Advanced Course on Higgs Physics

1st Graduate Week of the Quantum Universe Research School

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Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

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Outline of the lectures

- **Part 1: Why a Higgs boson is needed [Monday]**
- **Part 2: Connections between Higgs Physics and unanswered questions of Particle Physics (and possible solutions to them) [Monday]**
- **Part 3: What can be learnt from the Higgs boson at high-energy colliders – an overview [Yesterday]**
- **Part 4: The Higgs boson mass as a precision observable – calculations and interpretations [Partly yesterday/cont'd today]**
- **Part 5: The Higgs boson potential, its trilinear coupling, and relations with early-Universe evolution**

Part 4:
The Higgs boson mass,
a new precision observable

Measurements of the Higgs boson mass

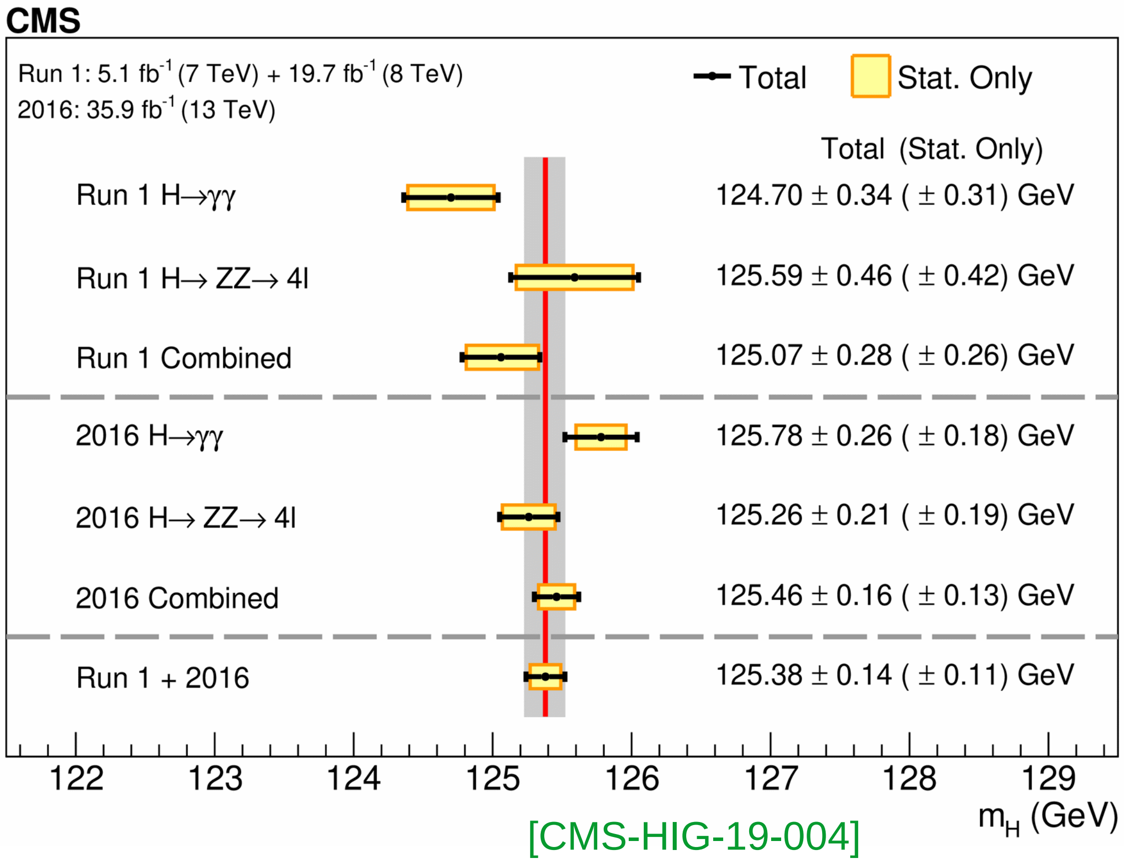
- **Higgs mass** already measured at sub-permille level! → new **precision observable**!

$$M_h = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\ \text{GeV}$$

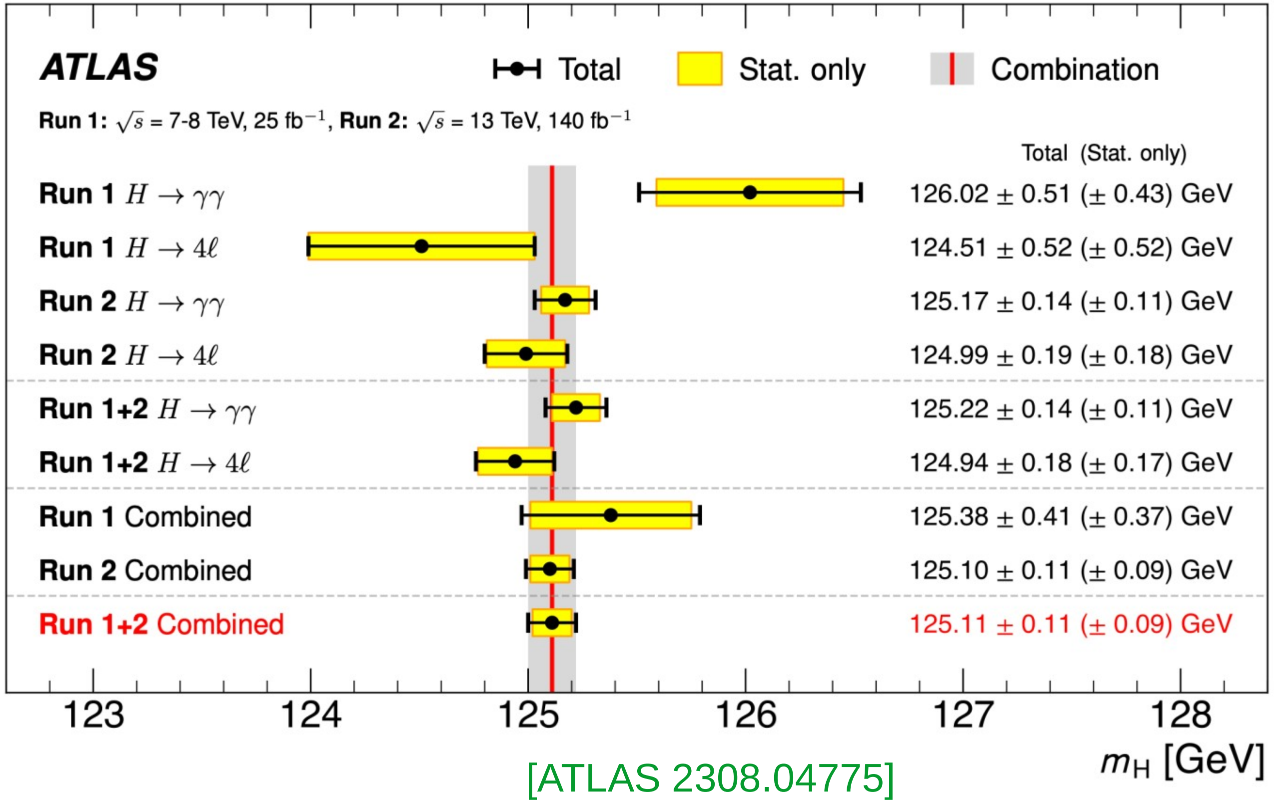
[ATLAS & CMS Run 1 combined, Moriond 2015]

$$M_h = 125.11 \pm 0.11(\text{stat.}) \pm 0.09(\text{syst.})\ \text{GeV}$$

[ATLAS 2308.04775 from Run1+Run2 in h→γγ and h→4l channels]



[CMS-HIG-19-004]



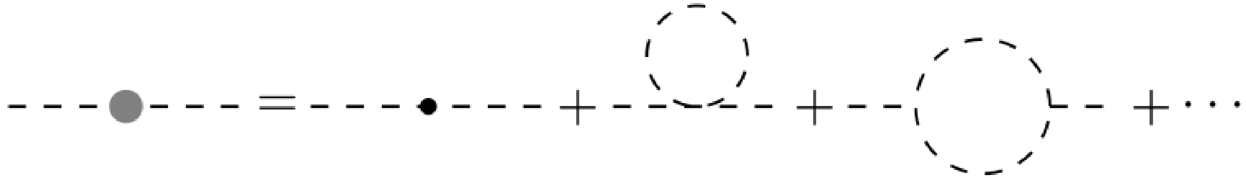
[ATLAS 2308.04775]

Radiative corrections to the Higgs boson mass

> *NB*: *other possible approach → **EFT** (more later)*

- **Feynman-diagrammatic** calculations, $i.e.$ solve for M_h^2

$$\underbrace{M_h^2}_{\text{pole mass}} = \underbrace{(m_h^2)^{\text{tree}}}_{\text{tree-level mass}} + \underbrace{\hat{\Sigma}_{hh}(p^2 = M_h^2)}_{\text{self-energy}}$$

$\leftrightarrow$ 

$\hat{\Sigma}_{hh}(p^2)$ computed order-by-order in perturbation theory as Feynman diagrams.
Difficulty: momentum dependence of self-energy diagrams <u>not always known</u> at two loops and higher + long numerical calculations

- **effective potential** approximation $V_{\text{eff}} = V^{(0)} + \Delta V$
  where ΔV are quantum corrections, computed as
  - **one loop**: supertrace formula
  - **two loops and beyond**: 1PI vacuum bubble diagrams

$$\left.\frac{\partial^2 V_{\text{eff}}}{\partial h^2}\right|_{\text{min}} \leftrightarrow \hat{\Sigma}_{hh}(0)$$

$\Rightarrow$ much simpler/faster calculations, but with lower accuracy

- ▷ **tadpole equation(s)** $\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{\text{min}} = 0$ are needed to properly relate all couplings

Two interpretations of Higgs mass calculations

- Higgs mass M_h is computed as a function of Lagrangian parameters, in particular quartic Higgs coupling λ

$$M_h = M_h(\lambda, \cdots)$$

- **Case 1: λ is a free parameter of the theory**
  e.g. in SM and many extensions (SSM, 2HDM, etc.)
  → one cannot predict M_h
  → but one can use the equation $M_h(\lambda, \cdots) = 125.09\ \mathrm{GeV}$ to **extract λ** and study the **high-scale behaviour of the theory**

- **Case 2: λ is predicted by the theory**
  e.g: - in SUSY, λ is related to other couplings (EW gauge couplings + eventually SUSY scalar couplings)
  - in (classical) scale invariant models, λ=0 at the scale at which the symmetry is imposed
  - but also the case in a non-SUSY extension of the SM taken as low-energy limit of a UV-model in which λ is predicted (*more on this later*)
  → **M_h can be predicted** as a function of the model parameters

$$(m_h^2)_{\text{tree}} \leq M_Z^2 c_{2\beta}^2, \qquad M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \ldots$$

  → Comparing computed and measured values of M_h → **constrain allowed BSM parameter space**

Case 2:
SUSY Higgs mass calculations

SUSY Higgs mass calculations – fixed-order calculation

- SUSY models contain extended scalar sectors → **physical masses** found as solutions for p^2 of equation

$$\det\left[p^2\delta_{ij} - (\mathcal{M}^2_{\rm tree})_{ij} - \Delta\mathcal{M}^2_{ij}(p^2)\right] = 0$$

- At tree level, $m_h \leq M_Z$, however, since early 1990's ([Okada, Yamaguchi, Yanagida '90], [Ellis, Ridolfi, Zwirner '90], [Haber, Hempfling '90]) it has been known that loop corrections can raise m_h to 125 GeV

$$M_h^2 \simeq (m_h^2)_{\rm tree} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M^2_{\rm SUSY}}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

tanβ: ratio of Higgs VEVs
X_t: stop mixing parameter
M_{SUSY}: SUSY-breaking scale
$\widehat{X}_t \equiv X_t/M_{\rm SUSY}$

- Since then, huge efforts to improve precision of SUSY Higgs mass calculations
  → summarised in recent report of "Precision SUSY Higgs Mass Calculation Initiative KUTS"
  [Slavich, Heinemeyer (eds.) et al 2012.15629]
  → for the MSSM, state-of-the-art is now almost full 2L in effective-potential approximation, + leading 2L momentum-dependent effect + leading 3L corrections
  → for the NMSSM and beyond (e.g. Dirac gaugino models), leading 2L corrections

  + reliable estimates of theoretical uncertainties (from missing higher orders & parametric uncertainties) → 1-3 GeV depending on point

- However, experimental searches now put lower bounds on stop (scalar partner of top quarks) masses beyond 1 TeV → fixed-order calculations start to suffer from **large logs**

SUSY Higgs mass calculations – the problem of large logs

- Scale of New Physics M_{NP} driven higher by experimental searches

$\Rightarrow$ in fixed-order calculations, **large logarithmic terms** $\propto \log \frac{M_{\text{NP}}}{m_{\text{EW}}}$ can spoil the accuracy, or even the validity, of the perturbative expansion, $e.g.$

$$\mathcal{O} = \underbrace{\alpha^0 a_0}_{\text{tree-level}} + \underbrace{\alpha(b_1 L + a_1)}_{\text{one-loop}} + \underbrace{\alpha^2(c_2 L^2 + b_2 L + a_2)}_{\text{two-loop}} + \cdots$$

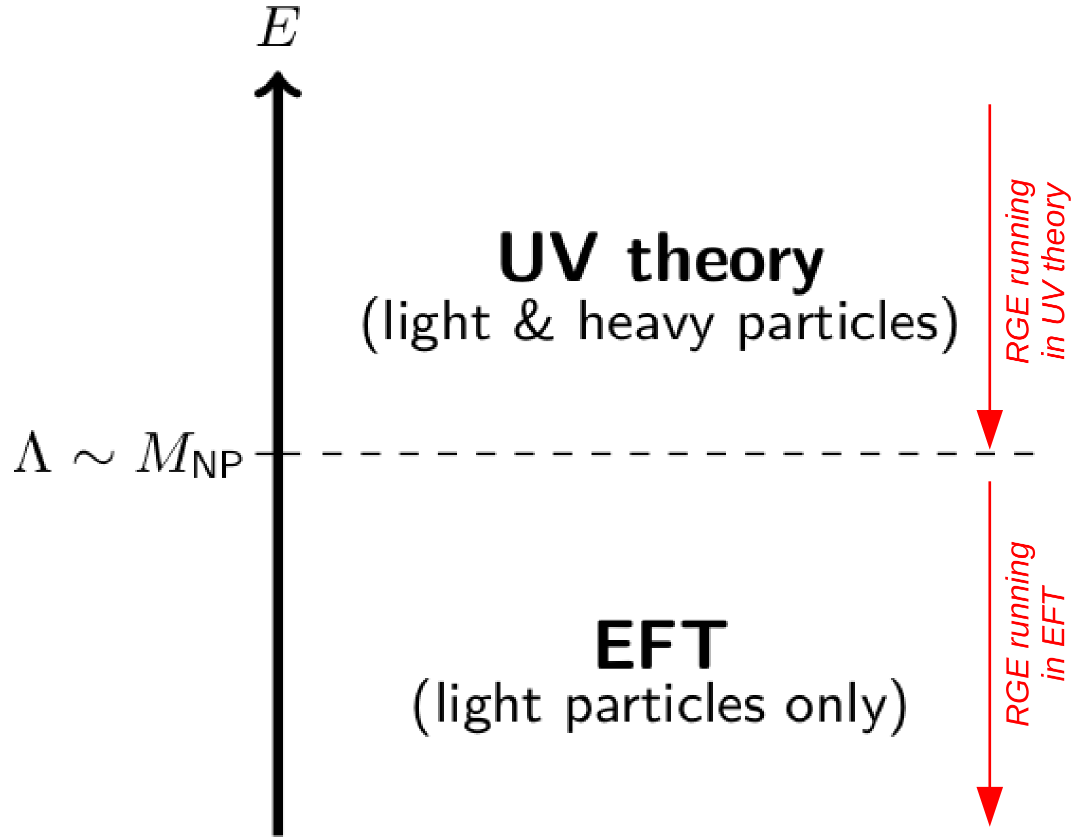
$\alpha \equiv (g/4\pi)^2$, $L \equiv \log M_{\text{NP}}/m_{\text{EW}}$, $a_i, b_i, c_i \in \mathbb{C}$.
Loss of perturbativity if

$$\alpha L \gtrsim 1 \ \cdots$$

The perturbative expansion must be reorganised $\rightarrow$ **EFT calculation**

Intermezzo: an EFT primer

- **Integrate out heavy fields** at some scale $\Lambda \sim M_{\text{NP}}$ and work in a low energy EFT below Λ
- Couplings in the EFT receive **threshold corrections** at the matching scale Λ



Coupling of the UV theory $\tilde{g}$

$\Downarrow$

Match effective actions computed in UV th. and EFT, at ℓ loops:

$$\tilde{\Gamma}^{\text{UV}}(Q=\Lambda) = \Gamma^{\text{EFT}}(Q=\Lambda)$$

$\Downarrow$

ℓ-loop threshold correction to coupling of the EFT g

$$g = \tilde{g} + \Delta g$$

- Use **RGEs** to run the couplings from the high input scale, to the low scale ($\ll M_{\text{NP}}$) at which the calculation is performed $\Rightarrow$ **large logs are resummed!**

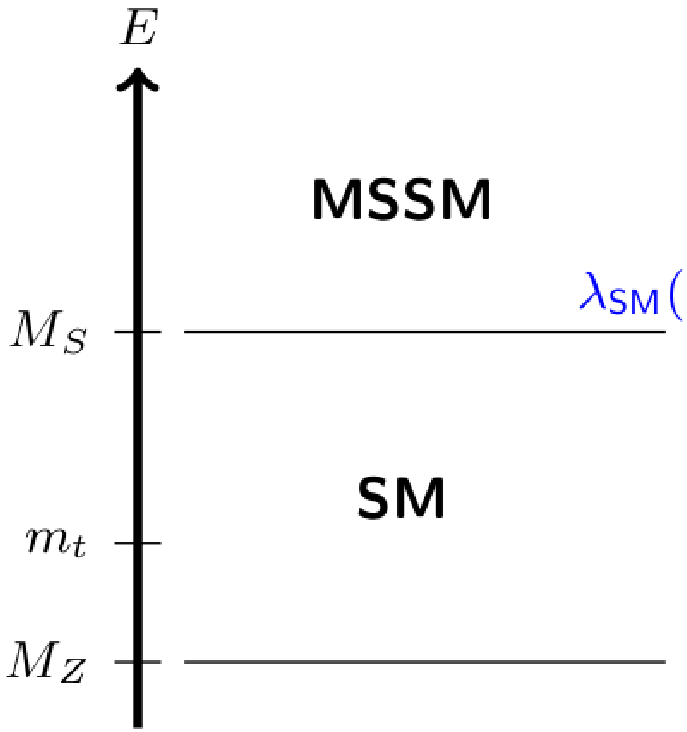
SUSY Higgs mass calculations – EFT approach

- *Simplest example:* **UV theory** $\to$ **MSSM**, and **EFT** $\to$ **SM**

see $e.g$ [Bernal, Djouadi, Slavich '07], [Draper, Lee, Wagner '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15], [Bagnaschi, Pardo Vega, Slavich '17], [Athron et al. '17], [Harlander, Klappert, Ochoa Franco, Voigt '18], etc.

More choices of EFTs also considered, see [KUTS report '20] *and refs. therein*

$e.g.$



MSSM inputs

$\Downarrow$

$$\lambda_{\mathsf{SM}}(M_S) = \frac{1}{4}(g^2(M_S) + g'^2(M_S))\cos^2 2\beta + \Delta\lambda^{1\ell}(M_S) + \Delta\lambda^{2\ell}(M_S) + \Delta\lambda^{3\ell}(M_S)$$

$\Downarrow$

$$M_h^2 = f(\lambda_{\mathsf{SM}}(m_t))$$ fixed-order calculation in the SM

$\Uparrow$

SM inputs

[Full 2ℓ RGEs in MSSM]

[Full 2ℓ + leading 3ℓ and 4ℓ RGE for λ_{SM} in SM]

- In many cases $M_S \gg v \Rightarrow$ effect of higher-dimensional operators $\propto v/M_S$ can be disregarded

NB: M_S = M_{SUSY}

SUSY Higgs mass calculations – hybrid approaches

- However, for lower M_{SUSY}, EFT calculations lose accuracy (because of v/M_{SUSY} effects)
- Can one **combine the advantages** of fixed-order (reliable for low M_{SUSY}) and EFT (reliable for high M_{SUSY})?
  → Yes!
- Different approaches

  1) FeynHiggs approach [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein PRL ‘13]

$$(M_h^2)_{\text{FH hyb.}} = (m_h^2)_{\text{tree}} + \underbrace{\hat{\Sigma}_{hh}^{\text{FO}}(M_h^2)}_{\text{fixed-order}} + \underbrace{\left[\lambda(M_t)v^2\right]_{\text{logs.}}}_{\text{EFT log. resum.}} - \underbrace{\left[\hat{\Sigma}_{hh}^{\text{FO}}(M_h^2)\right]_{\text{logs}}}_{\text{subtraction term}}$$

  2) FlexibleSUSY approach → “*pole mass matching*” [Athron et al ‘17]

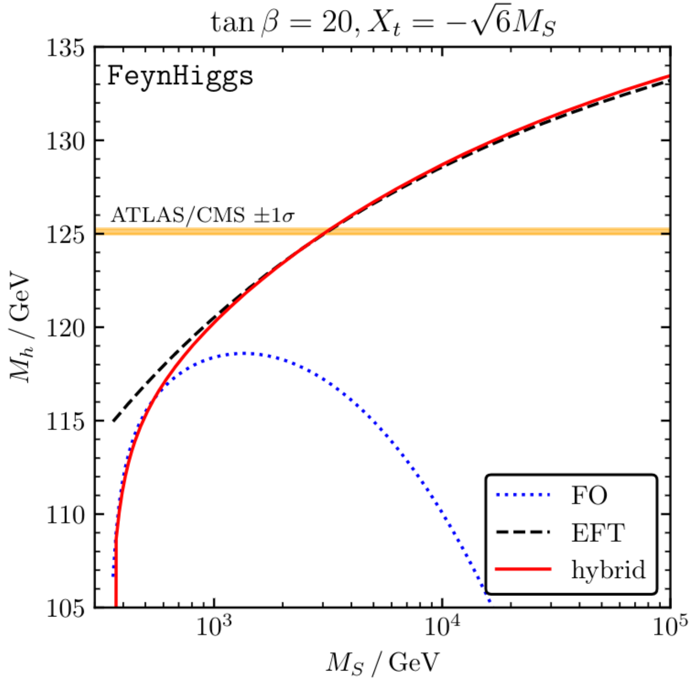
$$\lambda_{\text{SM}}(M_S) = \frac{1}{2\,v^2(M_S)}\left[(M_h^2)_{\text{HET}} - (\Delta M_h^2)_{\text{SM}}\right]$$

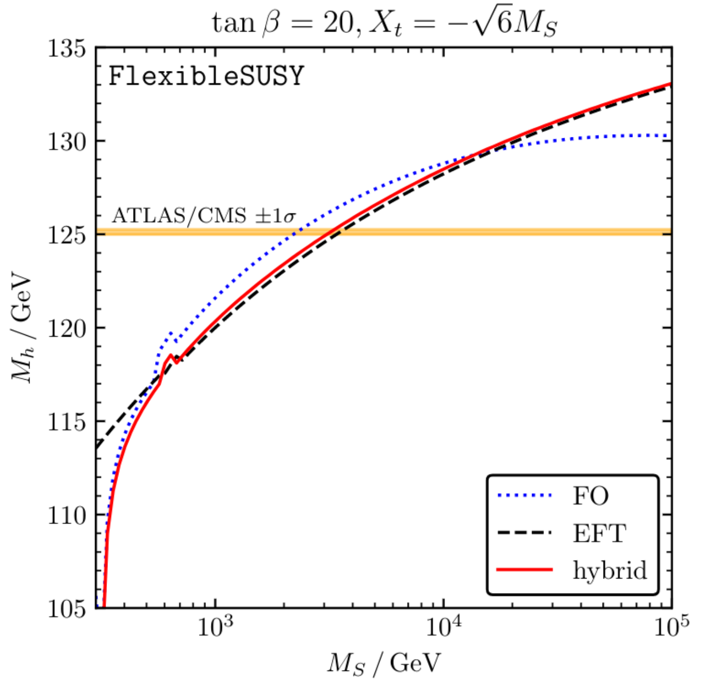
Fixed-order res. in full theory

(also included in SARAH/SPheno)

  3) Aachen group solution
  [Harlander, Klappert, Voigt ‘19]

$$(M_h^2)_{\text{hyb}} = (M_h^2)_{\text{EFT}} + \Delta_v^{0\ell+1\ell} + \Delta_v^{2\ell}$$





Different types of SUSY Higgs mass calculations – summary

$$M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M^2_{\text{SUSY}}}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

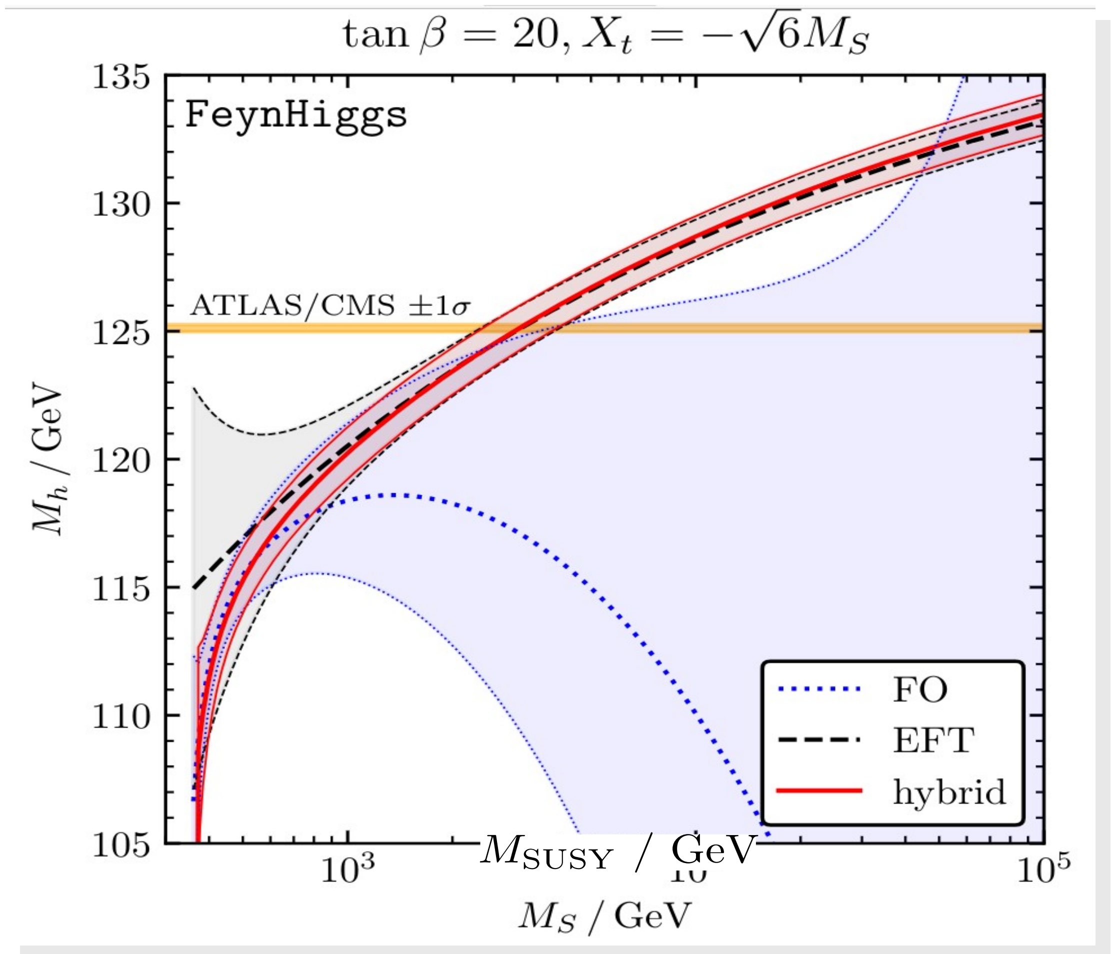
3 types of calculations for M_h:

- **Fixed-order approach**:
  + precise for low SUSY scales
  – but for high scales large logarithms $\log(M_{SUSY}/m_t)$ spoil convergence of perturbative expansion

- **Effective field theory approach**:
  + precise for high SUSY scales (since logarithms are resumed)
  – but for low scales $O(v/M_{SUSY})$ terms are missed if higher-dimensional operators are not included

- **Hybrid approach combing FO and EFT approaches**:
  ++ precise for both low and high SUSY scales.

