Advanced Course on Higgs Physics

1st Graduate Week of the Quantum Universe Research School

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Outline of the lectures

- Part 1: Why a Higgs boson is needed [Monday]
- Part 2: Connections between Higgs Physics and unanswered questions of Particle Physics (and possible solutions to them) [Monday]
- Part 3: What can be learnt from the Higgs boson at high-energy colliders – an overview [Yesterday]
- Part 4: The Higgs boson mass as a precision observable – calculations and interpretations [Partly yesterday/cont’d today]
- Part 5: The Higgs boson potential, its trilinear coupling, and relations with early-Universe evolution
Part 4: The Higgs boson mass, a new precision observable
Measurements of the Higgs boson mass

Higgs mass already measured at sub-permille level! → new precision observable!

\[ M_h = 125.09 \pm 0.21 \text{(stat.)} \pm 0.11 \text{(syst.)} \text{ GeV} \]

[ATLAS & CMS Run 1 combined, Moriond 2015]

\[ M_h = 125.11 \pm 0.11 \text{(stat.)} \pm 0.09 \text{(syst.)} \text{ GeV} \]

[ATLAS 2308.04775 from Run1+Run2 in \( h \to \gamma \gamma \) and \( h \to 4l \) channels]
Radiative corrections to the Higgs boson mass

- **Feynman-diagrammatic** calculations, *i.e.* solve for $M_h^2$

$$M_h^2 = \left( m_h^2 \right)_{\text{tree}} + \hat{\Sigma}_{hh}(p^2 = M_h^2)$$

$\hat{\Sigma}_{hh}(p^2)$ computed order-by-order in perturbation theory as Feynman diagrams.

**Difficulty**: momentum dependence of self-energy diagrams not always known at two loops and higher + long numerical calculations

- **Effective potential** approximation $V_{\text{eff}} = V^{(0)} + \Delta V$

where $\Delta V$ are quantum corrections, computed as
  - **one loop**: supertrace formula
  - **two loops and beyond**: 1PI vacuum bubble diagrams

$$\left. \frac{\partial^2 V_{\text{eff}}}{\partial h^2} \right|_{\text{min}} \leftrightarrow \hat{\Sigma}_{hh}(0)$$

⇒ much simpler/faster calculations, but with lower accuracy

- **tadpole equation(s)** $\left. \frac{\partial V_{\text{eff}}}{\partial h} \right|_{\text{min}} = 0$ are needed to properly relate all couplings
Two interpretations of Higgs mass calculations

- Higgs mass $M_h$ is computed as a function of Lagrangian parameters, in particular quartic Higgs coupling $\lambda$
  \[ M_h = M_h(\lambda, \cdots) \]

- **Case 1**: $\lambda$ is a free parameter of the theory
  - e.g. in SM and many extensions (SSM, 2HDM, etc.)
    → one cannot predict $M_h$
    → but one can use the equation $M_h(\lambda, \cdots) = 125.09$ GeV to **extract** $\lambda$ and study the **high-scale behaviour of the theory**

- **Case 2**: $\lambda$ is predicted by the theory
  - e.g: - in SUSY, $\lambda$ is related to other couplings (EW gauge couplings + eventually SUSY scalar couplings)
    - in (classical) scale invariant models, $\lambda=0$ at the scale at which the symmetry is imposed
    - but also the case in a non-SUSY extension of the SM taken as low-energy limit of a UV-model in which $\lambda$ is predicted (**more on this later**)
  → $M_h$ **can be predicted** as a function of the model parameters
    \[ (m_h^2)_{\text{tree}} \leq M_Z^2 c_{2\beta}, \quad M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2} \left( \ln \frac{M_{\text{SUSY}}^2}{m_t^2} + |\hat{X}_t|^2 - \frac{1}{12} |\hat{X}_t|^4 \right) + \ldots \]
  → Comparing computed and measured values of $M_h$ → **constrain allowed BSM parameter space**
Case 2: SUSY Higgs mass calculations
SUSY Higgs mass calculations – fixed-order calculation

- SUSY models contain extended scalar sectors → **physical masses** found as solutions for $p^2$ of equation

$$\det[p^2\delta_{ij} - (\mathcal{M}^2_{\text{tree}})_{ij} - \Delta \mathcal{M}^2_{ij}(p^2)] = 0$$

- At tree level, $m_h \leq M_Z$, however, since early 1990’s ([Okada, Yamaguchi, Yanagida '90], [Ellis, Ridolfi, Zwirner '90], [Haber, Hempfling '90]) it has been known that loop corrections can raise $m_h$ to 125 GeV

$$M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2v^2} \left( \ln \frac{M_{\text{SUSY}}^2}{m_t^2} + |\tilde{X}_t|^2 - \frac{1}{12} |\tilde{X}_t|^4 \right) + \ldots$$

- Since then, huge efforts to improve precision of SUSY Higgs mass calculations
  → summarised in recent report of “Precision SUSY Higgs Mass Calculation Initiative KUTS” ([Slavich, Heinemeyer (eds.) et al 2012.15629]
  → for the MSSM, state-of-the-art is now almost full 2L in effective-potential approximation, + leading 2L momentum-dependent effect + leading 3L corrections
  → for the NMSSM and beyond (e.g. Dirac gaugino models), leading 2L corrections

  + reliable estimates of theoretical uncertainties (from missing higher orders & parametric uncertainties) → 1-3 GeV depending on point

