Advanced Course on Higgs Physics

1st Graduate Week of the Quantum Universe Research School

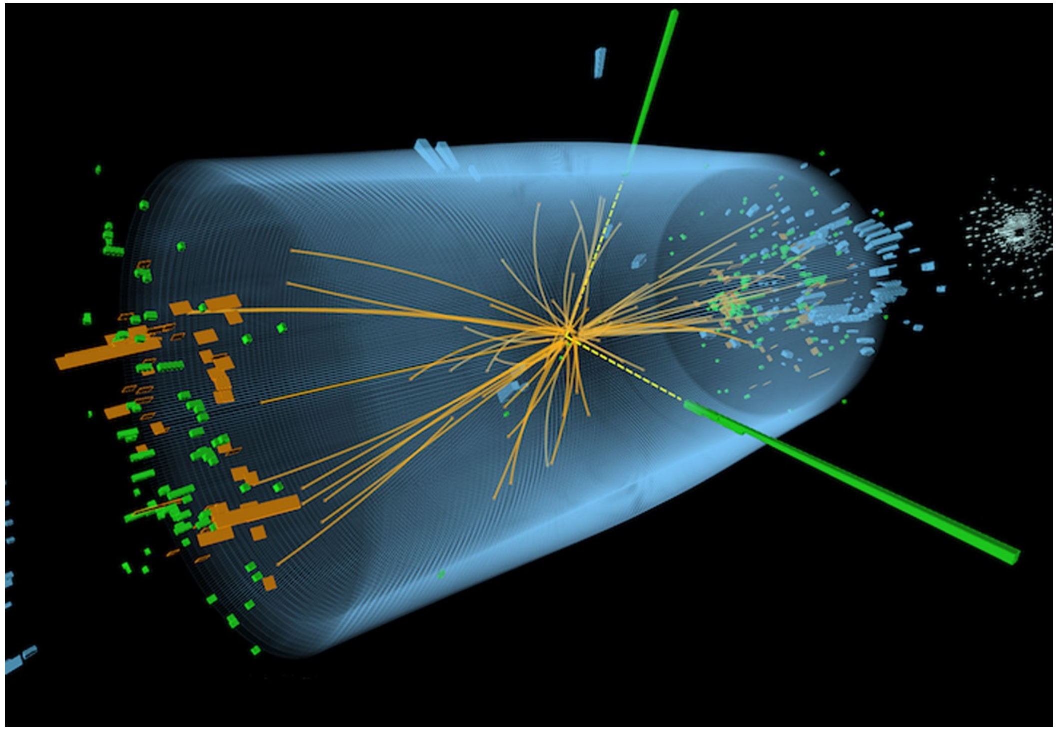
Johannes Braathen (DESY)

Hamburg, Germany | 5-8 February 2024

Outline of the lectures

- **Part 1: Why a Higgs boson is needed [Yesterday]**
- **Part 2: Connections between Higgs Physics and unanswered questions of Particle Physics (and possible solutions to them) [Yesterday]**
- **Part 3: What can be learnt from the Higgs boson at high-energy colliders – an overview**
- **Part 4: The Higgs boson mass as a precision observable – calculations and interpretations**
- **Part 5: The Higgs boson potential, its trilinear coupling, and relations with early-Universe evolution**

Part 3: Higgs coupling measurements and precision calculations

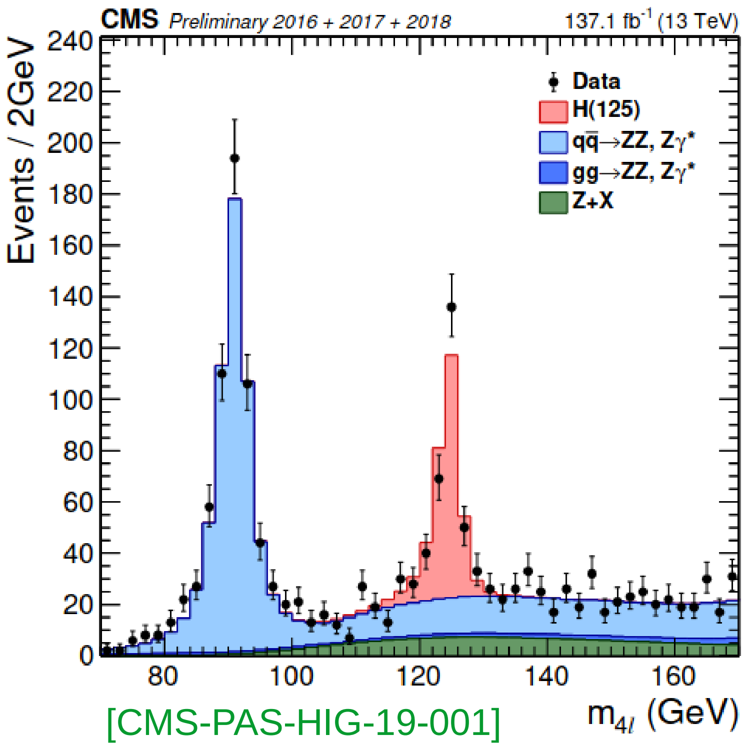


$h \to \gamma\gamma$ event at CMS

What we know and what remains to determine

What we know of the Higgs boson so far:

- Its mass **M_h=125 GeV**, to astonishing ~0.1% precision! (*more later*)
- The electroweak (EW) vacuum expectation value (vev) **v=246 GeV**
- Spin 0
- Not purely CP-odd



[CMS-PAS-HIG-19-001]

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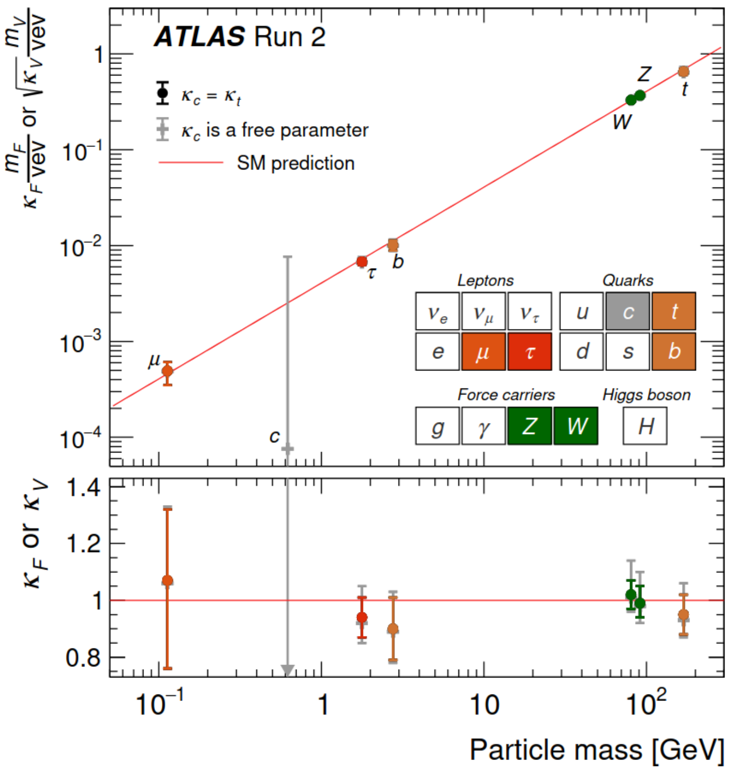

[ATLAS CERN-EP-2022-057]

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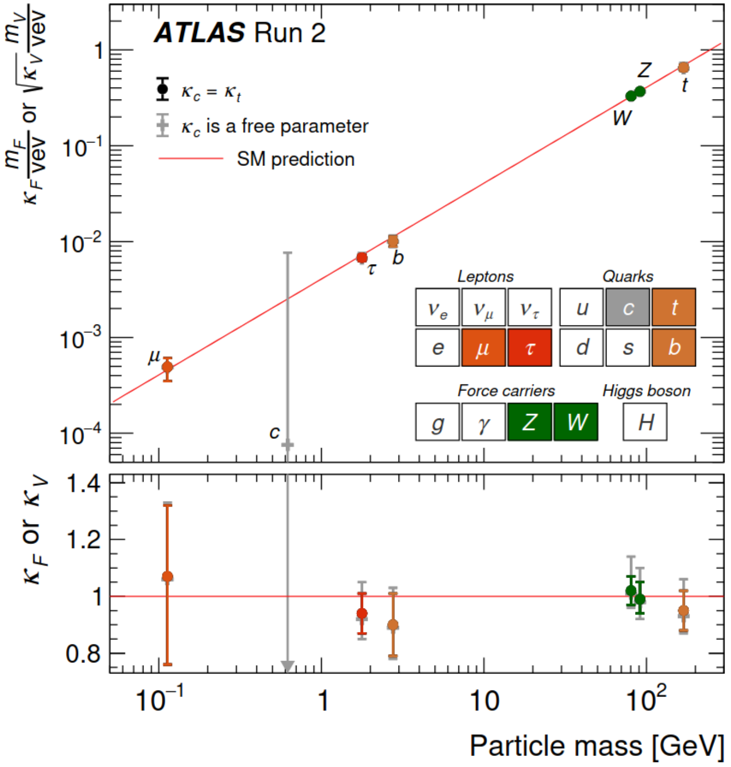

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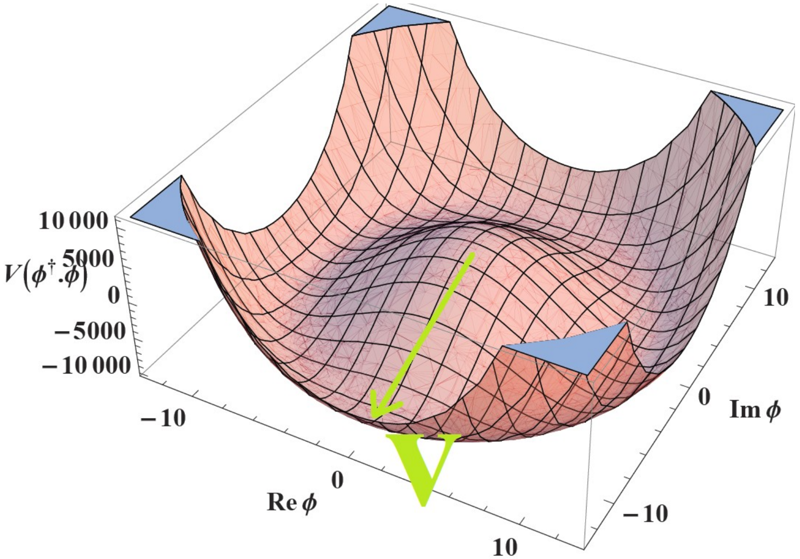