- Current status in FeynHiggs (*c.f. figure*)
  → FO: full 1L + 2L in gaugeless limit,
  → EFT: full leading-log (LL) + Next-to-LL (NLL) + NNLL + partial N^3LL in gaugeless limit



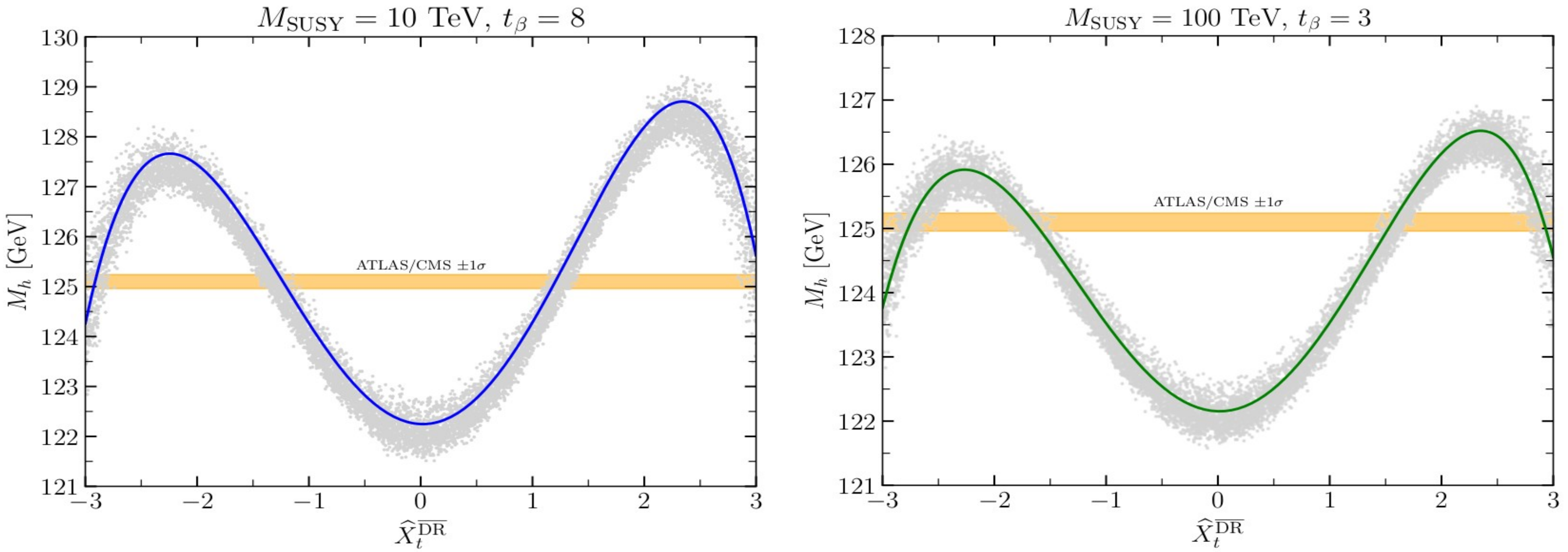
[KUTS report, Slavich, Heinemeyer et al. '20]

Accessing the stop mixing parameter X_t via the Higgs boson mass

- X_t enters prediction of M_h from 1L:

$$M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

- **Blue/green lines**: common mass scenarios, i.e. all non-SM masses $= M_{\text{SUSY}}$ and $A_{f\neq t} = 0$
- **Grey points**: scan over SUSY parameters (masses and trilinears) between $M_{\text{SUSY}}/2$ and $2\,M_{\text{SUSY}}$



[Bahl, JB, Weiglein ‘22] *with* `FeynHiggs 2.18.1`

- Significant dependence of M_h on X_t, even for high SUSY scale, at 10 or 100 TeV!
- If stop masses and tanβ known → **X_t can be extracted from M_h**

Automating Higgs mass computations

The motivation for automation

- Interest for non-minimal SUSY and non-SUSY models is growing, driven by experimental results, but in most cases Higgs mass calculations beyond one-loop are still missing
  $\longrightarrow$ huge uncertainties

- Computing corrections from the beginning for every new model would be extremely inefficient and time consuming!

- Idea:
  Do the **calculation for a general renormalisable theory** and then apply that result to the considered model
  $\longrightarrow$ can be automated, in public tools like `SARAH` [Staub '08-'15] or `FlexibleSUSY` [Athron et al. '14, '17]

Generic calculations of the Higgs boson mass – conventions

Write the most general renormalisable interactions with real scalars ϕ_i, Weyl fermions ψ_I and vector bosons A^a_μ:

$$\begin{aligned}
\mathcal{L}_S &= -\frac{1}{6}\lambda^{ijk}\phi_i\phi_j\phi_k - \frac{1}{24}\lambda^{ijkl}\phi_i\phi_j\phi_k\phi_l,\\
\mathcal{L}_{SF} &= -\frac{1}{2}y^{IJk}\psi_I\psi_J\phi_k + c.c.,\\
\mathcal{L}_{SV} &= -\frac{1}{2}g^{abi}A^a_\mu A^{\mu b}\phi_i - \frac{1}{4}g^{abij}A^a_\mu A^{\mu b}\phi_i\phi_j - g^{aij}A^a_\mu\phi_i\partial^\mu\phi_j\,,\\
\mathcal{L}_{FV} &= g^{aJ}_I\bar{\psi}^I\bar{\sigma}^\mu\psi_J A^a_\mu\,,\\
\mathcal{L}_{\text{gauge}} &= g^{abc}A^a_\mu A^b_\nu\partial^\mu A^{\nu c} - \frac{1}{4}g^{abe}g^{cde}A^{\mu a}A^{\nu b}A^c_\mu A^d_\nu + g^{abc}A^a_\mu\omega^b\partial^\mu\bar{\omega}^c
\end{aligned}$$

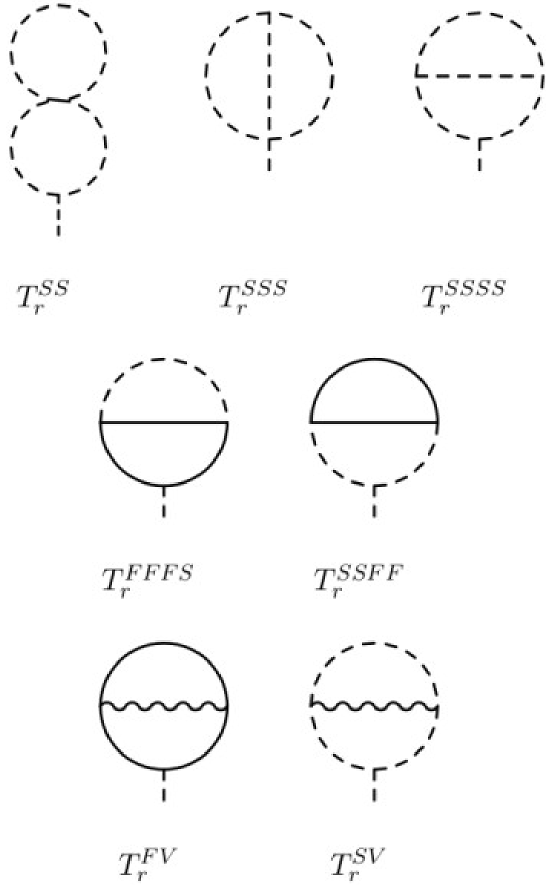
- Here, all fields are defined in mass-diagonal bases
  (some care needed to diagonalise scalar masses)

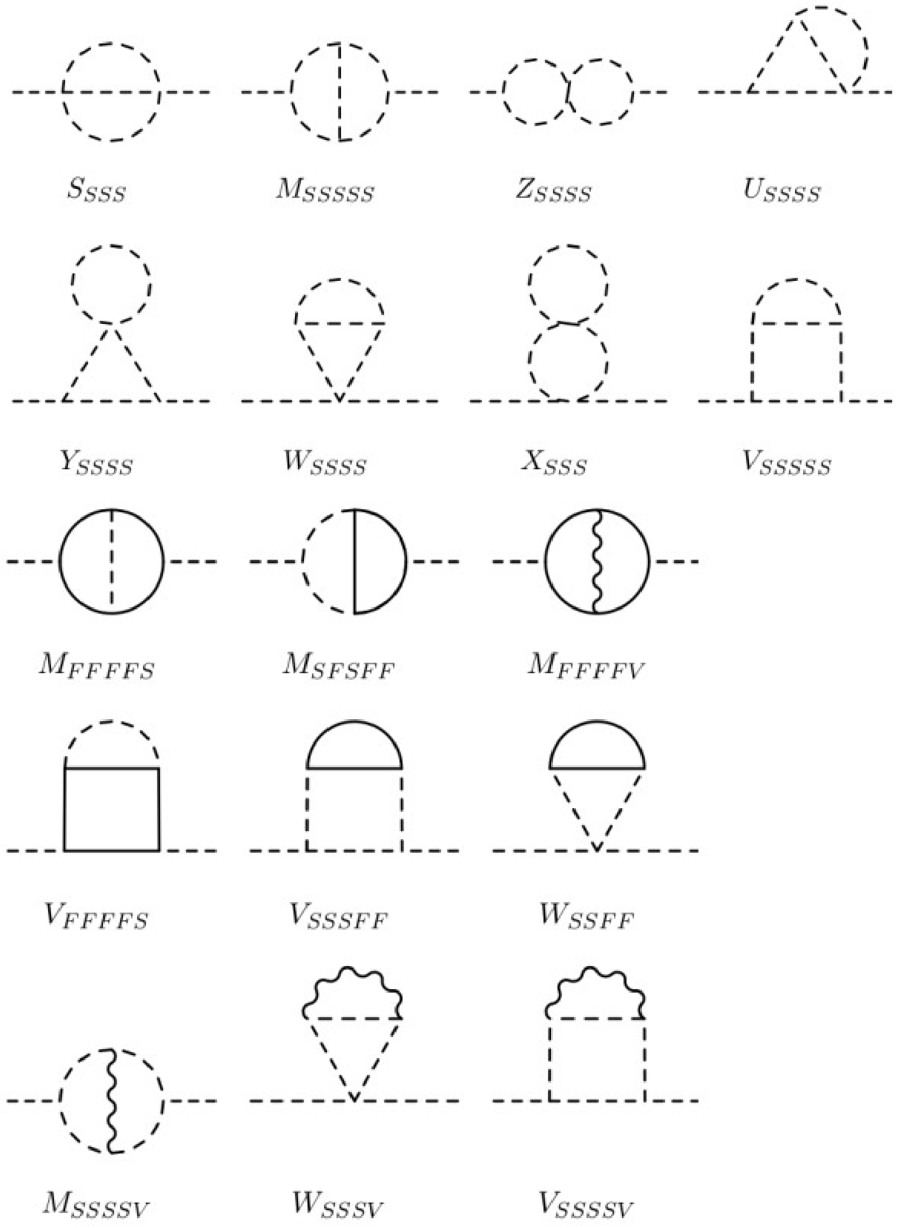
- Interactions between scalars and ghosts turned off by working in Landau gauge

- Parameters usually* renormalised in minimal subtraction schemes ($\overline{\text{MS}}$ or $\overline{\text{DR}}$)
  (: with one notable exception → anyH3 in Part 5)*

Generic calculations of the Higgs boson mass – diagrams

Then we need to compute loop diagrams for V_{eff}, the tadpole equations and the mass diagrams, *e.g.*





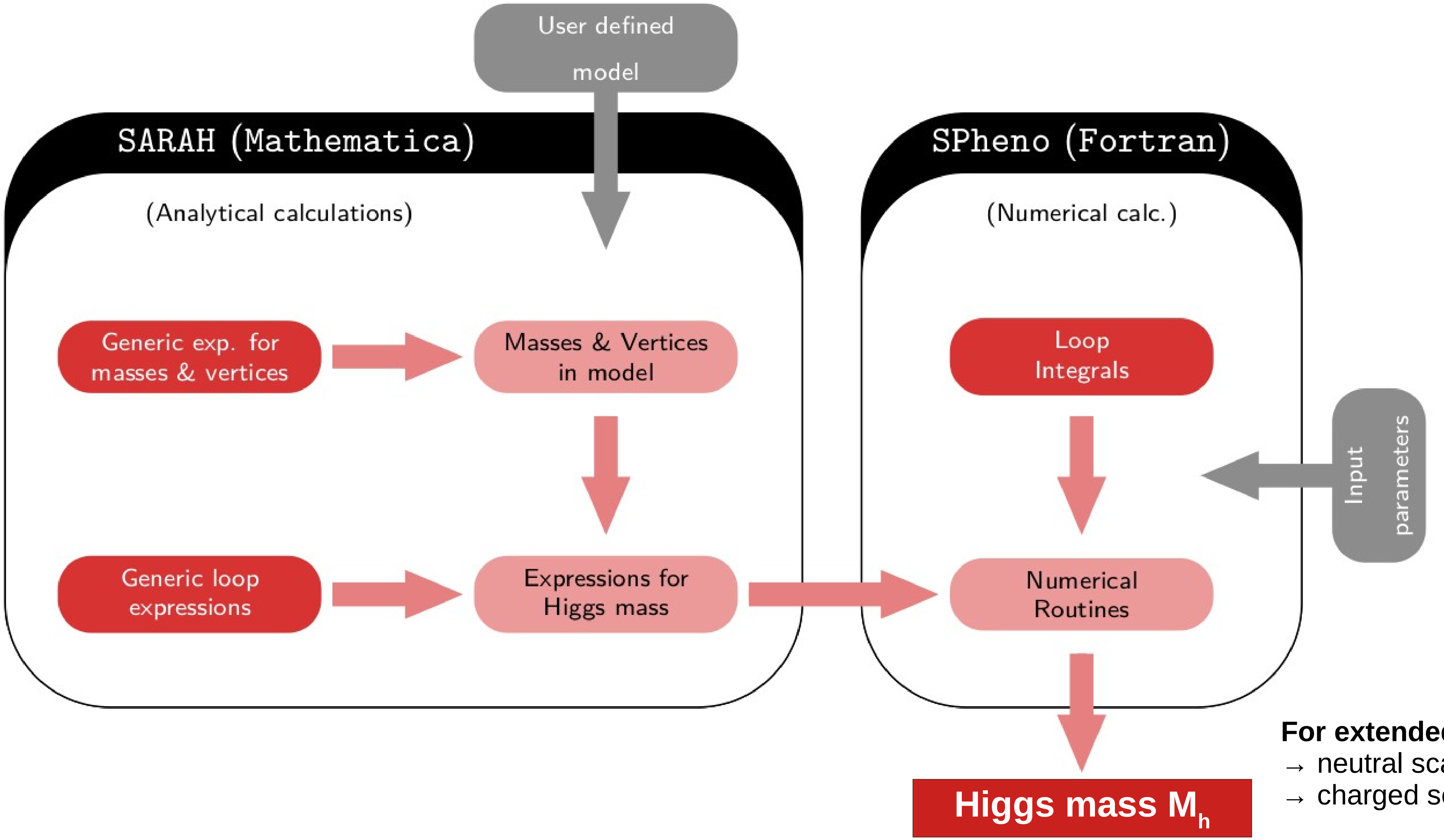
Generic 2L results available for:

- **V_{eff}**:
[Martin '01] (*Landau gauge*),
[Martin, Patel '18] (*general gauge*)
(3L V_{eff} in [Martin '17])

- **Tadpoles**:
[Goodsell, Killian, Staub '15]

- **Self-energies**:
[Martin '03, '05],
[Goodsell, Paßehr '19]

Generic calculations of the Higgs boson mass with SARAH/SPheno



Adapted from a figure by Florian Staub

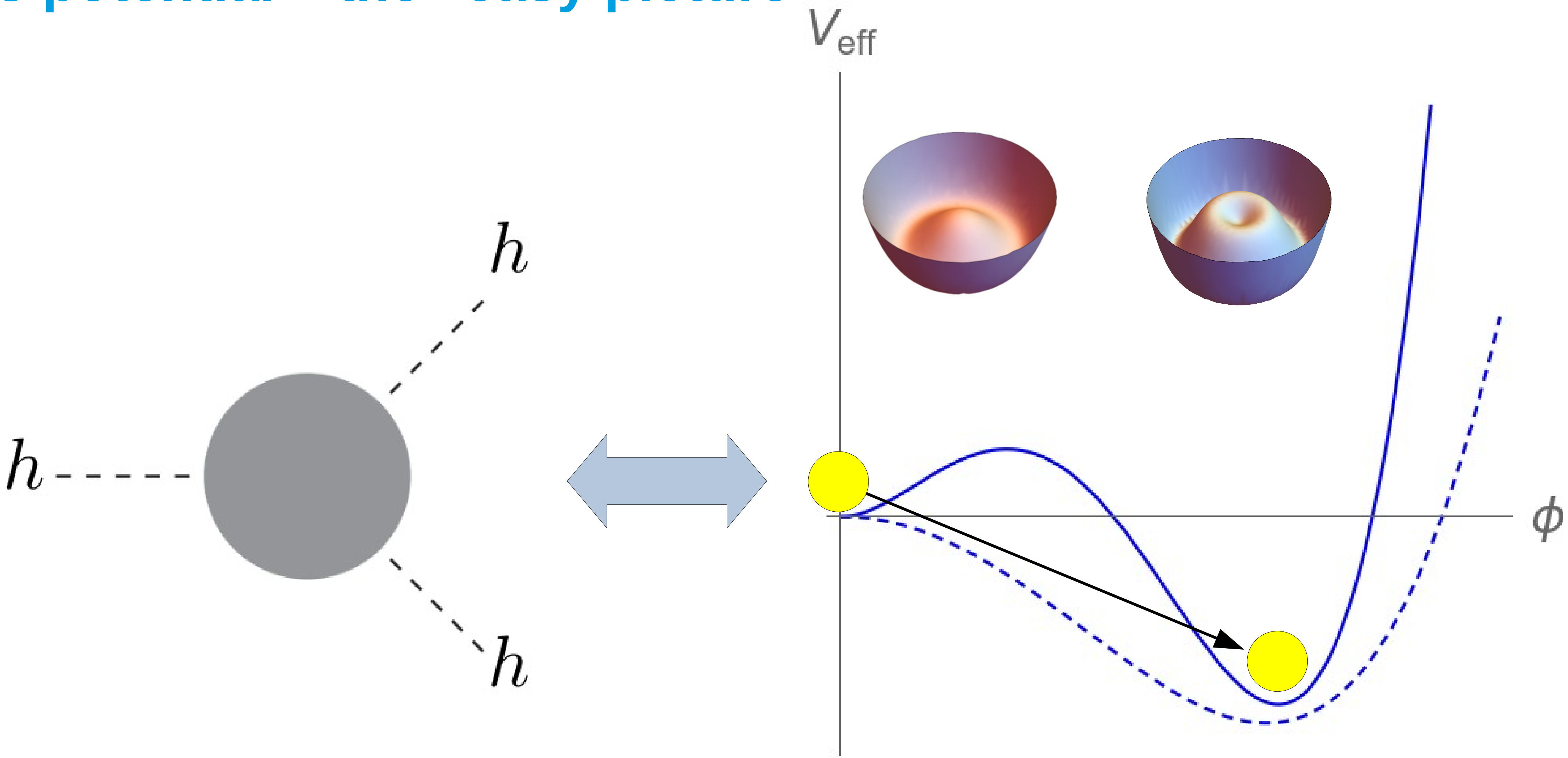
For extended scalar sectors:
→ neutral scalar masses @ **2L**;
→ charged scalar masses @ **1L**

Many other observables also available! (decays, STU, etc.)

Part 5:
Higgs potential,
trilinear Higgs coupling,
and early-Universe evolution

Higgs potential, trilinear Higgs coupling(s), and Electroweak Phase Transition

Higgs potential – the “easy picture”



→ a strong first-order phase transition (SFOEWPT), motivated in particular by EWBG, usually* correlates with a deviation in λ_{hhh} from its prediction in the SM

[*: if the EWPT occurs along the direction of the EW VEV in field space]

Kateryna's question: why is there such a correlation between λ_{hhh} (T=0) and the EWPT (T ~ 100 GeV)?

