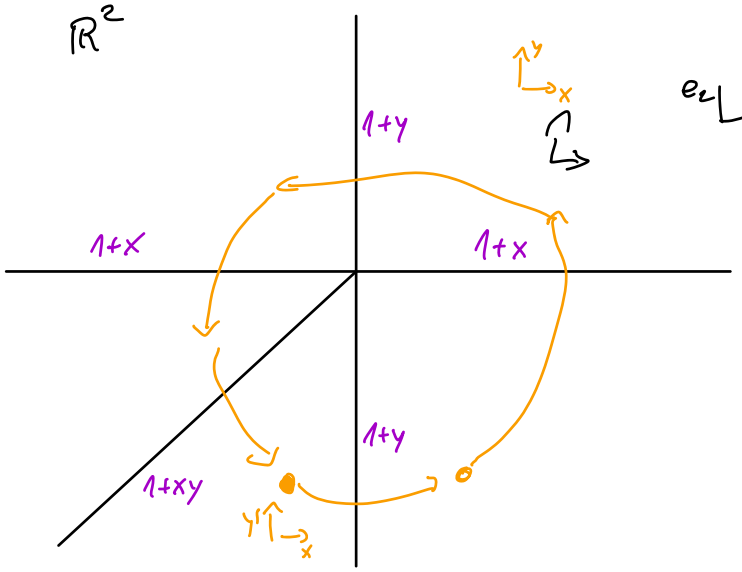
Recall the example of the A_2 cluster algebra

Variables x_m, $m \in \mathbb{Z}$

Relations $x_{m-1} \cdot x_{m+1} = 1 + x_m$

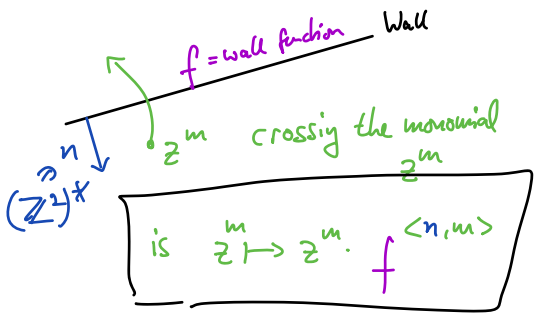
... computation yields $x_i = x_j$ if $i \equiv j \bmod 5$

Recall basic scattering diagram from last lecture

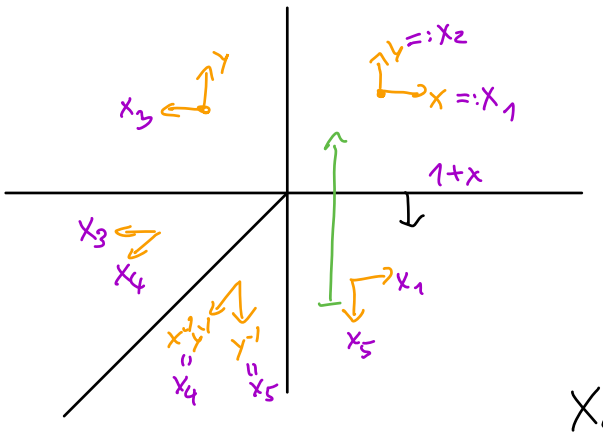


For $m \in \mathbb{Z}^2$, $m = (m_1, m_2)$
write $z^m := x^{m_1} \cdot y^{m_2}$

Wall crossing formula



Last time: composition of 5 crossings is identity



$$y^{-1} \cdot (1+x) = x_2^{-1}(1+x_1)$$

$$x_5 = z^{(0,-1)} = y^{-1}$$

$$x_5 = x_2^{-1}(1+x_1)$$

$$\boxed{x_2 x_6 = (1+x_1)}$$

GHKK

$\underbrace{Bl_5 \mathbb{P}^2_{\mathbb{C}}}_{X} \supset \underbrace{\Delta}_{D}$