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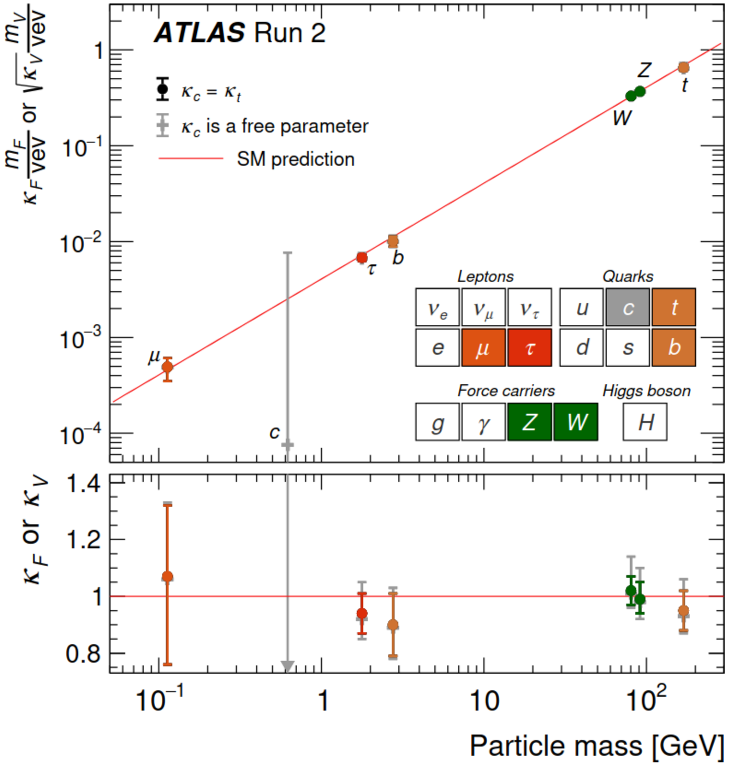
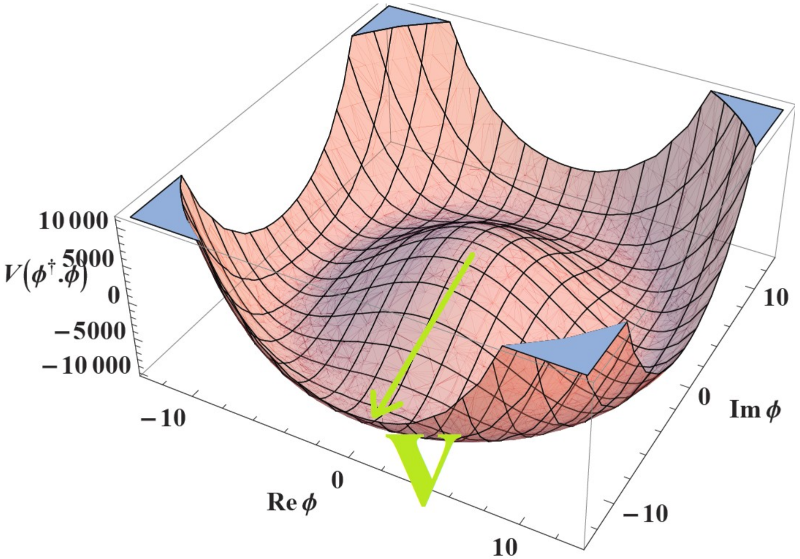


[ATLAS CERN-EP-2022-057]

What we still don't know:

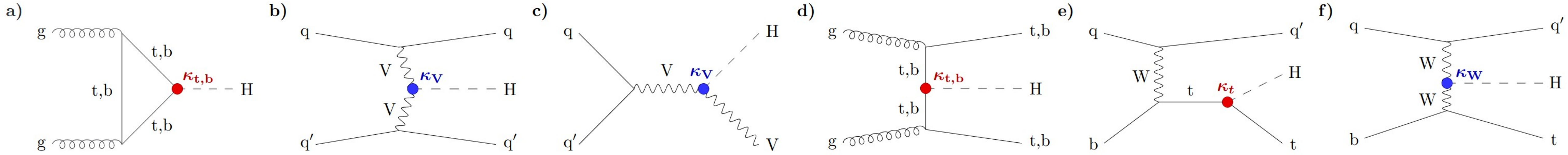
- Its couplings to 1st and 2nd gen. fermions
- Its total width; BR(h→inv.) < 9%
- Its CP nature
- Its fundamental nature? (elementary or composite)
- The structure of the Higgs sector? (minimal or extended)
- The form of the Higgs potential? *(more in Part 5)*

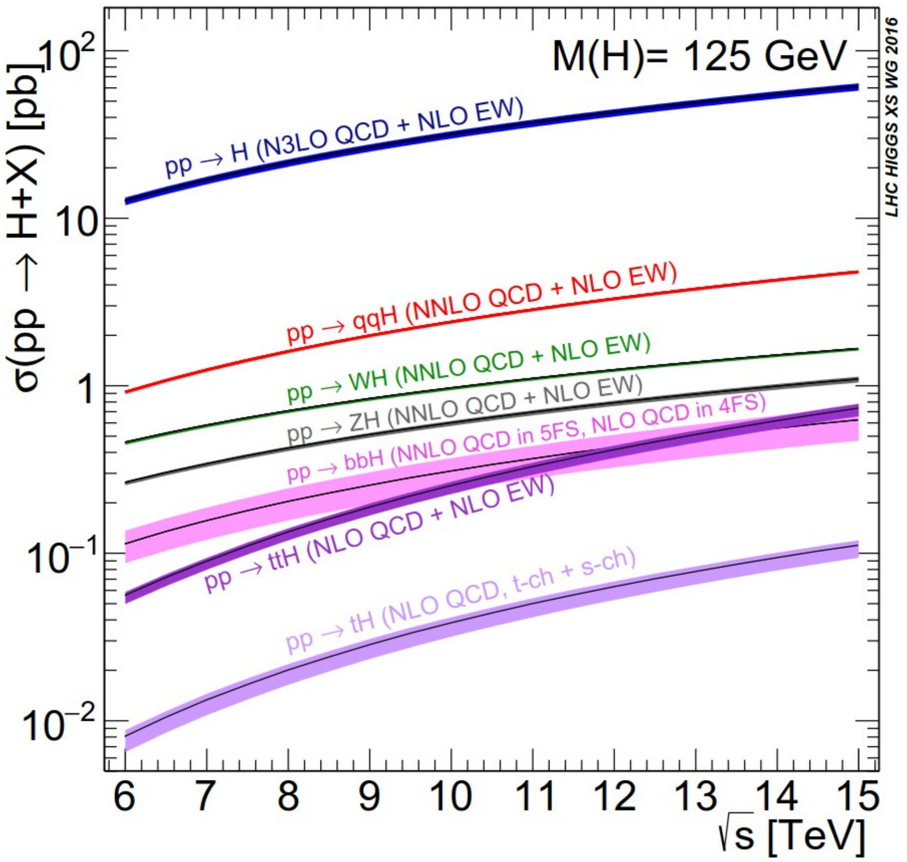


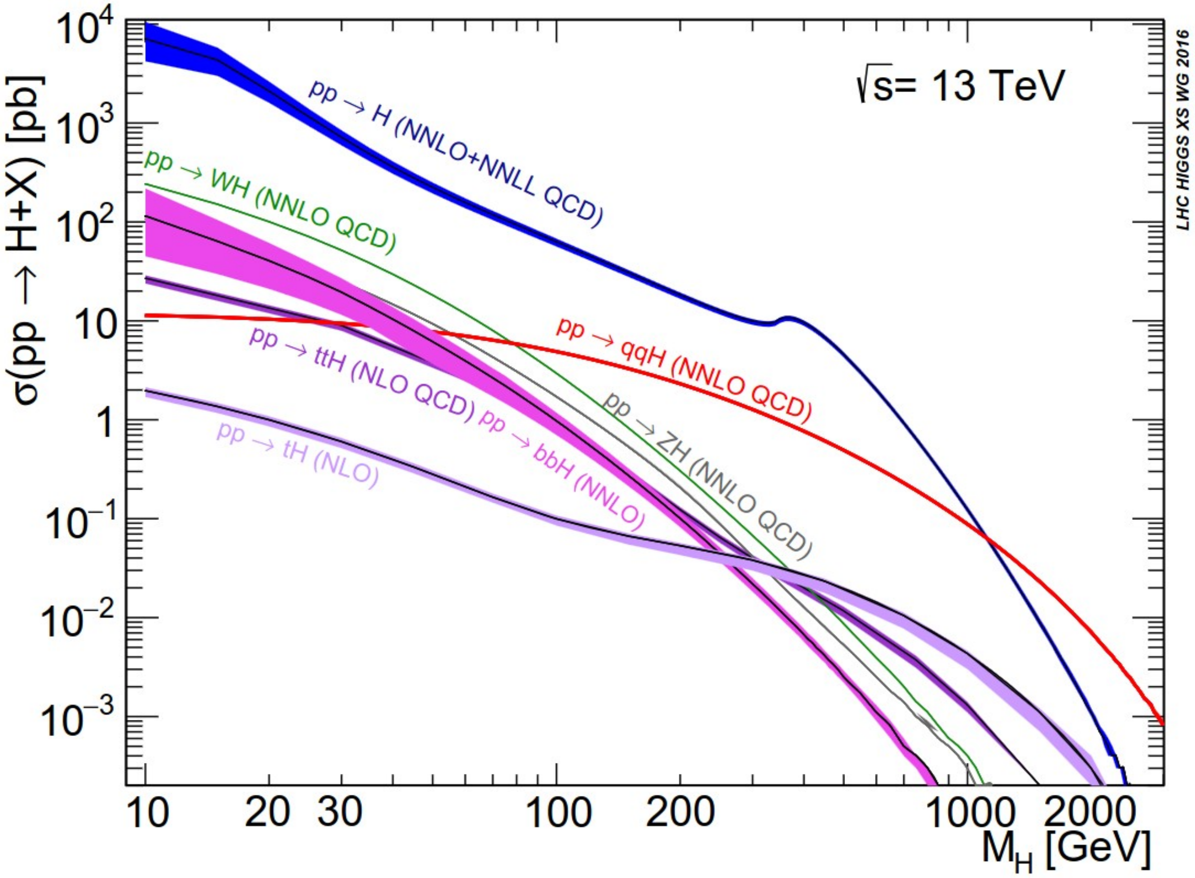
Higgs production at LHC

Diagrams from [CMS Nature ‘22],
Plots from [LHC Higgs WG ‘16]
See also reviews of [Djouadi ‘05]

