Why we want to measure transversity at HERMES

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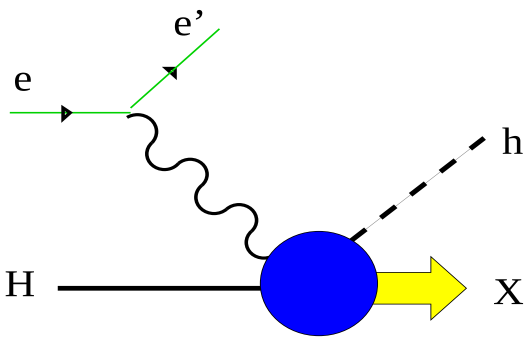
- Formulation of DIS in the light cone gauge
- Distribution Functions
- Definition and meaning of $Twist$
- What is the structure function $h_1(x, Q^2)$
- What are the predictions for $h_1(x, Q^2)$
- Fragmentation Functions
- How to measure $h_1(x, Q^2)$
- Observing h_1 in Electroproduction
- *... HERMES results (next talk!!!)*

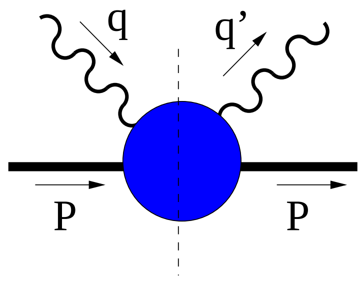
DESY Hamburg, 16 July 1999

Formulation of DIS

$$\sigma \propto l^{\mu\nu} W_{\mu\nu}$$

$l^{\mu\nu}$ = leptonic tensor
$W_{\mu\nu}$ = hadronic tensor





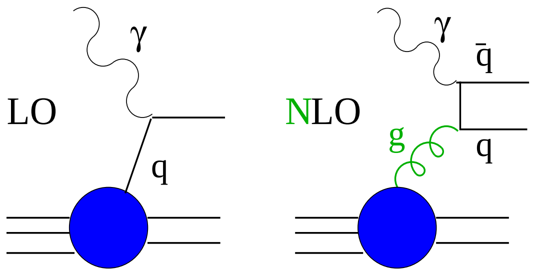
$$2\pi W^{\mu\nu} = Im T^{\mu\nu}$$

$T^{\mu\nu}$ = forward virtual Compton scattering amplitude

Light-cone coordinates

$P^\mu = (P_0, 0, 0, P_4)$

$q^\mu = (q_0, 0, 0, q_4)$ $\Longrightarrow$ $A^+ = 0$
(no link and no gluon diagrams at LO)



Dirac algebra

- redefinition of γ^5, γ^μ
- usual algebra satisfied: $\gamma^\mu, \gamma^\nu = 2g^{\mu\nu}$
  $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$
- two possible basis:
  - diagonalize $\gamma^5, \Sigma^3 \Rightarrow$ *chirality or helicity*
  - diagonalize $\gamma^1 \Rightarrow$ *transversity*
- light-cone projectors: $P_\pm = \frac{1}{2}\gamma^\pm\gamma^\mp$
  $\gamma^\pm = \frac{1}{\sqrt{2}}(\gamma^0 \pm \gamma^3)$

Dirac field: $\Phi \equiv P_+\Psi$ "good" component
$\chi \equiv P_-\Psi$ "bad" component

Helicity and Chirality

$$\Sigma_3 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}$$

measures the helicity of a particle with momentum in the $\hat{e}_3$-direction

"good" and "bad" components of Ψ carry helicity

$$\Psi = \begin{pmatrix} \Phi_+ \\ \Phi_- \\ \chi_+ \\ \chi_- \end{pmatrix}$$

$$\gamma_5 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}$$

measures the chirality

"good" components have same helicity and chirality,
"bad" components have chirality opposite to helicity

$$\gamma_5 \Psi = \begin{pmatrix} +\Phi_+ \\ -\Phi_- \\ -\chi_+ \\ +\chi_- \end{pmatrix}$$