- However, experimental searches now put lower bounds on stop (scalar partner of top quarks) masses beyond 1 TeV → **fixed-order calculations start to suffer from large logs**

\[\tan \beta: \text{ratio of Higgs VEVs} \]
\[X_t: \text{stop mixing parameter} \]
\[M_{\text{SUSY}}: \text{SUSY-breaking scale} \]
\[\tilde{X}_t \equiv X_t/M_{\text{SUSY}} \]
SUSY Higgs mass calculations – the problem of large logs

- Scale of New Physics $M_{\text{NP}}$ driven higher by experimental searches

 $\Rightarrow$ in fixed-order calculations, **large logarithmic terms** $\propto \log \frac{M_{\text{NP}}}{m_{\text{EW}}}$ can spoil the accuracy, or even the validity, of the perturbative expansion, *e.g.*

$$\mathcal{O} = \underbrace{\alpha^0 a_0}_{\text{tree-level}} + \underbrace{\alpha (b_1 L + a_1)}_{\text{one-loop}} + \underbrace{\alpha^2 (c_2 L^2 + b_2 L + a_2)}_{\text{two-loop}} + \cdots$$

$\alpha \equiv (g/4\pi)^2$, $L \equiv \log \frac{M_{\text{NP}}}{m_{\text{EW}}}$, $a_i, b_i, c_i \in \mathbb{C}$.

**Loss of perturbativity** if

$$\alpha L \gtrsim 1 \quad \cdots$$

The perturbative expansion must be **reorganised** $\rightarrow$ **EFT calculation**
Intermezzo: an EFT primer

- **Integrate out heavy fields** at some scale $\Lambda \sim M_{\text{NP}}$ and work in a low energy EFT below $\Lambda$
- **Couplings in the EFT** receive **threshold corrections** at the matching scale $\Lambda$

![Diagram](image)

**UV theory**
(light & heavy particles)

$\Lambda \sim M_{\text{NP}}$

**EFT**
(light particles only)

Coupling of the UV theory $\tilde{g}$

$\tilde{\Gamma}^{\text{UV}}(Q = \Lambda) = \Gamma^{\text{EFT}}(Q = \Lambda)$

$\ell$-loop threshold correction to coupling of the EFT $g$

$g = \tilde{g} + \Delta g$

- **Use RGEs** to run the couplings from the high input scale, to the low scale ($\ll M_{\text{NP}}$) at which the calculation is performed  
  ⇒ **large logs are resummed**!
SUSY Higgs mass calculations – EFT approach

Simplest example: \( \text{UV theory } \rightarrow \text{MSSM, and EFT } \rightarrow \text{SM} \)

see e.g. [Bernal, Djouadi, Slavich '07], [Draper, Lee, Wagner '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15], [Bagnaschi, Pardo Vega, Slavich '17], [Athron et al. '17], [Harlander, Klappert, Ochoa Franco, Voigt '18], etc.

More choices of EFTs also considered, see [KUTS report '20] and refs. therein

e.g.

\[
\begin{align*}
E & \\
\downarrow & \\
\text{MSSM inputs} & + \Delta \lambda^1 (M_S) + \Delta \lambda^2 (M_S) + \Delta \lambda^3 (M_S) \\
& \downarrow \\
M^2_h = f(\lambda_{\text{SM}}(m_t)) & \quad \text{fixed-order calculation} \\
& \uparrow \\
\text{SM inputs} & \\
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& \\nb:\text{M}_S = M^{\text{SUSY}}
\]
SUSY Higgs mass calculations – hybrid approaches

- However, for lower $M_{\text{SUSY}}$, EFT calculations lose accuracy (because of $v/M_{\text{SUSY}}$ effects)
- Can one combine the advantages of fixed-order (reliable for low $M_{\text{SUSY}}$) and EFT (reliable for high $M_{\text{SUSY}}$)?
  → Yes!
- Different approaches
  1) FeynHiggs approach \[ (M_h^2)_{\text{FH hyb.}} = (m_h^2)_{\text{tree}} + \hat{\Sigma}^{\text{FO}}_{hh} (M_h^2)_{\text{hyb.}} + \left[ \lambda(M_t)v^2 \right]_{\text{logs}} - \left[ \hat{\Sigma}^{\text{FO}}_{hh} (M_h^2) \right]_{\text{logs}} \]
  2) FlexibleSUSY approach → “pole mass matching” \[ \lambda_{\text{SM}}(M_S) = \frac{1}{2v^2(M_S)} \left[ (M_h^2)_{\text{HET}} - (\Delta M_h^2)_{\text{SM}} \right] \]
  (also included in SARAH/SPheno)
  3) Aachen group solution \[ (M_h^2)_{\text{hyb.}} = (M_h^2)_{\text{EFT}} + \Delta_{\text{EFT}}^{\ell+1\ell} + \Delta_{\text{EFT}}^{2\ell} \]
Different types of SUSY Higgs mass calculations – summary

\[ M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2} \left( \ln \frac{M_{\text{SUSY}}^2}{m_t^2} + |\hat{X}_t|^2 - \frac{1}{12} |\hat{X}_t|^4 \right) + \ldots \]

3 types of calculations for \( M_h \):

- **Fixed-order approach:**
  + precise for low SUSY scales
  - but for high scales large logarithms \( \log(M_{\text{SUSY}}/m_t) \) spoil convergence of perturbative expansion

- **Effective field theory approach:**
  + precise for high SUSY scales (since logarithms are resumed)
  - but for low scales \( O(v/M_{\text{SUSY}}) \) terms are missed if higher-dimensional operators are not included

- **Hybrid approach combing FO and EFT approaches:**
  ++ precise for both low and high SUSY scales.