[Kanemura, Okada, Senaha '05]
(see also [Grojean, Servant, Wells '04])

- **Dynamics of EWPT controlled by finite-temperature effective potential**

$$V_{\rm eff}(\varphi,T)=V^{(0)}(\varphi)+\Delta V^{T=0}(\varphi)+\Delta V^{T}(\varphi,T)\xrightarrow{T\gg m}D(T^2-T_0^2)\varphi^2+ET\varphi^3+\frac{1}{4}\lambda(T)\varphi^4+\cdots$$

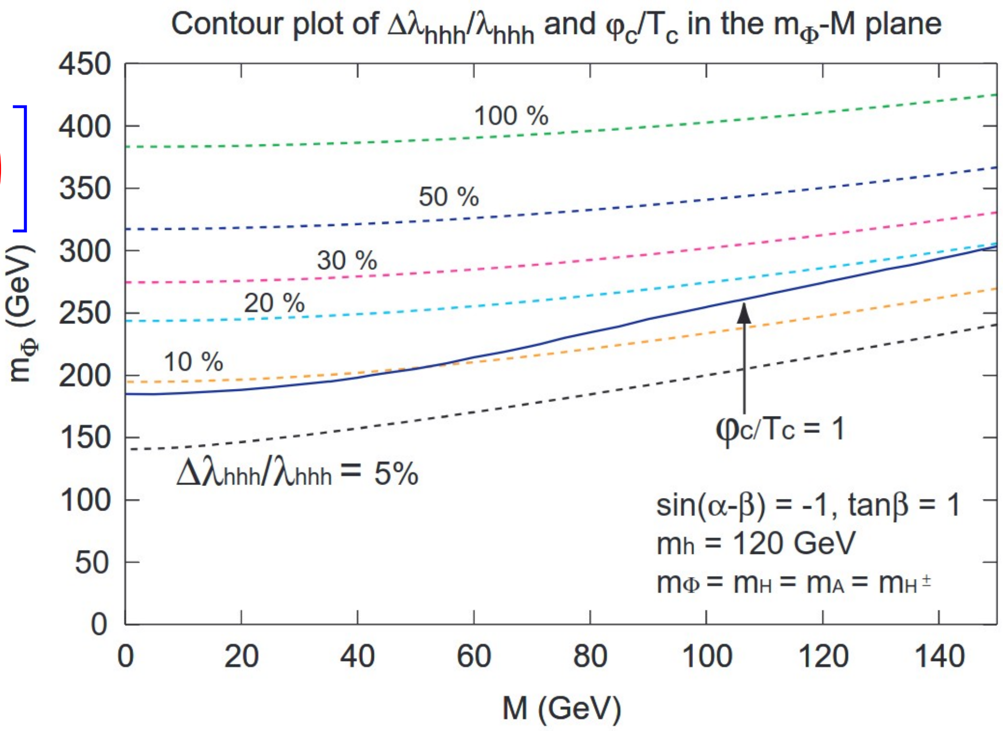
- At critical temperature T_c : 2 degenerate minima at φ = 0, and $\varphi_c \sim 2ET_c/\lambda(T_c)$, so that the sphaleron decoupling condition (to ensure a *strong* FOEWPT) becomes $\dfrac{\varphi_c}{T_c}=\dfrac{2E}{\lambda(T_c)}\gtrsim 1$

- In a model with an extended (and aligned) Higgs sector,:
(Φ: additional scalars, M: BSM mass scale, n_Φ: no. of d.o.f of scalar Φ)

$$\frac{\varphi_c}{T_c}\simeq\frac{1}{3\pi v m_h^2}\left[6m_W^3+3m_Z^3+\sum_\Phi n_\Phi m_\Phi^3\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3\left(1+\frac{3\mathcal{M}^2}{2m_\Phi^2}\right)\right]$$

while the corrections to λ_{hhh} (*more later*) are

$$\lambda_{hhh}\simeq\frac{3m_h^2}{v}-\frac{1}{16\pi^2}\left[-\frac{48m_t^4}{v^3}+\sum_\Phi\frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3\right]$$



Higgs potential – a more realistic BSM picture

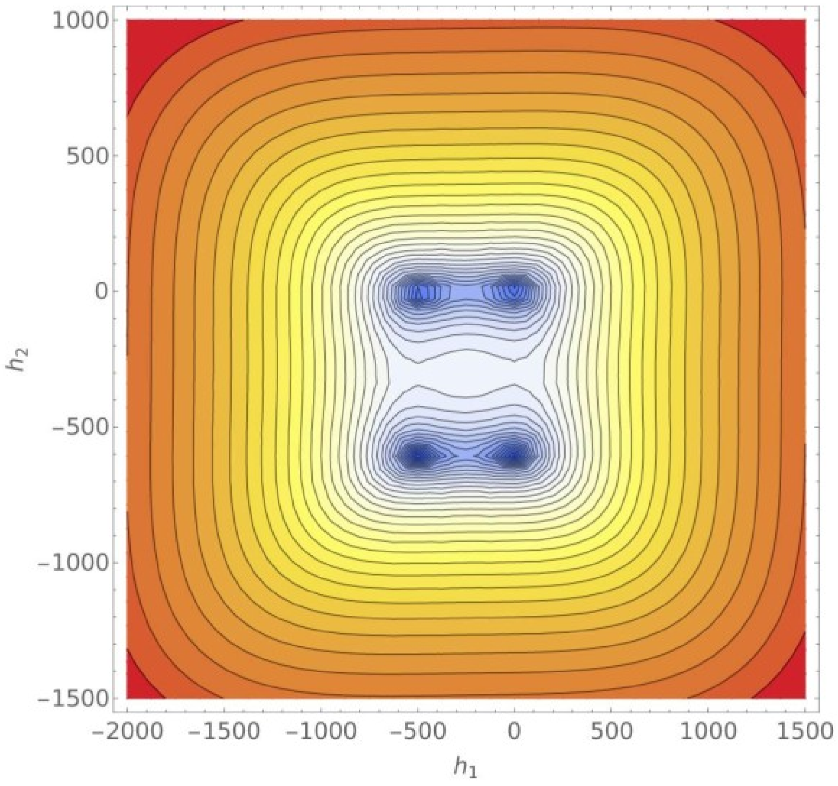
[Bosse, JB, Gabelmann, Hannig, Weiglein '23]

- For instance, for a Z_2SSM where the Z_2 symmetry is **spontaneously broken** → S gets a **VEV v_S**

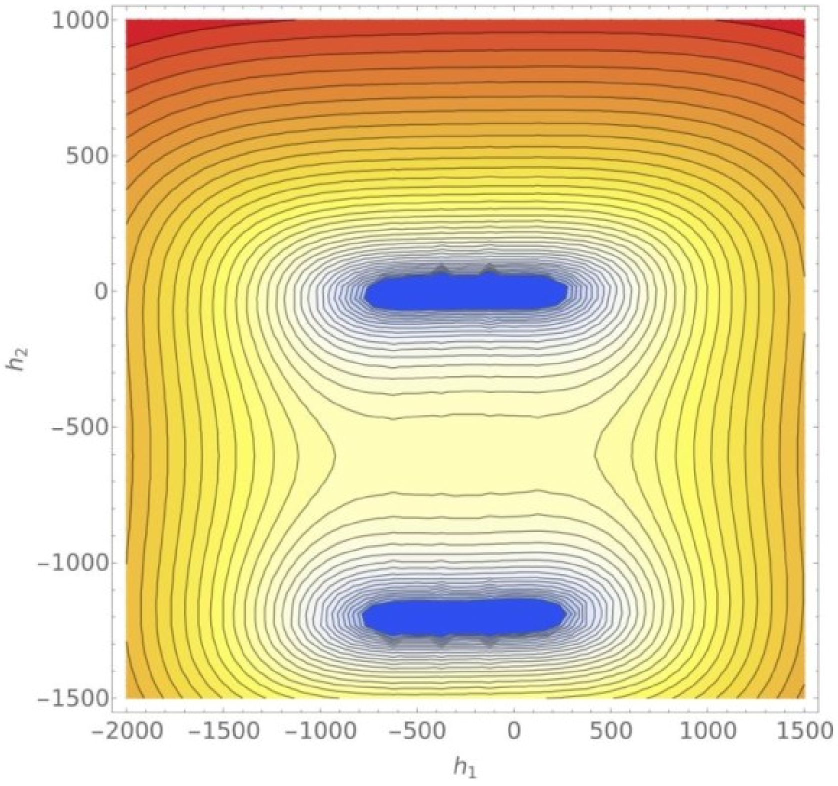
$$S = s + v_S,$$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}G^+ \\ v + h + iG \end{pmatrix}$$

$$V(\Phi, S) = \mu^2|\Phi|^2 + \frac{\lambda_H}{2}|\Phi|^4 + \frac{m_S^2}{2}S^2 + \frac{\lambda_S}{2}S^4 + \frac{\lambda_{SH}}{2}S^2|\Phi|^2$$



$$m_{h_2} = v_S = 300 \text{ GeV}, \alpha = -0.01$$



$$m_{h_2} = 900 \text{ GeV}, v_S = 600 \text{ GeV}, \alpha = -0.01$$

$$\lambda_H = \frac{1}{2v^2}\left(m_{h_1}^2 + m_{h_2}^2 + \left(m_{h_1}^2 - m_{h_2}^2\right)\cos(2\alpha)\right),$$

$$\lambda_S = \frac{1}{8v_S^2}\left(m_{h_1}^2 + m_{h_2}^2 - \left(m_{h_1}^2 - m_{h_2}^2\right)\cos(2\alpha)\right),$$

$$\lambda_{SH} = \frac{1}{2vv_S}\left(m_{h_1}^2 - m_{h_2}^2\right)\sin(2\alpha),$$

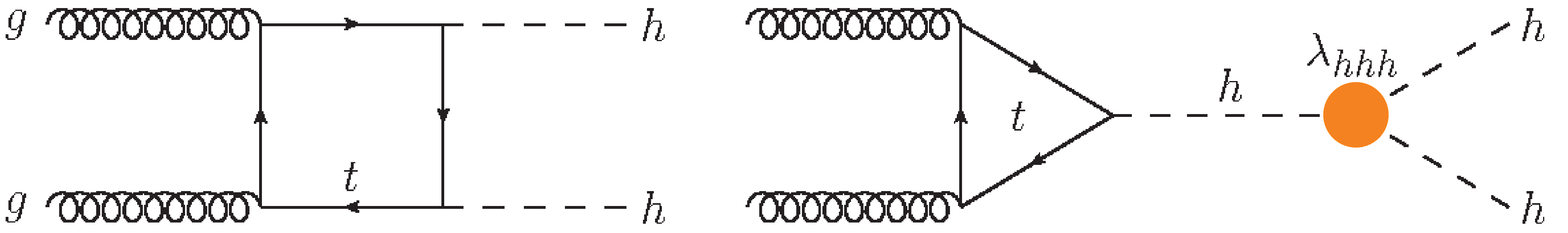
Slide elements by M. Hannig

Experimental probes of the trilinear Higgs coupling

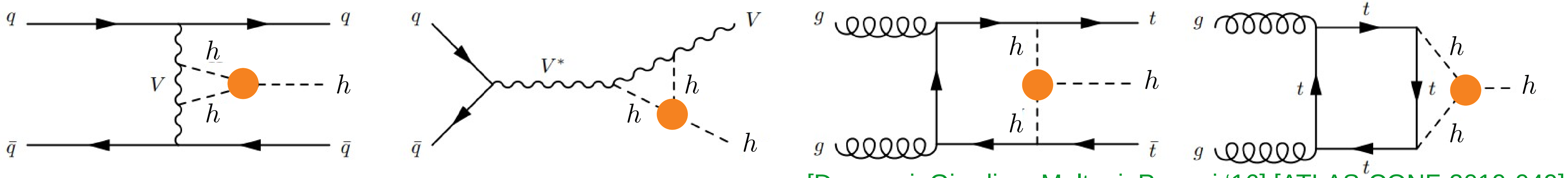
Experimental probes of λ_{hhh}

[NB: triple-Higgs production in a few slides]

- **Double-Higgs production** → λ_{hhh} enters at <u>leading order (LO)</u> → **most direct probe!**

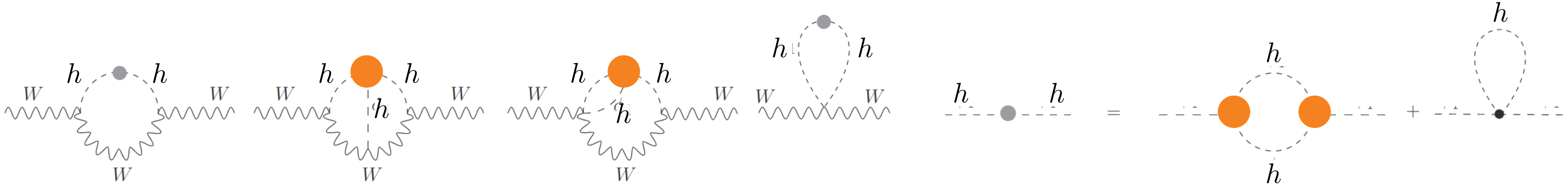


- **Single-Higgs production** → λ_{hhh} enters at <u>NLO</u> (i.e. indirect probe)



[Degrassi, Giardino, Maltoni, Pagani '16] [ATLAS-CONF-2019-049]

- **Electroweak Precision Observables** (EWPOs) → λ_{hhh} enters at <u>NNLO</u> (i.e. indirect probe)

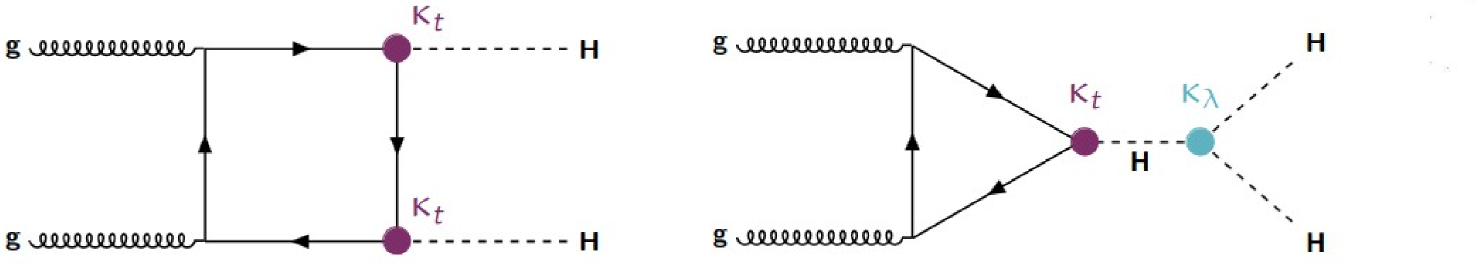


[Degrassi, Fedele, Giardino '17]

Probing λ_{hhh} via double-Higgs production

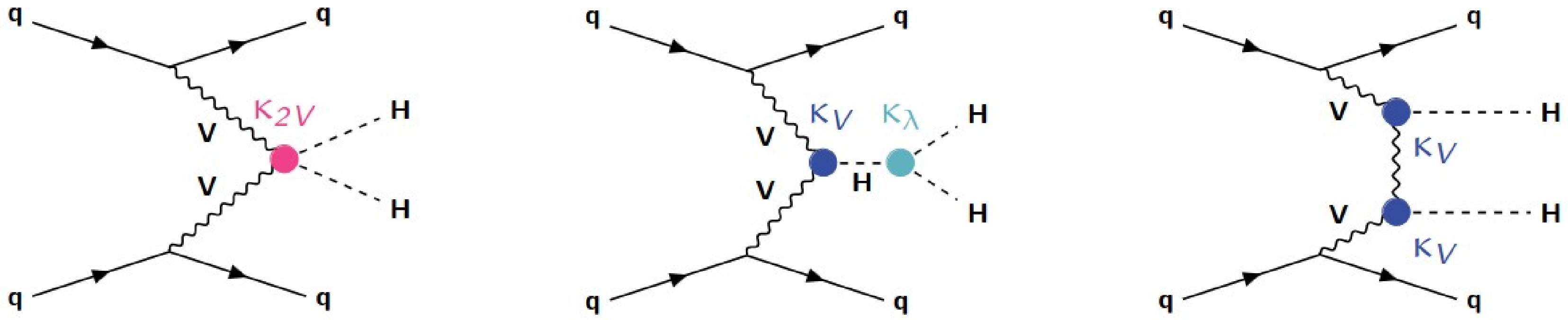
gluon-fusion

$\sigma_{ggF}(pp \to HH) = 31.05$ fb



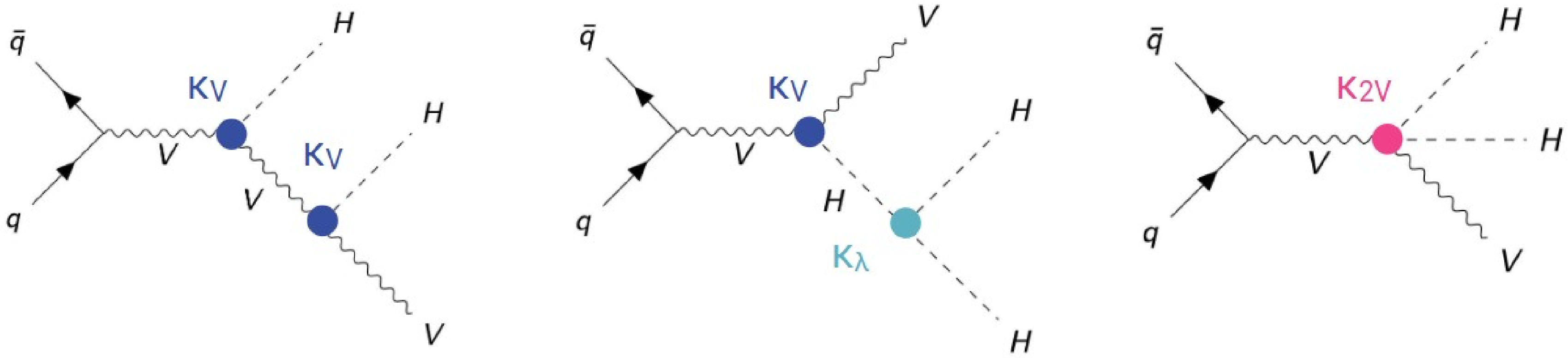
VBF

$\sigma_{VBF}(pp \to HH) = 1.726$ fb



VHH

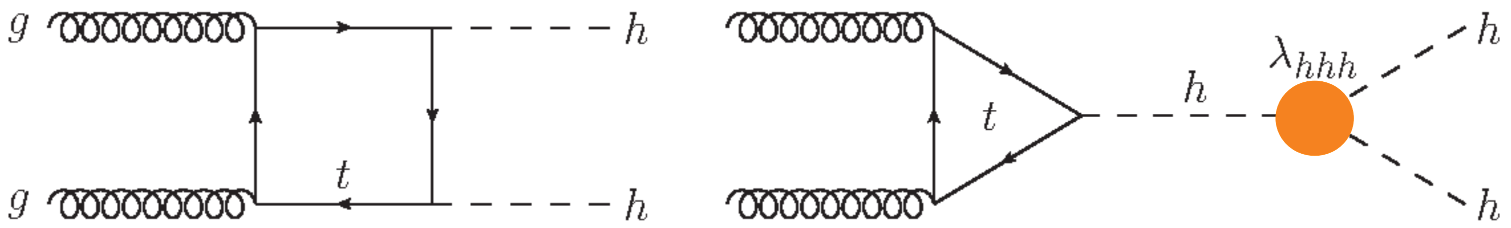
$\sigma_{VHH}(pp \to HH) = 0.86$ fb



Slide by K. Leney @ HiggsDays 23

Probing λ_{hhh} via double-Higgs production

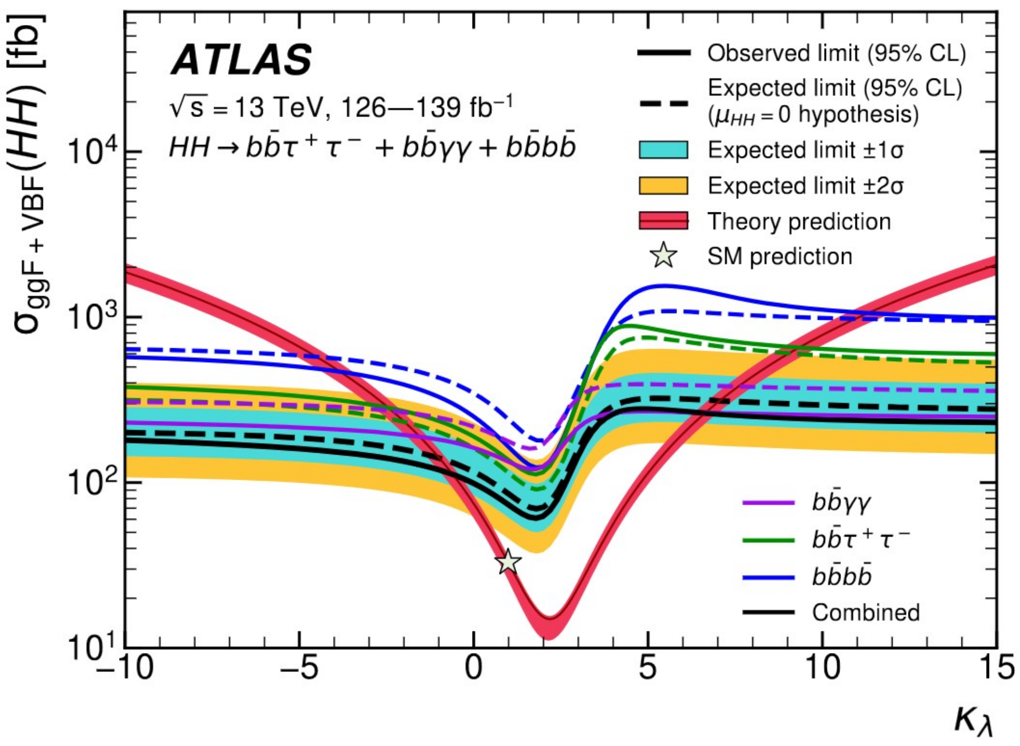
- **Double-Higgs production** → λ_{hhh} enters at LO → **most direct probe of λ_{hhh}**



- Box and triangle diagrams **interfere destructively**
→ small prediction in SM
→ BSM deviation in λ_{hhh} can **significantly enhance double-Higgs production!**

- Search limits on double-Higgs production
→ **limits on effective coupling $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$**

- Current best limits: **$-0.4 < \kappa_\lambda < 6.3$** (95% CL) [ATLAS PLB '23]
(including information from single-Higgs production)
$-1.4 < \kappa_\lambda < 6.1$ (95% CL) [ATLAS PLB '23]
(including information from single-Higgs production + κ_t floating)
$-1.2 < \kappa_\lambda < 6.5$ (95% CL) [CMS '22]



Probing λ_{hhh} via double-Higgs production

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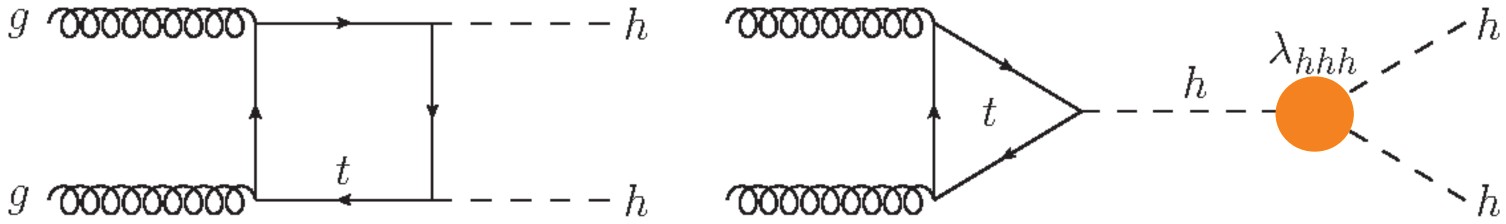

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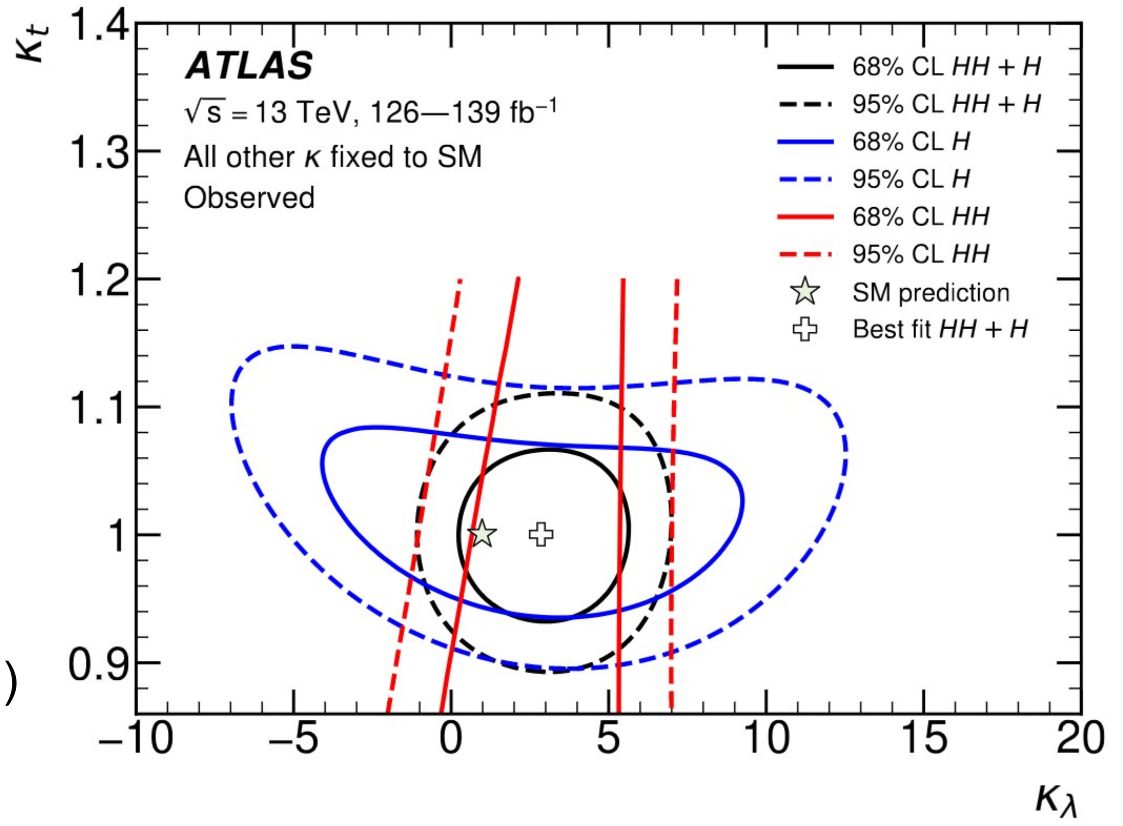
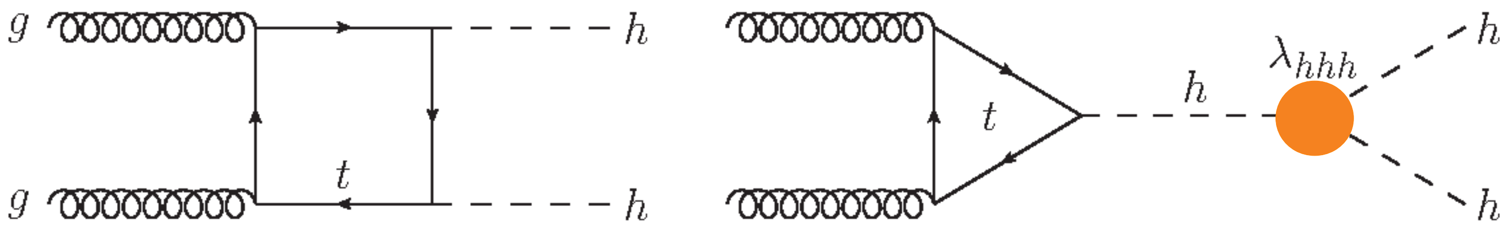


Probing λ_{hhh} via double-Higgs production – HL-LHC prospects

- **Double-Higgs production** → λ_{hhh} enters at LO → **most direct probe of λ_{hhh}**



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  → BSM deviation in λ_{hhh} can **significantly enhance double-Higgs production!**

- Search limits on double-Higgs production
  → **limits on effective coupling $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$**

- Prospects at HL-LHC: **$0.1 < \kappa_\lambda < 2.3$** (95% CL) with ATLAS+CMS
  [Cepeda et al. '19]

  $0.0 < \kappa_\lambda < 2.7$ (95% CL) with ATLAS alone
  [ATL-PHYS-PUB-2022-053]

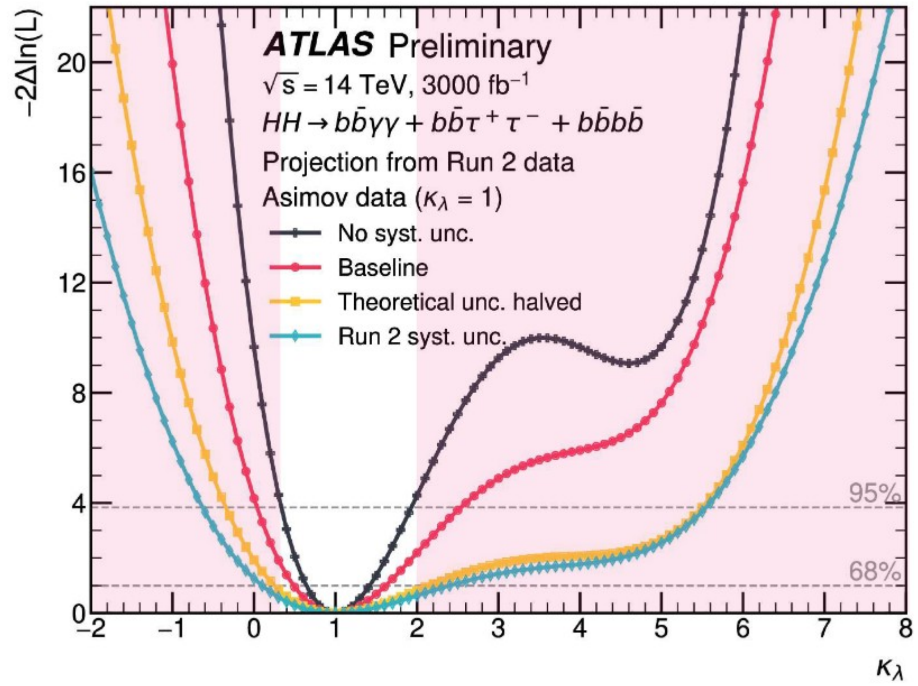


Figure adapted from [ATL-PHYS-PUB-2022-053]

Direct probes of λ_{hhh} at e^+e^- colliders

- **Double-Higgs production, either in $e^+e^- \to Zhh$ or $e^+e^- \to \nu\bar{\nu}hh$**
- Relies however on being above the Zhh threshold!