- (Single*)**Higgs production channels at LHC** [*: di- and tri-Higgs production → Part 5]







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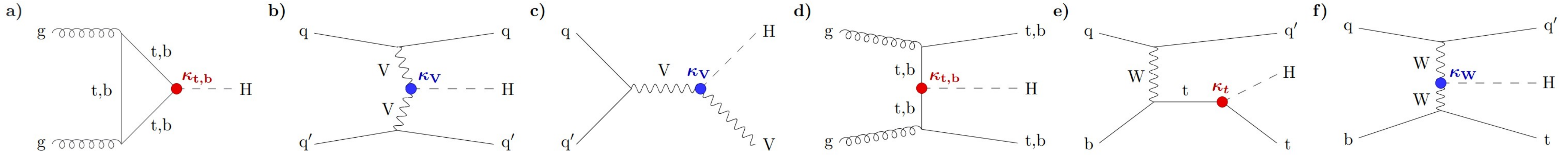
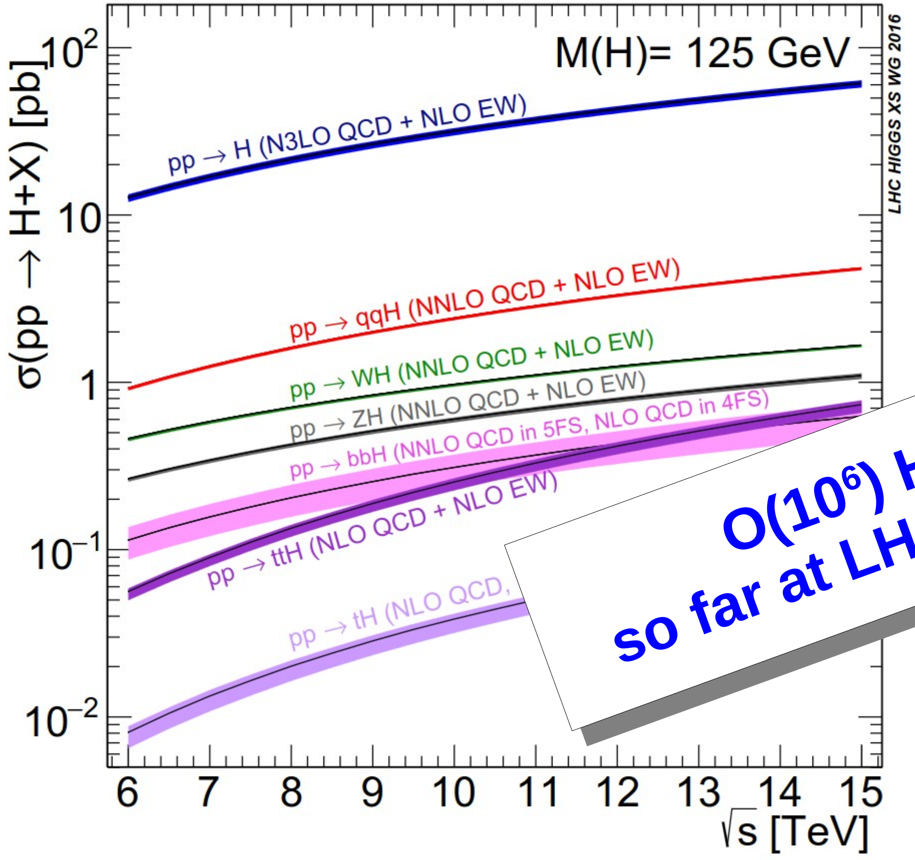
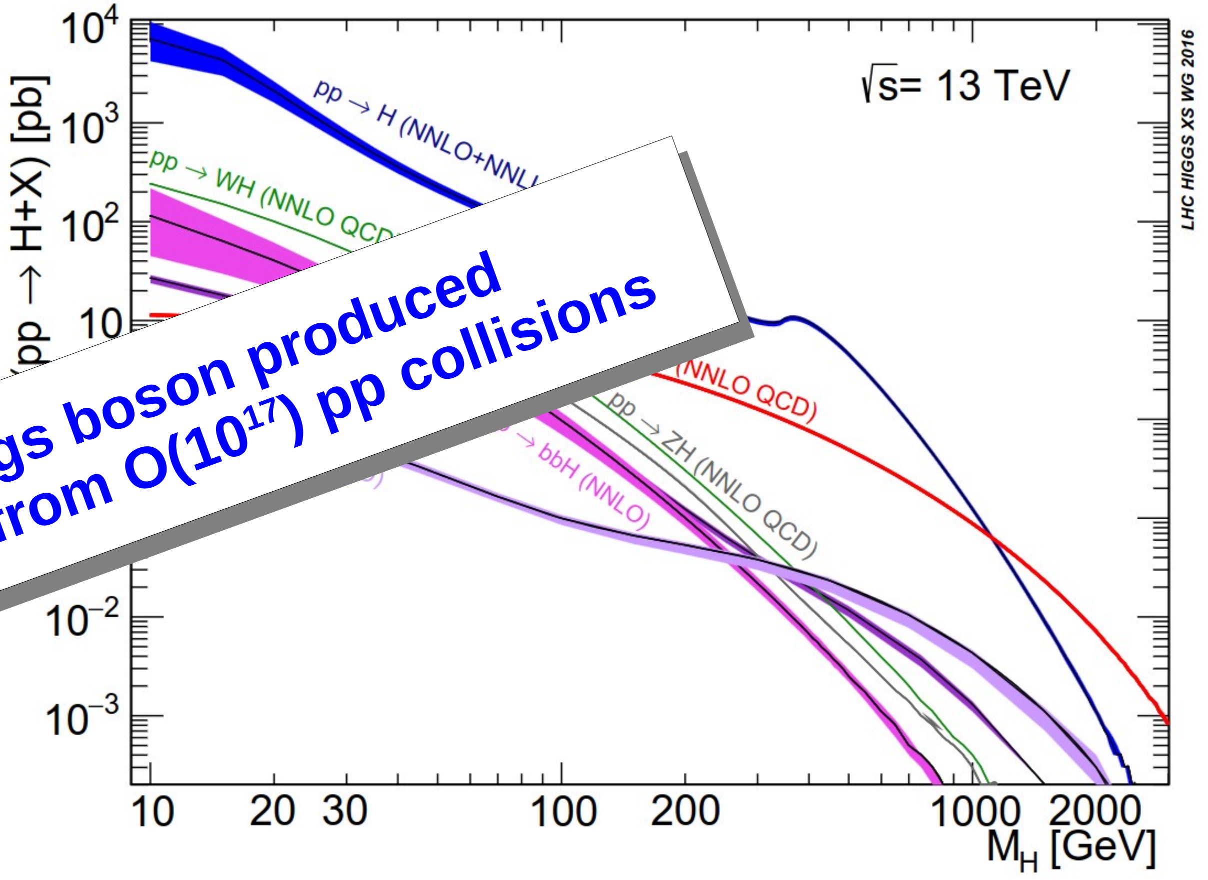




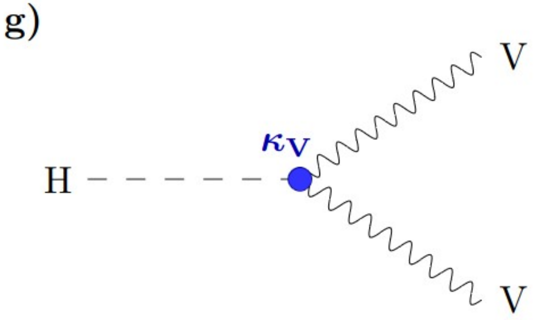


O(10^6) Higgs boson produced so far at LHC, from O(10^17) pp collisions

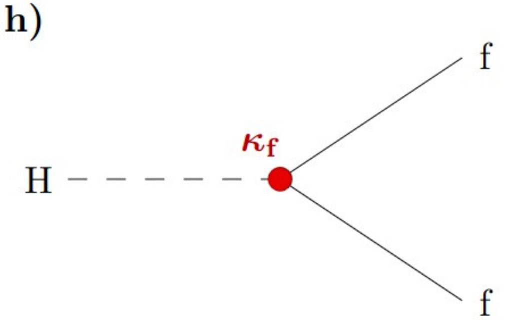
Higgs decay channels

- **Decay channels**

Tree-level decays:

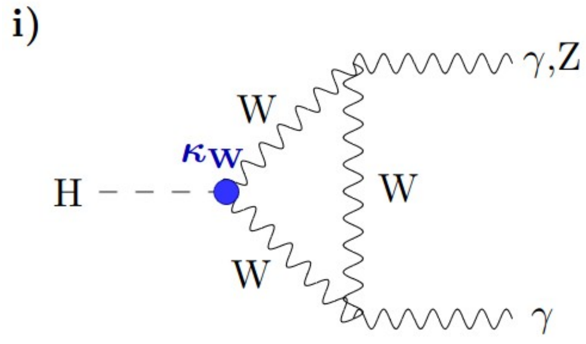


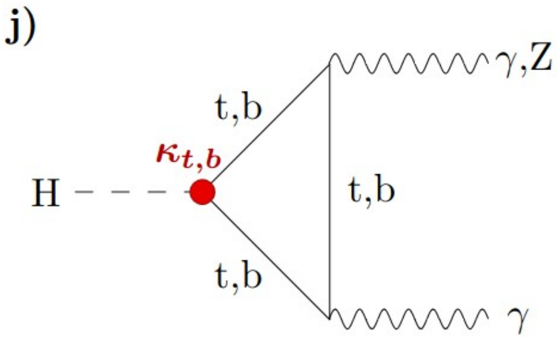
(V=W, Z)



