) → 5 Mpc Distance Between Galaxies

[1] https://upload.wikimedia.org/wikipedia/commons/thumb/d/db/History_of_the_Universe.svg/1280px-History_of_the_Universe.svg.png

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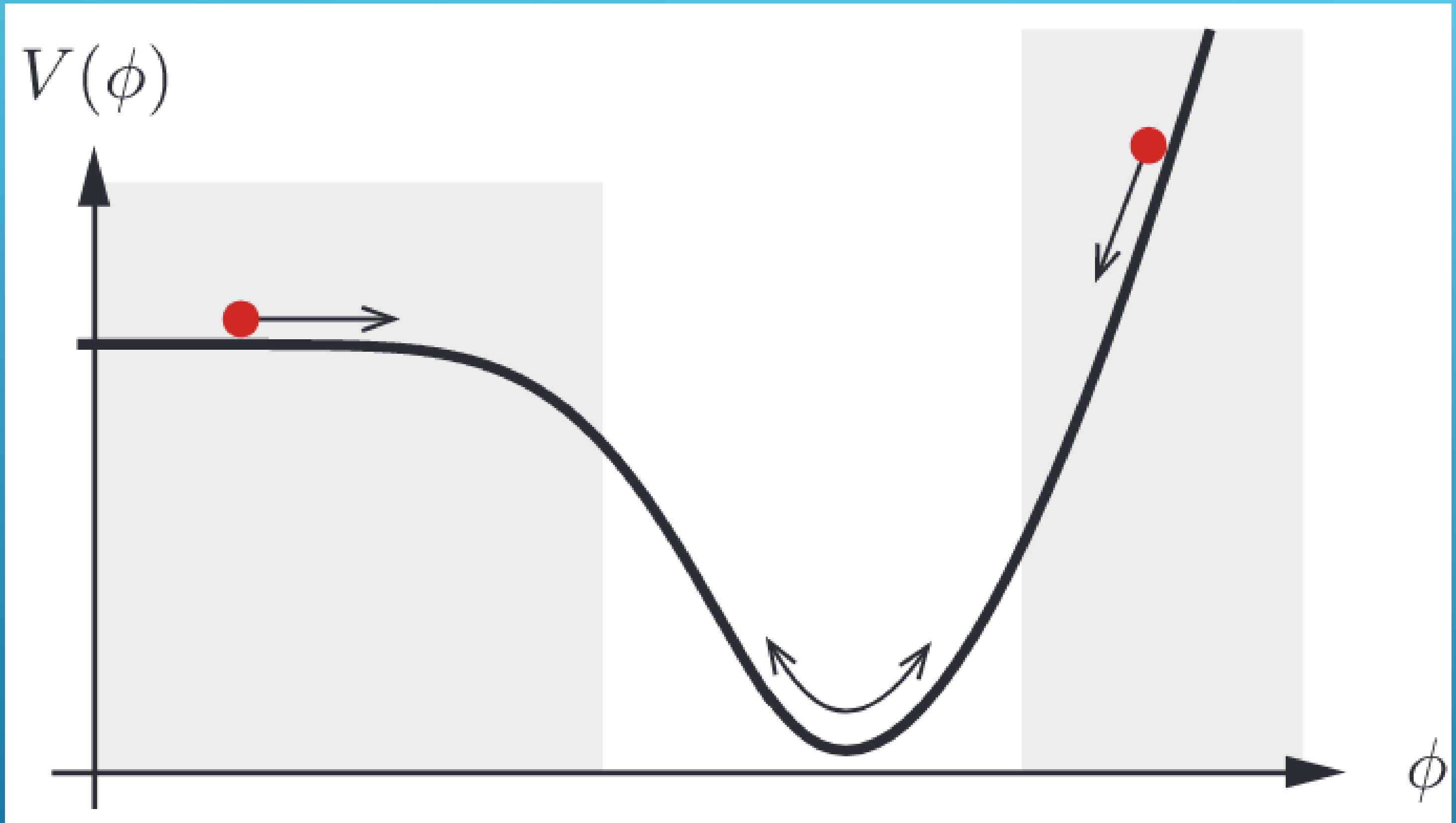
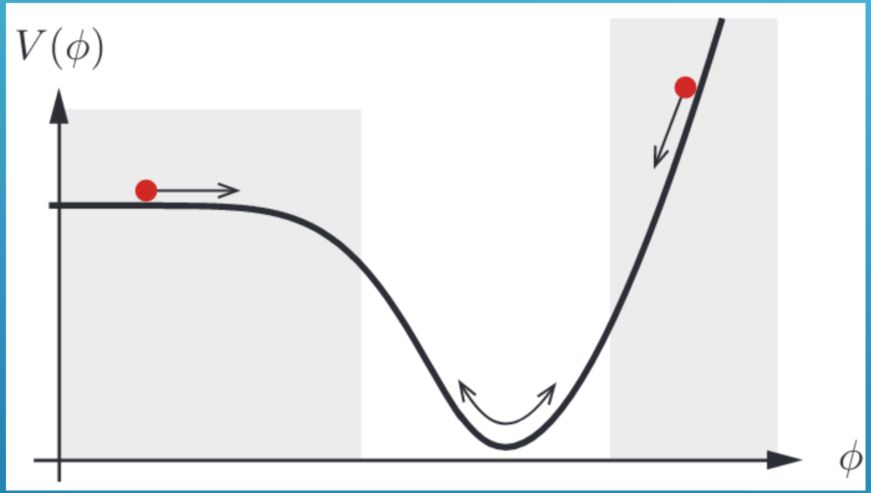