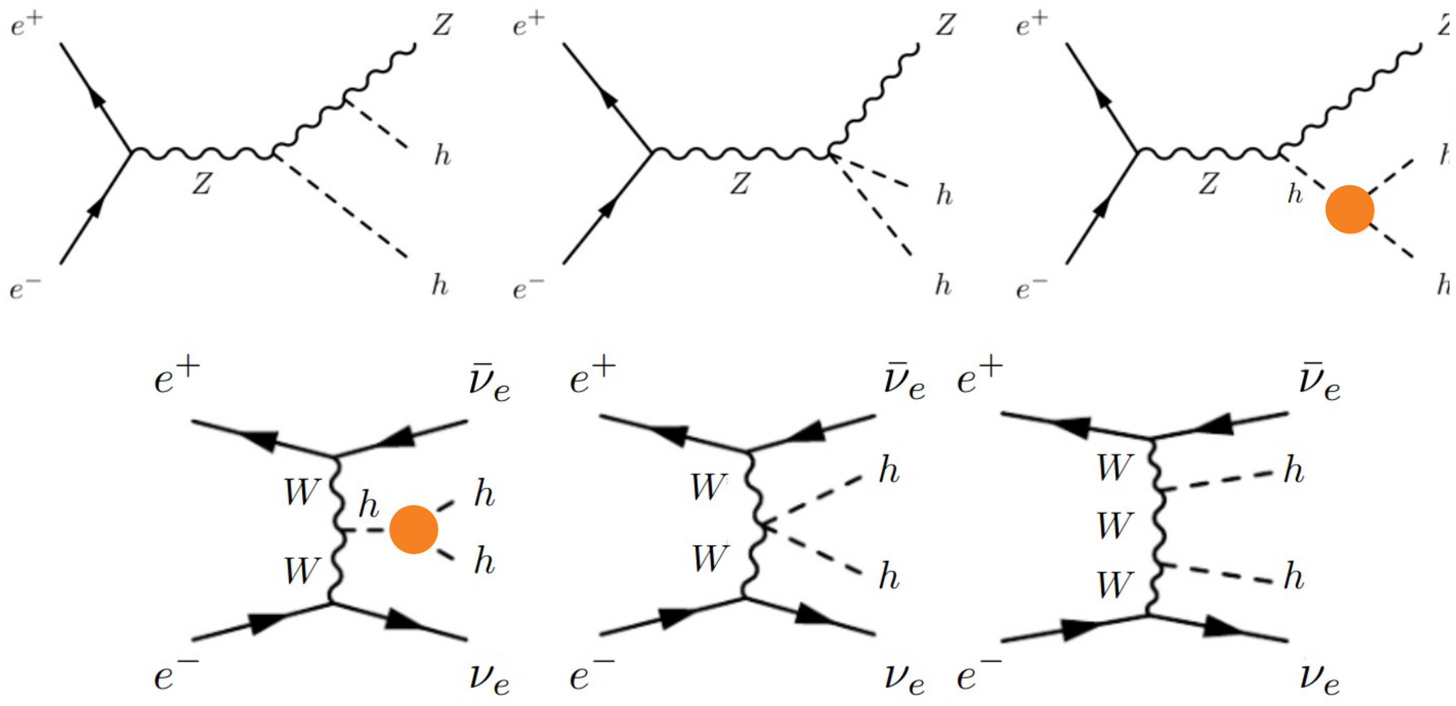
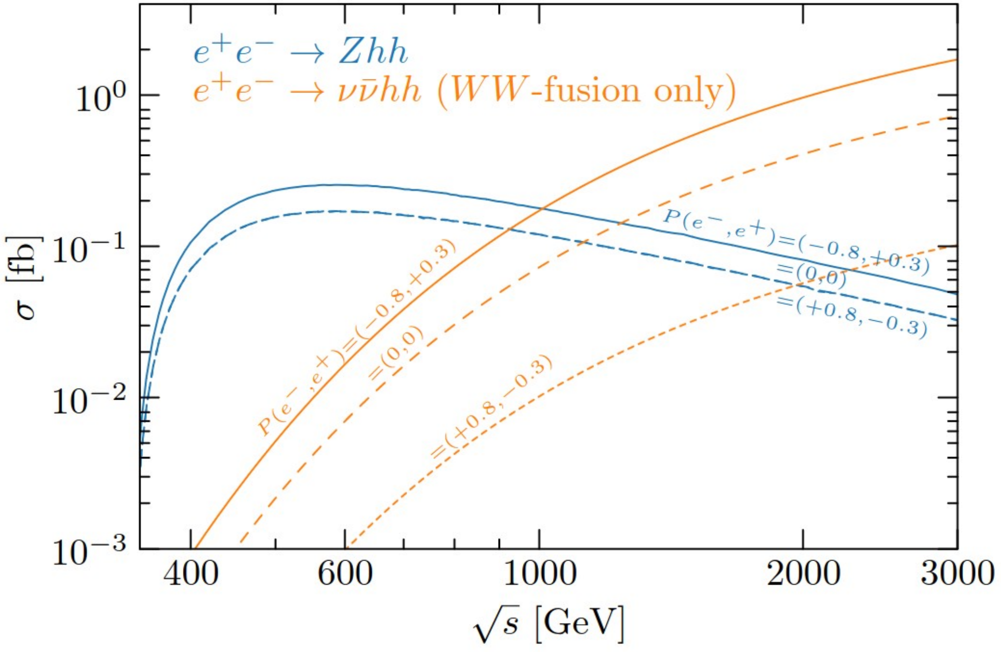


Figure from [De Blas et al. 1905.03764]

Figure from [De Blas et al. 1812.02093]



- $e^+e^- \to Zhh$ better at $\sqrt{s}$~500 GeV
- $e^+e^- \to \nu\bar{\nu}hh$ better for larger $\sqrt{s}$

Indirect probes of λ_{hhh} at e^+e^- colliders

- Below the Zhh threshold, λ_{hhh} can still be investigated through its (indirect) effect in quantum corrections to single-Higgs production

- In particular, λ_{hhh} enters NLO corrections to $e^+e^- \to Zh$
  First pointed out in [McCullough '13], numerous works since (also with global analyses in EFT setting)

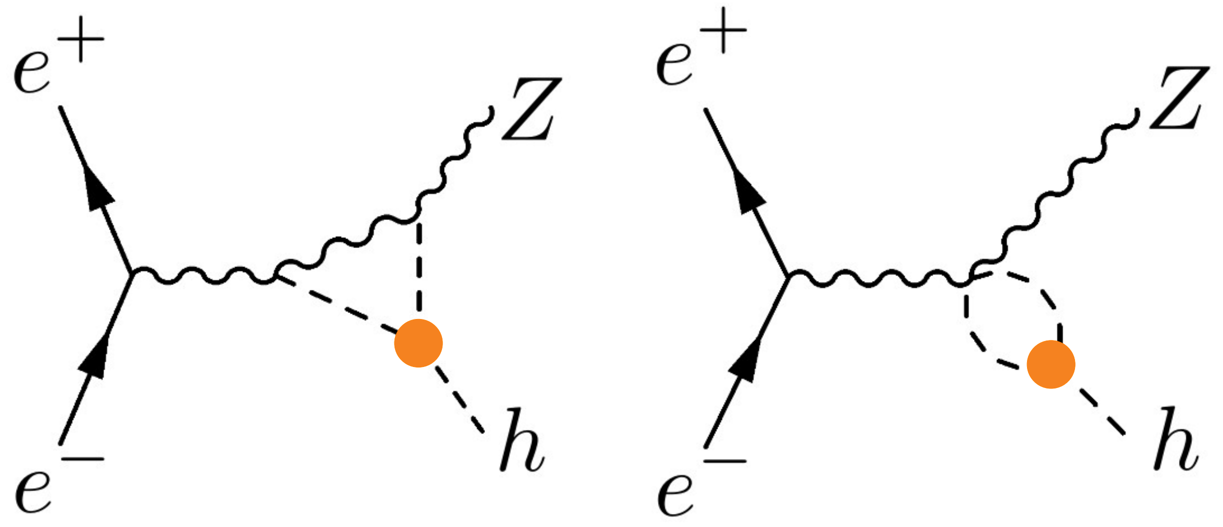


Figure adapted from [McCullough 1312.3322]

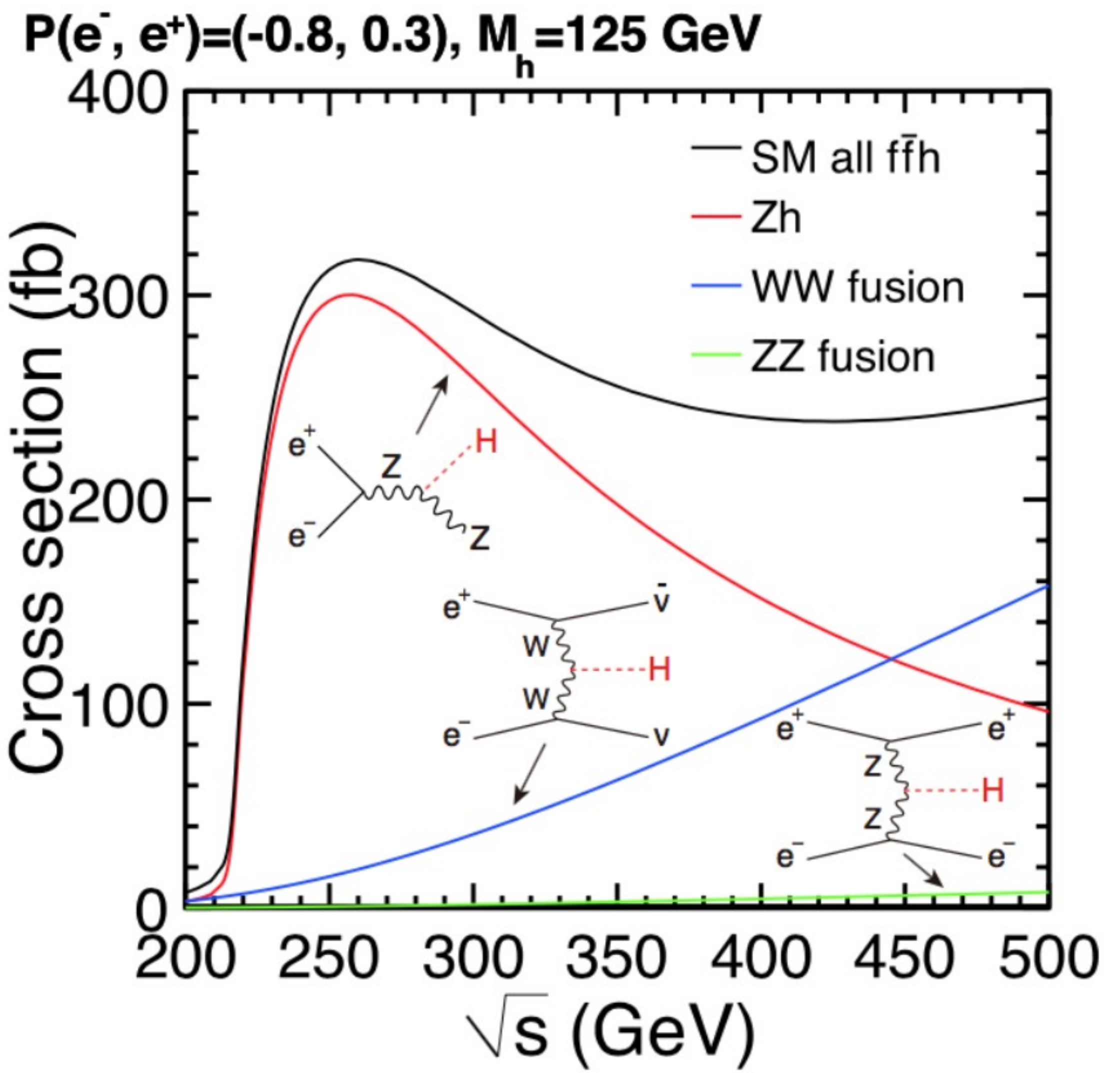


Figure from [Fujii et al. 1710.07621]

Future determination of λ_{hhh}

Expected sensitivities in literature, assuming $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

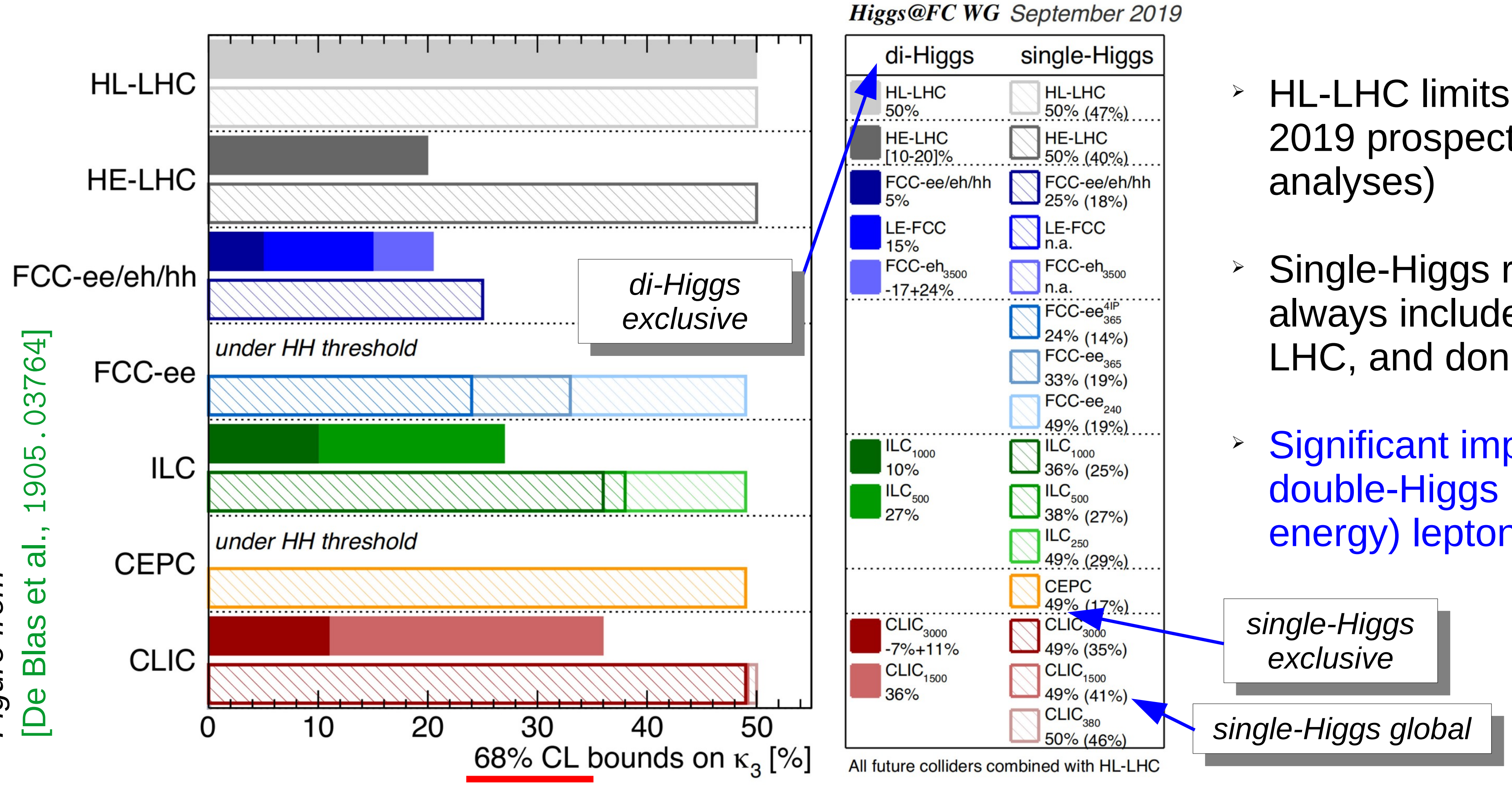


Figure from [De Blas et al., 1905.03764]

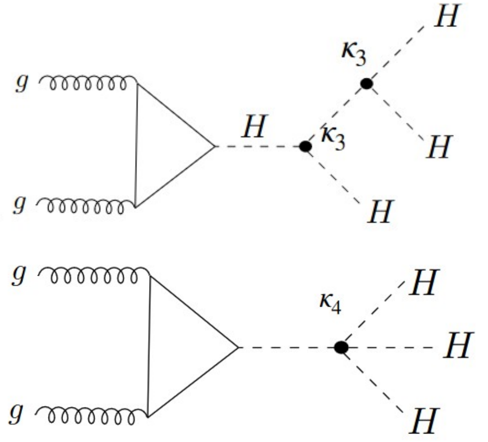
- HL-LHC limits will likely outperform 2019 prospects (even with global analyses)
- Single-Higgs results at lepton colliders always include information from HL-LHC, and don't improve much (if at all)
- Significant improvements only with double-Higgs production at (high-energy) lepton colliders or FCC-hh

see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], *etc.*

New investigations via triple-Higgs production

Constraining the trilinear and quartic Higgs couplings at the same time

Hadron collider



$\kappa_3=\kappa_\lambda$: trilinear coupling modifier
κ_4 : quartic coupling modifier

Lepton collider

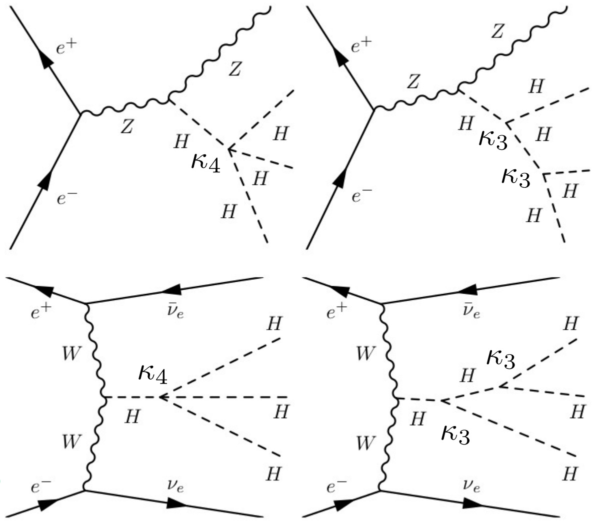
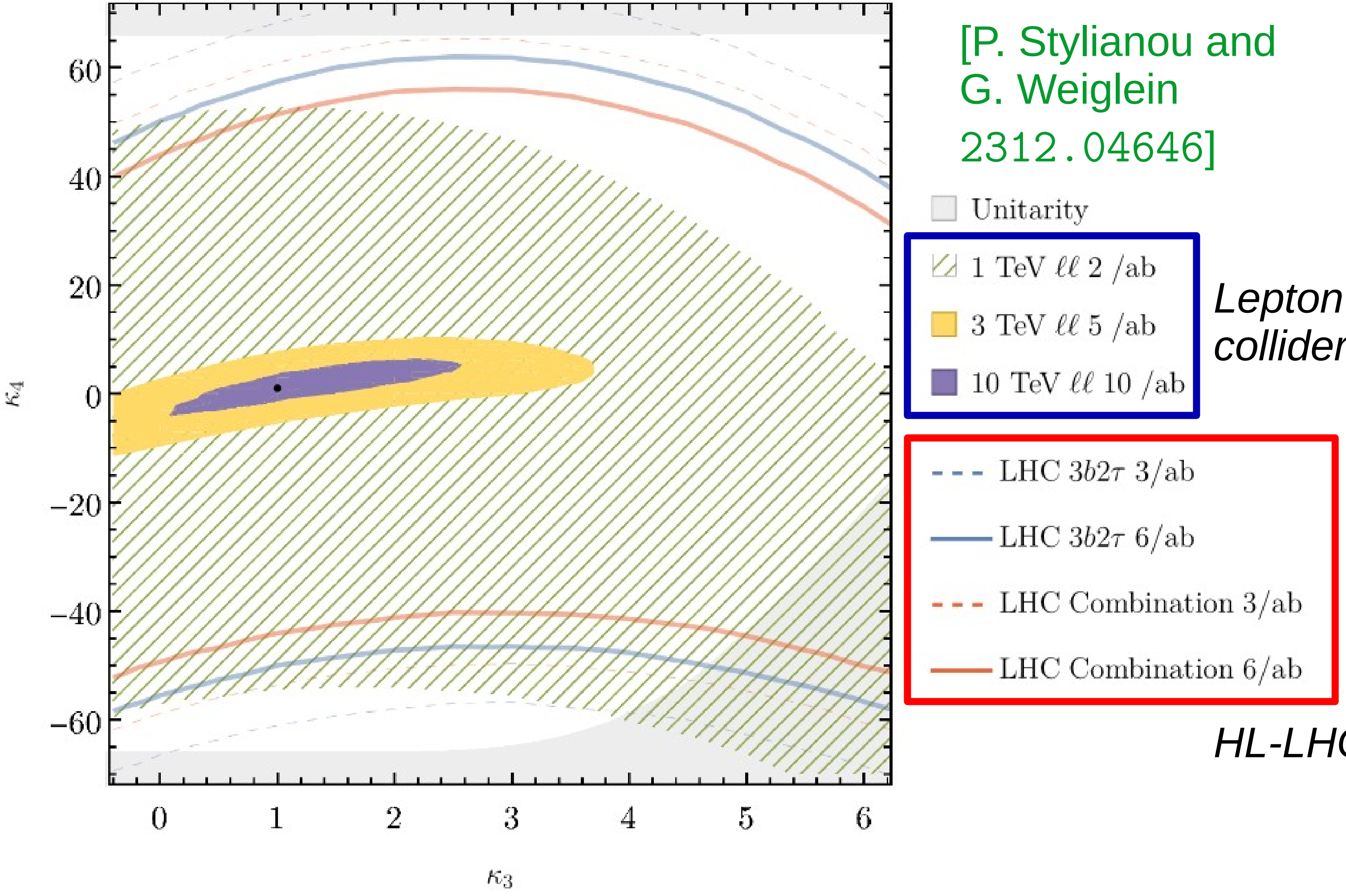


Figure adapted from
[Maltoni, Pagani, Zhao 1802.07616]

[P. Stylianou and G. Weiglein 2312.04646]

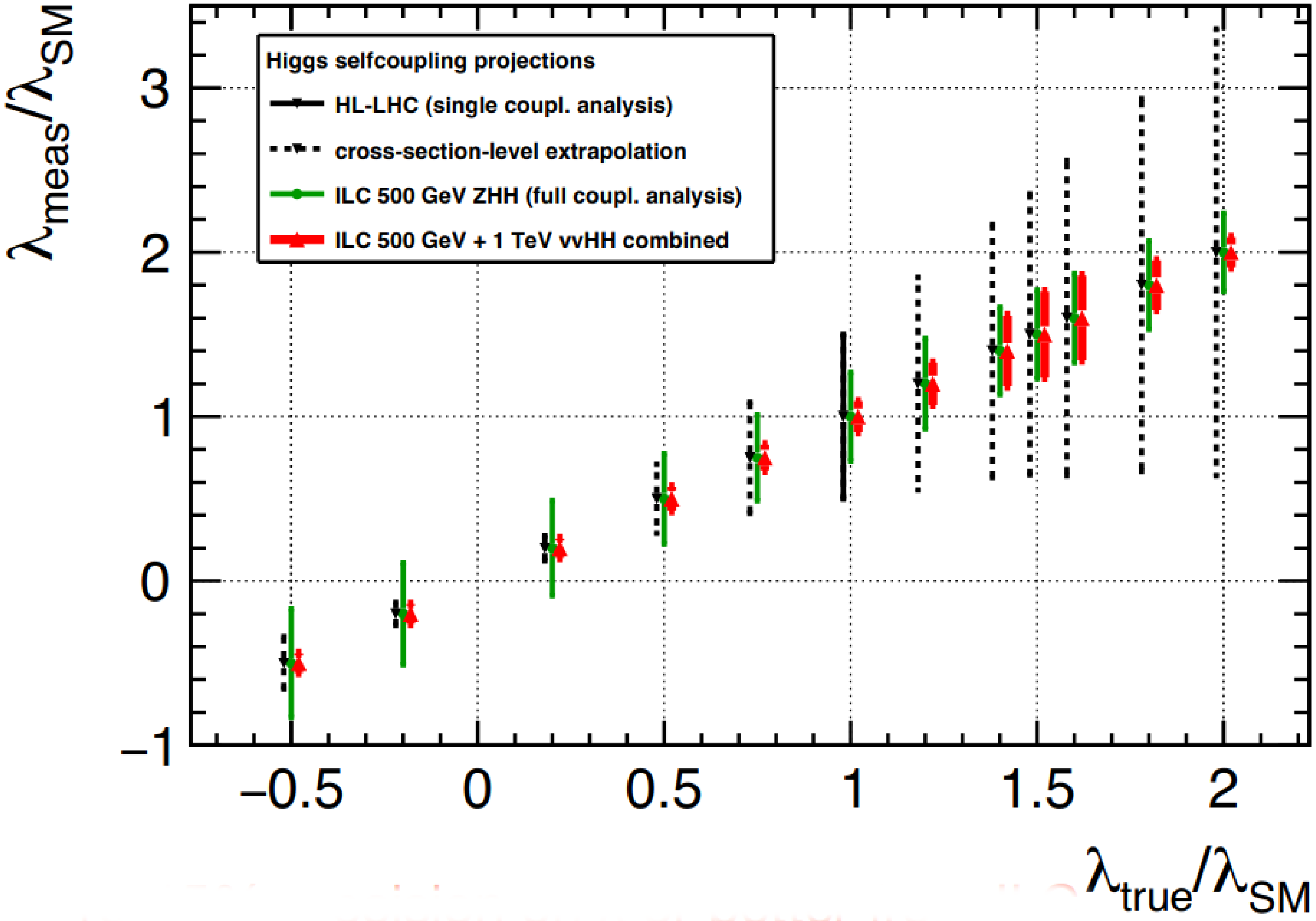


Lepton colliders

HL-LHC

Future determination of λ_{hhh}

Achieved accuracy actually depends on the value of λ_{hhh}



[J. List et al. '21]