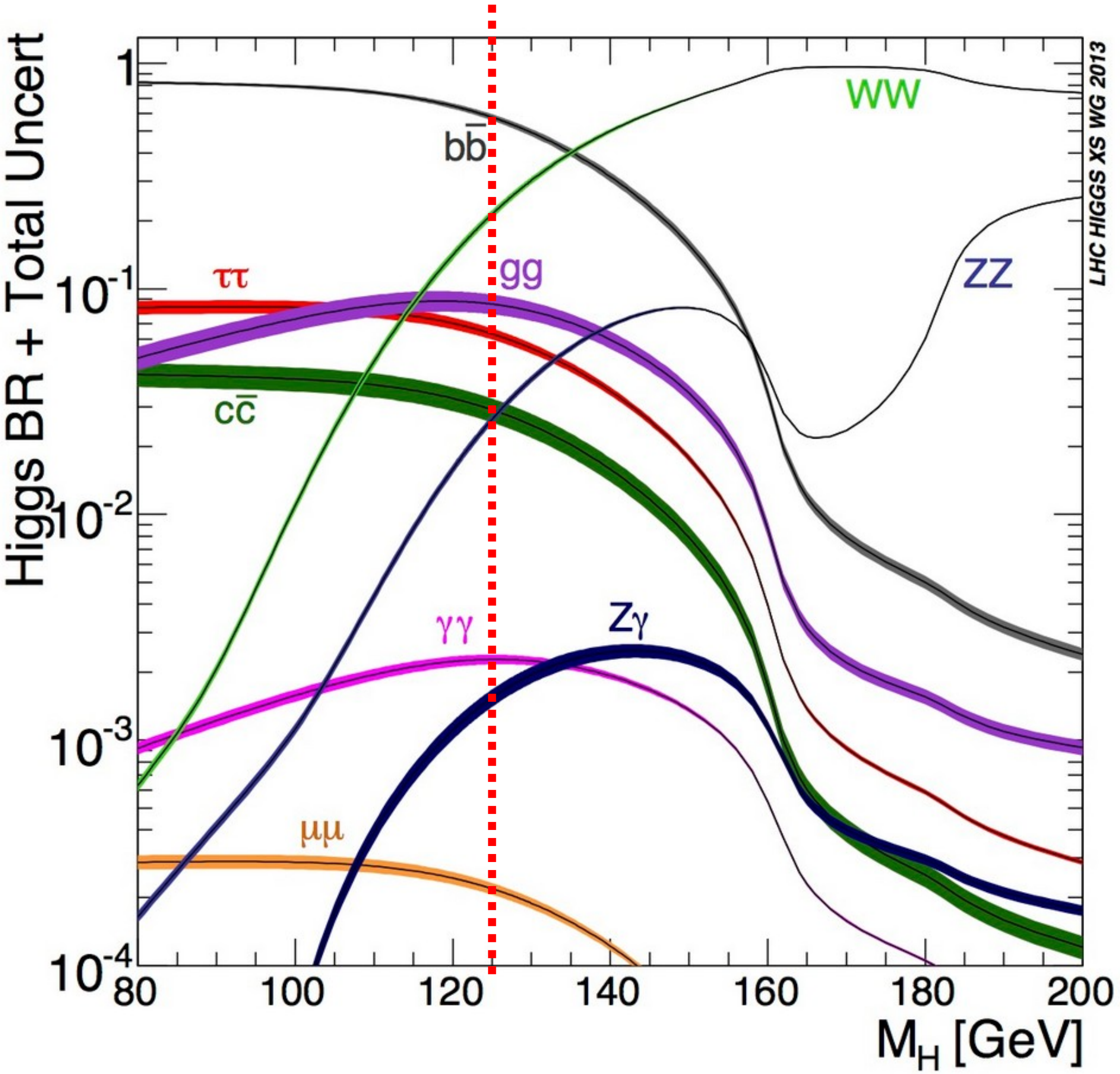
(f=b, τ, μ, ...)

Loop-induced decays:





Diagrams from
[CMS Nature ‘22]

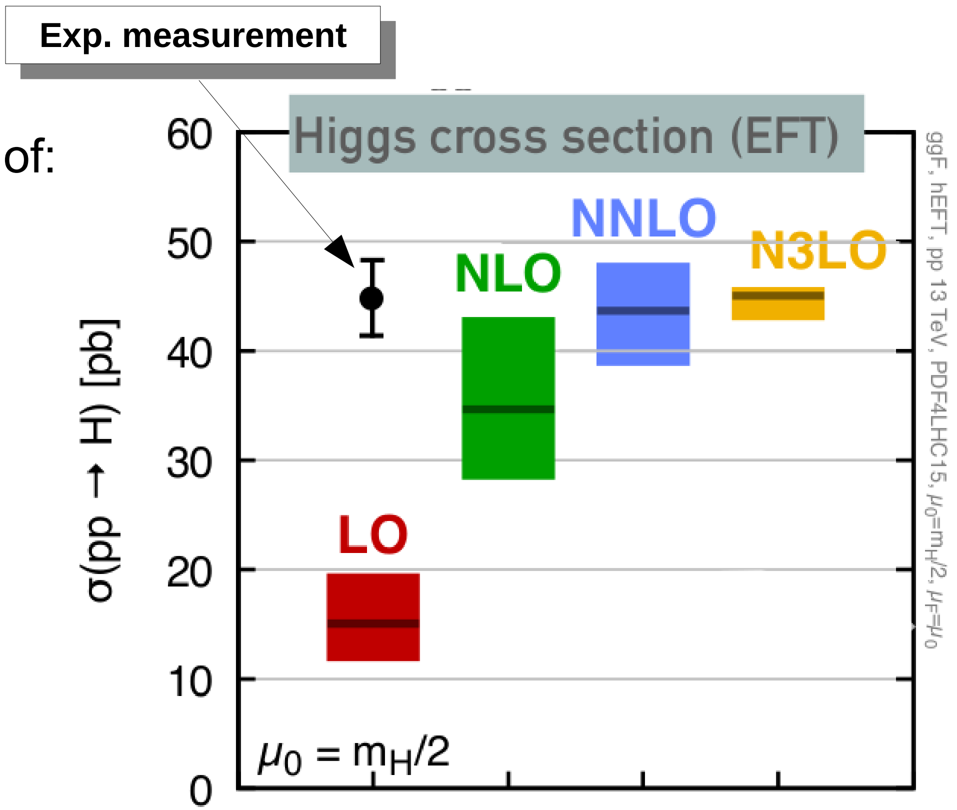


Interpreting experimental results

- Comparison between experiment and theory carried out at the level of:
  - Signal strengths
  - κ parameters (signal strength modifiers)
  - Simplified Template Cross-Sections (STXS)
  - Fiducial cross sections
  - Coefficients of EFT operators

- Requires **high-precision theoretical predictions** (with level of accuracy at least matching that of experimental results)
  → both in SM and BSM theories
  → huge efforts from precision calculation communities (QCD, EW, BSM)

- **Public tools to confront experimental results with model predictions**:
  - HiggsSignals (signal strengths, STXS) [Bechtle et al '13, '20] → now included in HiggsTools [Bahl et al '22]
  - Lilith (signal strengths) [Bernon, Dumont '15], [Kraml et al '19], [Bertrand et al '20]



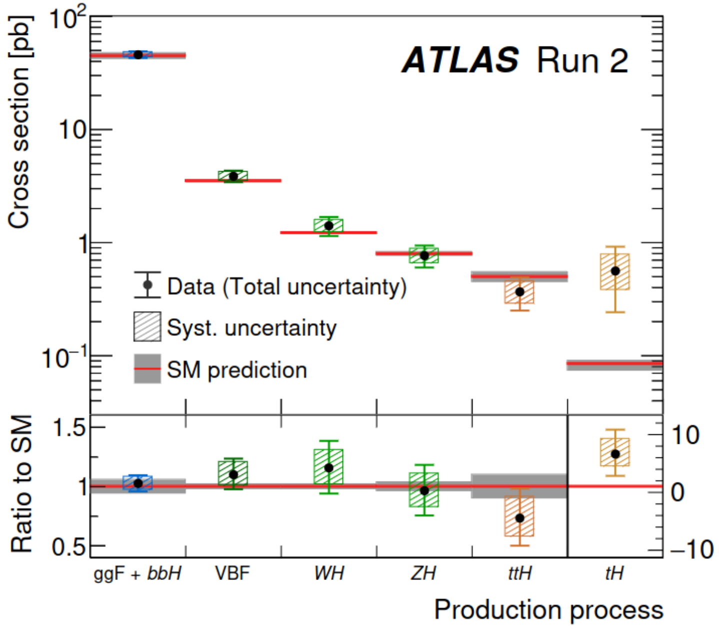
Total cross section for (inclusive) single-Higgs production, in heavy top limit ($m_t \to +\infty$)

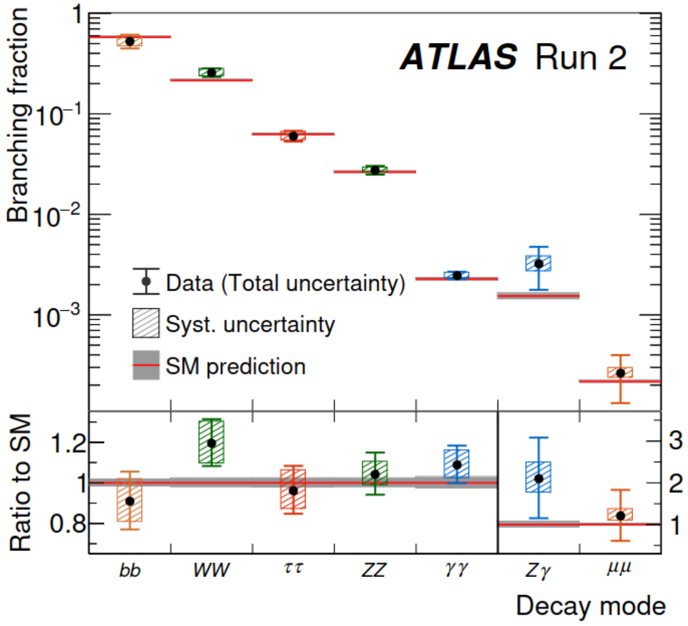
Figure taken from [Weiglein '22], itself from [Wiesemann '22], based on results from [Anastasiou et al. '15], [Mistlberger '18]