[1]

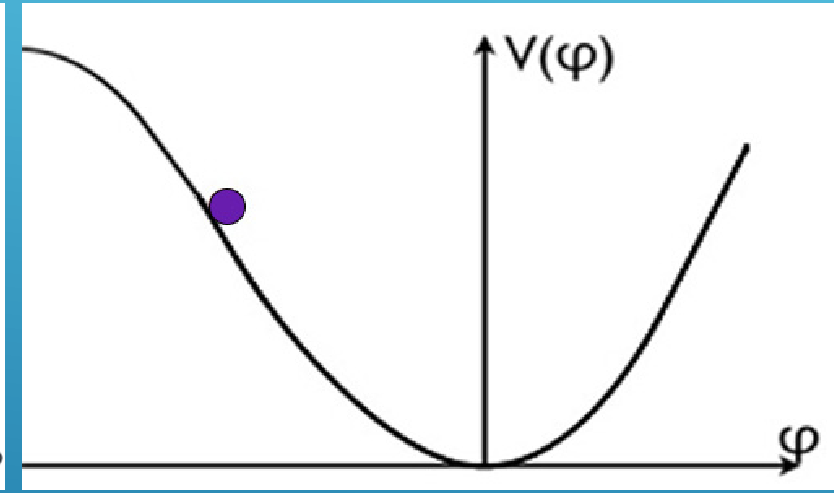
Inflation is driven by a scalar field, the Inflaton field
Has to happen over an extended period of time → slow roll Inflation

[1] Bauman Cosmology lecture notes 07.01.2025

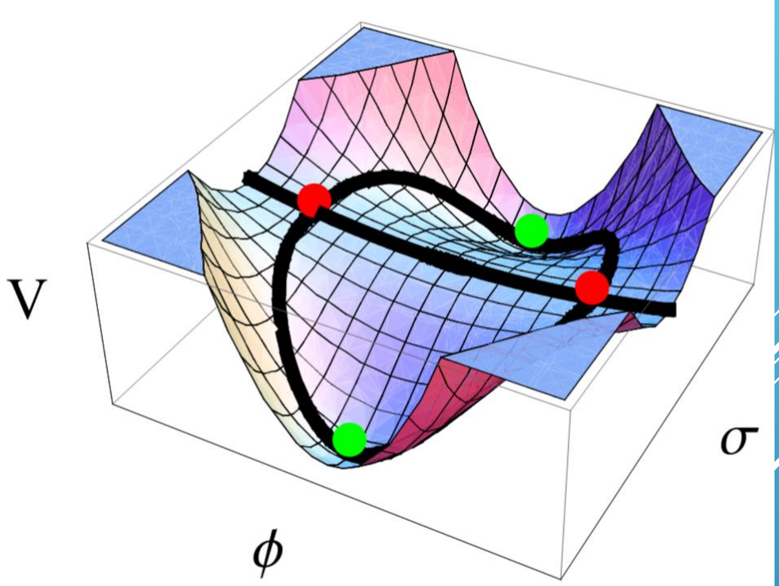
INFLATION



Small Field Inflation



Large Field Inflation



Hybrid Inflation

Depending on the model, the change of the scale factor may vary, we will here assume de Sitter Inflation, meaning $a(t) \propto e^{Ht}$ and $H = const$