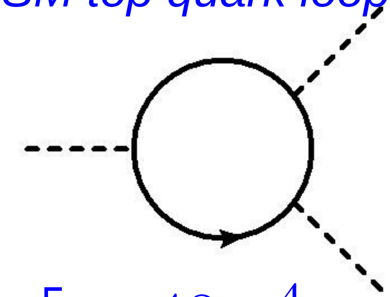
See also [Dürig, DESY-THESIS-2016-027]

Calculating λ_{hhh} in models with extended scalar sectors

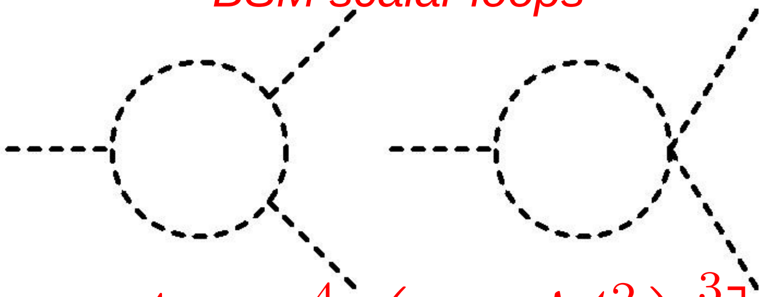
One-loop mass-splitting effects

- **Leading one-loop** corrections to λ_{hhh} in models with extended sectors (like 2HDM):

SM top quark loop



BSM scalar loops



$$\delta^{(1)}\lambda_{hhh} \supset \frac{1}{16\pi^2}\left[-\frac{48m_t^4}{v^3} \qquad + \sum_{\Phi} \frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3\right]$$

First found in 2HDM:
[Kanemura, Kiyoura, Okada, Senaha, Yuan '02]

$\mathcal{M}$: **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM
n_Φ : # of d.o.f of field Φ

- Size of new effects depends on how the BSM scalars acquire their mass: $m_\Phi^2 \sim \mathcal{M}^2 + \tilde{\lambda}v^2$

$$\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3 \longrightarrow \begin{cases} 0, \text{ for } \mathcal{M}^2 \gg \tilde{\lambda}v^2 \\ 1, \text{ for } \mathcal{M}^2 \ll \tilde{\lambda}v^2 \end{cases}$$

→ **Huge BSM effects possible!**

One-loop mass-splitting effects

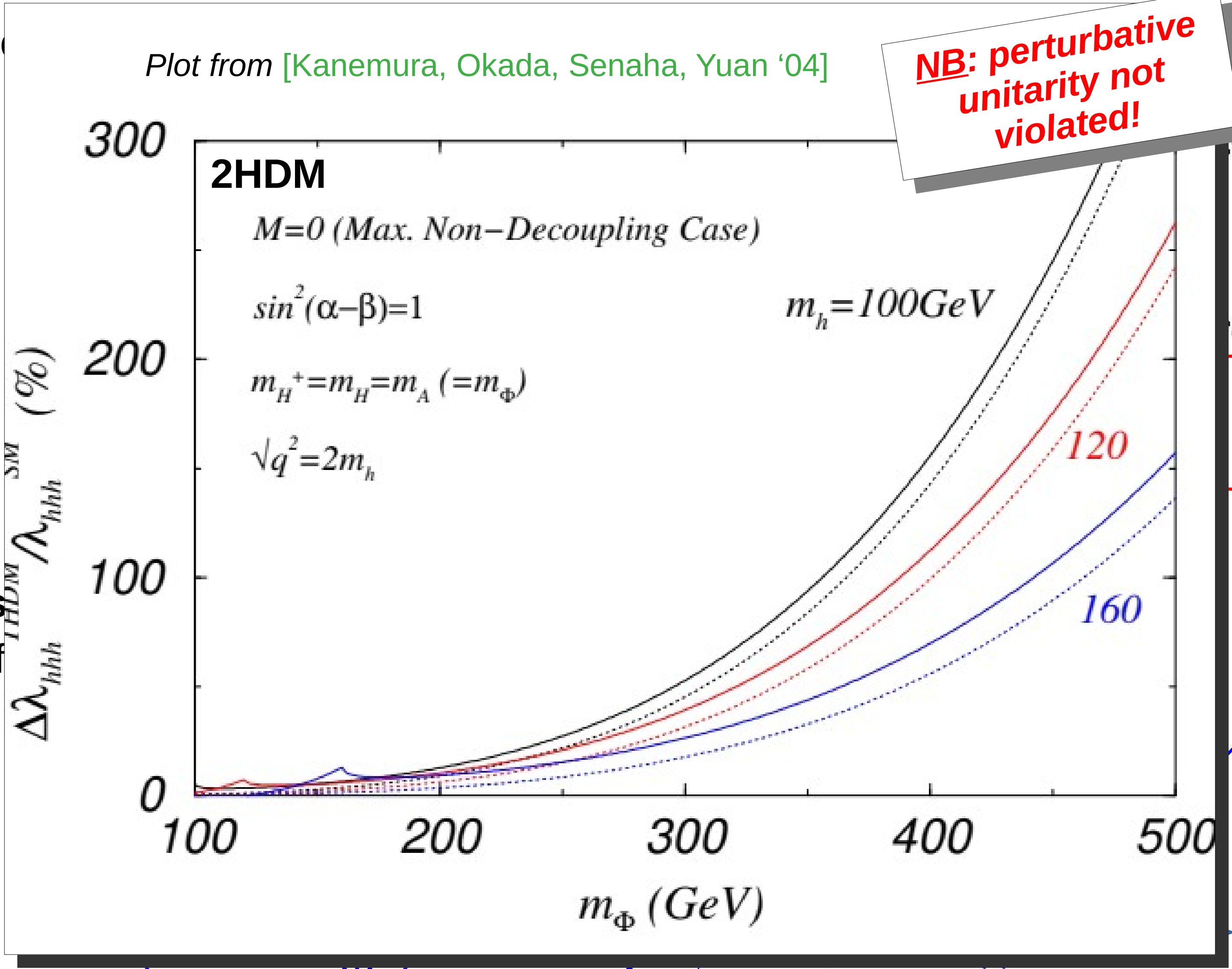
- Leading one-loop c

$\delta^{(1)}\lambda_{hhh} \supset$

$\mathcal{M}$: **BSM mass**

n_Φ : # of d.o.f of

- Size of new effects



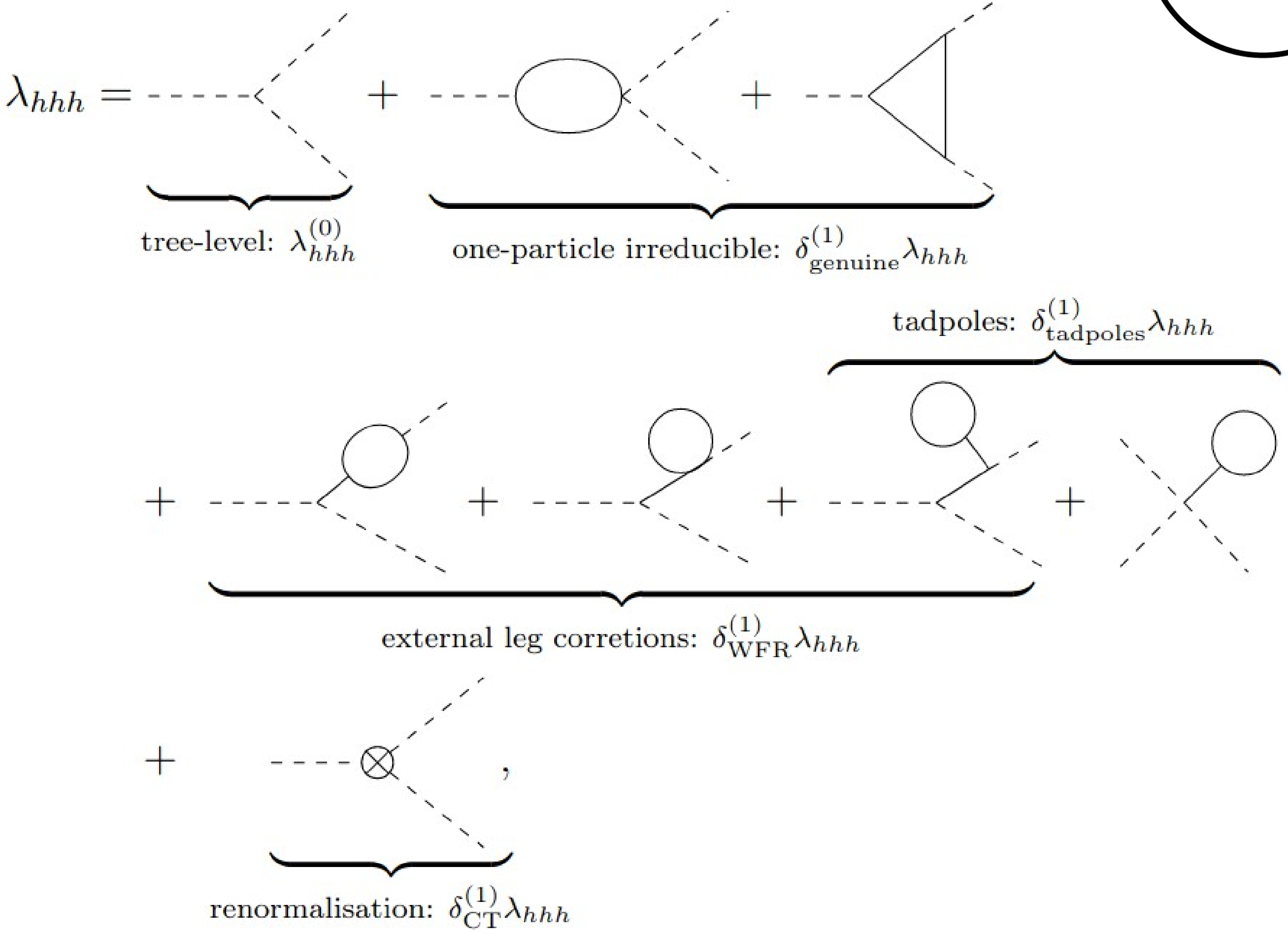
***NB*: perturbative unitarity not violated!**

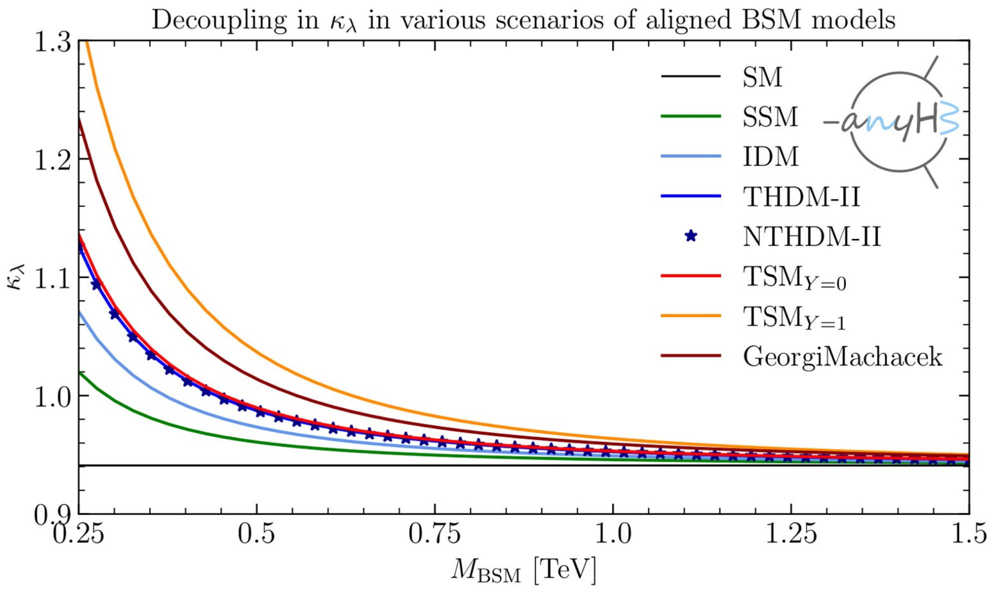
$\Lambda^2 + \tilde{\lambda} v^2$

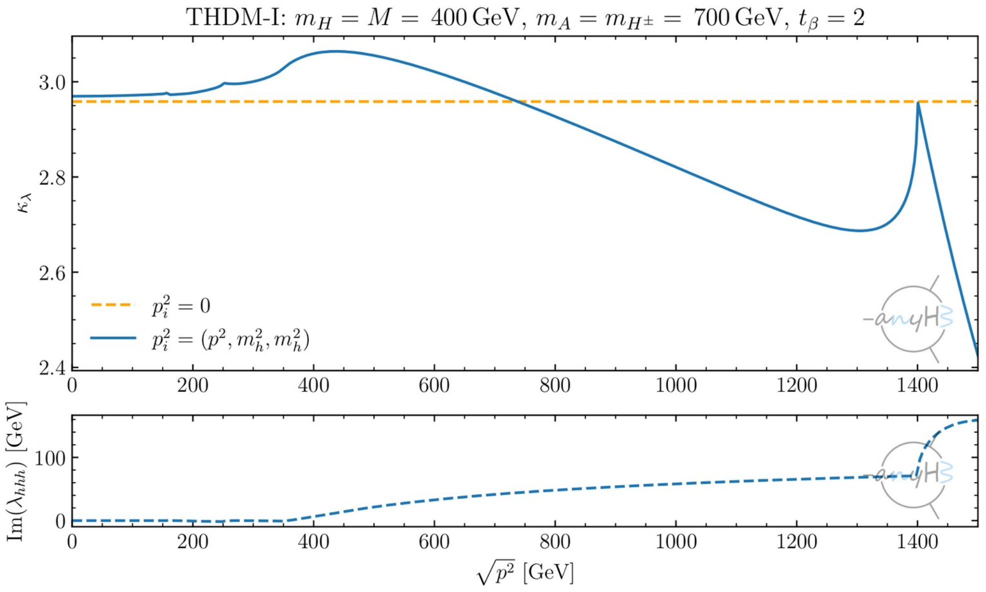
First found in 2HDM:
[Kanemura, Kiyoura, Okada, Senaha, Yuan ‘02]

Huge BSM effects possible!

anyH3: full 1L calculation of λ_{hhh} in any renormalisable model







anyH3: mass-splitting effects in various BSM models

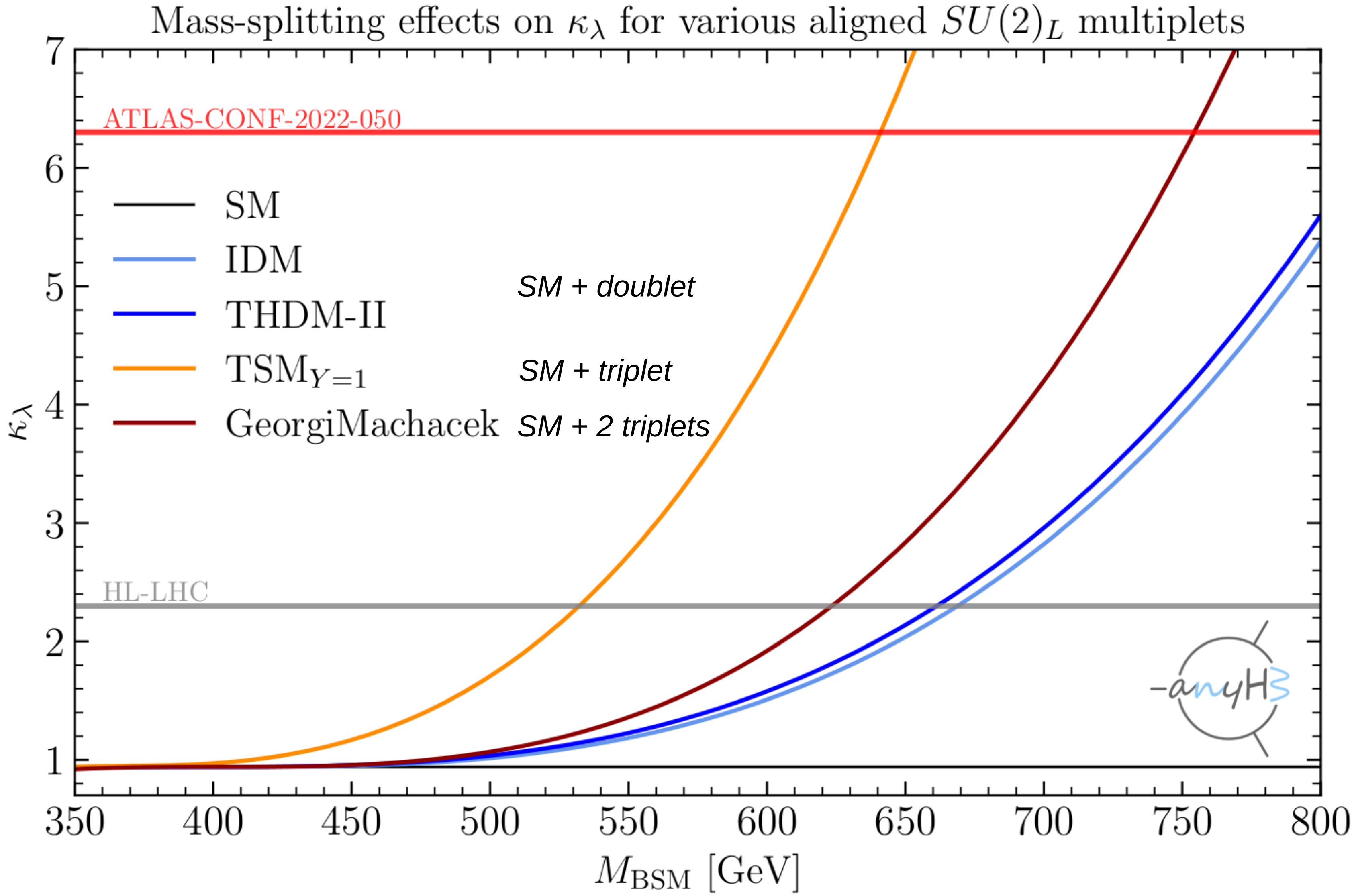
- Consider the non-decoupling limit in several BSM models

$$M^2_{\mathrm{BSM}} = \mathcal{M}^2 + \tilde{\lambda} v^2$$

- Increase M_{BSM}, keeping $\mathcal{M}$ fixed
  → large mass splittings
  → **large BSM effects!**

- Perturbative unitarity checked with `anyPerturbativeUnitarity`

- **Constraints on BSM parameter space!**



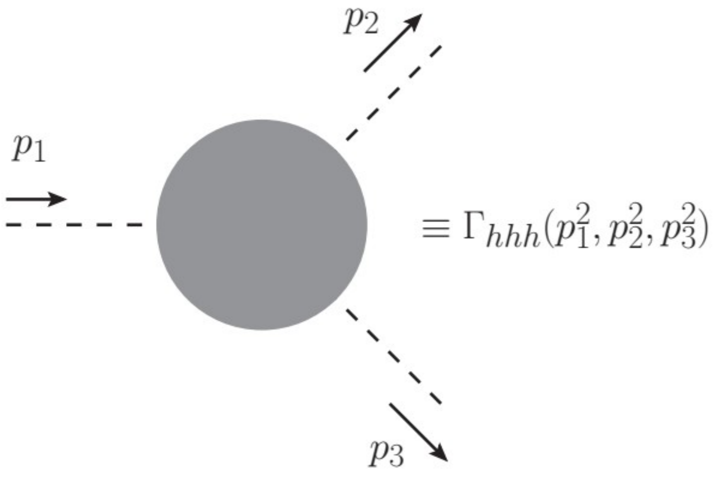
Here: scenarios with lightest BSM scalar mass & BSM mass param. at 400 GeV; other BSM scalar masses = M_{BSM}

Two-loop calculation of λ_{hhh}

Goal: How large can the two-loop corrections to λ_{hhh} become?

An effective Higgs trilinear coupling

- In principle: consider 3-point function Γ_{hhh}
  but this is momentum dependent → very difficult beyond one loop



- Instead, consider an **effective trilinear coupling**

$$\lambda_{hhh} \equiv \left.\frac{\partial^3 V_{\text{eff}}}{\partial h^3}\right|_{\text{min}}$$

entering the coupling modifier

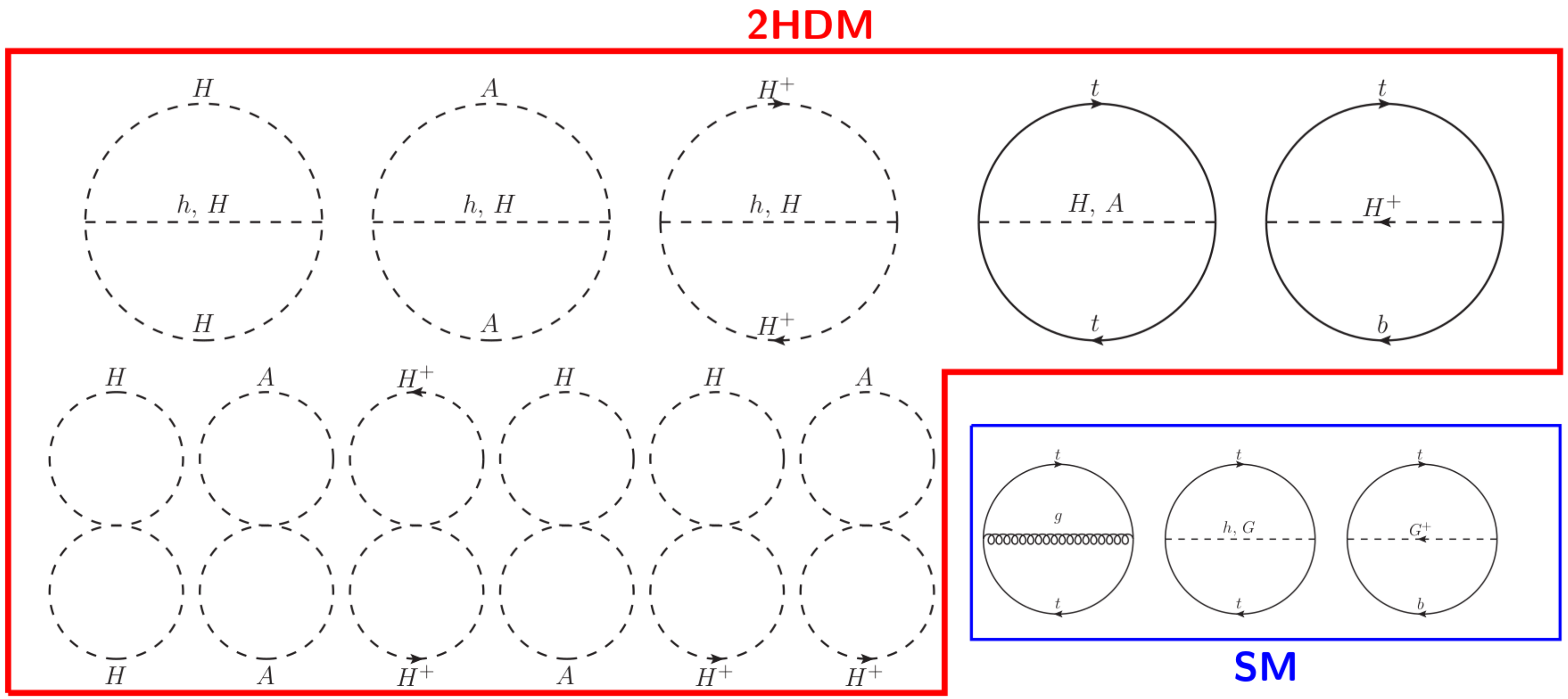
$$\kappa_\lambda = \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})^{\text{SM}}} \qquad \text{with } (\lambda_{hhh}^{(0)})^{\text{SM}} = \frac{3m_h^2}{v}$$

constrained by experiments (*applicability of this assumption discussed later*)

Effective-potential calculation

[JB, Kanemura '19]

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)}$ ($\overline{\text{MS}}$ result)
  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to* $V^{(2)}$ = diagrams involving **heavy BSM scalars and top quark**
  - **Neglect masses of light states** (SM-like Higgs, light fermions, ...)