A quantum of χ is a composite of a transfer gluon (h=1, c=0) and a quantum of Φ (with opposite h for conservation of angular momentum)

Distribution Functions

$$\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} < PS|\bar{\Psi}(0)\Gamma\Psi(\lambda x)|PS >$$

Γ = q-q density matrix (or q-q-g d.m.)
S = nucleon momentum vector
P = nucleon spin vector

$$\Gamma = \left\{ \begin{array}{cccc} \gamma_\mu & \Longrightarrow & f_1(x) + O(1/Q^2) & CHIRAL \\ \gamma_\mu\gamma_5 & \Longrightarrow & g_1(x) + g_T(x) + O(1/Q^2) & EVEN \\ 1 & \Longrightarrow & e(x) & CHIRAL \\ \sigma_{\mu\nu} i\gamma_5 & \Longrightarrow & h_1(x) + h_L(x) + O(1/Q^2) & ODD \end{array} \right.$$

In DIS chiral-odd distributions are suppressed (small quark mass effect)

Meaning of Twist

Using good and bad components of Dirac field:

$$f_1(x) \propto \int \frac{d\lambda}{2\pi} e^{i\lambda x} < PS|\Phi^\dagger(0)\Phi(\lambda n)|PS >$$

$$e(x) \propto \int \frac{d\lambda}{2\pi} e^{i\lambda x} < P|\Phi^\dagger(0)\lambda^0\chi(\lambda n) + \chi^\dagger(\lambda n)\lambda^0\Phi(\lambda n)|P >$$

$e(x)$ is a quark-gluon correction in the light cone gauge.

$$\begin{array}{ccc} Twist-2 & \Longleftrightarrow & \Phi^\dagger\Phi \\ Twist-3 & \Longleftrightarrow & \Phi^\dagger\chi \\ Twist-4 & \Longleftrightarrow & \chi^\dagger\chi \end{array}$$

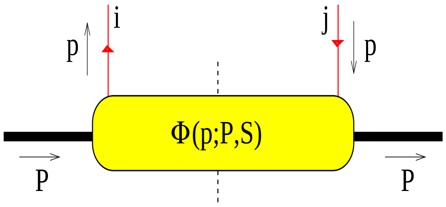
Twist ordering scheme

Twist = $t = J + \frac{1}{2}$ $\Longrightarrow$ twist order $(1/Q)^{t-2}$

$J = j_\gamma + j_q + nj_g = 1 + 1/2 + n1$

$\boxed{t = 2}$

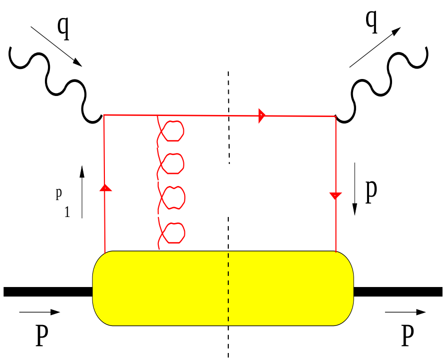
$$\Phi(x) \sim (Af_1 + Bg_1 + Ch_1) + O(\tfrac{M}{Q})$$



First order approx. = SF of order $(1/Q)^0$

$\boxed{t = 3}$

$$\Phi(x) \sim (Af_1 + Bg_1 + Ch_1) + (De + Eg_T + Fh_L) + O(\tfrac{M}{Q^2})$$



Second order approx. = SF of order $(1/Q)^1$

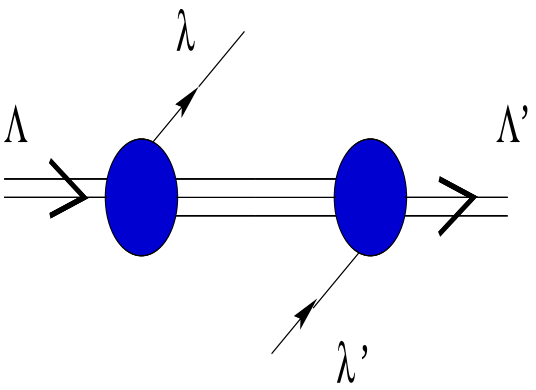
$\boxed{t = 4}$

$$\Phi(x) \sim (Af_1 + Bg_1 + Ch_1) + (De + Eg_T + Fh_L) + (Gf_3 + Hg_3 + Ih_3) + O(\tfrac{M}{Q^3})$$