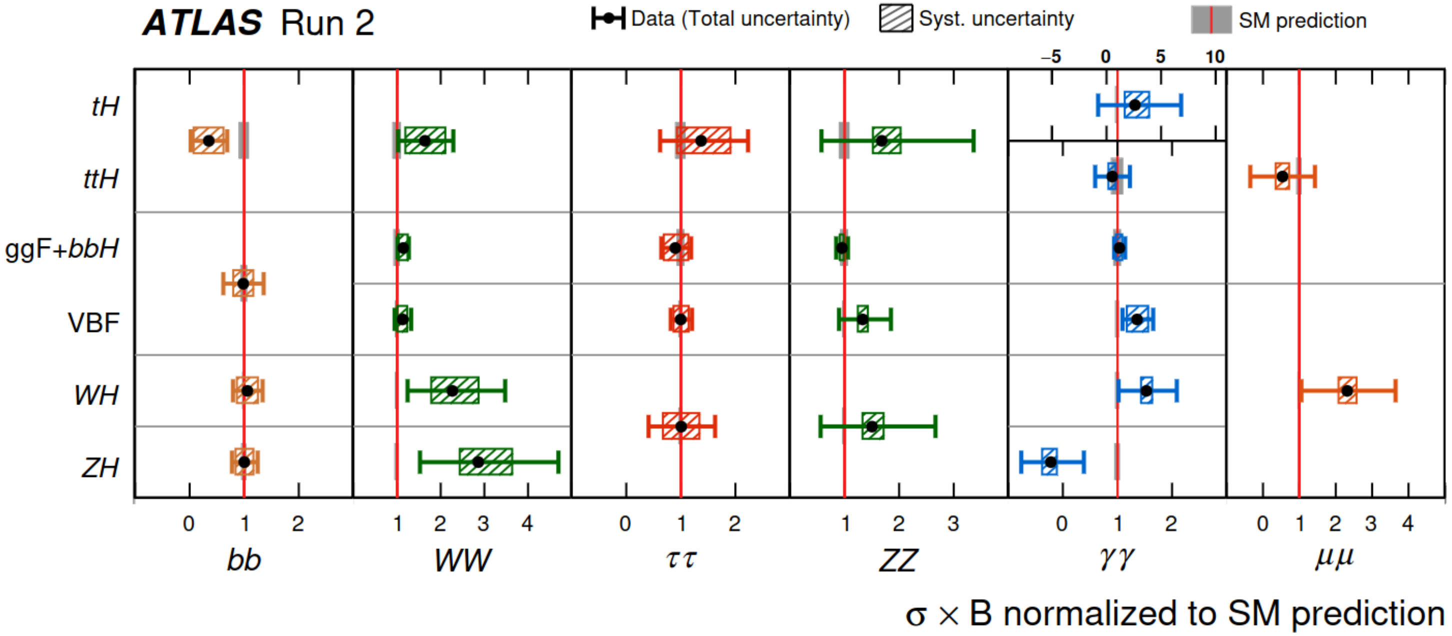
Interpreting experimental results

Cross sections and signal strengths

e.g. [ATLAS Nature '22]

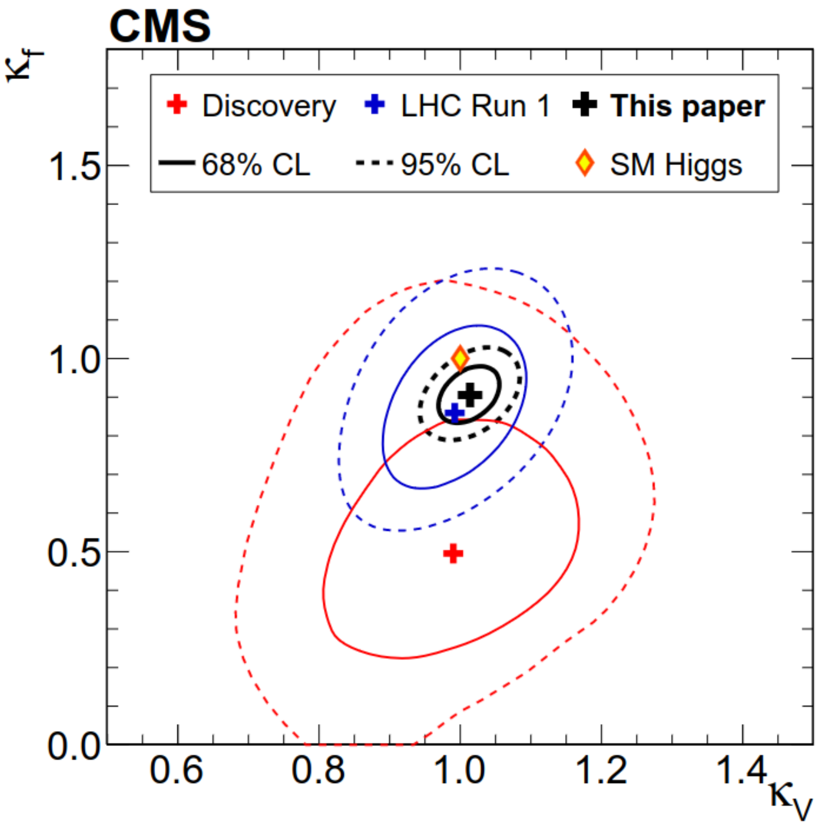


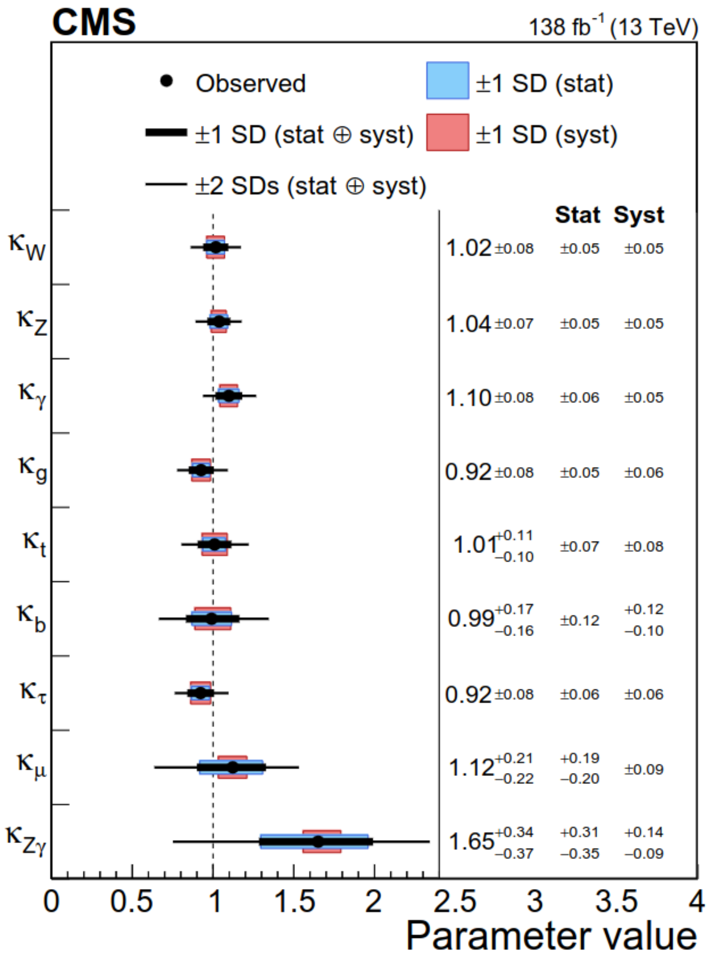


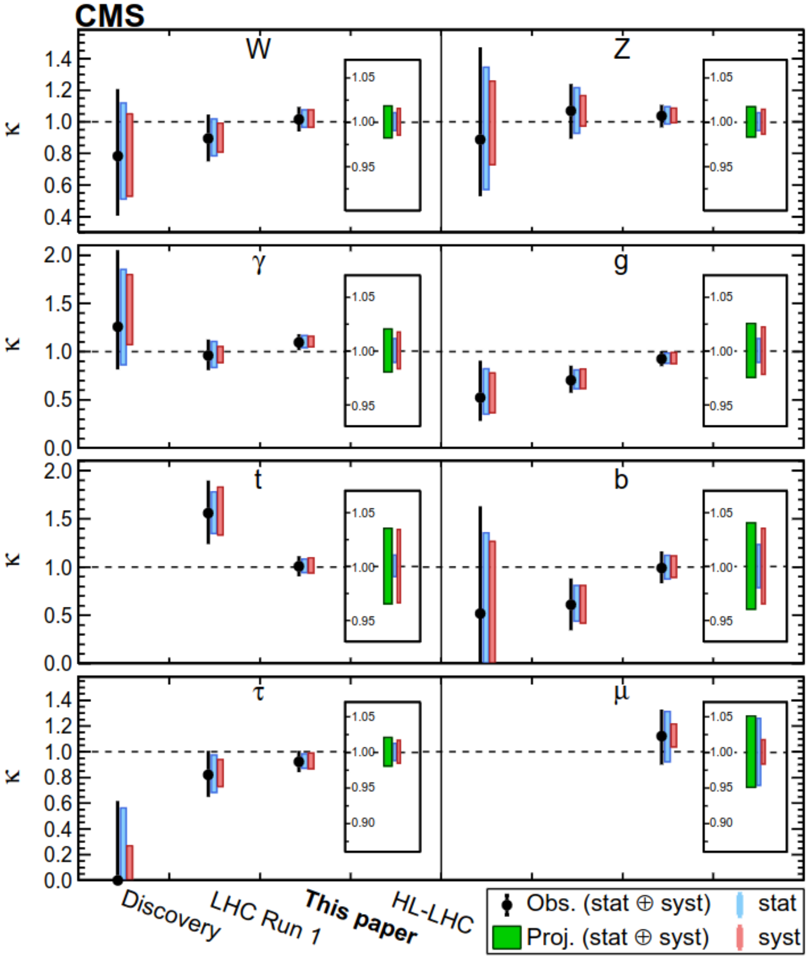


Interpreting experimental results

Coupling modifiers
(κ framework)
e.g. [CMS Nature ‘22]