Current status in FeynHiggs *(c.f. figure)*

- FO: full 1L + 2L in gaugeless limit,
- EFT: full leading-log (LL) + Next-to-LL (NLL) + NNLL + partial \( N^3\)LL in gaugeless limit

[KUTS report, Slavich, Heinemeyer et al. ‘20]
Accessing the stop mixing parameter $X_t$ via the Higgs boson mass

- $X_t$ enters prediction of $M_h$ from 1L:

$$M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2} \left( \ln \frac{M_{\text{SUSY}}^2}{m_t^2} + |\tilde{X}_t|^2 - \frac{1}{12} |\tilde{X}_t|^4 \right) + \ldots$$

- **Blue/green lines**: common mass scenarios, i.e. all non-SM masses $= M_{\text{SUSY}}$ and $A_{\text{at}} = 0$

- **Grey points**: scan over SUSY parameters (masses and trilinears) between $M_{\text{SUSY}}/2$ and $2 M_{\text{SUSY}}$

- Significant dependence of $M_h$ on $X_t$, even for high SUSY scale, at 10 or 100 TeV!

- If stop masses and $\tan\beta$ known → $X_t$ can be extracted from $M_h$

[Bahl, JB, Weiglein '22] with FeynHiggs 2.18.1
Automating Higgs mass computations
The motivation for automation

- Interest for non-minimal SUSY and non-SUSY models is growing, driven by experimental results, but in most cases Higgs mass calculations beyond one-loop are still missing → huge uncertainties

- Computing corrections from the beginning for every new model would be extremely inefficient and time consuming!

- **Idea:**
  Do the **calculation for a general renormalisable theory** and then apply that result to the considered model → can be automated, in public tools like SARAH [Staub ‘08-‘15] or FlexibleSUSY [Athron et al. ‘14, ‘17]
Generic calculations of the Higgs boson mass – conventions

Write the most general renormalisable interactions with real scalars $\phi_i$, Weyl fermions $\psi_I$ and vector bosons $A^{a\mu}_\mu$:

$$\mathcal{L}_S = -\frac{1}{6} \lambda^{ijk} \phi_i \phi_j \phi_k - \frac{1}{24} \lambda^{ijkl} \phi_i \phi_j \phi_k \phi_l,$$

$$\mathcal{L}_{SF} = -\frac{1}{2} g^{IJK} \psi_I \psi_J \phi_K + \text{c.c.},$$

$$\mathcal{L}_{SV} = -\frac{1}{2} g^{abi} A^{a\mu}_\mu A^{b\nu}_\nu \phi_i - \frac{1}{4} g^{abij} A^{a\mu}_\mu A^{b\nu}_\nu \phi_i \phi_j - g^{aij} A^{a\mu}_\mu \phi_i \partial^\mu \phi_j,$$

$$\mathcal{L}_{FV} = g^{aJ}_I \psi_I \sigma^\mu \psi_J A^{a}_\mu,$$

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2} g^{abc} A^{a\mu}_\mu A^{b\nu}_\nu \partial^\mu A^{c\nu}_\nu - \frac{1}{4} g^{abc} g^{ced} A^{a\mu}_\mu A^{b\nu}_\nu A^{c}_\mu A^{d}_\nu + g^{abc} A^{a}_\mu \omega^b \partial^\mu \omega^c$$

➢ Here, all fields are defined in mass-diagonal bases
  (some care needed to diagonalise scalar masses)

➢ Interactions between scalars and ghosts turned off by working in Landau gauge

➢ Parameters usually* renormalised in minimal subtraction schemes ($\overline{\text{MS}}$ or $\overline{\text{DR}}$)
  (*: with one notable exception $\rightarrow$ anyH3 in Part 5)
Generic calculations of the Higgs boson mass – diagrams

Then we need to compute loop diagrams for $V_{\text{eff}}$, the tadpole equations and the mass diagrams, e.g.

Generic 2L results available for:

- $V_{\text{eff}}$:
  [Martin ‘01] (Landau gauge),
  [Martin, Patel ‘18] (general gauge)
  (3L $V_{\text{eff}}$ in [Martin ‘17])

- Tadpoles:
  [Goodsell, Killian, Staub ‘15]

- Self-energies:
  [Martin ‘03, ‘05],
  [Goodsell, Paßehr ‘19]
Generic calculations of the Higgs boson mass with **SARAH/SPheno**

For extended scalar sectors:
- neutral scalar masses @ 2L;
- charged scalar masses @ 1L

Many other observables also available! (decays, STU, etc.)
Part 5: Higgs potential, trilinear Higgs coupling, and early-Universe evolution
Higgs potential, trilinear Higgs coupling(s), and Electroweak Phase Transition
Higgs potential – the “easy picture”

→ a strong first-order phase transition (SFOEWPT), motivated in particular by EWBG, usually* correlates with a deviation in $\lambda_{hhh}$ from its prediction in the SM

[*: if the EWPT occurs along the direction of the EW VEV in field space]
Kateryna’s question: why is there such a correlation between $\lambda_{hhh}(T=0)$ and the EWPT ($T \sim 100$ GeV)?