Third order approx. = SF of order $(1/Q)^2$

Helicity Amplitudes

For each twist there are 3 different distribution structure functions... *why?*



$A_{\Lambda\lambda,\Lambda'\lambda'}$ = helicity amplitude

$$A_{\Lambda\lambda,\Lambda'\lambda'} = A_{-\Lambda-\lambda,-\Lambda'-\lambda'} \quad \text{PARITY}$$
$$A_{\Lambda\lambda,\Lambda'\lambda'} = A_{\Lambda'\lambda',\Lambda\lambda} \quad \text{TIME REV.}$$

Parity, time reversal and angular momentum conservation leave 3 independent amplitudes:

$A_{++,++} + A_{+-,+-}$ spin average
$A_{++,++} - A_{+-,+-}$ helicity asymmetry
$A_{+-,-+}$ helicity flip

Integrated over transverse momenta

DISTRIBUTIONS (only T-even)			
$\Phi^{[\Gamma]}(x)$		chirality even	chirality odd
twist 2	U	f_1	
	L	g_1	
	T		h_1
twist 3	U		e
	L		h_L
	T	g_T	

FRAGMENTATION (T-even & T-odd)			
$\Delta^{[\Gamma]}(z)$		chirality even	chirality odd
twist 2	U	D_1	
	L	G_1	
	T		H_1
twist 3	U		E H
	L		H_L E_L
	T	G_T D_T	

Nucleon Spin Structure at Twist-2

$$f_1(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} < P|\bar{\Psi}(0)\ \not{n}\Psi(\lambda x)|P >$$

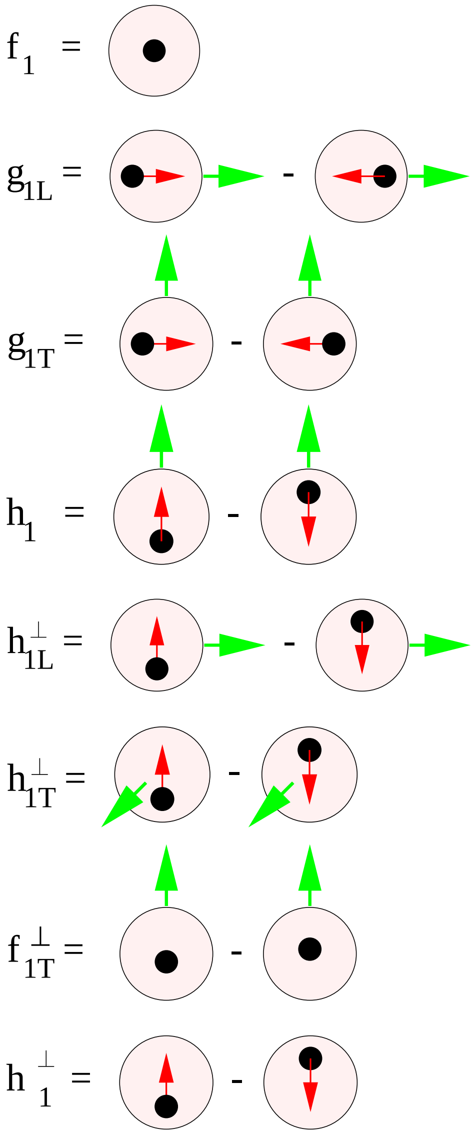
$$g_1(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} < PS_{\parallel}|\bar{\Psi}(0)\ \not{n}\gamma_5\Psi(\lambda x)|PS_{\parallel} >$$

$$h_1(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} < PS_{\perp}|\bar{\Psi}(0)[\not{S}_{\perp}, \not{n}]\gamma_5\Psi(\lambda x)|PS_{\perp} >$$

give a full description of the nucleon's quark spin substructure at leading order.