Interpreting experimental results

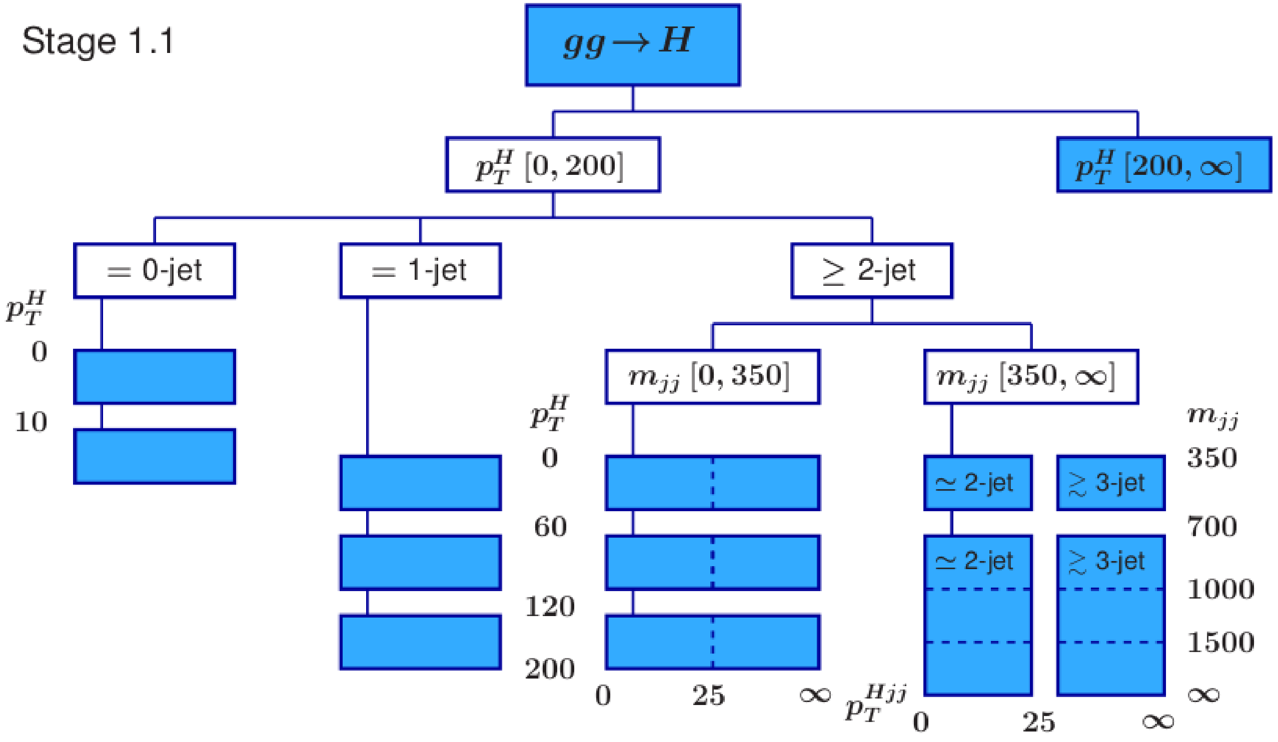
STXS
e.g. [Berger et al., LHC Higgs WG '19]

Measurement | Interpretation

Data → Simplified Cross Sections → κ_i, g_k → Lagrangian parameters

Minimize theory dependence

Direct theory dependence

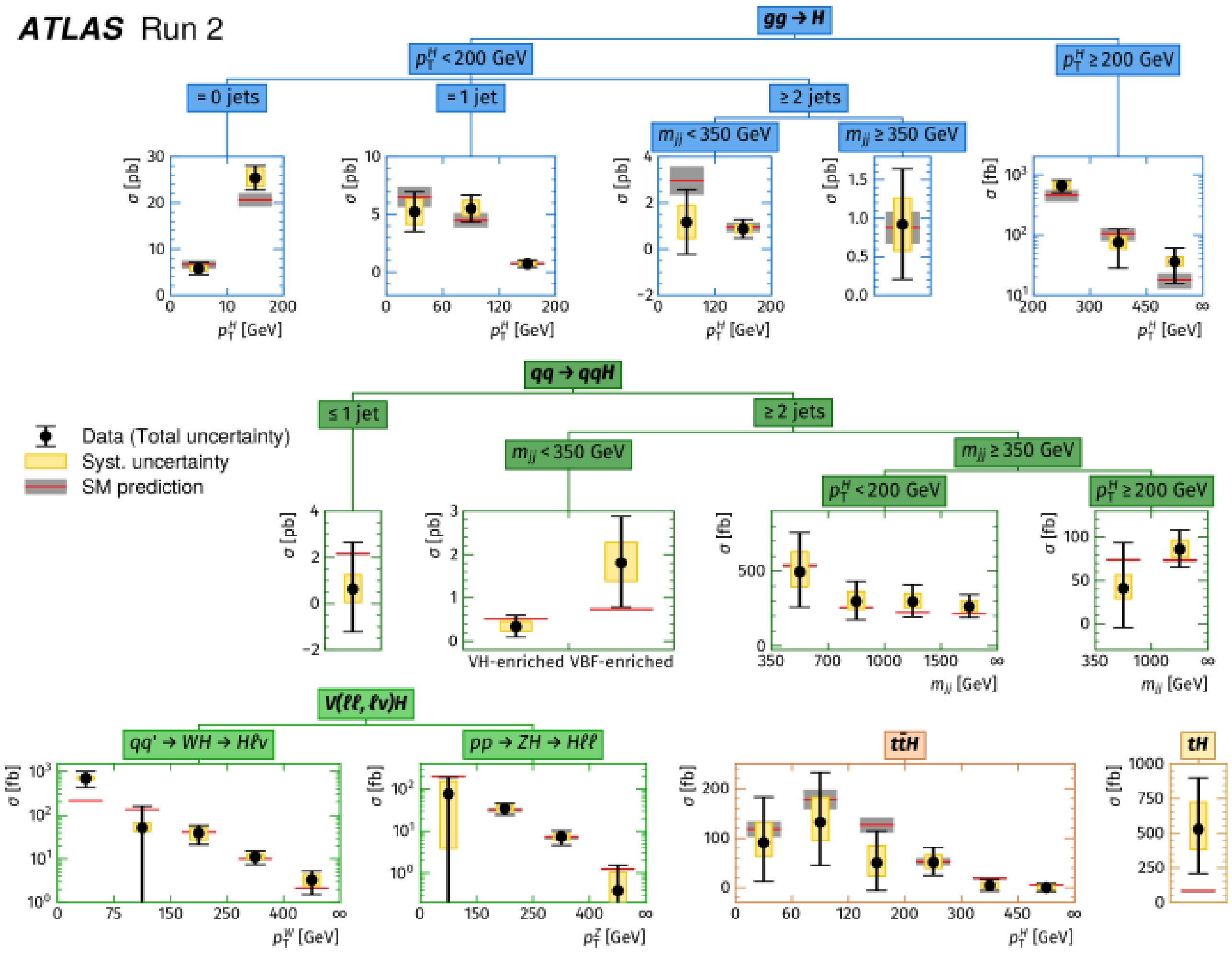


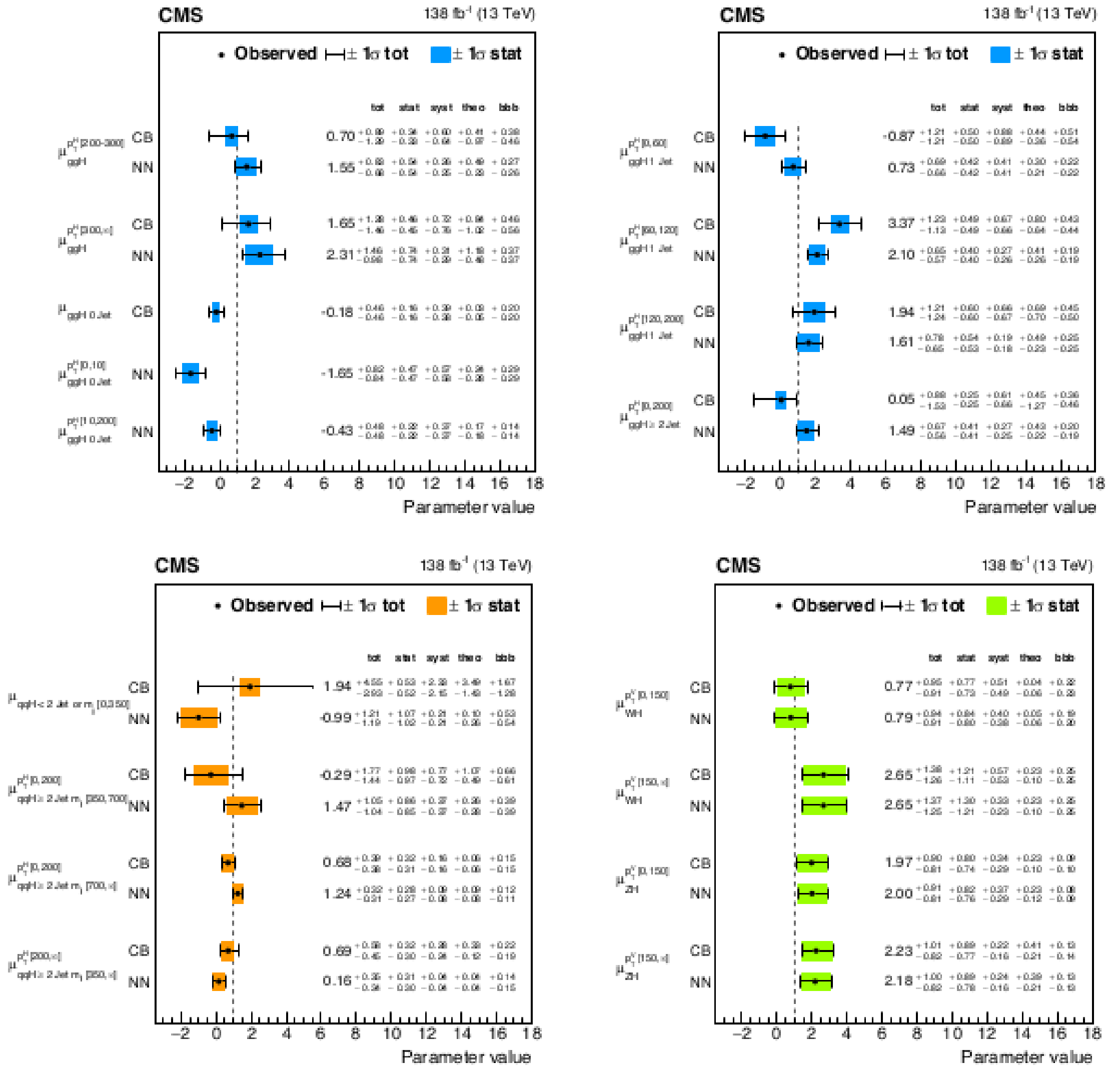
⇒ Extraction of differential information, adapted to available data