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  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to $V^{(2)}$* = diagrams involving **heavy BSM scalars and top quark**

- **Step 2**: derive an effective trilinear coupling

$$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \left[\frac{\partial^3}{\partial h^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial h^2} - \frac{1}{v}\frac{\partial}{\partial h}\right)\right]\Delta V\bigg|_{\text{min.}}$$

($\overline{\text{MS}}$ result too)

Express tree-level result in terms of effective-potential Higgs mass

Effective-potential calculation

[JB, Kanemura '19]

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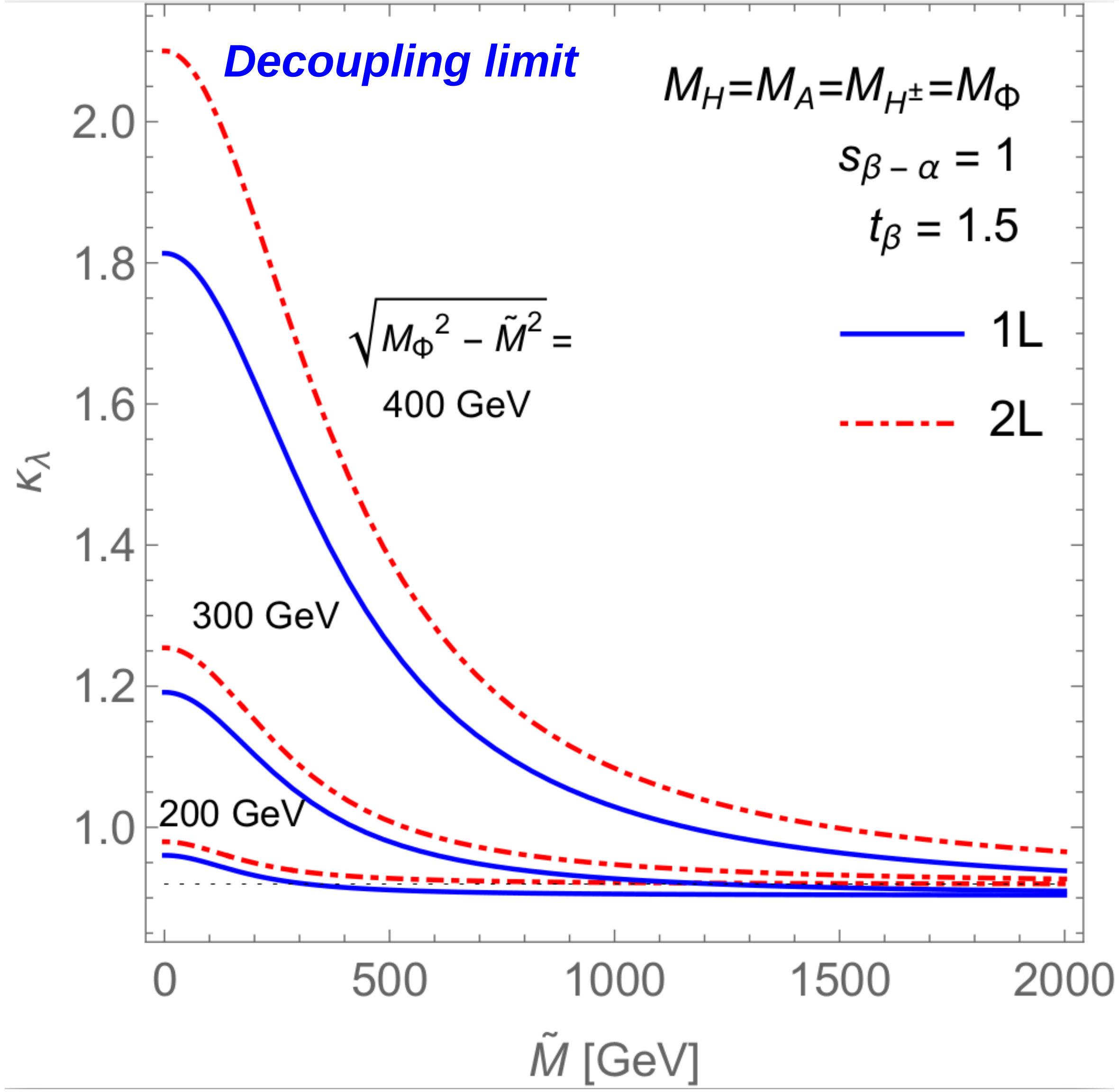
  ($\overline{\text{MS}}$ result too)

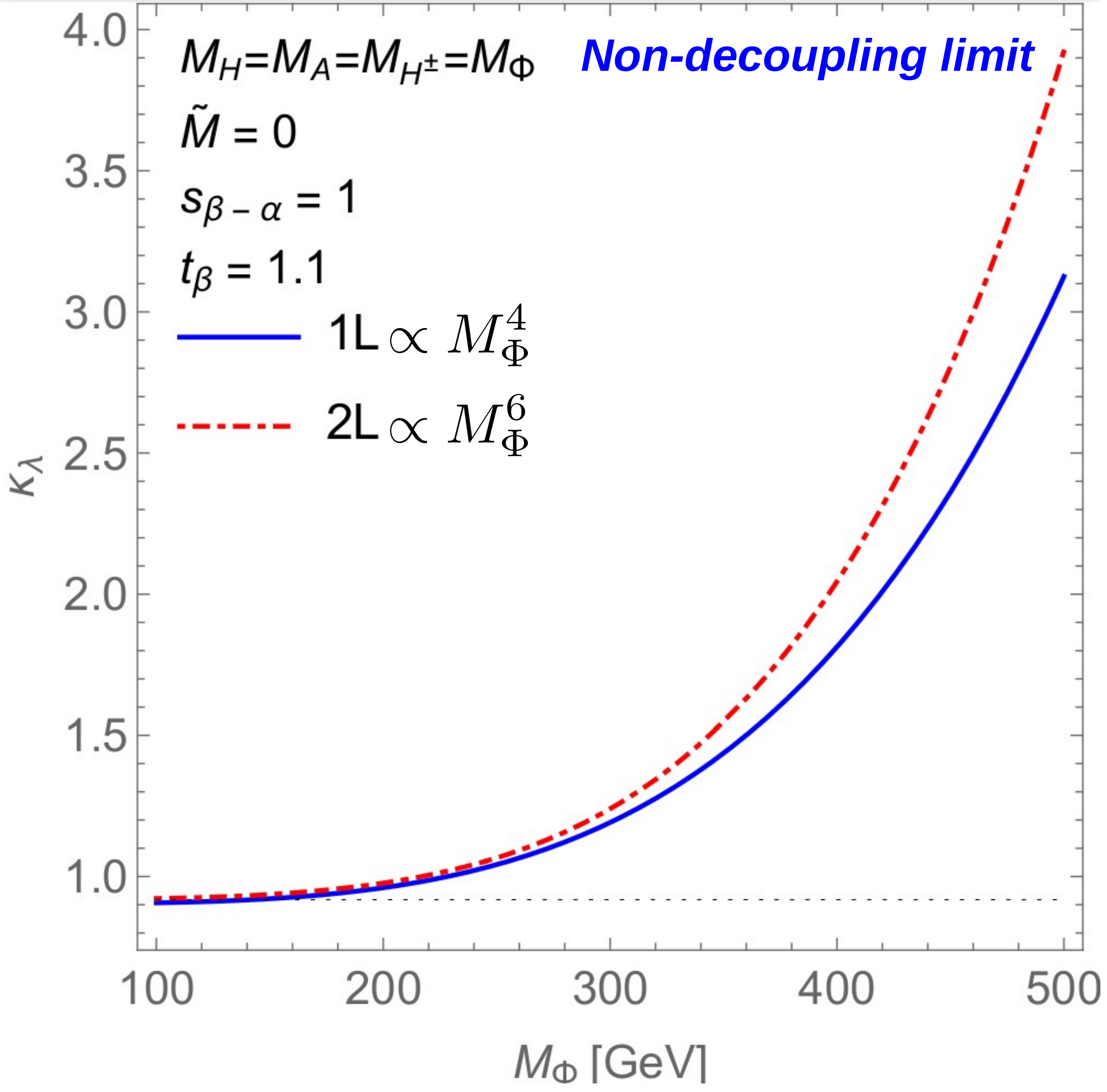
- **Step 3**: conversion from $\overline{\text{MS}}$ to OS scheme
  - Express result in terms of **pole masses**: M_t, M_h, M_Φ (Φ=H,A,H$^\pm$); OS Higgs VEV $v_{\text{phys}} = \frac{1}{\sqrt{\sqrt{2}G_F}}$
  - Include **finite WFR**: $\hat{\lambda}_{hhh} = (Z_h^{\text{OS}}/Z_h^{\overline{\text{MS}}})^{3/2}\lambda_{hhh}$
  - Prescription for M to ensure **proper decoupling** with $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

Our results in the aligned 2HDM

[JB, Kanemura '19]

Taking degenerate BSM scalar masses: $M_\Phi = M_H = M_A = M_{H^\pm}$





Constraining BSM models with λ_{hhh}

i. Can we apply the limits on κ_λ, extracted from experimental searches for di-Higgs production, for BSM models?

ii. Can large BSM deviations occur for points still allowed in light of theoretical and experimental constraints? If so, how large can they become?

As a concrete example, we consider a 2HDM

A benchmark scenario in the aligned 2HDM

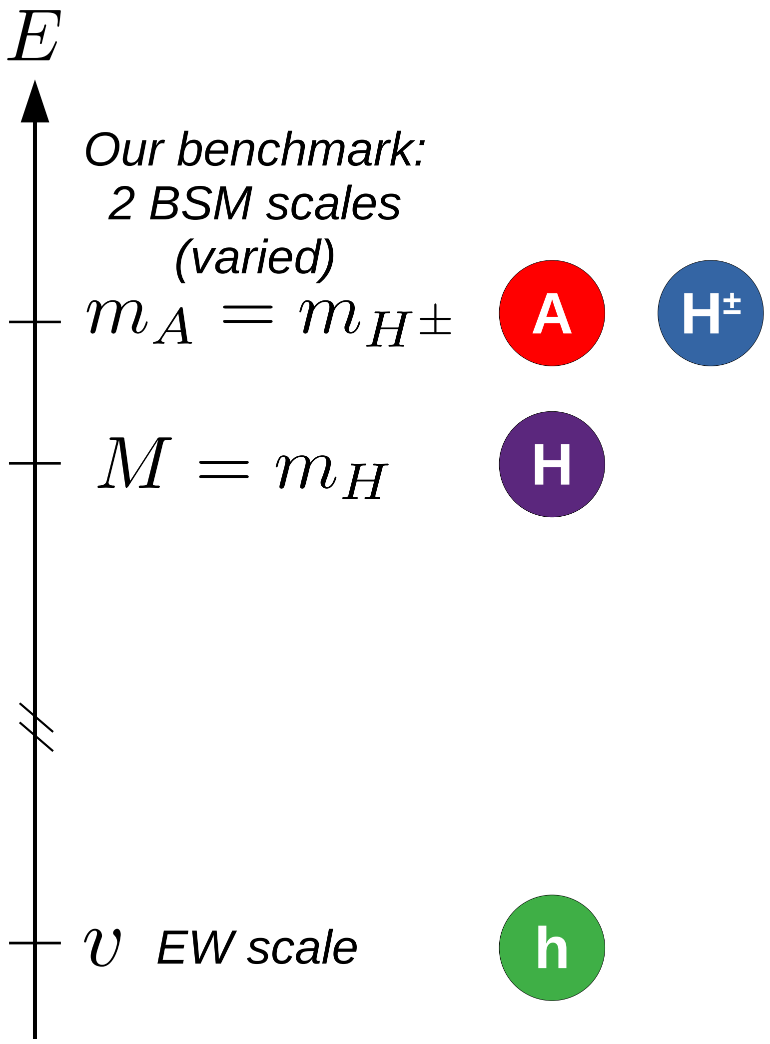
- **Two-Higgs-Doublet Model (2HDM)**:
  add a 2nd scalar doublet to the SM

  Here: CP conservation assumed, Yukawa couplings of type I

- Mass eigenstates:
  - 2 CP-even Higgs bosons

    h (125-GeV Higgs), H
  - CP-odd Higgs boson A
  - Charged Higgs bosons $H^{\pm}$
  - M: new BSM mass term in 2HDM

- Scenario with **alignment**: couplings of h are SM-like at tree level



Can we apply di-Higgs results for the aligned 2HDM?

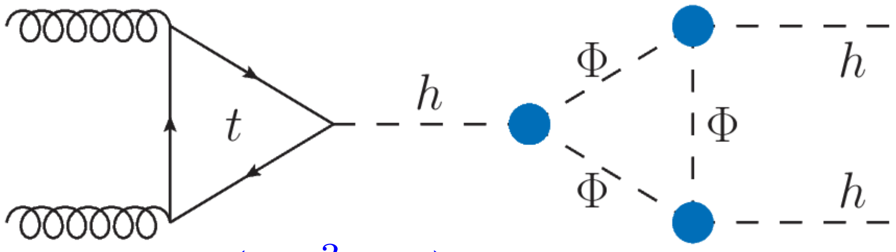
- Current strongest limit on κ_λ are from ATLAS double- (+ single-) Higgs searches

  $-0.4 < \kappa_\lambda < 6.3$ [ATLAS-CONF-2022-050]

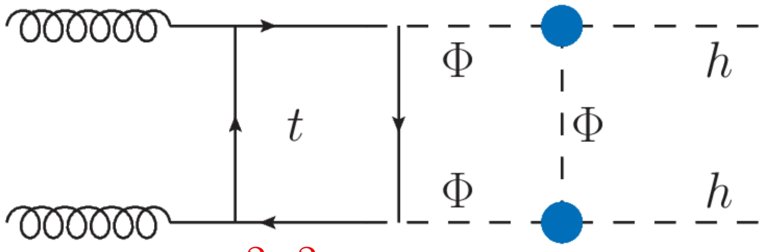
  [where $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$]

- What are the *assumptions* for the ATLAS limits?
  - All other Higgs couplings (to fermions, gauge bosons) are SM-like

    → this is **ensured by the alignment** ✓
  - The modification of λ_{hhh} is the only source of deviation of the *non-resonant Higgs-pair production cross section* from the SM



$\propto \mathcal{O}(y_t g^3_{hh\Phi\Phi})$ ***included***

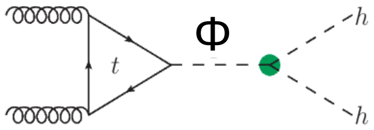


$\propto \mathcal{O}(y_t^2 g^2_{hh\Phi\Phi})$ *not included*

→ We **correctly include all leading BSM effects to di-Higgs production, in powers of $g_{hh\Phi\Phi}$, up to NNLO!** ✓

- **We can apply the ATLAS limits to our setting!**

  (Note: BSM resonant Higgs-pair production cross section also suppressed at LO, thanks to alignment)

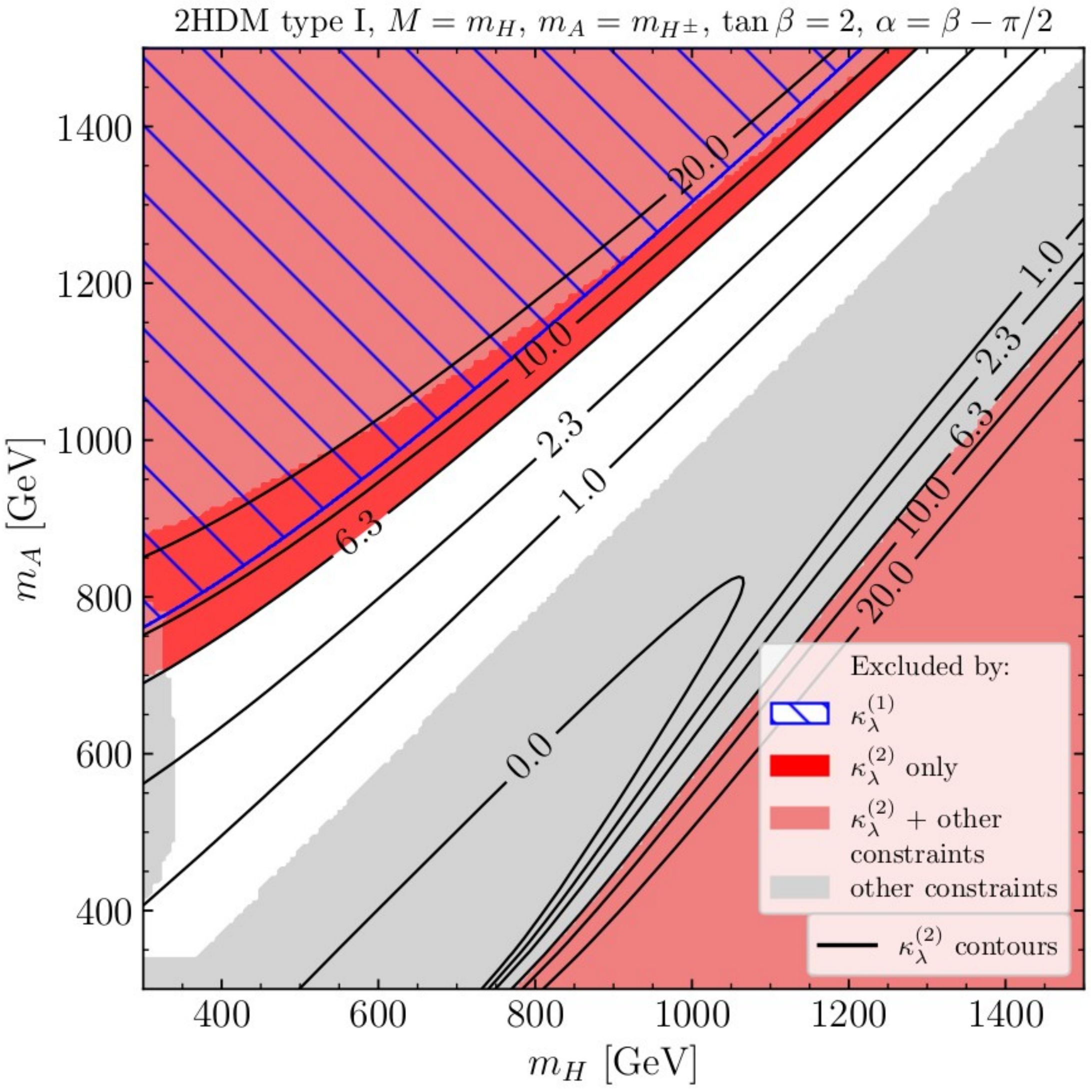


A benchmark scenario in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)

We take $m_A = m_{H\pm}$, $M = m_H$, $\tan\beta = 2$



- ***Grey area:*** area excluded by other constraints, in particular:
  - BSM Higgs searches,
  - boundedness-from-below (BFB),
  - perturbative unitarity (at NLO)
- ***Light red area:*** area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)} > 6.3$ [in region where $\kappa_\lambda^{(2)} < -0.4$ the calculation isn't reliable]
- ***Dark red area:*** new area that is **excluded ONLY by $\kappa_\lambda^{(2)} > 6.3$**. Would otherwise not be excluded!
- ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3$ → impact of including 2L corrections is significant!

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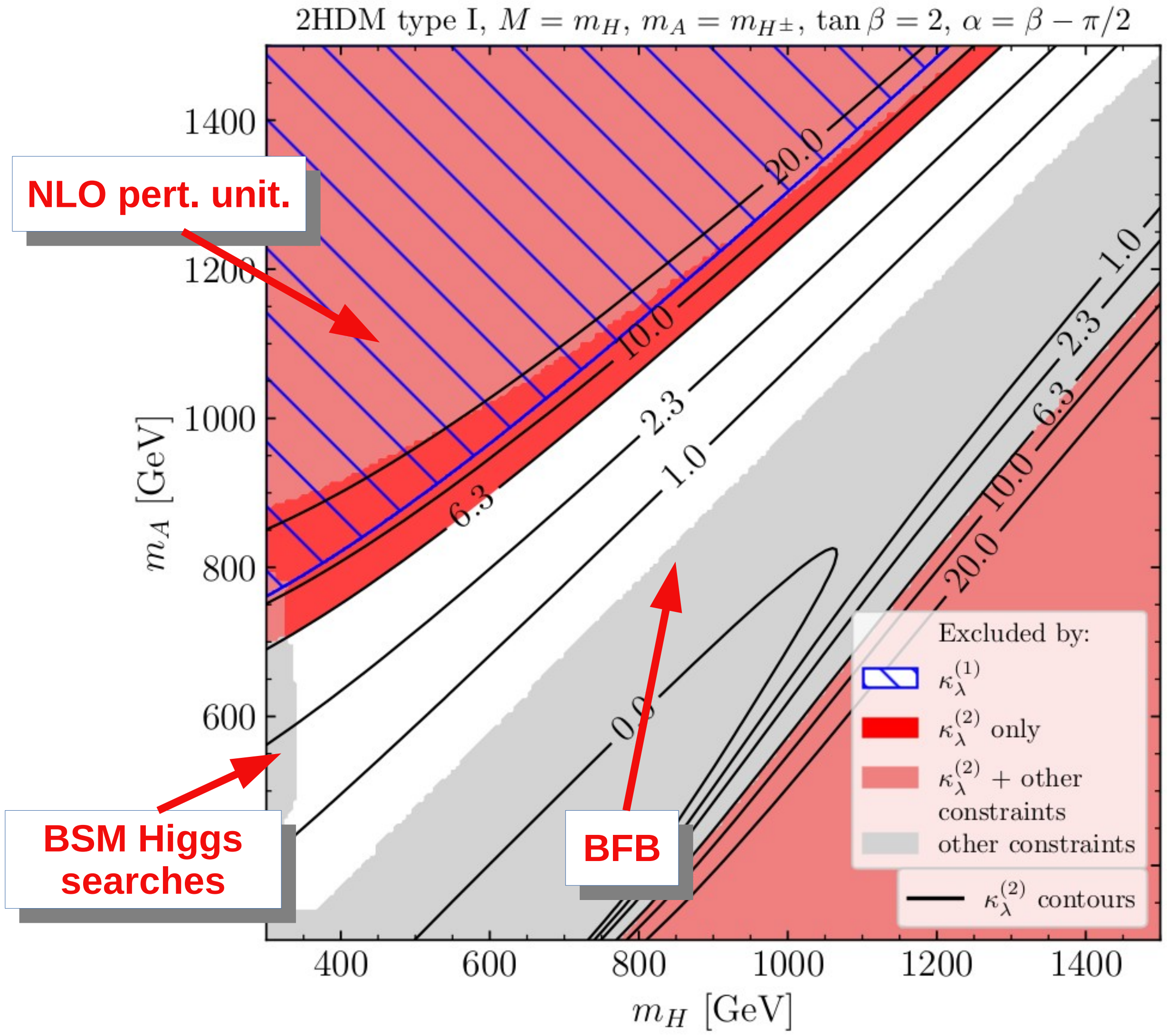

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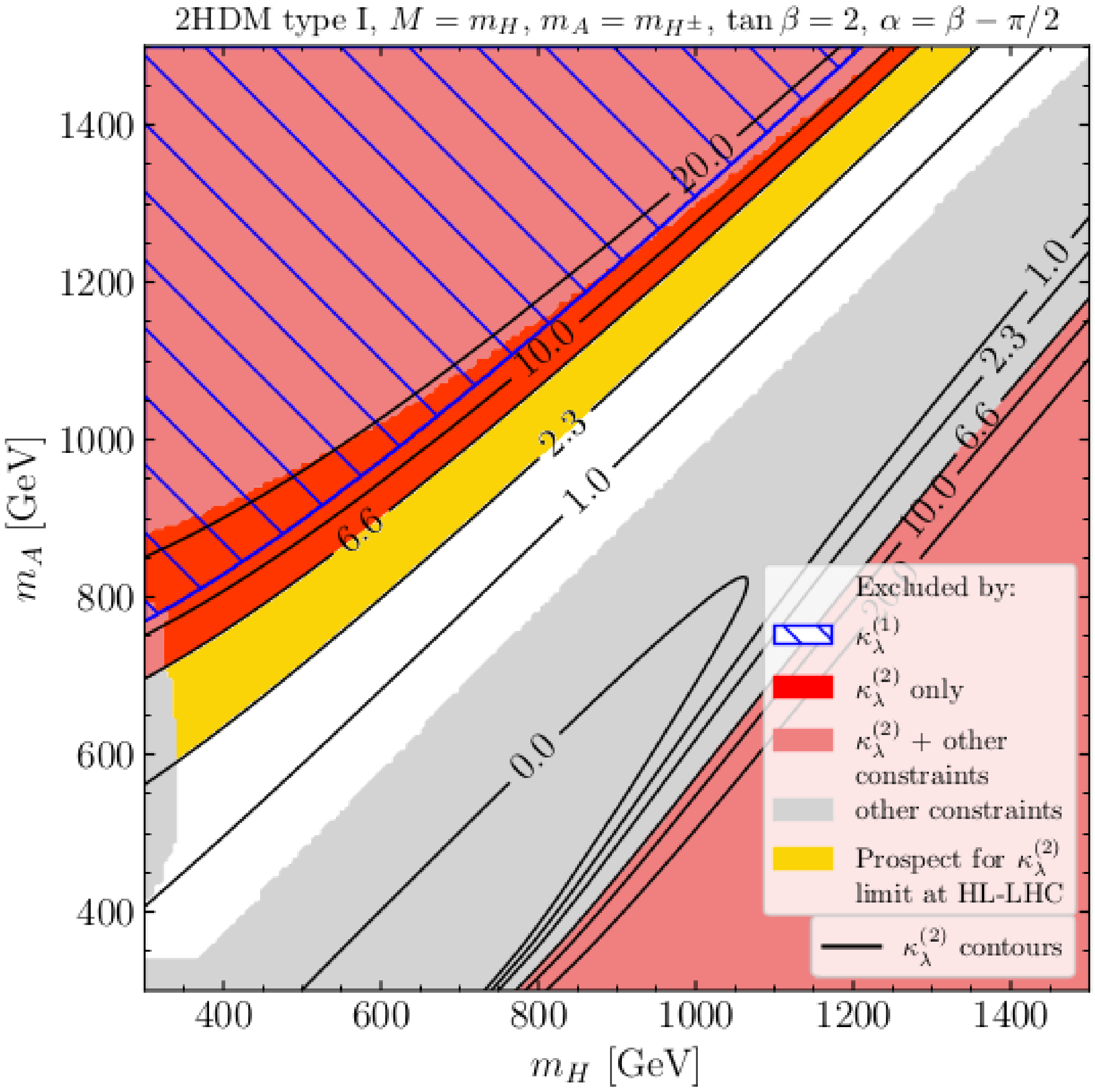
- ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3$ → impact of including 2L corrections is significant!