$h_1(x, Q^2)$ measures TRANSVERSITY.
Chiral odd, spin dependent, not directly related to quark or nucleon spin.

All these functions can be interpreted as momentum densities:



Longitudinally polarized quarks in a transversely polarized target

Transversely polarized quarks in a longitudinally polarized target

Unpolarized quarks in a transversely polarized target (T-odd)

Transversely polarized quarks in an unpolarized target (T-odd)

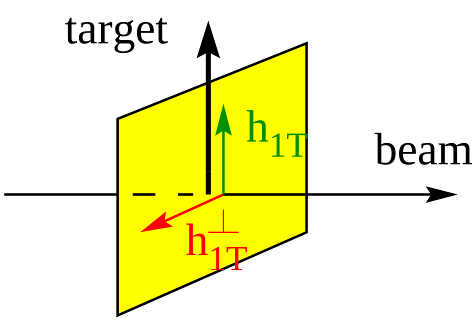
What is exactly $h_1(x, Q^2)$

- In the *chirality or helicity basis*:

  after integrating over k_T

$$h_1 = h_{1T} + \frac{k_T^2}{2M^2} h_{1T}^{\perp}$$

  it mixes R and L handed quarks.



- In the *transversity basis*:

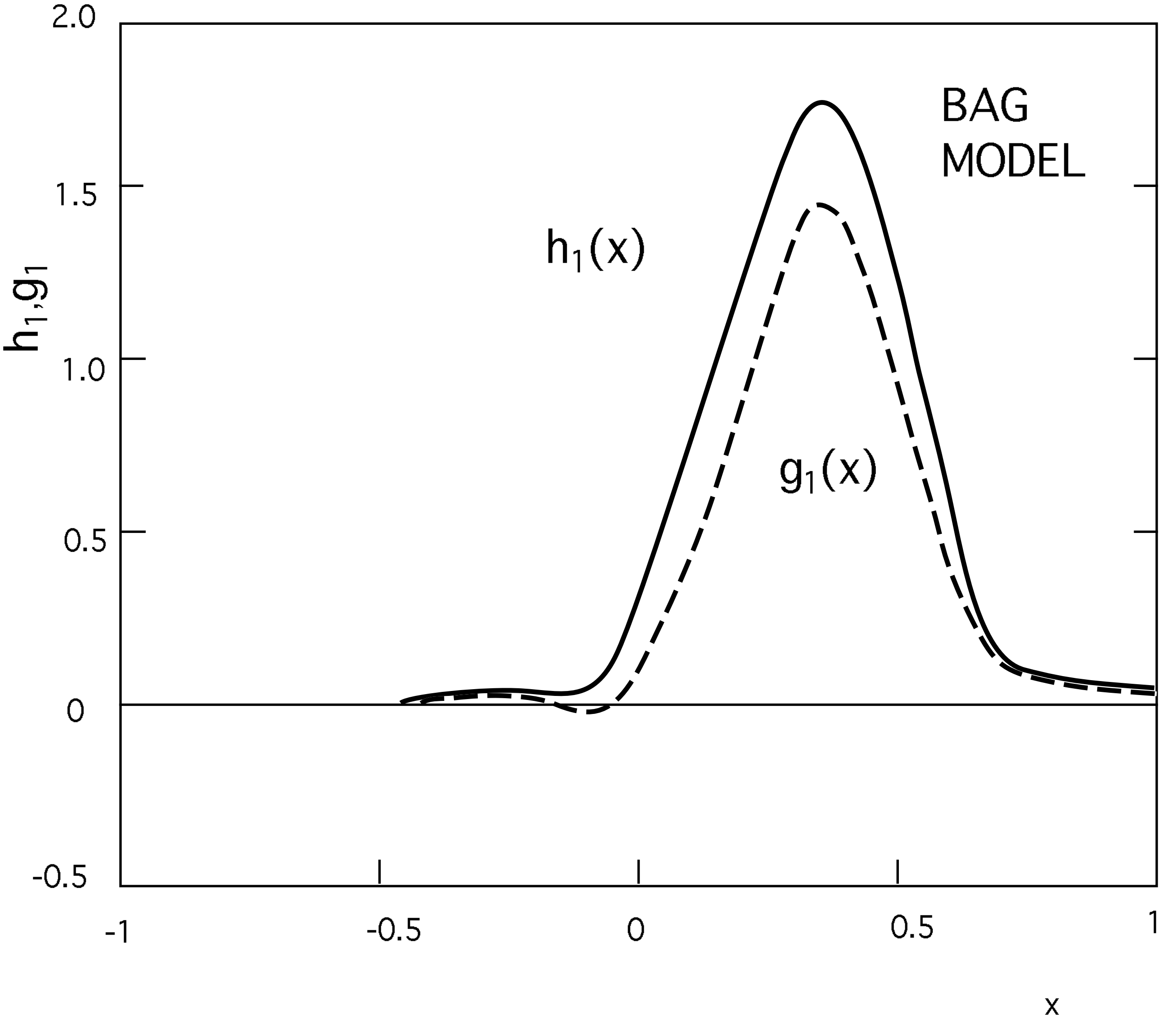
  h_1 = probability to find q with spin polarized along the transverse spin of a polarized nucleon minus the probability to find it polarized oppositely.

