Musterlösung Aufgabe 2

ME Compton-Streuung: (Schmüser S.69)

$M = -ie^2(2\pi)^4\,\delta(p'+k'-p-k)\;\cdot$ (Impulse wie Abb. 5.7., Schmüser)

$$\bar{u}(p')\left[\not\varepsilon'\,\frac{1}{\not k+\not p-m}\,\not\varepsilon + \not\varepsilon\,\frac{1}{\not p-\not k'-m}\,\not\varepsilon'\right]u(p)$$

ersetze $\not\varepsilon \to \not k$ und füge ein
$$\left.\begin{aligned}(\not p-m)\,u(p) &= 0\\ \text{und}\quad \bar{u}(p')(-\not p'+m) &= 0\end{aligned}\right\}\ \text{Dirac-Gl}$$

$$\Rightarrow M = \dots\cdot\bar{u}(p')\left[\not\varepsilon'\,\frac{\not k+(\not p-m)}{\not k+\not p-m} + \frac{\not k+(-\not p'+m)}{\not p-\not k'-m}\,\not\varepsilon'\right]u(p)$$

es gilt $k-p' = k'-p$ (wg. $p+k = p'+k'$, E-p-Erhaltung)

$$\Rightarrow M = \dots\;\bar{u}(p')\left[\not\varepsilon'\cdot 1 - 1\,\not\varepsilon'\right]u(p) = 0.$$

NB zur Dirac-Gleichung: aus $\boxed{(\not p-m)\,u = 0}$ folgt:

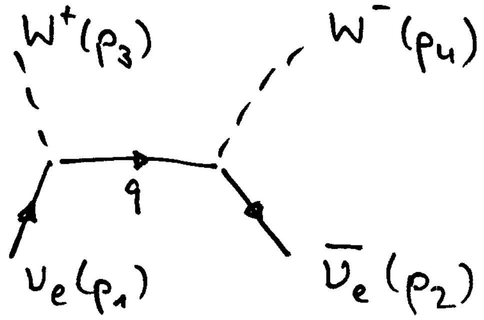
$$((\not p-m)u)^+ = u^+(\not p^+-m) = u^+\gamma^0(\gamma^0\not p^+-\gamma^0 m) =$$

$$\bar{u}(\not p\gamma^0 - m\gamma^0) = \bar{u}(\not p-m)\gamma^0 = 0 \Rightarrow \boxed{\bar{u}(\not p-m) = \bar{u}(-\not p+m) = 0}$$

Dabei haben wir benutzt, daß:

$\bar{u} := u^+\gamma^0$, $\gamma^{0+} = \gamma^0$, $\gamma^{i+} = -\gamma^i$ $(i=1\dots3)$, $\gamma^0\gamma^0 = 1$, $\gamma^0\gamma^i = -\gamma^i\gamma^0$

Musterlösung Aufgabe 3



$\nu_e + \bar{\nu}_e \to W^+ W^-$

a) Aufstellen des Matrixelements:

Regel: Fermion-linien entgegen dem Pfeil aufschreiben

$$M = \Big[\underbrace{(\text{Einl.})}_{E}\underbrace{(\text{Vtx})}_{V}\underbrace{(\text{Ausl.})}_{A}\Big]\underbrace{\text{Propagator}}_{P}\Big[\underbrace{(\text{Ausl.})}_{A}\underbrace{(\text{Vtx})}_{V}\underbrace{(\text{Einl.})}_{E}\Big]$$

Einl. $\bar{\nu}_e$: $\bar{v}(p_2)$

Einl. ν_e : $u(p_1)$

Ausl. W^+ : $\varepsilon^*_{\mu +}(p_3)$

Ausl. W^- : $\varepsilon^*_{\mu -}(p_4)$

Vtx : $-i\frac{e}{\sqrt{2}\sin\theta_W}\gamma^\mu\frac{1-\gamma^5}{2}$ f. gelad. schw. WW

Propagator : $i\frac{\not{q}+m_e}{q^2-m_e^2}$

$\frac{e}{\sin\theta_W} = g \qquad \frac{g^2}{8} = G_F M_W^2/\sqrt{2}$

also:

$$M = \underbrace{\bar{v}(p_2)}_{E}\cdot\underbrace{\frac{(-i)g}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)}_{V}\cdot\underbrace{\varepsilon^*_{-\mu}}_{A}\cdot\underbrace{\frac{i(\not{q}+m_e)}{q^2-m_e^2}}_{P}\cdot\underbrace{\varepsilon^*_{+\nu}}_{A}\underbrace{\frac{(-i)g}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)}_{V}\underbrace{u(p_1)}_{E}$$

$$\Rightarrow M = -i\frac{g^2}{8}\,\bar{v}(p_2)\,\not{\varepsilon}^*_-(1-\gamma^5)\frac{\not{q}}{q^2}\not{\varepsilon}^*_+(1-\gamma^5)\,u(p) \qquad (m_e \approx 0)$$