$X \setminus D \simeq \mathrm{Spec}\ A_2\text{-cluster-algebra}$

$\bigcup$ 5 cluster tori $= X \setminus (D \cup 2 \text{ points})$

$Bl_3 \mathbb{P}^2$ 

Thm Collins, Jacob, Lin

$\mathbb{P}^2 \setminus$ smooth elliptic curve has special Lagr. torus fibration

$\mathbb{C}$-integral affine structure $\simeq$ CPS affine manifold

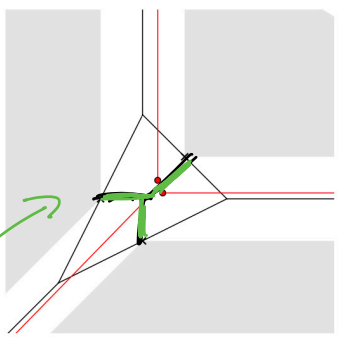
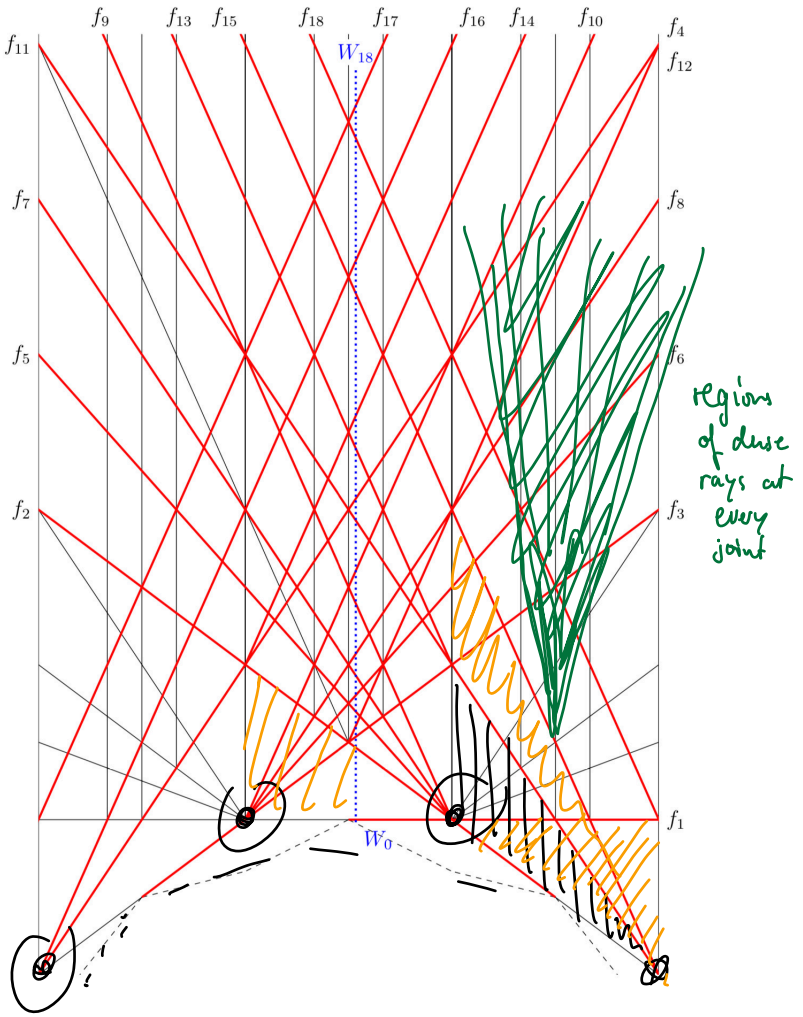


FIGURE 6.1. Broken lines in the central region

take the universal cover of the complement of Y-shaped graph to arrive at picture on the next page.

Thomas Prince: https://arxiv.org/pdf/1703.07620.pdf

Scattering diagram in the base of $\mathbb{P}^2 \setminus$ smooth ell. curve



Gräfnitz-Ruddat-Zaslow: https://arxiv.org/abs/2204.12249