Predictions for $h_1(x, Q^2)$

Inequities: $|g_1(x, Q^2)| \le f_1(x, Q^2)$

$|h_1(x, Q^2)| \le f_1(x, Q^2)$

Models:



Why to measure h_1

between the three twist-2 distribution functions, h_1 is the most pure measurement of the spin structure of the nucleon:

- *No gluon operator contribute* to h_1 in any order (chiral odd function)

- The first moment of the distribution gets *no contribution from the sea quarks*:

$$\delta q^a(Q^2) = \int_0^1 dx(h_1^a(x,Q^2) - h_1^{\bar{a}}(x,Q^2))$$

$$\left(\Delta q^a(Q^2) = \int_0^1 dx(g_1^a(x,Q^2) + g_1^{\bar{a}}(x,Q^2))\right)$$

One more reason to measure h_1

SLAC is measuring g_2:

$$g_2 = g_2^{WW} - \int_x^1 \frac{\partial}{\partial y}\left(\frac{m_q}{M} h_1(y,Q^2) + e(y,Q^2)\right)\frac{d}{dy}$$

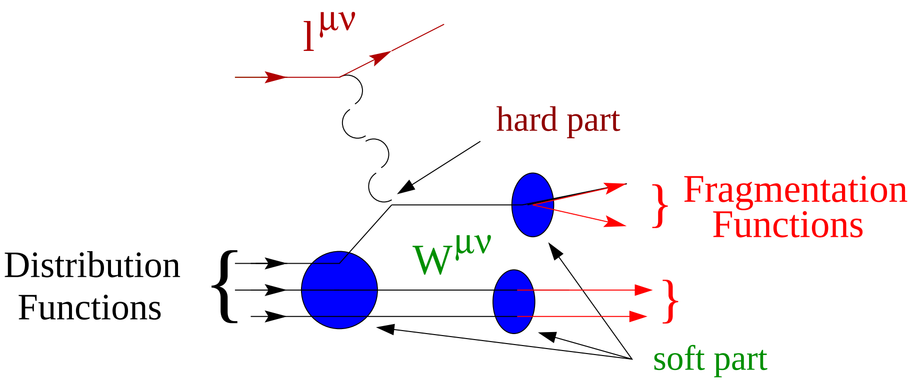
$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2)$$