- Dynamics of EWPT controlled by finite-temperature effective potential

$$V_{\text{eff}}(\varphi, T) = V^{(0)}(\varphi) + \Delta V^{T=0}(\varphi) + \Delta V^T(\varphi, T) \xrightarrow{T \gg m} D(T^2 - T^2_0)\varphi^2 + ET\varphi^3 + \frac{1}{4}\lambda(T)\varphi^4 + \ldots$$

- At critical temperature $T_c$: 2 degenerate minima at $\varphi = 0$, and $\varphi_c \sim 2ET_c/\lambda(T_c)$, so that the sphaleron decoupling condition (to ensure a strong FOEWPT) becomes

$$\frac{\varphi_c}{T_c} = \frac{2E}{\lambda(T_c)} \gtrsim 1$$

- In a model with an extended (and aligned) Higgs sector,

$$(\Phi: \text{additional scalars}, M: \text{BSM mass scale}, n_{\Phi}: \text{no. of d.o.f of scalar } \Phi)$$

$$\frac{\varphi_c}{T_c} \simeq \frac{1}{3\pi v m_h^2} \left[ 6m_W^3 + 3m_Z^3 + \sum_{\Phi} n_{\Phi} m_{\Phi}^3 \left( 1 - \frac{M^2}{m_{\Phi}^2} \right)^3 \left( 1 + \frac{3M^2}{2m_{\Phi}^2} \right) \right]$$

while the corrections to $\lambda_{hhh}$ (more later) are

$$\lambda_{hhh} \simeq \frac{3m_h^2}{v} - \frac{1}{16\pi^2} \left[ -\frac{48m_t^4}{v^3} + \sum_{\Phi} \frac{4n_{\Phi} m_{\Phi}^4}{v^3} \left( 1 - \frac{M^2}{m_{\Phi}^2} \right)^3 \right]$$

[Kanemura, Okada, Senaha '05]
(see also [Grojean, Servant, Wells '04])
Higgs potential – a more realistic BSM picture

For instance, for a $\mathbb{Z}_2$ SSM where the $\mathbb{Z}_2$ symmetry is spontaneously broken $\rightarrow$ $S$ gets a VEV $v_S$

\[
S = s + v_S, \\
\Phi = \frac{1}{\sqrt{2}} \left( \sqrt{2}G^+ \right) \\
\phi = \frac{1}{\sqrt{2}} \left( v + h + iG \right)
\]

\[
V(\Phi, S) = \mu^2 |\Phi|^2 + \frac{\lambda_H}{2} |\Phi|^4 + \frac{m_S^2}{2} S^2 + \frac{\lambda_S}{2} S^4 + \frac{\lambda_{SH}}{2} S^2 |\Phi|^2
\]

\[
\begin{align*}
\lambda_H &= \frac{1}{2v^2} \left( m_{h_1}^2 + m_{h_2}^2 + (m_{h_1}^2 - m_{h_2}^2) \cos(2\alpha) \right), \\
\lambda_S &= \frac{1}{8v_S^2} \left( m_{h_1}^2 + m_{h_2}^2 - (m_{h_1}^2 - m_{h_2}^2) \cos(2\alpha) \right), \\
\lambda_{SH} &= \frac{1}{2vv_S} \left( m_{h_1}^2 - m_{h_2}^2 \right) \sin(2\alpha),
\end{align*}
\]

\[
\begin{align*}
m_{h_2} &= v_S = 300 \text{ GeV}, \alpha = -0.01 \\
m_{h_2} &= 900 \text{ GeV}, v_S = 600 \text{ GeV}, \alpha = -0.01
\end{align*}
\]
Experimental probes of the trilinear Higgs coupling
Experimental probes of $\lambda_{hhh}$

- **Double-Higgs production** $\rightarrow \lambda_{hhh}$ enters at **leading order (LO)** $\rightarrow$ most direct probe!

- **Single-Higgs production** $\rightarrow \lambda_{hhh}$ enters at **NLO** (i.e. indirect probe)

- **Electroweak Precision Observables** (EWPOs) $\rightarrow \lambda_{hhh}$ enters at **NNLO** (i.e. indirect probe)

[NB: triple-Higgs production in a few slides]

[Degrassi, Fedele, Giardino '17] [Degrassi, Giardino, Maltoni, Pagani '16] [ATLAS-CONF-2019-049]

[Degrassi, Fedele, Giardino '17]
Probing $\lambda_{hhh}$ via double-Higgs production

**gluon-fusion**

$\sigma_{ggF}(pp \to HH) = 31.05 \text{ fb}$

**VBF**

$\sigma_{VBF}(pp \to HH) = 1.726 \text{ fb}$

**VHH**

$\sigma_{VHH}(pp \to HH) = 0.86 \text{ fb}$

*Slide by K. Leney @ HiggsDays 23*
Probing $\lambda_{hhh}$ via double-Higgs production

- **Double-Higgs production** $\rightarrow \lambda_{hhh}$ enters at LO $\rightarrow$ **most direct probe of $\lambda_{hhh}$**

- **Box and triangle diagrams** interfere destructively $\rightarrow$ small prediction in SM $\rightarrow$ BSM deviation in $\lambda_{hhh}$ can **significantly enhance double-Higgs production**!