A benchmark scenario in the aligned 2HDM – future prospects

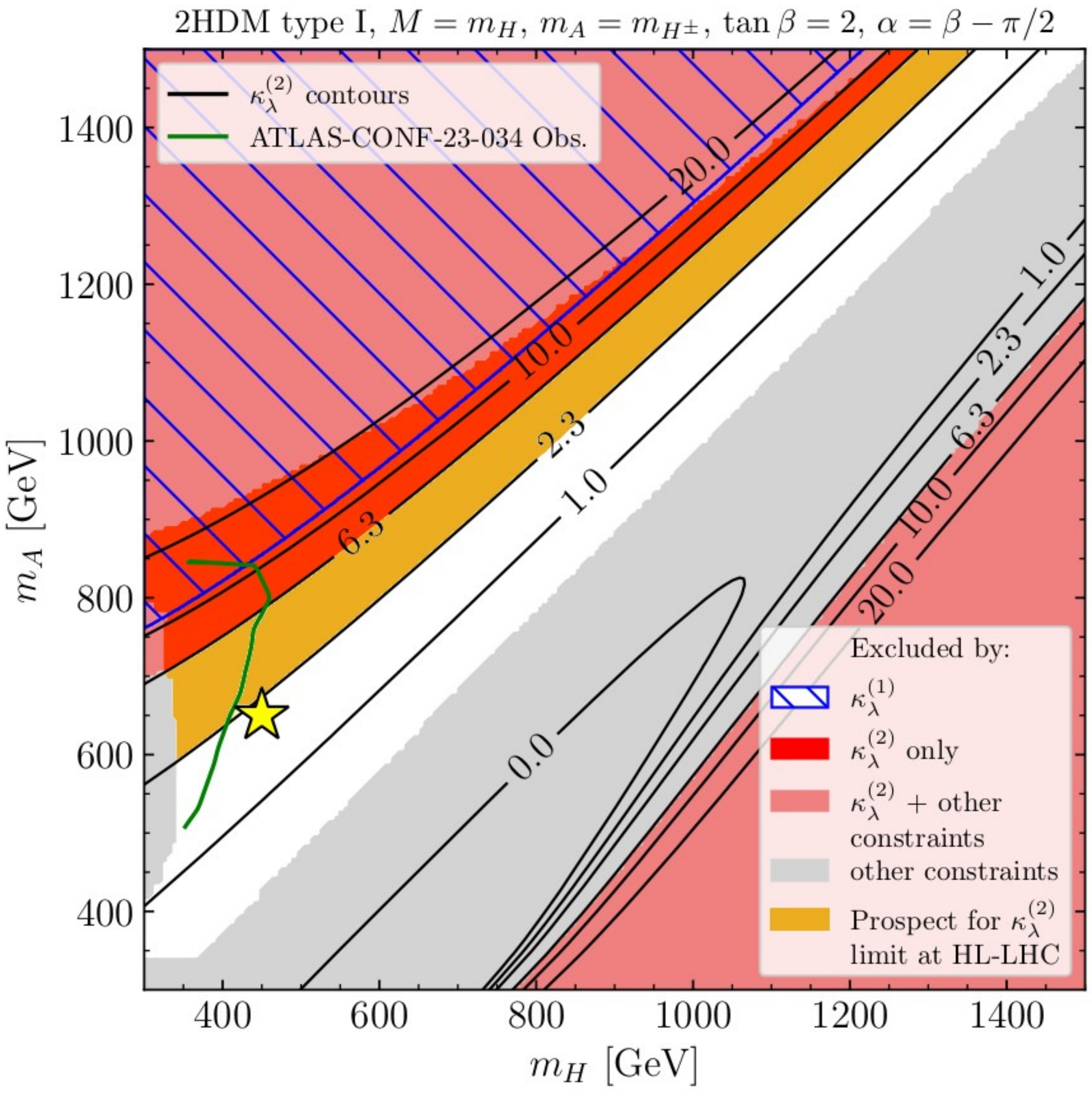
Suppose for instance the upper bound on κ_λ becomes $\kappa_\lambda < 2.3$



- ***Golden area:*** additional exclusion if the limit on κ_λ becomes $\kappa_\lambda^{(2)} < 2.3$ (achievable at HL-LHC)
- Of course, **prospects even better with an e^+e^- collider!**
- Experimental constraints, such as Higgs physics, may also become more stringent, however **not** theoretical constraints (like BFB or perturbative unitarity)

A benchmark scenario in the aligned 2HDM

In view of recent ATLAS-CONF-23-034



- ***Green line:*** additional exclusion from direct searches for heavy Higgs bosons, via
  A → Z H
  with full LHC-Run2 data
  [ATLAS-CONF-23-034]

- ***Small excess*** *(2.9 σ)* for **m_H ~ 450 GeV** and **m_A ~ 650 GeV**
  → near region probed by κ_λ at HL-LHC
  → complementarity between direct and indirect searches!

λ_{hhh} in relation to thermal history of the EWPT

At 1L

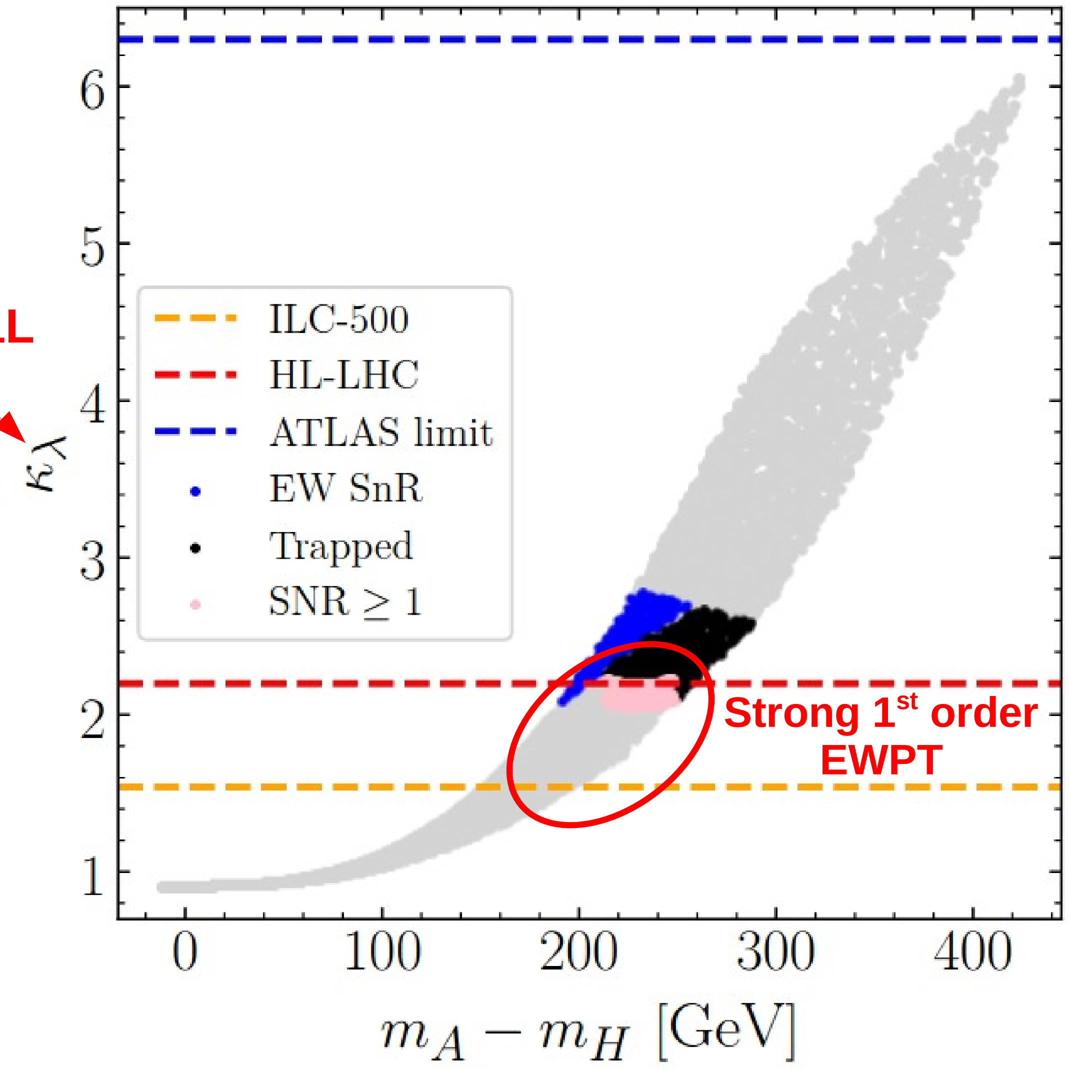


Figure from [Biekötter et al., 2208.14466]

- Corrections to λ_{hhh} correlate with the thermal history of the EWPT
  - If potential barrier is too high, the EWPT cannot occur → **vacuum trapping** (black region)
  - Conversely, it can occur that the **EW symmetry is not restored at high T** (blue region)
  - Strong 1st order EWPT, with gravitational waves (produced by bubble collisions) observable at LISA in pink
  - Impact of 2L corrections likely **strong**
- **Sphaleron decoupling condition**

$$\frac{v_n}{T_n} \text{ rather than } \frac{v_c}{T_c}, \qquad \frac{v_c}{T_c} \gtrsim 1$$

All receive quantum corrections!

Cosmological relics of a strong first-order phase transition

Gravitational waves from first-order phase transitions

[Grojean, Servant '06], [Caprini et al. '15, '19]

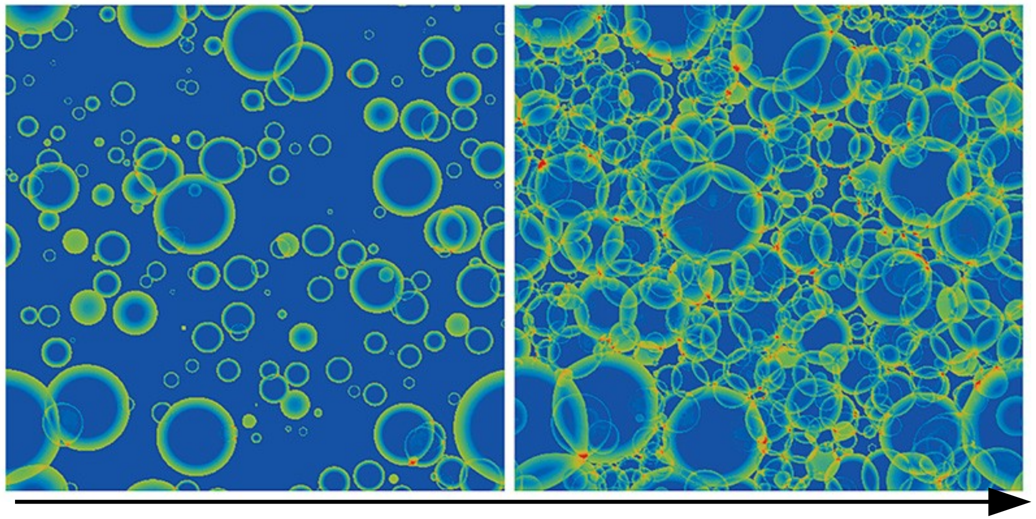
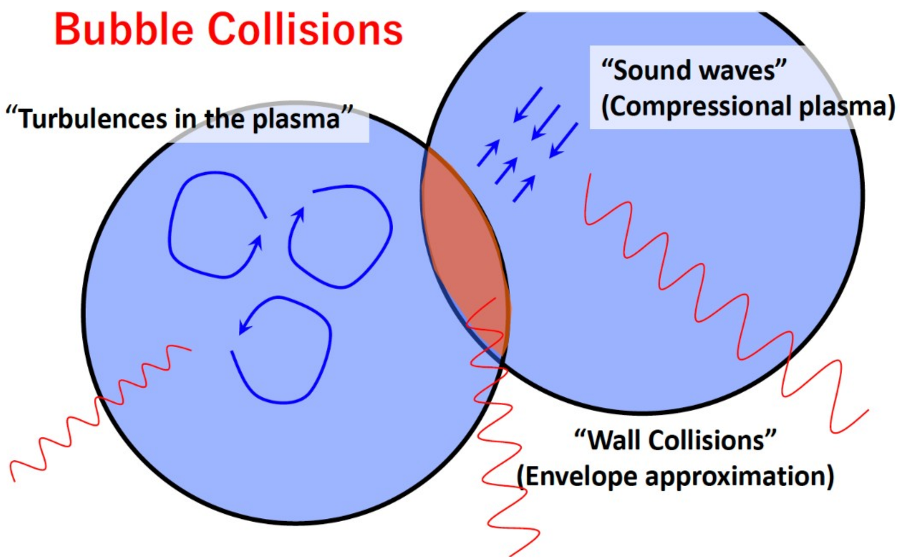


Figure adapted from [Jinno, Konstandin, Rubira '20] *and* [Servant '22]

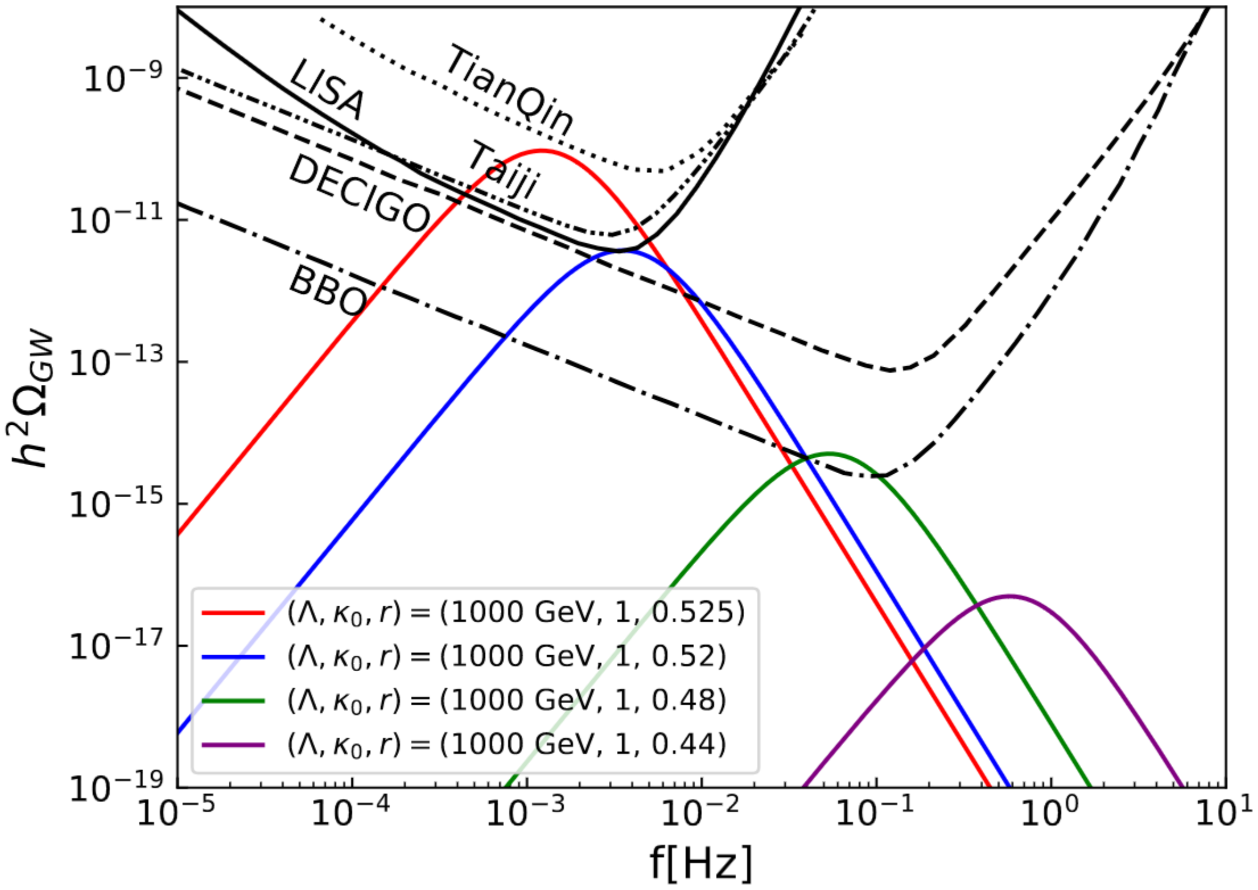


$$h^2\Omega_{\rm GW} \simeq h^2\Omega_{\phi\ \rm env} + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}$$

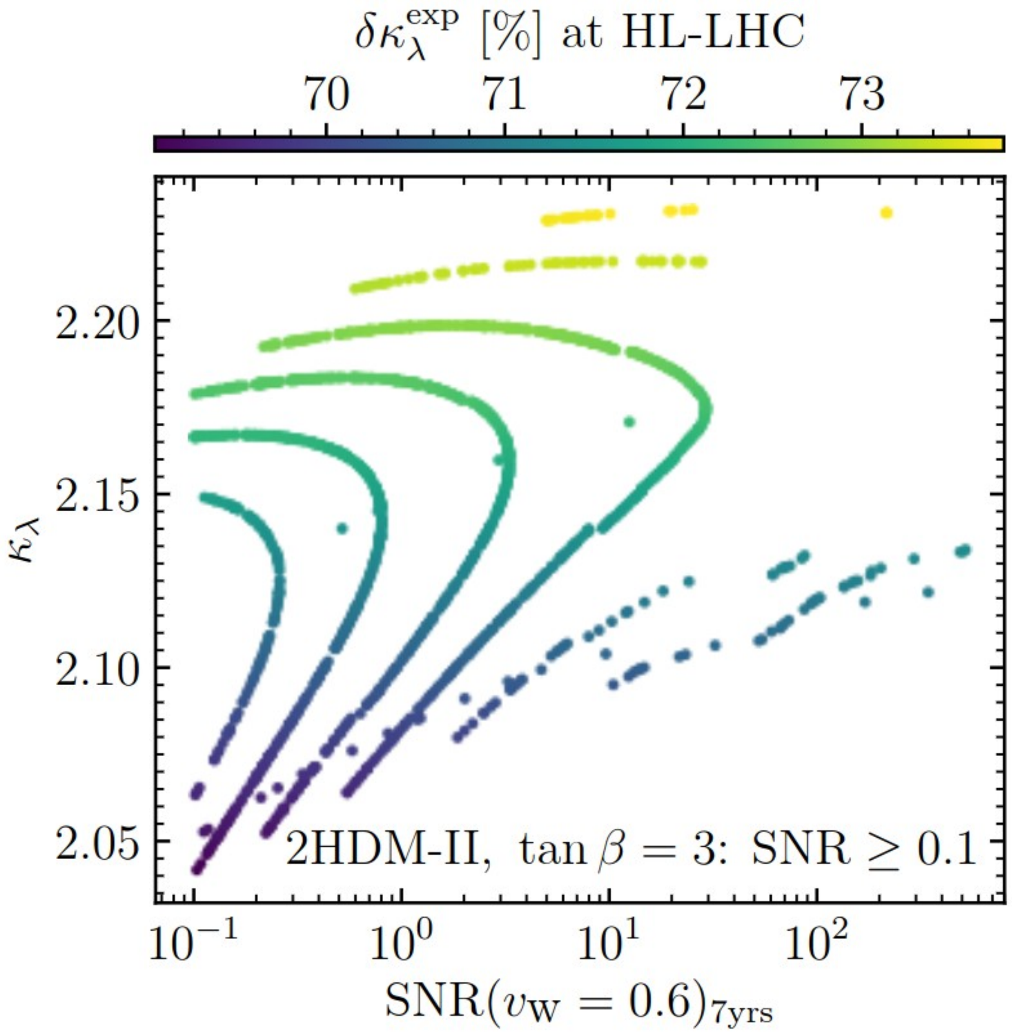
- For each contributions, results/estimates exist, which depends mostly on:
  - Assumptions for spectral shapes for different types of GW sources
  - **α**: "latent heat", ratio of vacuum energy density released in the transition to radiation bath density
    $\rightarrow \alpha \sim \rho_{vac}/\rho_{rad}{}^*$
  - **β/H_*** where β is (approx.) the inverse duration of the PT, and H_* is the Hubble parameter at T_* (temperature when GW are produced)
    $$\beta = -\frac{dS_E}{dt}\bigg|_{t=t_*} \simeq \frac{1}{\Gamma_{\rm nuc}}\frac{d\Gamma_{\rm nuc}}{dt}\bigg|_{t=t_*}$$
    S_E: Euclidian action of critical bubble
    Γ_{nuc}: bubble nucleation rate
  - **v_w**: bubble wall velocity
    (often taken as an assumption, but see workshop at DESY/UHH "How fast does the bubble grow?")

Probing scenarios of SFOEWPT with gravitational waves

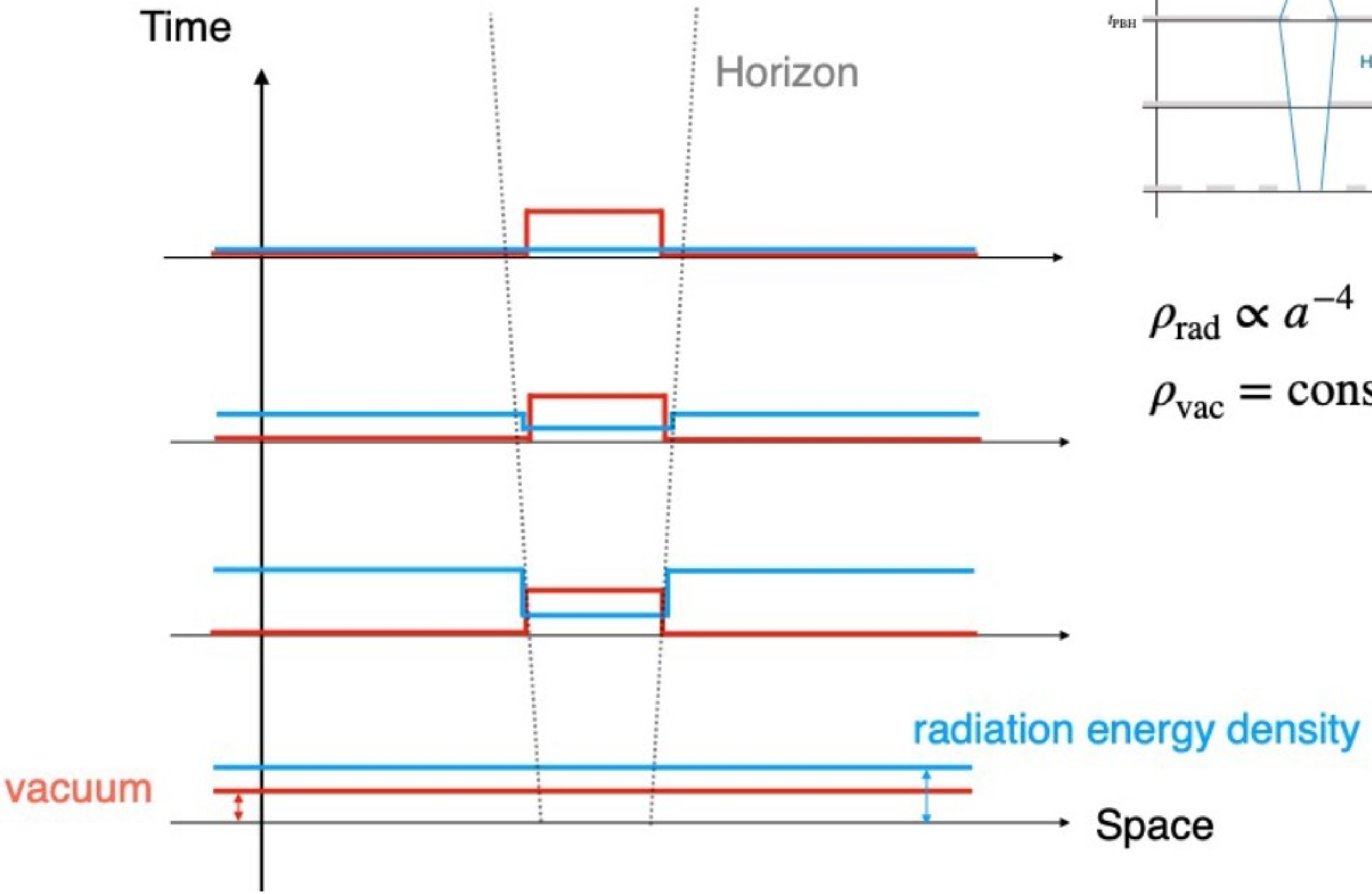
- *Exemple 1*: spectra of GW produced by the EWPT in the **near-aligned Higgs EFT** [Kanemura, Nagai '21], [Kanemura, Nagai, Tanaka '22]
  Λ: mass of BSM state(s); κ_0: no. of BSM d.o.f;
  r: "non-decouplingness" $\Lambda^2 = \mathcal{M}^2 + \tilde{\lambda} v^2 \,, r \equiv \frac{\tilde{\lambda} v^2}{\Lambda^2}$

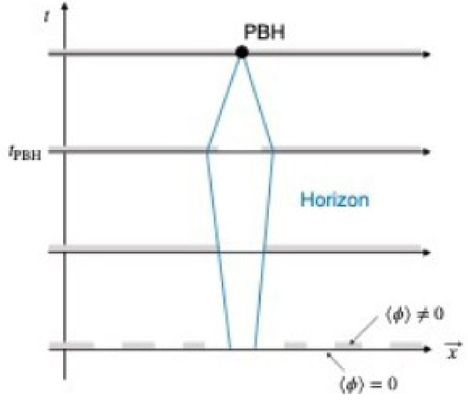


- *Exemple 2*: correlation of κ_λ and signal-to-noise ratio (SNR) of GW at LISA for **2HDM scenarios with SFOEWPT** [Biekötter et al. '22]



Primordial black holes from first-order phase transitions





$$\rho_{\mathrm{rad}} \propto a^{-4}$$

$$\rho_{\mathrm{vac}} = \mathrm{const}$$

Figure from [Kanemura ‘23]

Primordial black holes from first-order phase transitions

[Gouttenoire, Volansky PRL ‘23]

- Patches of Universe in which EWPT is (randomly) delayed can lead to **overdensities** sourcing **primordial black holes** (PBH)

- PBH formation if: $$\delta = \frac{\rho_{\text{over.}} - \rho_{\text{bkgd}}}{\rho_{\text{bfgd}}} \gtrsim \delta_c \sim 0.45$$

[Hawking ‘71], [Hawking, Carr, ‘74], [Harada, Yoo, Kohri ‘13]