[1]Baumann Comology lecture notes 07.01.2025
[2] https://blogger.googleusercontent.com/img/b/R29vZ2xl/AVvXsEjD-Xzx6RHob6Q1Nhz9BypuVQdVLG4dmws3UcJ8OtL6QfNupP6yC1GtMfcMmuJEG6L-cM4D7WRzdPLv0peMFu1uwYJZ1o88_7cR-zOeFLQM53GmjrfcbtU6KoWTyCs6DvDxhjCepb9beX9V/s1600/inflatonpotential.jpg
[3]Dufaux et al. Gravity Waves from Tachyonic Preheating after Hybrid Inflation (2009) http://arxiv.org/abs/0812.2917

GRAVITATIONAL WAVES FROM INFLATION

The Equation that describes the propagation of gravitational waves in the T T gauge is defined as:

$$\ddot{h}_{\mu\nu} - \Delta h_{\mu\nu} = 16\pi G T_{\mu\nu}$$

In Friedman cosmology with $\frac{d}{a d\tau} = \frac{d}{dt}$ and $x_{cm} = ax$ the wave equation becomes:

$$h''_{\mu\nu} + \frac{2a'}{a} h_{\mu\nu}' - \Delta h_{\mu\nu} = 16\pi G T_{\mu\nu}$$

Buchmüller et al. The Gravitational Wave Spectrum from Cosmological B−L Breaking 2013
http://arxiv.org/abs/1305.3392

GRAVITATIONAL WAVES FROM INFLATION

$$ah''_{\mu\nu} + 2a'h'_{\mu\nu} - a\Delta h_{\mu\nu} = 16\pi a G T_{\mu\nu}$$

Fourier Transform, written in terms of $\tilde{h}_{\mu\nu} = a h_{\mu\nu}$:

$$\tilde{h}''_{\mu\nu} + \left(k^2 - \frac{a''}{a}\right)\tilde{h}_{\mu\nu} = 16\pi a G T_{\mu\nu}$$

Without sources:

$$\tilde{h}''_{\mu\nu} + \left(k^2 - \frac{a''}{a}\right)\tilde{h}_{\mu\nu} = 0$$

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GRAVITATIONAL WAVES FROM INFLATION

$$\tilde{h}''_{\mu\nu} + \left(k^2 - \frac{a''}{a}\right)\tilde{h}_{\mu\nu} = 0$$

using $\dot{H} + H^2 = \frac{\ddot{a}}{a} = H^2 \rightarrow \frac{a''}{a} = (aH)^2$ in de Sitter:

$$\tilde{h}''_{\mu\nu} + (k^2 - (aH)^2)\tilde{h}_{\mu\nu} = 0$$

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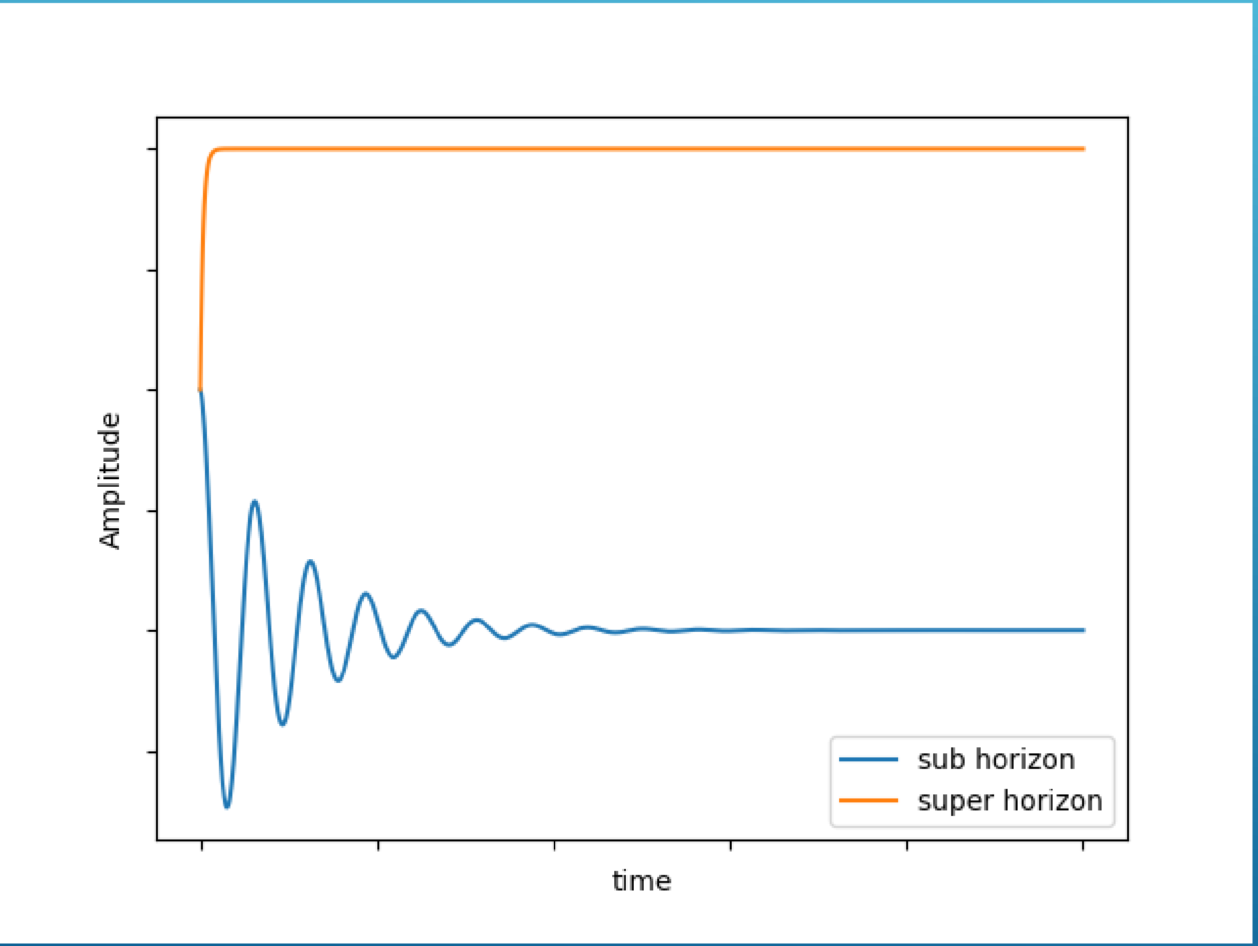
GRAVITATIONAL WAVES FROM INFLATION



$$\tilde{h}''_{\mu\nu} + (k^2 - (aH)^2)\tilde{h}_{\mu\nu} = 0$$

For $k^2 \gg (aH)^2 \; : \tilde{h}_{\mu\nu} \propto \cos(\omega t)$

$\rightarrow h_{\mu\nu} \propto \frac{1}{a}\cos(\omega t)$

(sub horizon)

For $k^2 \ll (aH)^2 : ah''_{\mu\nu} + 2a'h_{\mu\nu}' = 0$

$\rightarrow h = A + B\int_{\tau_i}^{\tau_f} \frac{d\tau}{a(\tau)^2}$

(super horizon)