- **Search limits on double-Higgs production** $\rightarrow$ limits on effective coupling $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{\text{SM}}$

- **Current best limits:** -0.4 < $\kappa_\lambda$ < 6.3 (95% CL) [ATLAS PLB ‘23] (including information from single-Higgs production)
  - -1.4 < $\kappa_\lambda$ < 6.1 (95% CL) [ATLAS PLB ‘23]
  - including information from single-Higgs production + $\kappa_t$ floating
  - -1.2 < $\kappa_\lambda$ < 6.5 (95% CL) [CMS ‘22]
Probing $\lambda_{hhh}$ via double-Higgs production

- Double-Higgs production $\rightarrow \lambda_{hhh}$ enters at LO $\rightarrow$ most direct probe of $\lambda_{hhh}$

- Box and triangle diagrams interfere destructively $\rightarrow$ small prediction in SM
  $\rightarrow$ BSM deviation in $\lambda_{hhh}$ can significantly enhance double-Higgs production!

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- Current best limits: $-0.4 < \kappa_\lambda < 6.3$ (95% CL) [ATLAS PLB ‘23]
  (including information from single-Higgs production)
  $-1.4 < \kappa_\lambda < 6.1$ (95% CL) [ATLAS PLB ‘23]
  (including information from single-Higgs production + $\kappa_t$ floating)
  $-1.2 < \kappa_\lambda < 6.5$ (95% CL) [CMS ‘22]
Probing $\lambda_{hhh}$ via double-Higgs production – HL-LHC prospects

- **Double-Higgs production** → $\lambda_{hhh}$ enters at LO → **most direct probe of $\lambda_{hhh}$**

- Box and triangle diagrams **interfere destructively** → small prediction in SM → BSM deviation in $\lambda_{hhh}$ can **significantly enhance double-Higgs production!**

- Search limits on double-Higgs production → **limits on effective coupling** $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})_{SM}$

- Prospects at HL-LHC: $0.1 < \kappa_\lambda < 2.3$ (95% CL) with ATLAS+CMS
  
  \[0.0 < \kappa_\lambda < 2.7\] (95% CL) with ATLAS alone

  \[\text{[Cepeda et al. '19]}\]

  \[\text{[ATL-PHYS-PUB-2022-053]}\]

Figure adapted from [ATL-PHYS-PUB-2022-053]
Direct probes of $\lambda_{hhh}$ at $e^+e^-$ colliders

- Double-Higgs production, either in $e^+e^- \rightarrow Zhh$ or $e^+e^- \rightarrow \nu\bar{\nu}hh$

- Relies however on being above the Zhh threshold!

Figure from [De Blas et al. 1905.03764]

- $e^+e^- \rightarrow Zhh$ better at $\sqrt{s} \sim 500$ GeV
- $e^+e^- \rightarrow \nu\bar{\nu}hh$ better for larger $\sqrt{s}$

Figure from [De Blas et al. 1812.02093]
Indirect probes of $\lambda_{hhh}$ at $e^+e^-$ colliders

- Below the Zhh threshold, $\lambda_{hhh}$ can still be investigated through its (indirect) effect in quantum corrections to single-Higgs production.

- In particular, $\lambda_{hhh}$ enters NLO corrections to $e^+e^- \rightarrow Zh$
  First pointed out in [McCullough '13], numerous works since (also with global analyses in EFT setting)

![Diagram](image-url)

Figure adapted from [McCullough 1312.3322]
Future determination of $\lambda_{hhh}$

Expected sensitivities in literature, assuming $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

- HL-LHC limits will likely outperform 2019 prospects (even with global analyses)

- Single-Higgs results at lepton colliders always include information from HL-LHC, and don't improve much (if at all)

- Significant improvements only with double-Higgs production at (high-energy) lepton colliders or FCC-hh

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see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], etc.
New investigations via triple-Higgs production

Constraining the trilinear and quartic Higgs couplings at the same time

\[ \kappa_3 = \kappa_\lambda : \text{trilinear coupling modifier} \]
\[ \kappa_4 : \text{quartic coupling modifier} \]

Figure adapted from [Maltoni, Pagani, Zhao 1802.07616]
Future determination of $\lambda_{hhh}$

Achieved accuracy actually depends on the value of $\lambda_{hhh}$

See also [Dürig, DESY-THESIS-2016-027]

[J. List et al. ‘21]
Calculating $\lambda_{hhh}$ in models with extended scalar sectors
One-loop mass-splitting effects

- **Leading one-loop** corrections to $\lambda_{hhh}$ in models with extended sectors (like 2HDM):

$$\delta^{(1)} \lambda_{hhh} \supset \frac{1}{16\pi^2} \left[ -\frac{48m_t^4}{v^3} + \sum_{\Phi} \frac{4n_{\Phi} m_{\Phi}^4}{v^3} \left(1 - \frac{M^2}{m_{\Phi}^2}\right)^3 \right]$$

- $M$ : BSM mass scale, e.g. soft breaking scale $M$ of $Z_2$ symmetry in 2HDM
- $n_{\Phi}$ : # of d.o.f of field $\Phi$

- Size of new effects depends on how the BSM scalars acquire their mass: $m_{\Phi}^2 \sim M^2 + \tilde{\lambda} v^2$

$$\left(1 - \frac{M^2}{m_{\Phi}^2}\right)^3 \rightarrow \begin{cases} 0, & \text{for } M^2 \gg \tilde{\lambda} v^2 \\ 1, & \text{for } M^2 \ll \tilde{\lambda} v^2 \end{cases}$$

Huge BSM effects possible!