Slide adapted from G. Weiglein

Interpreting experimental results

STXS e.g. [ATLAS Nature '22], [CMS Nature '22]

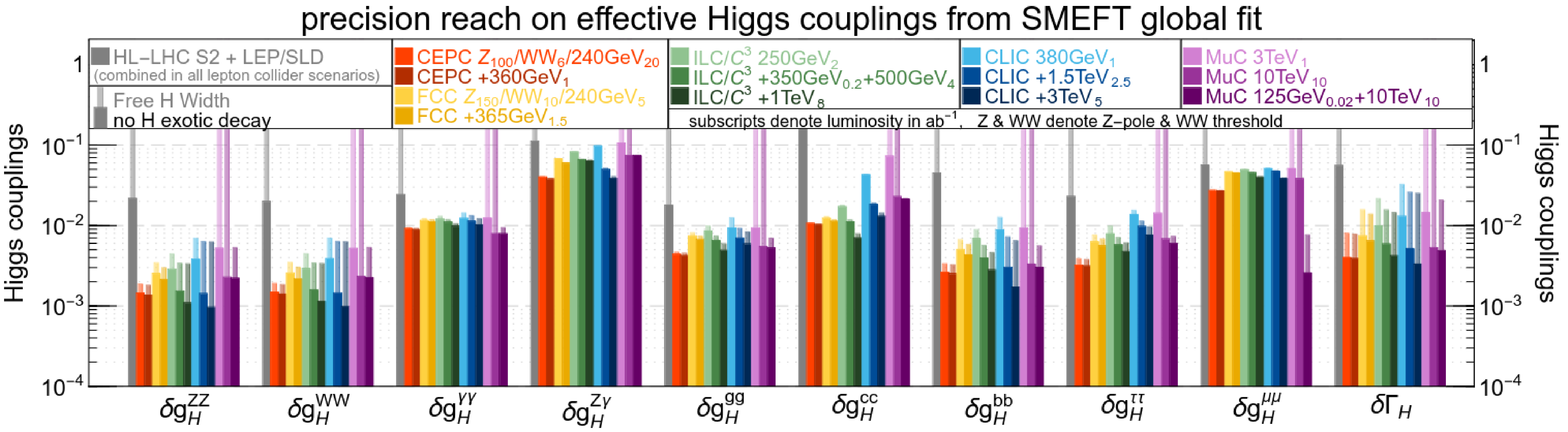




Slide adapted from G. Weiglein

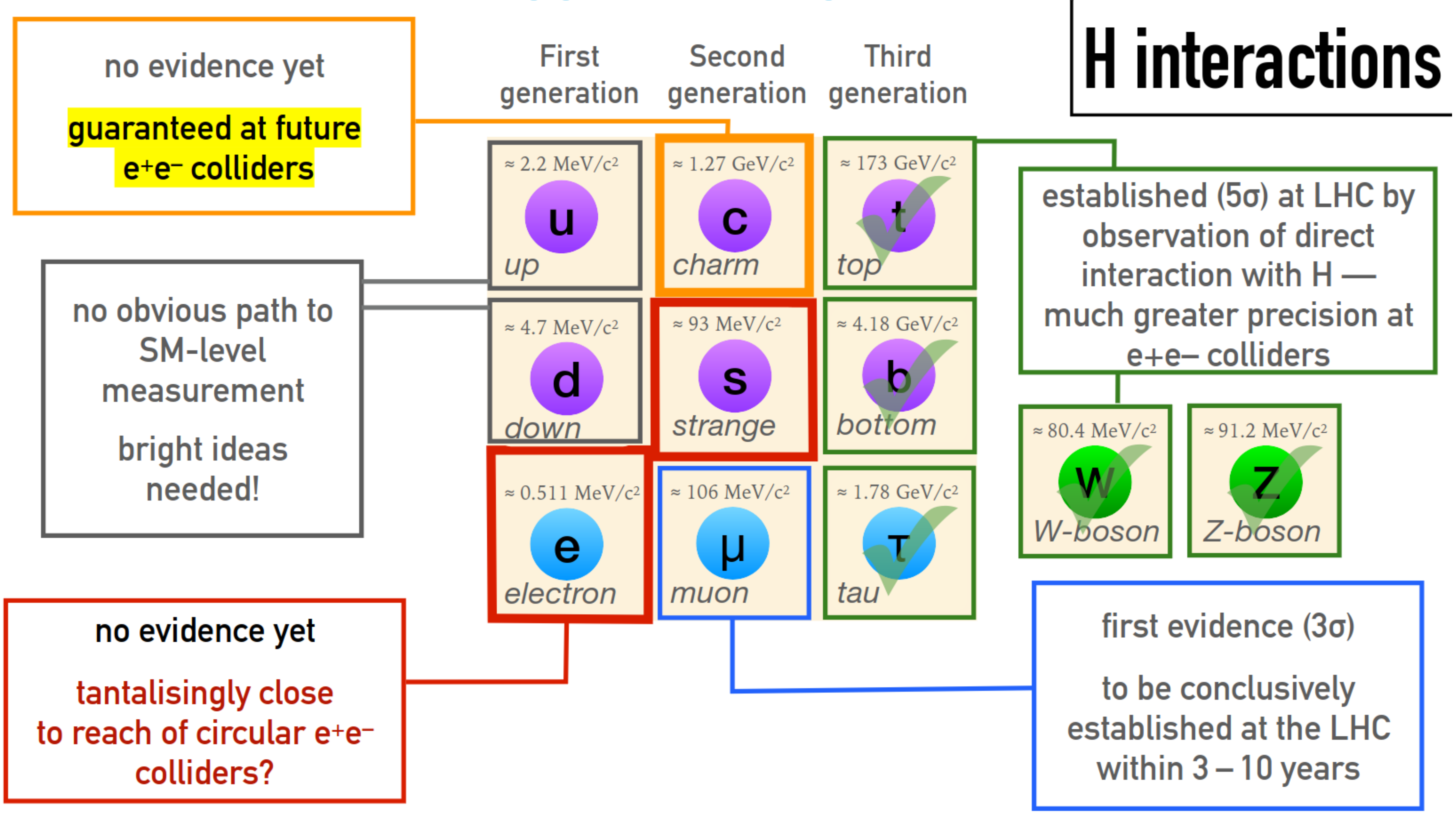
Future projections for Higgs coupling measurements

Global fit in SMEFT, using Higgs data,
EW precision observables, di-boson data
e.g. [Snowmass Higgs topical report '22]



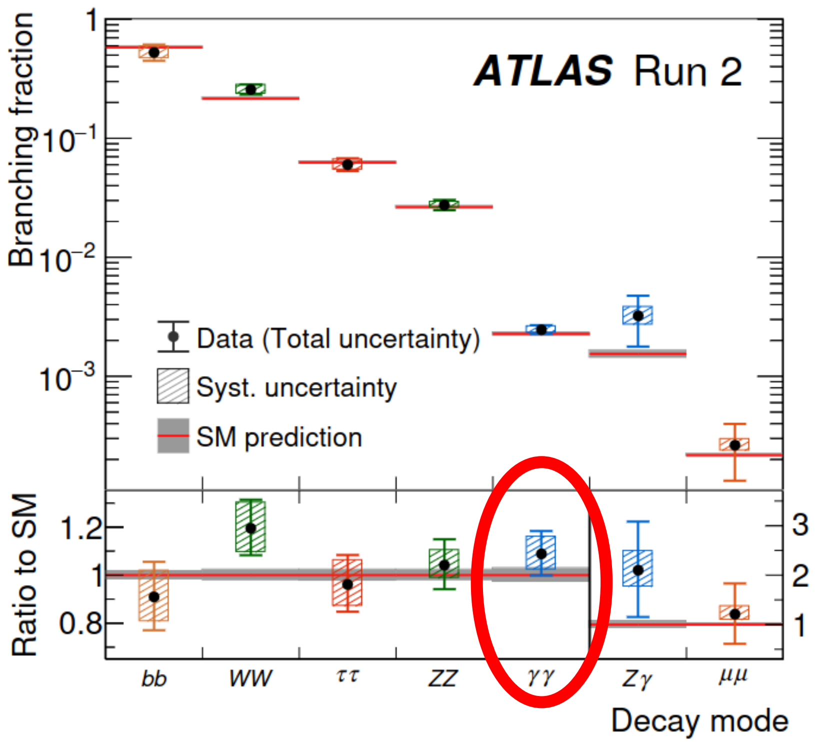
→ **important to properly assess prospects at future colliders**

Future prospects for Higgs coupling measurements



Slide from [G. Salam '23]

An example calculation of Higgs properties in a BSM model: leading two-loop corrections to Γ(h→ γγ) in the Inert Doublet Model



The Inert Doublet Model

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge ½

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG) \end{pmatrix} \qquad \text{and} \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}$$

- **Unbroken Z_2 symmetry** $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$

$$\begin{aligned} V^{(0)}_{\text{IDM}} = &\ \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 \\ &+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_2^\dagger \Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger \Phi_1)^2 + \text{h.c.}\Big) \end{aligned}$$

- **Model parameters**:
  3 BSM masses m_H, m_A, $m_{H\pm}$, BSM mass scale μ_2, inert doublet quartic self-coupling λ_2