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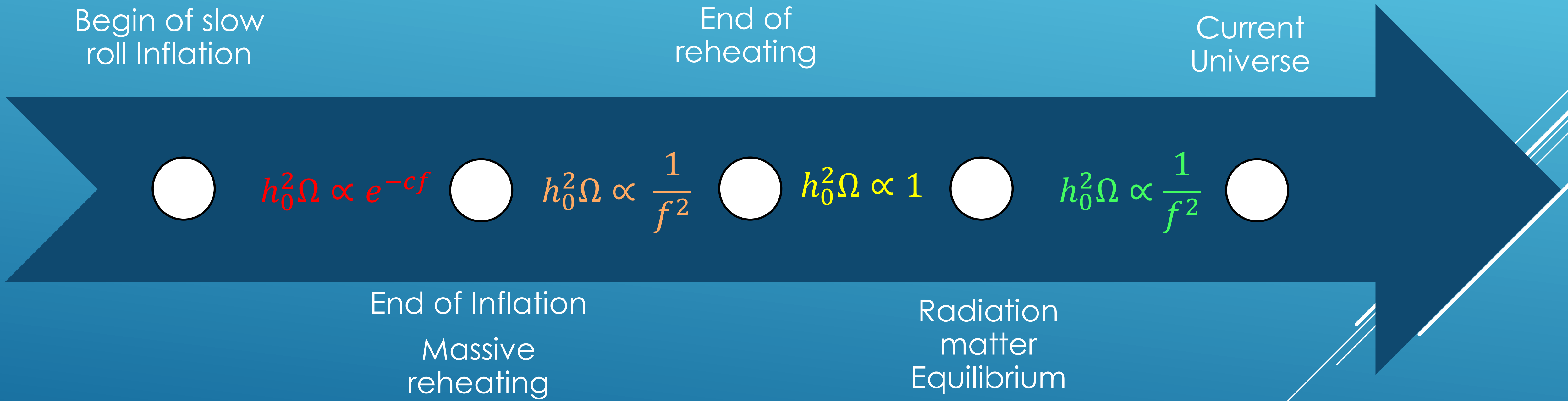
GRAVITATIONAL WAVES FROM INFLATION



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TRANSFER FUNCTION

- The transfer function calculates how a mode changed throughout the history of the universe (expansion) → what we expect to see today
- It can be derived from Friedman equations & parameters from measurements of the CMB

This roughly could get us:

$$h_0^2\Omega = 6.73 \cdot 10^{-18} \cdot \left(\frac{f_{eq}}{f}\right)^2 \quad \text{for } f \ll f_{eq}$$

$$= 5.12 \cdot 10^{-18} \quad \text{for } f_{eq} \ll f \ll f_{end}$$

$$= 5.12 \cdot 10^{-18} \cdot \left(\frac{f_{RH}}{f}\right)^2 \quad \text{for } f_{RH} \ll f \ll f_{RHend}$$