First found in 2HDM: [Kanemura, Kiyoura, Okada, Senaha, Yuan '02]
One-loop mass-splitting effects

- Leading one-loop corrections to $\lambda_{hhh}$ in models with extended sectors (e.g. 2HDM):
  - SM top quark loop
  - BSM scalar loops

- Size of new effects depends on how the BSM scalars acquire their mass:
  - First found in 2HDM: [Kanemura, Kiyoura, Okada, Senaha, Yuan ’02]
  - Huge BSM effects possible!

Plot from [Kanemura, Okada, Senaha, Yuan ’04]

$\delta^{(1)} \lambda_{hhh} \supset$

$\mathcal{M}$: BSM mass

$n_\Phi$: # of d.o.f of

$\lambda^{2} + \tilde{\lambda} v^{2}$
anyH3: full 1L calculation of $\lambda_{hhh}$ in any renormalisable model
anyH3: mass-splitting effects in various BSM models

- Consider the non-decoupling limit in several BSM models

\[ M_{BSM}^2 = \mathcal{M}^2 + \tilde{\lambda} v^2 \]

- Increase \( M_{BSM} \), keeping \( \mathcal{M} \) fixed
  → large mass splittings
  → large BSM effects!

- Perturbative unitarity checked with anyPerturbativeUnitarity

- Constraints on BSM parameter space!

Here: scenarios with lightest BSM scalar mass & BSM mass param. at 400 GeV; other BSM scalar masses = \( M_{BSM} \)
Two-loop calculation of $\lambda_{hhh}$

Goal: How large can the two-loop corrections to $\lambda_{hhh}$ become?
An effective Higgs trilinear coupling

➢ In principle: consider 3-point function $\Gamma_{hhh}$
   but this is momentum dependent $\rightarrow$ very difficult beyond one loop

➢ Instead, consider an effective trilinear coupling

$$\lambda_{hhh} \equiv \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{\text{min}}$$

entering the coupling modifier

$$\kappa_\lambda = \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})^{SM}}$$

with $(\lambda_{hhh}^{(0)})^{SM} = \frac{3m_h^2}{v}$

constrained by experiments (applicability of this assumption discussed later)
Effective-potential calculation

\textbf{Step 1:} compute \( V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2} V^{(1)} + \frac{1}{(16\pi^2)^2} V^{(2)} \) \hspace{1em} (\text{MS result})

\( V^{(2)}: \) 1PI vacuum bubbles

\textit{Dominant BSM contributions to} \( V^{(2)} = \) diagrams involving \textbf{heavy BSM scalars and top quark}

\textbf{Neglect masses of light states} (SM-like Higgs, light fermions, ...)

\[ \text{2HDM} \]

\[ \text{SM} \]
Effective-potential calculation

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2} V^{(1)} + \frac{1}{(16\pi^2)^2} V^{(2)}$ (MS result)
  
  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to $V^{(2)}$* = diagrams involving heavy BSM scalars and top quark

- **Step 2**: derive an effective trilinear coupling

\[
\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \bigg|_{\text{min.}} = \frac{3}{\nu} [M_h^2] V_{\text{eff}} \left[ \frac{\partial^3}{\partial h^3} - \frac{3}{\nu} \left( \frac{\partial^2}{\partial h^2} - \frac{1}{\nu} \frac{\partial}{\partial h} \right) \right] \Delta V \bigg|_{\text{min.}}
\]

*Express tree-level result in terms of effective-potential Higgs mass*
Effective-potential calculation

- **Step 1:** compute \( V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2} V^{(1)} + \frac{1}{(16\pi^2)^2} V^{(2)} \) (\( \overline{\text{MS}} \) result)

  - \( V^{(2)} \): 1PI vacuum bubbles
  - **Dominant BSM contributions to** \( V^{(2)} \) **= diagrams involving heavy BSM scalars and top quark**

- **Step 2:**
  \[
  \lambda_{hhh} \equiv \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{\text{min.}} = 3[M_h^2]_{V_{\text{eff}}} + \left[ \frac{\partial^3}{\partial h^3} - \frac{3}{v} \left( \frac{\partial^2}{\partial h^2} - \frac{1}{v} \frac{\partial}{\partial h} \right) \right] \Delta V \bigg|_{\text{min.}}
  \]
  (\( \overline{\text{MS}} \) result too)

- **Step 3:** conversion from \( \overline{\text{MS}} \) to OS scheme
  - **Express result in terms of pole masses:** \( M_t, M_h, M_\Phi \) (\( \Phi=H,A,H^\pm \)); OS Higgs VEV \( v_{\text{phys}} = \frac{1}{\sqrt{\sqrt{2}G_F}} \)
  - Include **finite WFR:** \( \hat{\lambda}_{hhh} = (Z_{h}^{\text{OS}} / Z_{h}^{\overline{\text{MS}}})^{3/2} \lambda_{hhh} \)
  - Prescription for \( M \) to ensure **proper decoupling** with \( M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2 \) and \( \tilde{M} \to \infty \)

[JB, Kanemura '19]
Our results in the aligned 2HDM

Taking degenerate BSM scalar masses: $M_\Phi = M_H = M_A = M_H^\pm$

[JB, Kanemura '19]

$\sqrt{M_\Phi^2 - \tilde{M}^2} = 400$ GeV

$\tilde{M} = 0$

$M_H = M_A = M_H^\pm = M_\Phi$

$\tilde{M} = 0$

$1L \propto M_\Phi^4$

$2L \propto M_\Phi^6$
Constraining BSM models with $\lambda_{hhh}$

i. Can we apply the limits on $\kappa_{\lambda}$, extracted from experimental searches for di-Higgs production, for BSM models?

ii. Can large BSM deviations occur for points still allowed in light of theoretical and experimental constraints? If so, how large can they become?