"Neglecting h_1" contribution,

$$\bar{g}_2(x,Q^2) \equiv g_2(x,Q^2) - g_2^{WW}(x,Q^2)$$

gives the first measurement of the Twist-3 SF $e(y,Q^2)$

Fragmentation Functions



$\sigma_{ab\to cd} \sim l^{\mu\nu} W_{\mu\nu}$ convolution of:

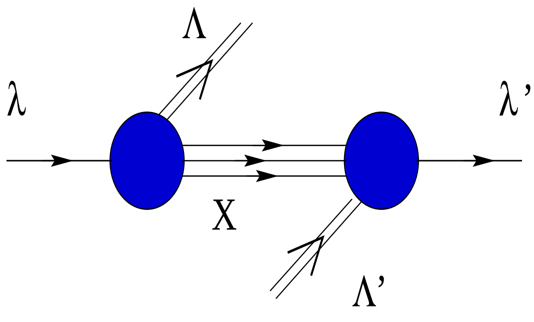
- "hard part" = scattering among elementary constituents
- "soft part" = q produced by the initial h or final h produced from q resulting from the scattering

$\Downarrow$

Fragmentation Function

$D_1^a(z_h) \Rightarrow$ density of h with momentum fraction z_h resulting from the fragmentation of q^a

for polarized processes $\Rightarrow$ classification of FF in a helicity basis:



$A_{\Lambda\lambda,\Lambda'\lambda'} = A_{-\Lambda-\lambda,-\Lambda'-\lambda'}$

NO time rev. invariance

$\Downarrow$

higer number of indipendent functions (T-odd)

How to measure h_1

- Both chiral odd soft functions are distribution functions:
  Drell-Yan process

$$p_T + p_T \to l + \bar{l} + X$$
$$h_1^a(x_1) \otimes h_1^{\bar{a}}(x_2)$$

- One is a distribution function, the other is a fragmentation function:
  semi-inclusive DIS (as Λ_T or π production)

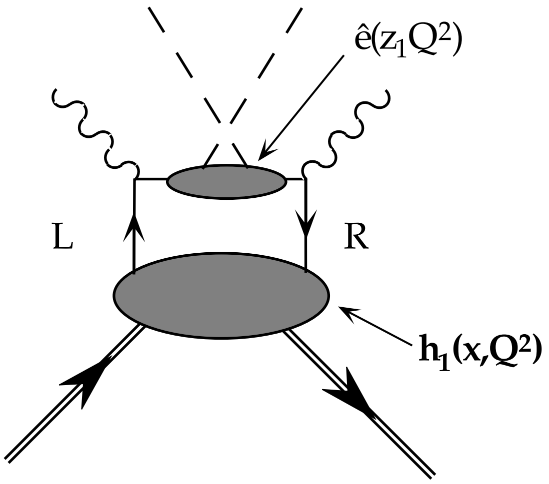
$$e + p_T \to e'\Lambda_T + X$$
$$h_{1,N}(x) \otimes H_{1,\Lambda}(z)$$

!! chirally-odd SF are suppressed in totally inclusive DIS!!

Observing h_1 in Electroproduction

measuring a chirally-odd FF enables a measurement of h_1

$$e_L + p_T \rightarrow e + \pi + X$$



For chirality conservation at the hard vertex:
LO $\Longrightarrow$ no contribution to the cross section
NLO $\Longrightarrow$ two contributions suppressed by $1/Q$

$$h_1(x) \otimes E(z) \oplus g_T(x) \otimes D_1(z)$$

Extraction of h_1

$h_1(x) \otimes E(z)$ is the product of two unknowns.

The chiral quark model (X. Ji and Z.-K. Zhu) predicts:

$$E(z) = zD(z)\frac{m_q}{M} \sim \frac{1}{3} zD(z)$$