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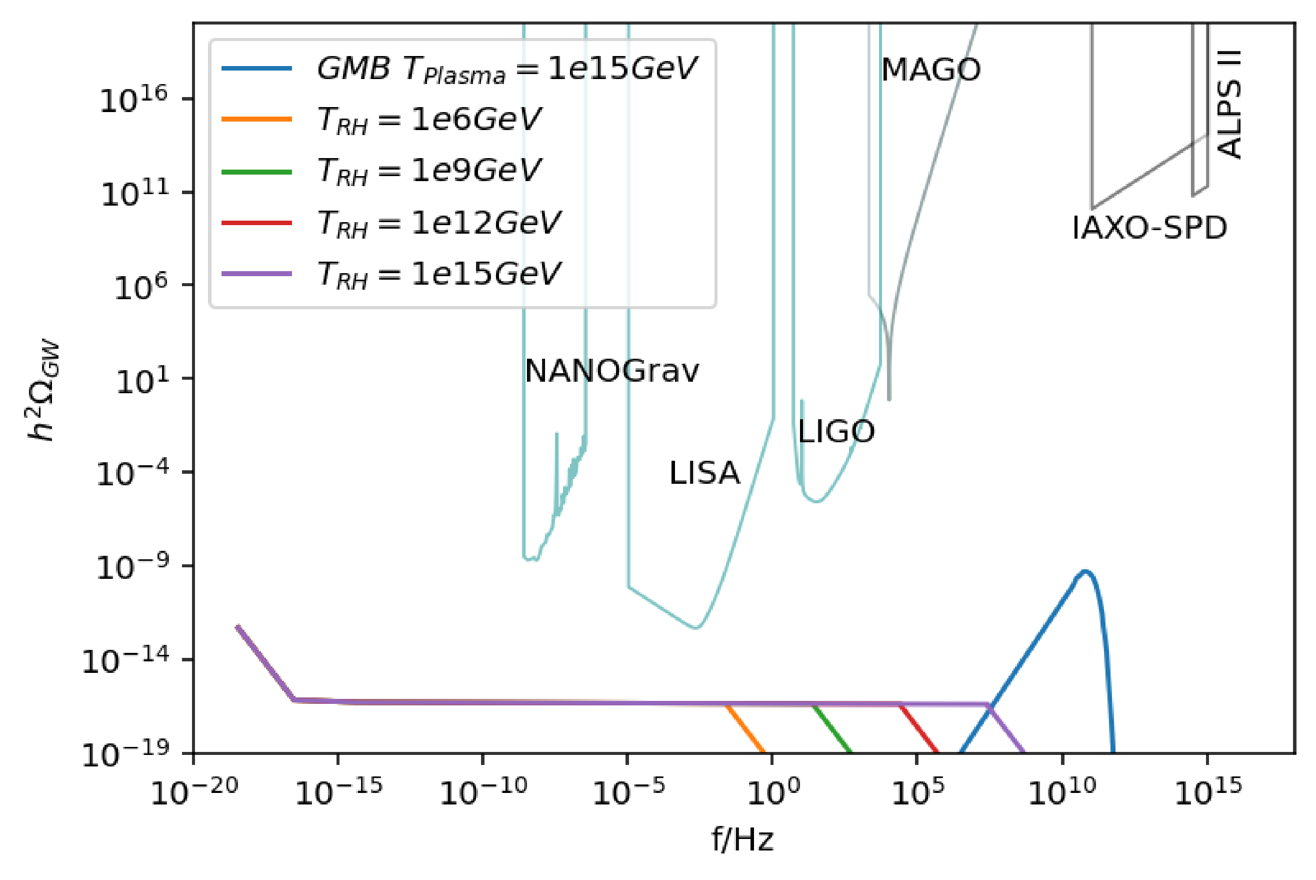
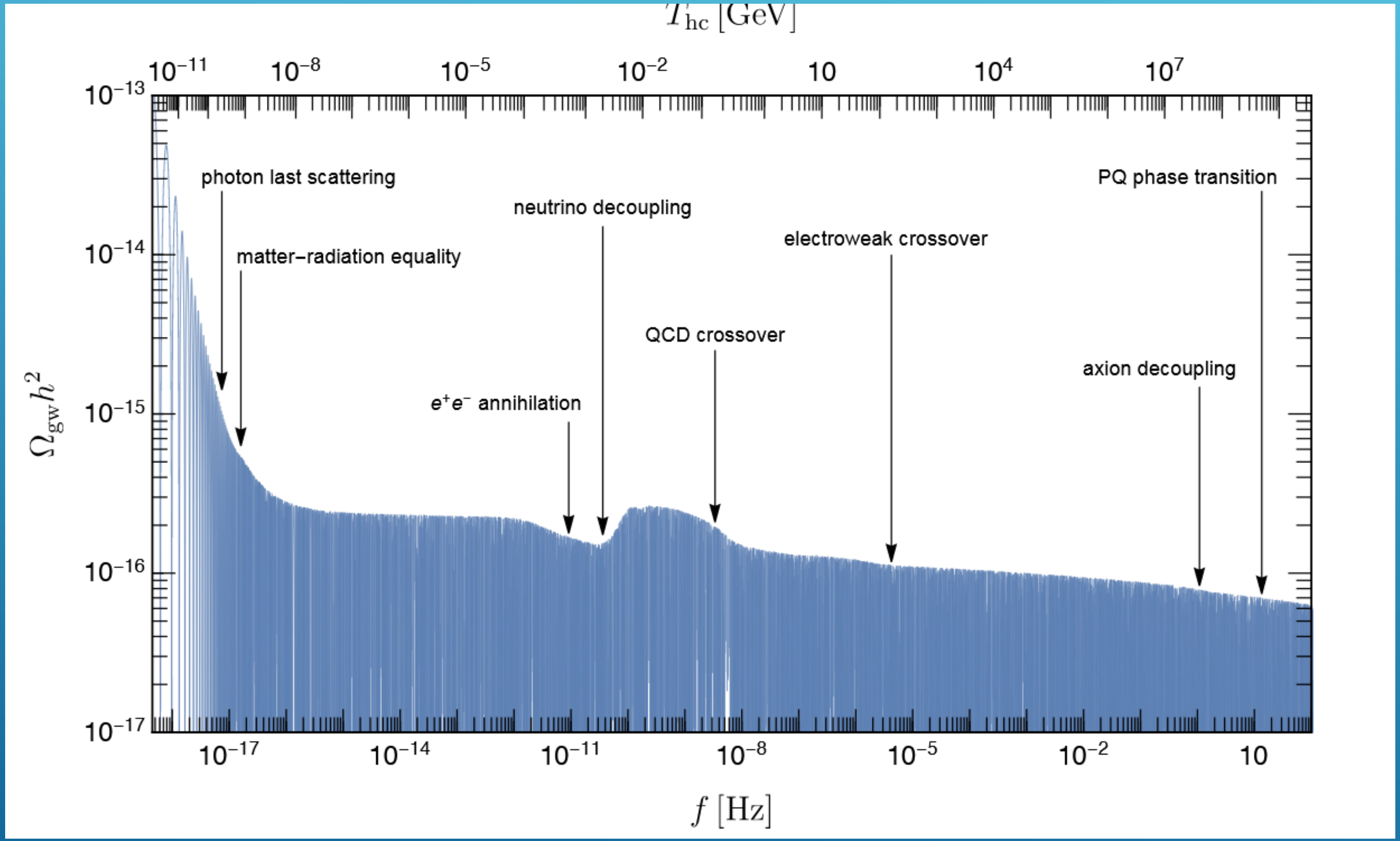
DIFFERENTIATING BETWEEN DIFFERENT MODELS