As a concrete example, we consider a 2HDM
A benchmark scenario in the aligned 2HDM

- **Two-Higgs-Doublet Model (2HDM):**
  add a 2nd scalar doublet to the SM
  
  *Here: CP conservation assumed, Yukawa couplings of type I*

- **Mass eigenstates:**
  - 2 CP-even Higgs bosons
    - \( h \) (125-GeV Higgs), \( H \)
  - CP-odd Higgs boson \( A \)
  - Charged Higgs bosons \( H^\pm \)
  - \( M \): new BSM mass term in 2HDM

- Scenario with **alignment**: couplings of \( h \) are SM-like at tree level
Can we apply di-Higgs results for the aligned 2HDM?

➢ Current strongest limit on $\kappa_\lambda$ are from ATLAS double- (+ single-) Higgs searches

$$-0.4 < \kappa_\lambda < 6.3 \ [\text{ATLAS-CONF-2022-050}]$$

➢ What are the assumptions for the ATLAS limits?
  • All other Higgs couplings (to fermions, gauge bosons) are SM-like
    → this is **ensured by the alignment** ✓
  • The modification of $\lambda_{hhh}$ is the only source of deviation of the non-resonant Higgs-pair production cross section from the SM
    → We **correctly include all leading BSM effects to di-Higgs production, in powers of $g_{hh\phi\Phi}$, up to NNLO!** ✓

➢ We can apply the ATLAS limits to our setting!

(Note: BSM resonant Higgs-pair production cross section also suppressed at LO, thanks to alignment)
A benchmark scenario in the aligned 2HDM

Results shown for aligned 2HDM of type-I, similar for other types (available in backup)

We take $m_A = m_{H^+}$, $M = m_{H^0}$, $\tan \beta = 2$

- **Grey area**: area excluded by other constraints, in particular:
  - BSM Higgs searches,
  - boundedness-from-below (BFB),
  - perturbative unitarity (at NLO)

- **Light red area**: area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)} > 6.3$ [in region where $\kappa_\lambda^{(2)} < -0.4$ the calculation isn’t reliable]

- **Dark red area**: new area that is excluded **ONLY** by $\kappa_\lambda^{(2)} > 6.3$. Would otherwise not be excluded!

- **Blue hatches**: area excluded by $\kappa_\lambda^{(1)} > 6.3 \rightarrow$ impact of including 2L corrections is significant!

A benchmark scenario in the aligned 2HDM

Results shown for aligned 2HDM of type-I, similar for other types (available in backup)

We take $m_A = m_{H^0}$, $M = m_{H^+}$, $\tan\beta = 2$

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- **Dark red area**: new area that is excluded ONLY by $\kappa^{(2)}_{\lambda} > 6.3$. Would otherwise not be excluded!

- **Blue hatches**: area excluded by $\kappa^{(1)}_{\lambda} > 6.3$ → impact of including 2L corrections is significant!
A benchmark scenario in the aligned 2HDM – future prospects

Suppose for instance the upper bound on $\kappa_\lambda$ becomes $\kappa_\lambda < 2.3$

- **Golden area:** additional exclusion if the limit on $\kappa_\lambda$ becomes $\kappa_\lambda^{(2)} < 2.3$ (achievable at HL-LHC)

- Of course, **prospects even better with an e+e- collider!**

- Experimental constraints, such as Higgs physics, may also become more stringent, however **not** theoretical constraints (like BFB or perturbative unitarity)
A benchmark scenario in the aligned 2HDM

In view of recent ATLAS-CONF-23-034

- **Green line:** additional exclusion from direct searches for heavy Higgs bosons, via $A \rightarrow Z \, H$ with full LHC-Run2 data [ATLAS-CONF-23-034]

- **Small excess** ($2.9 \, \sigma$) for $m_H \sim 450 \, \text{GeV}$ and $m_A \sim 650 \, \text{GeV}$
  - near region probed by $\kappa_A$ at HL-LHC
  - complementarity between direct and indirect searches!
\( \lambda_{hhh} \) in relation to thermal history of the EWPT

- Corrections to \( \lambda_{hhh} \) correlate with the thermal history of the EWPT
  - If potential barrier is too high, the EWPT cannot occur → vacuum trapping (black region)
  - Conversely, it can occur that the EW symmetry is not restored at high T (blue region)
  - Strong 1\(^{st}\) order EWPT, with gravitational waves (produced by bubble collisions) observable at LISA in pink
  - Impact of 2L corrections likely strong

\[ \frac{v_n}{T_n} \text{ rather than } \frac{v_c}{T_c} , \quad \frac{v_c}{T_c} \gtrsim 1 \]  

All receive quantum corrections!