long. polarisierte W's: $\varepsilon_+^* \approx \frac{\cancel{p}_3}{M_W}$ $\quad \varepsilon_-^* \approx \frac{\cancel{p}_4}{M_W}$

benutze $\frac{g^2}{8} = G_F M_W^2 / \sqrt{2}$

und ersetze $\cancel{p}_3$ durch $\cancel{p}_3 - \cancel{p}_1 = -\cancel{q}$ (wg. $\cancel{p}_1 u(p_1) = 0$ Dirac)

und $\cancel{p}_4$ durch $\cancel{p}_4 - \cancel{p}_2 = \cancel{q}$ (wg. $\bar{v}(p_2)\cancel{p}_2 = 0$ ")

$$\Rightarrow M = i \frac{G_F}{\sqrt{2}} \bar{v}(p_2) \cancel{q} (1-\gamma^5) \frac{\cancel{q}\cancel{q}}{q^2} (1-\gamma^5) u(p_1)$$

es gilt $\cancel{q}\cancel{q} = q^2$ und $(1-\gamma^5)^2 = 2(1-\gamma^5)$

$$\Rightarrow \boxed{M = i G_F \sqrt{2}\, \bar{v}(p_2) \cancel{q} (1-\gamma^5) u(p_1)}$$

b) <u>Spinmittelung:</u>

$$|M|^2 = 2G_F \cdot \sum_{i=1}^{2} \sum_{j=1}^{2} |\bar{v}_i \cancel{q} (1-\gamma^5) u_j|$$

$$= 2G_F \cdot \sum_i \sum_j \bar{v}_i \cancel{q} (1-\gamma^5) u_j u_j^+ \underbrace{\gamma^0 \gamma^0}_{1 \text{ eingefügt}} (1-\gamma^5)^+ \cancel{q}^+ \underbrace{\gamma^0 v_i}_{=(v^+\gamma^0)^+}$$

$$= 2G_F \cdot \sum_i \sum_j \bar{v}_i \cancel{q} (1-\gamma^5) u_j \bar{u}_j (1+\gamma^5) \cancel{q} \underbrace{\gamma^{0^2}}_{=1} v_i$$

$$= 2G_F \sum_i \sum_j \bar{v}_i \cancel{q} (1-\gamma^5) \underbrace{u_j \bar{u}_j}_{= \cancel{p}_1} \cancel{q} (1-\gamma^5) v_i$$

$= \cancel{p}_1$ (wg $\sum u_j \bar{u}_j = \cancel{p} + m$)

also $|M|^2 = 2 G_F \sum_i \underset{\substack{\uparrow \\ \text{Spinor}^+}}{\bar{v}_i} \underbrace{\not{q}(1-\gamma^5)\not{p}_1 \not{q}(1-\gamma^5)}_{\text{Matrix}} \underset{\substack{\uparrow \\ \text{Spinor}}}{v_i}$

Schreibe in Komponenten:

$$|M|^2 = 2G_F \sum_i (\bar{v}_i)_\alpha \left[\not{q}(1-\gamma^5)\not{p}_1 \not{q}(1-\gamma^5)\right]_{\alpha\beta} (v_i)_\beta$$

in Komponenten kann man vertauschen:

$$|M|^2 = 2 G_F \underbrace{\sum_i (\bar{v}_i)_\alpha (v_i)_\beta}_{(\not{p}_2)_{\beta\alpha}} [\ldots]_{\alpha\beta}$$

$$= 2G_F \left\{ \not{p}_2 [\;\;] \right\}_{\beta\beta} = 2G_F \, Sp\left(\not{p}_2 \not{q}(1-\gamma^5)\not{p}_1 \not{q}(1-\gamma^5)\right)$$

$$= 2G_F 2 Sp\left(\not{p}_2 \not{q}(1-\gamma^5)\not{p}_1 \not{q}\right)$$ ($1-\gamma^5$ an zwei $/$'s vorbeiziehen, dann $(1-\gamma^5)^2 = 2(1-\gamma^5)$)

$$= 4G_F \, Sp\left(\not{p}_2 \not{q} \not{p}_1 \not{q} - \gamma^5 \not{p}_2 \not{q} \not{p}_1 \not{q}\right)$$

Es ist $Sp(\gamma^5 \not{p}_2 \not{q} \not{p}_1 \not{q}) = 0$ und $Sp(\not{a}\not{b}\not{c}\not{d}) = 4(ab)(cd) + 4(ad)(bc) - 4(ac)(bd)$

$$\Rightarrow \boxed{\overline{|M|^2} = 16 G_F \left(2(p_1 q)(p_2 q) - (p_1 p_2) q^2\right)}$$