Ringwald et al. Gravitational Waves as a Big Bang Thermometer 2021 http://arxiv.org/abs/2011.04731

A CLOSER LOOK



Ringwald et al. Primordial gravitational waves in a minimal model of particle physics and cosmology(2021) http://arxiv.org/abs/2009.02050

CONCLUSIONS

- If there was a period of Inflation, one way to prove it would be the relic GW background
- The expected Spectrum is created through expanded QF
- The defining Characteristic is the Cutoff frequency, it gives us the Hubble radius at the end of Inflation, though other backgrounds overshadow is
- A precise measurement could show the impact of events in RD on expansion

Outlook

In case of Hybrid Inflation the pre and reheating process gives rise to classical source terms. Further differences as such can be found in the reheating spectra

THANK YOU FOR YOUR ATTENTION

GRAVITATIONAL WAVES FROM INFLATION

The Amplitude of the Background can be estimated through measurements of the CMB

- For $f \ll f_{eq}$:

$$h_0^2\Omega = 1.87 \cdot 10^{-16}\left(\frac{r}{0.1}\right)\left(\frac{A_R}{2.14 \cdot 10^{-9}}\right)\left(\frac{f_{eq}}{f}\right)^2\left(\frac{f}{f_R}\right)^{n_T}$$

- For $f_{eq} \ll f \ll f_{RH}$:

$$h_0^2\Omega = 1.44 \cdot 10^{-16}\left(\frac{r}{0.1}\right)\left(\frac{A_R}{2.14 \cdot 10^{-9}}\right)\left(\frac{f}{f_R}\right)^{n_T}$$

- For $f_{RH} < f$:

$$h_0^2\Omega = 1.44 \cdot 10^{-16}\left(\frac{r}{0.1}\right)\left(\frac{A_R}{2.14 \cdot 10^{-9}}\right)\left(\frac{f_{RH}}{f}\right)^2\left(\frac{f}{f_R}\right)^{n_T}$$

$$r = 0.036$$
$$A_R = 2.14 \cdot 10^{-9}$$
$$n_T = -\frac{r}{8}$$
$$f_R = 7.73 \cdot 10^{-17}\ \text{Hz}$$
$$f_{RH} = 0.027 \cdot \left(\frac{T_{RH}}{10^6 GeV}\right)\text{Hz}$$

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INFLATION

Slow roll conditions:

In inflation, the co-moving Hubble radius decreases

That means:

$$\frac{d}{dt}\left(\frac{1}{aH}\right) = -\left(\frac{\dot{a}H + a\dot{H}}{(aH)^2}\right) = -\frac{1}{a}\left(1 + \frac{\dot{H}}{H^2}\right) \rightarrow \epsilon = -\frac{\dot{H}}{H^2} < 1$$

(0 in de Sitter Phase)

With $H^2 = \frac{\rho}{3M_{pl}}$ and $\dot{\rho} = -3H(\rho + P)$:

$$\dot{H} + H^2 = -\frac{1}{6M_{pl}^2}(\rho + 3P) = -\frac{H^2}{2}(1 + \frac{3P}{\rho})$$

[1] Bauman Cosmologylecture notes 07.01.2025

INFLATION

$$\dot{H} + H^2 = -\frac{1}{6M_{pl}^2}(\rho + 3P) = -\frac{H^2}{2}(1 + \frac{3P}{\rho})$$

$$\frac{\dot{H}}{H^2} + 1 = -\frac{1}{2}(1 + \frac{3P}{\rho})$$

$\frac{\dot{H}}{H^2} = -\frac{3}{2}\left(1 + \frac{P}{\rho}\right) = -\epsilon$ →pressure has to be negative

[1] Bauman Cosmology lecture notes 07.01.2025

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INFLATION

$$\dot{H} + H^2 = -\frac{H^2}{2}(1 + \frac{3P}{\rho}) \text{ with } \dot{\rho} = -3H(\rho + P)$$

$$\rho\left(\frac{\dot{H}}{H^2} + 1\right) = -\frac{3}{2}(\rho + 3P)$$

$$\frac{\dot{H}}{H} = \frac{1}{2}\frac{\dot{\rho}}{\rho} = \frac{1}{2}\frac{d}{dt}\ln(\rho)$$

$$\text{With } H = \frac{\dot{a}}{a} = \frac{d}{dt}\ln(a)$$

$$\frac{\dot{H}}{H^2} = \frac{1}{2}\frac{\text{dln}(\rho)}{\text{dln}(a)} \rightarrow \left|\frac{1}{2}\frac{d\ \ln(\rho)}{\text{dln}(a)}\right| = \epsilon < 1$$ → density stays almost constant

[1] Bauman Cosmology lecture notes 07.01.2025

WAVE EQUATION

For linear perturbations $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

Becomes

$$\Box \bar{h}_{\mu\nu} + \eta_{\mu\nu}\, \partial_\rho \partial_\sigma\, \bar{h}_{\rho\sigma} - \partial_\rho \partial_\nu\, \bar{h}_{\mu\sigma} - \partial_\rho \partial_\mu\, \bar{h}_{\nu\rho} = -16\pi G\, T_{\mu\nu}$$

Which in Lorentz T T Gauge is

$$h_{\mu\nu} = 16\pi G T_{\mu\nu}$$

GRAVITATIONAL WAVES FROM INFLATION

- Given the fact that the source are expanded QF, the
- The Important question regarding a mode becomes when does it enter the horizon:
  - Before entering it's constant
  - After it is suppressed with $\frac{1}{a}$
- We can now differentiate between different Intervals:
  - $[k_0, k_{eq})$, $h_0^2\Omega \propto \frac{1}{f^2}$
  - (k_{eq}, k_{RHf}) , (k_{RHf}, k_{RH}),
  - $k > k_{PH}$ exponentially suppressed, cutoff rüberkopieren

GRAVITATIONAL WAVES FROM INFLATION

We can now differentiate between super and sub horizon modes

For sub horizon modes with $k^2 \gg (aH)^2$: $\tilde{h}_{\mu\nu} \propto \cos(\omega t)$→$h_{\mu\nu} \propto \frac{1}{a}\cos(\omega t)$

→exponentially suppressed

For super horizon modes with $k^2 \ll (aH)^2$: $ah''_{\mu\nu} + 2a'h'_{\mu\nu} = 0$

→$h = A + B\int_{\tau_i}^{\tau_f} \frac{d\tau}{a(\tau)^2}$ →has a constant component