Figure from [Biekötter et al., 2208.14466]
Cosmological relics of a strong first-order phase transition
Gravitational waves from first-order phase transitions

\[ h^2 \Omega_{GW} \simeq h^2 \Omega_{\phi \text{ env}} + h^2 \Omega_{\text{sw}} + h^2 \Omega_{\text{turb}} \]

- For each contributions, results/estimates exist, which depends mostly on:
  - Assumptions for spectral shapes for different types of GW sources
  - \( \alpha \): “latent heat”, ratio of vacuum energy density released in the transition to radiation bath density
    \[ \rightarrow \alpha \sim \rho_{\text{vac}} / \rho_{\text{rad}}^* \]
  - \( \beta / H^* \), where \( \beta \) is (approx.) the inverse duration of the PT, and \( H^* \) is the Hubble parameter at \( T^* \) (temperature when GW are produced)
    \[ \beta = - \left. \frac{dS_E}{dt} \right|_{t=t^*} \simeq \frac{1}{\Gamma_{\text{nuc}}} \left. \frac{d\Gamma_{\text{nuc}}}{dt} \right|_{t=t^*} \]
  - \( v_w \): bubble wall velocity
    (often taken as an assumption, but see workshop at DESY/UHH “How fast does the bubble grow?”)
Probing scenarios of SFOEWPT with gravitational waves

Example 1: spectra of GW produced by the EWPT in the near-aligned Higgs EFT [Kanemura, Nagai '21], [Kanemura, Nagai, Tanaka '22]

$\Lambda$: mass of BSM state(s); $\kappa_0$: no. of BSM d.o.f;
$r$: “non-decouplingness”

$$\Lambda^2 = M^2 + \tilde{\lambda} v^2, \ r \equiv \frac{\tilde{\lambda} v^2}{\Lambda^2}$$

Example 2: correlation of $\kappa_\lambda$ and signal-to-noise ratio (SNR) of GW at LISA for 2HDM scenarios with SFOEWPT [Biekötter et al. '22]
Primordial black holes from first-order phase transitions

Figure from [Kanemura ‘23]

\[ \rho_{\text{rad}} \propto a^{-4} \]
\[ \rho_{\text{vac}} = \text{const} \]
Primordial black holes from first-order phase transitions

- Patches of Universe in which EWPT is (randomly) delayed can lead to **overdensities** sourcing **primordial black holes** (PBH)

- PBH formation if:

\[ \delta = \frac{\rho_{\text{over.}} - \rho_{\text{bkgd}}}{\rho_{\text{bkgd}}} \gtrsim \delta_c \sim 0.45 \]

**Figure from** [Kanemura '23], [Tanaka '23]

**Figure from** [Gouttenoire, Volansky PRL '23]

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Searching for primordial black holes

$M_{\text{PBH}}$ [g]

$\Omega_{\text{PBH}} / \Omega_{\text{DM}}$

$T_{\text{eq}} = 10^7$ GeV

$T_{\text{eq}} = 10^5$ GeV

$T_{\text{eq}} = 100$ GeV

$T_{\text{eq}} = 150$ MeV

BBN

CMB (evaporation)

Cosmic Rays

(Micro) lensing

HSC

OGLE

LIGO

VIRGO

CMB (accretion)

[Gottenoire, Volansky PRL ‘23]
Searching for primordial black holes

$M_{\text{PBH}} \text{ [g]}$

$M_{\text{PBH}} \approx \Omega_{\text{PBH}} / \Omega_{\text{DM}}$

- BBN
- CMB
- Cosmic Rays
- (evaporation)
- (accretion)

$T_{\text{eq}} = 10^7 \text{ GeV}$

$T_{\text{eq}} = 10^5 \text{ GeV}$

$T_{\text{eq}} = 100 \text{ GeV}$

LIGO

VIRGO

OGLE

PBHs produced during a SFOEWPT can be constrained by micro-lensing!

[Gouttenoire, Volansky PRL ‘23]
Complementary probes of SFOEWPT with PBHs

- Production of PBHs and GW in near-aligned Higgs EFT
  \(\Lambda\): mass of BSM state(s); \(\kappa_0\): no. of BSM d.o.f;
  \(r\): “non-decouplingness”

\[
\Lambda^2 = \mathcal{M}^2 + \bar{\lambda}v^2, \quad r \equiv \frac{\bar{\lambda}v^2}{\Lambda^2}
\]

[Hashino, Kanemura, Takahashi, Tanaka ‘23]

\(r = 1\)

\(0.3 < r < 1\)

\(f_{\text{PBH}} > 10^{-4}\)

\(r = 0.3\)

\(\kappa_0 = 1\)

\(\Gamma/H^4 < 1\)

\(v_n/T_n \geq 1\)

\(\Delta \lambda_{hh} / \lambda_{SM}^{\text{SM}} = 20\%\)
Summary

The vision for the future of particle physics must acknowledge the central role of the Higgs field. The Higgs field is a crucial part of the Standard Model. It is our ignorance about this field that keeps us from solving the remaining mysteries that the Standard Model cannot address. To make progress, we must remedy this. We need to make clear (with apologies to Red Sanders and Vince Lombardi): “Higgs isn’t everything; it’s the only thing.” A vision for particle physics that is not built on this idea cannot address the most profound questions for our field or realize its greatest opportunities.

[Peskin, Vision for Elementary Particle Physics 2302.05472]
Thank you very much for your attention!

Contact

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