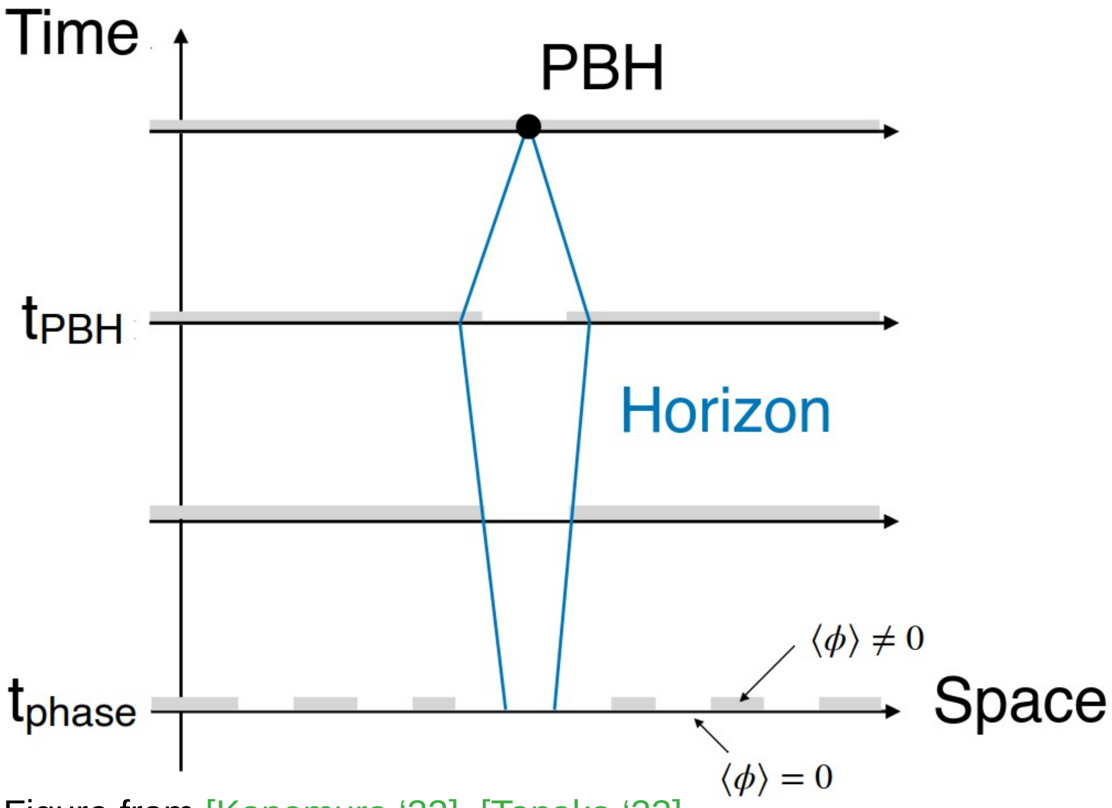


Figure from [Kanemura ‘23], [Tanaka ‘23]

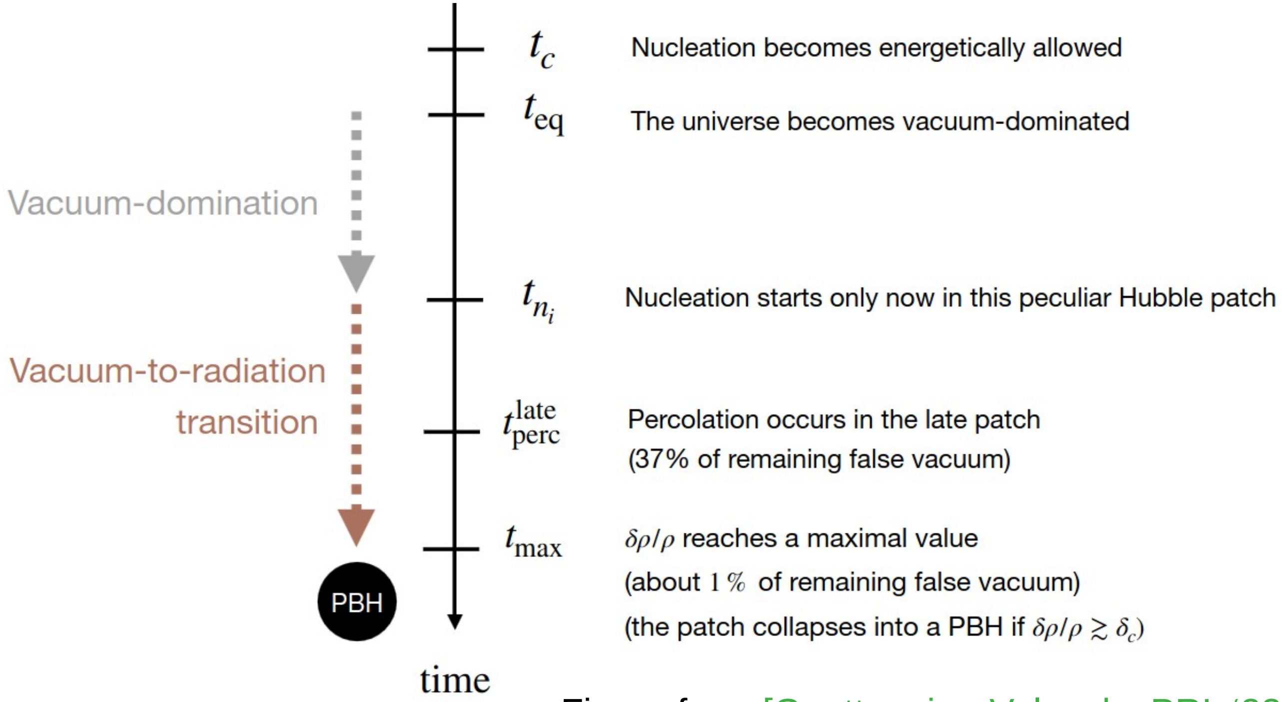
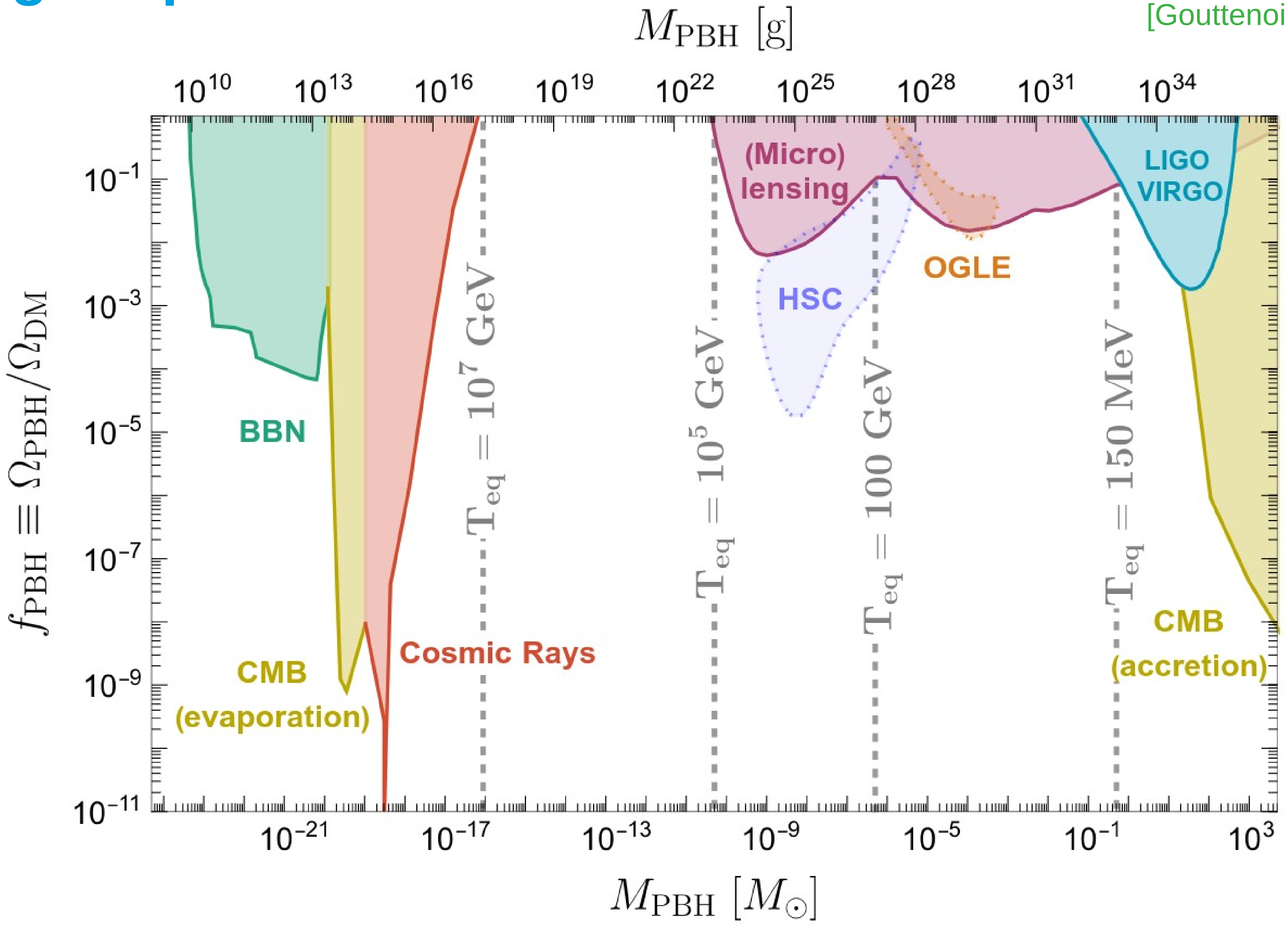


Figure from [Gouttenoire, Volansky PRL ‘23]

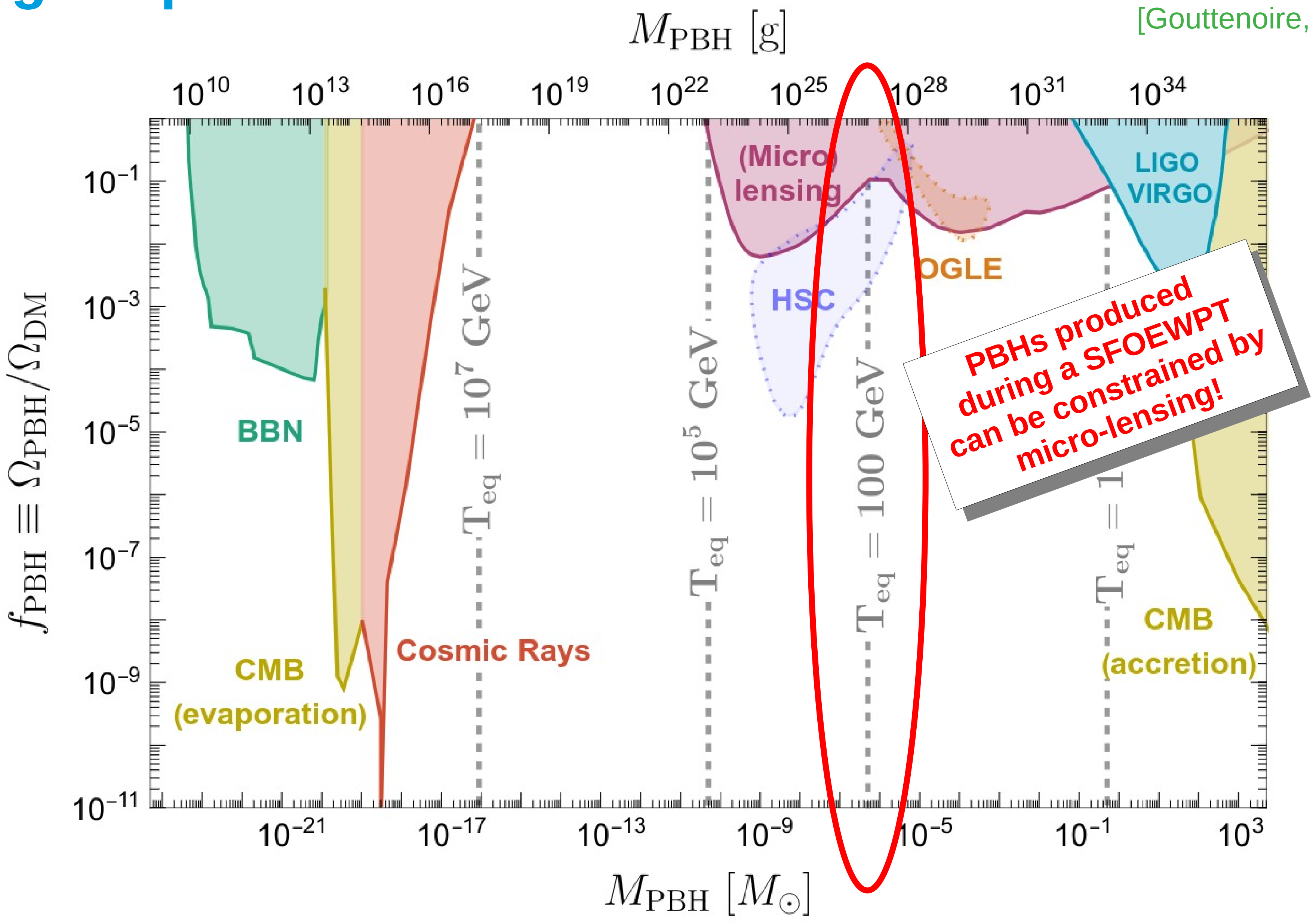
Searching for primordial black holes

[Gouttenoire, Volansky PRL ‘23]



Searching for primordial black holes

[Gouttenoire, Volansky PRL '23]

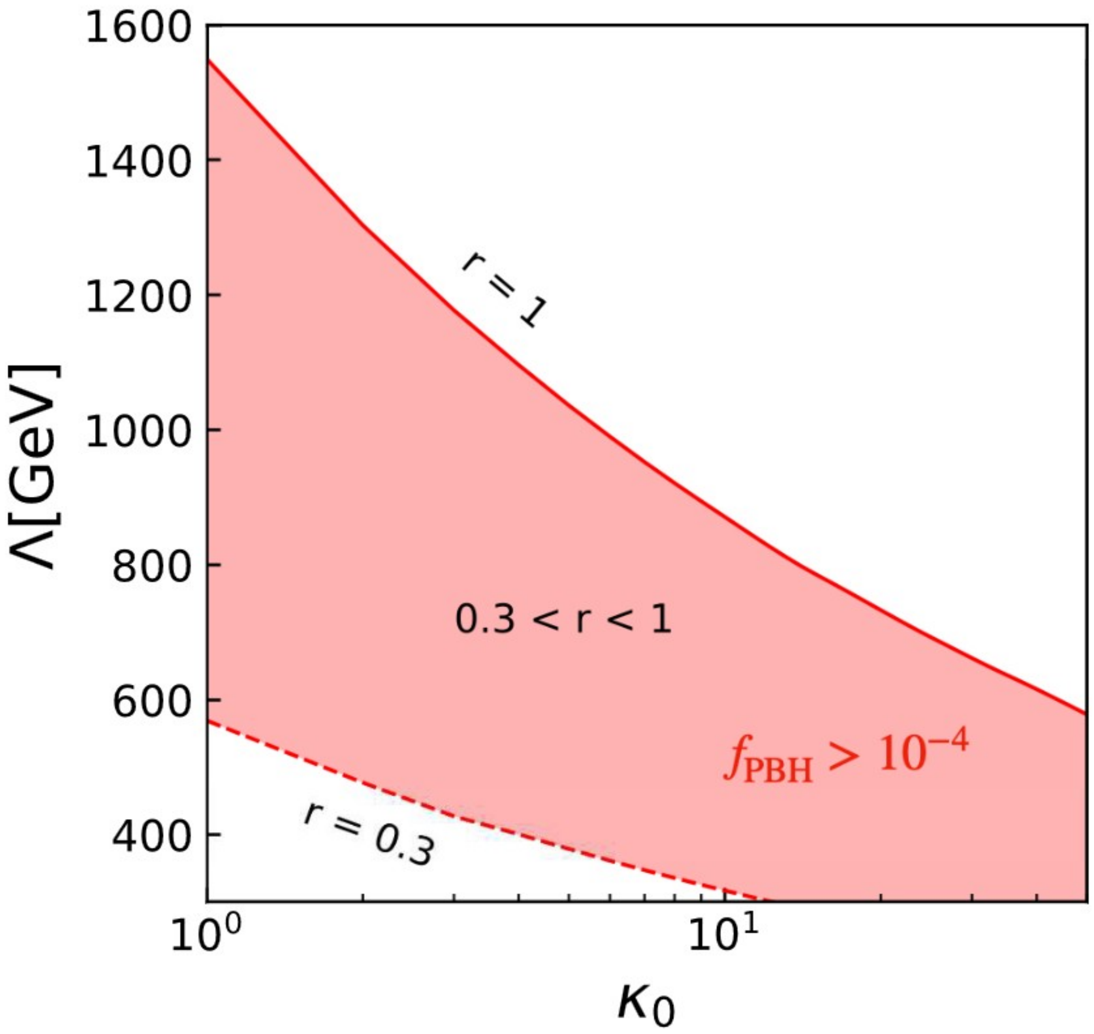


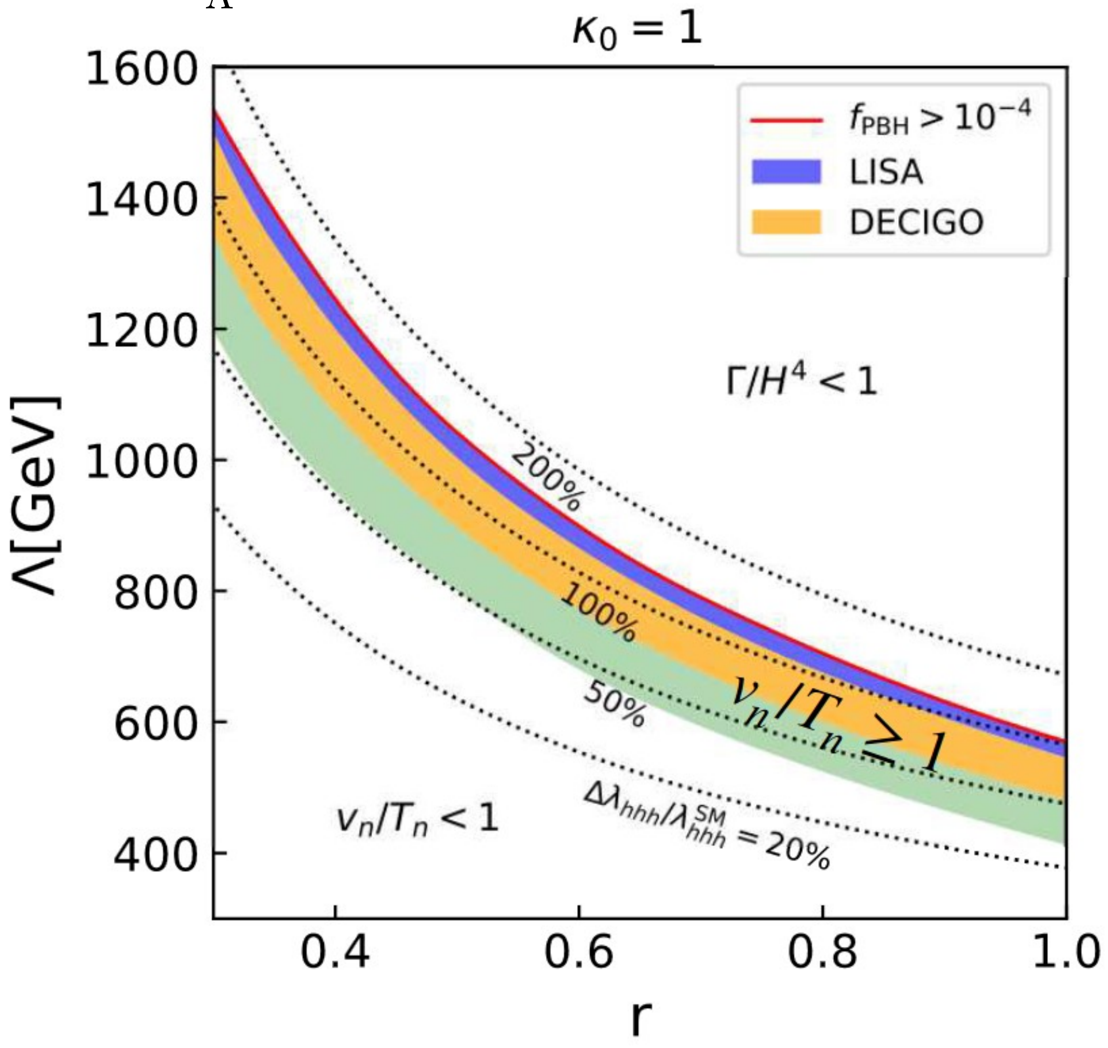
Complementary probes of SFOEWPT with PBHs

- **Production of PBHs and GW in near-aligned Higgs EFT**
  Λ: mass of BSM state(s); κ_0: no. of BSM d.o.f;
  r: "non-decouplingness"

[Hashino, Kanemura, Takahashi, Tanaka '23]

$$\Lambda^2 = \mathcal{M}^2 + \tilde{\lambda} v^2\,, r \equiv \frac{\tilde{\lambda} v^2}{\Lambda^2}$$





Summary

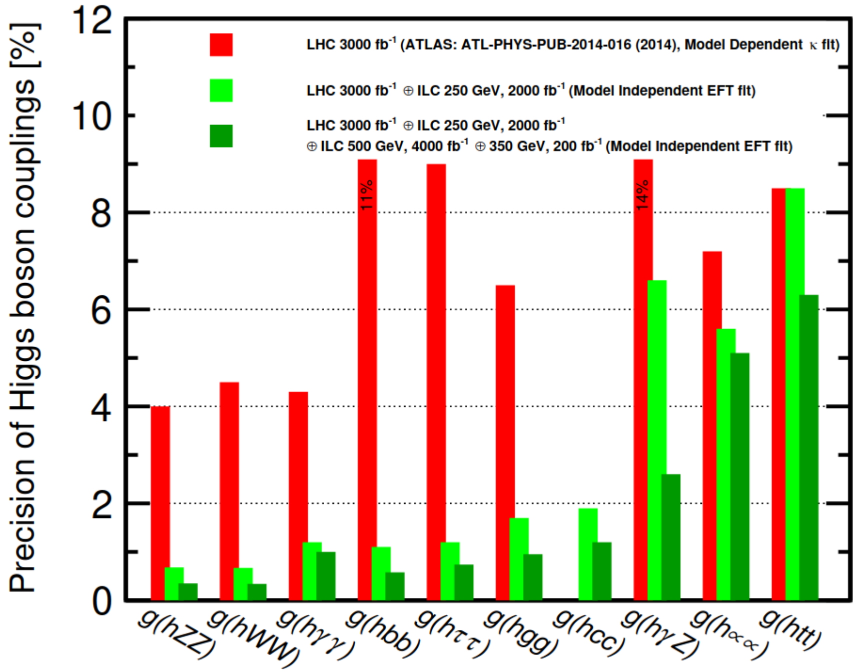
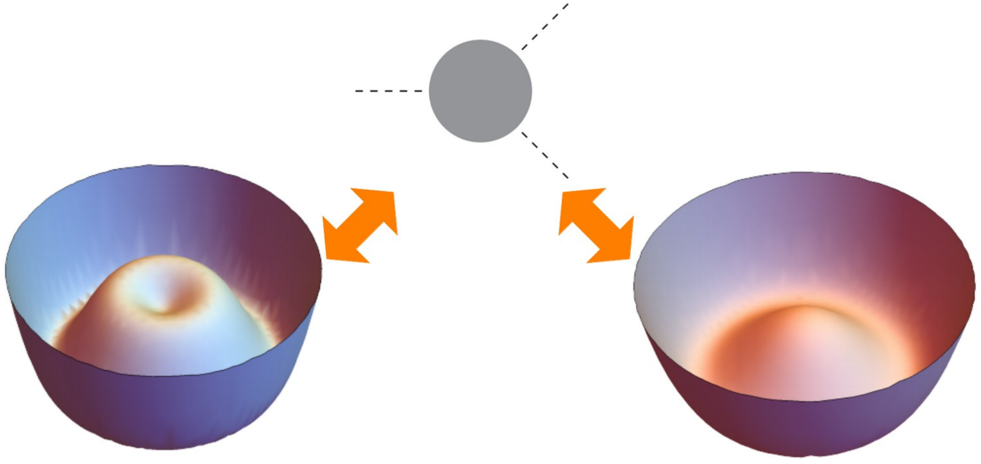
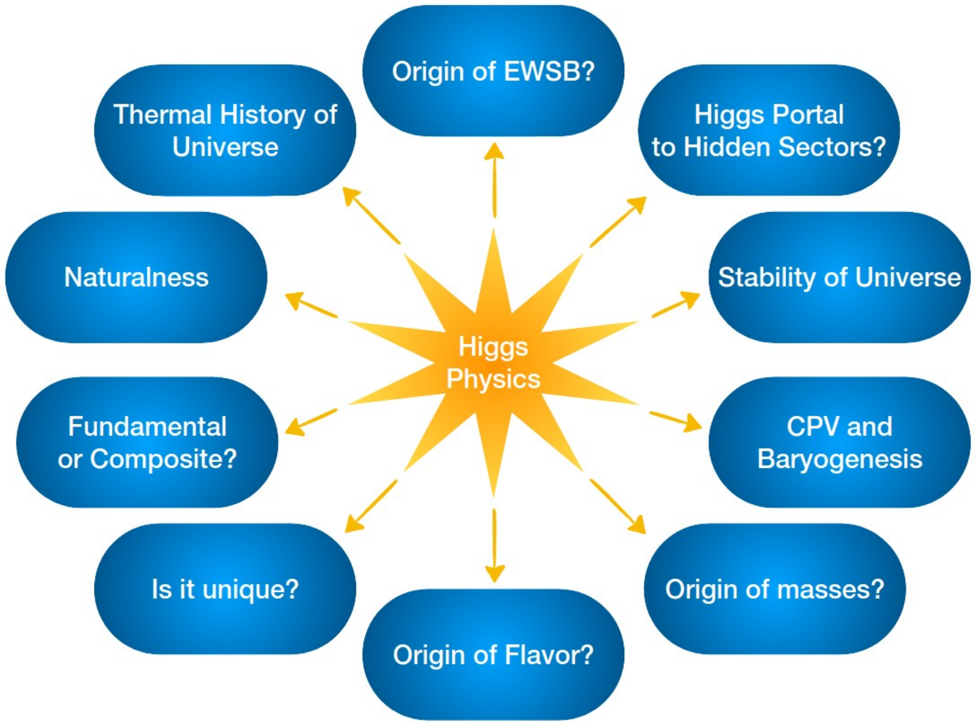


Figure from [ILC250 Physics case, 1710.07621]



The vision for the future of particle physics must acknowledge the central role of the Higgs field. The Higgs field is a crucial part of the Standard Model. It is our ignorance about this field that keeps us from solving the remaining mysteries that the Standard Model cannot address. To make progress, we must remedy this. We need to make clear (with apologies to Red Sanders and Vince Lombardi): **"Higgs isn't everything; it's the only thing."** A vision for particle physics that is not built on this idea cannot address the most profound questions for our field or realize its greatest opportunities.

[Peskin, Vision for Elementary Particle Physics 2302.05472]



[Snowmass report on Higgs Physics 2209.07510]

Thank you very much for your attention!

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