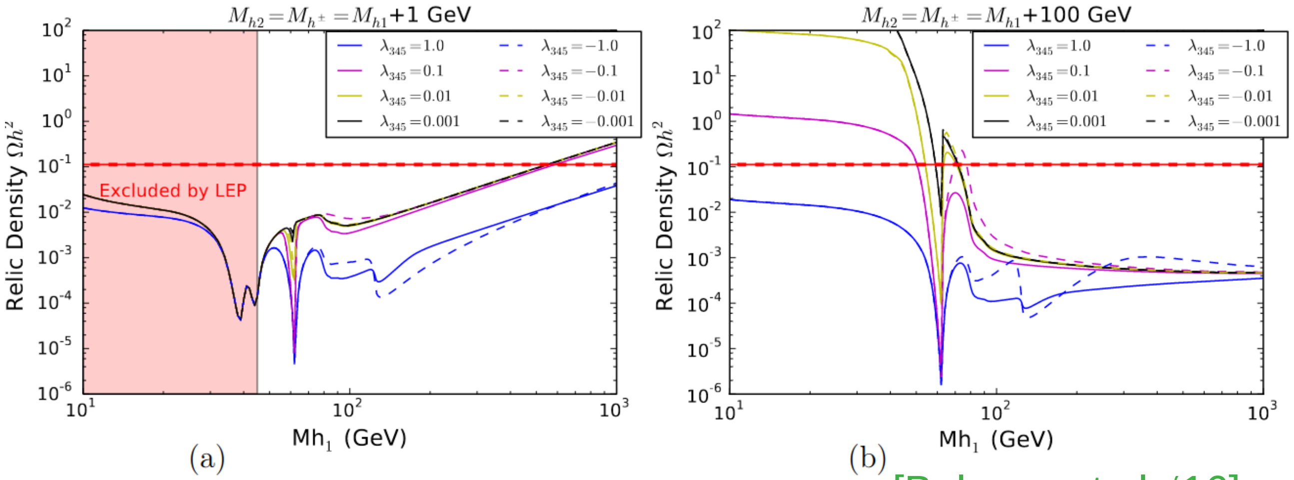
- **BSM-scalar masses** take form

$$m_H^2 = \mu_2^2 + \lambda_H v^2\,, \quad m_A^2 = \mu_2^2 + \lambda_A v^2\,, \quad m_{H^\pm}^2 = \mu_2^2 + \lambda_3 v^2\,,$$

$$\text{with } \lambda_{H,A} = \lambda_3 + \lambda_4 \pm \lambda_5$$

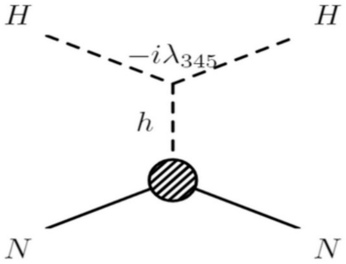
Dark Matter in the Inert Doublet Model

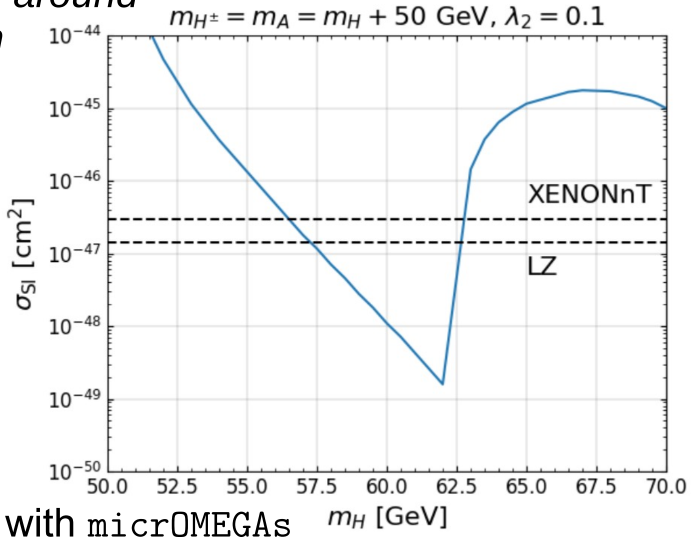
- Inert scalars: charged under Z_2 symmetry (Z_2-odd)
- Lightest inert scalar = **Dark Matter candidate**
  → *assume H here*
- DM relic density obtained via freeze-out mechanism, while evading current detection bounds
- 2 possible scenarios:
  → **"Higgs resonance scenario" $m_H \sim m_h/2$**
  → **"Heavy Higgs scenario" $m_H \geq 500$ GeV**
- IDM testable at current and future experiments via
  - DM direct and indirect searches
  - direct searches at colliders
  - *precision/indirect tests*
    → properties of 125-GeV Higgs boson



[Belyaev et al. '16]

Direct detection bounds around Higgs resonance region





Plot made with `micrOMEGAs`
[Bélanger et al. '18]

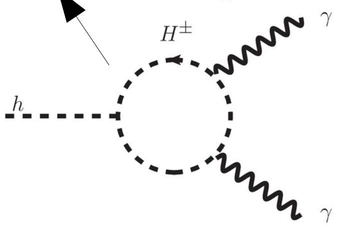
Higgs decay to two photons: existing one-loop results

- DM scenarios of IDM investigated via Higgs properties at one loop (1L) in [Kanemura, Kikuchi, Sakurai '16]

- Additional charged inert Higgs → Higgs decay to 2 photons especially important!

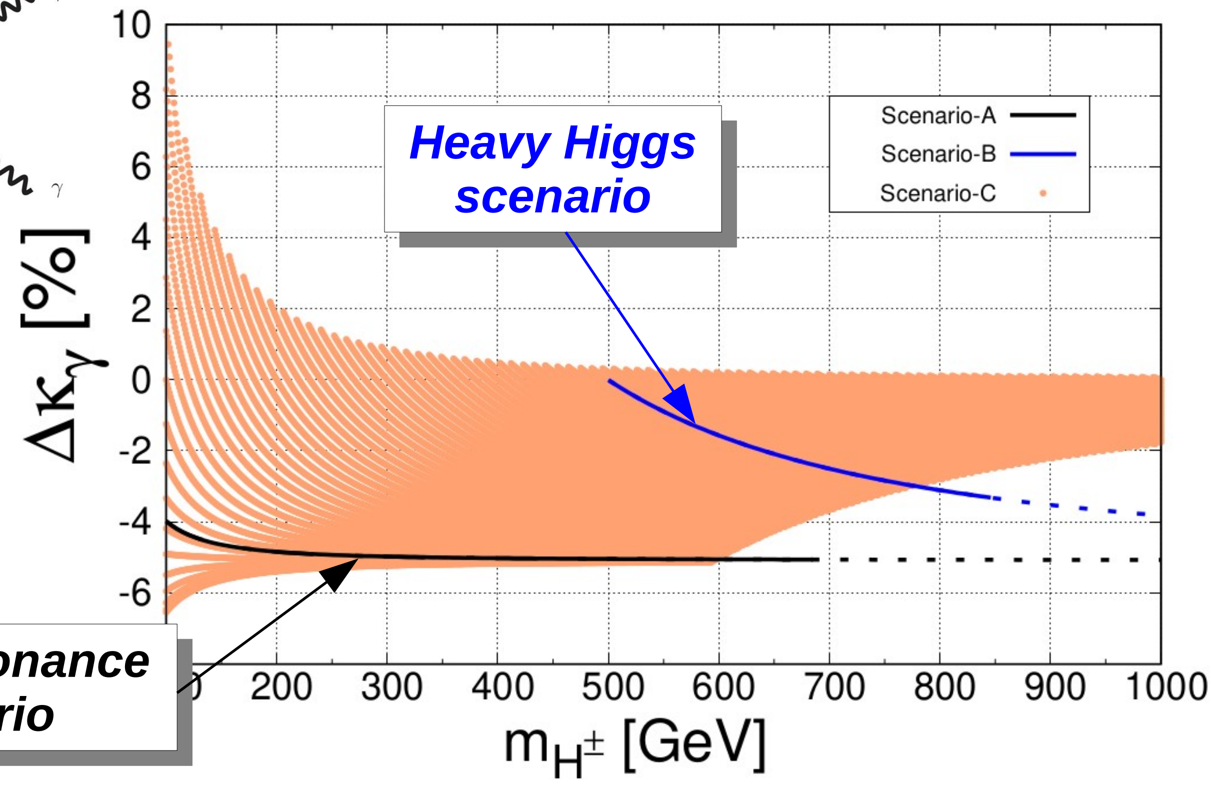
$$\Gamma[h \to \gamma\gamma] \underset{m_h \ll m_{H^\pm}}{\simeq} \frac{\sqrt{2} G_F \alpha_{\rm EM}^2 m_h^3}{64\pi^3} \left| -\frac{1}{6}\left(1 - \frac{\mu_2^2}{m_{H^\pm}^2}\right) + \sum_f Q_f^2 N_c^f I_f[m_h^2] + I_W[m_h^2] \right|^2$$

I_f, I_W: fermion/W-boson loops (SM-like)



- **Charged Higgs contribution:**
  Compensation between mass dependence of coupling ($\lambda_3=2(m_{H\pm}^2-\mu_2^2)/v^2$) and of loop function ($C_0\sim 1/m_{H\pm}^2$)
  → does not decouple when increasing $m_{H\pm}$!

- h→γγ is a loop-induced decay, i.e. ***1L is only leading order (LO)***
  → **What about 2L (NLO) corrections?**



Higgs Low-Energy Theorem

- Calculation of 2L 3-point functions with external momenta not possible in general

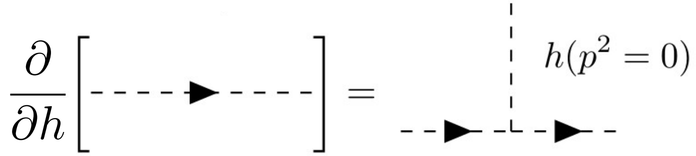
- Assuming m_h << heavy BSM scalar masses, we can employ a **Higgs Low-Energy Theorem** (see e.g. [Kniehl, Spira '95])
- Compute **effective Higgs-photon coupling** $C_{h\gamma\gamma}$ of the form

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} C_{h\gamma\gamma} h F^{\mu\nu} F_{\mu\nu}$$

  by taking derivative of (unrenormalised) photon self-energy w.r.t Higgs field

$$C_{h\gamma\gamma} = \frac{\partial}{\partial h} \Pi_{\gamma\gamma}(p^2 = 0)\Big|_{h=0} \qquad \text{where } \Sigma^{\mu\nu}_{\gamma\gamma}(p^2) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\Pi_{\gamma\gamma}(p^2)$$

- Schematically:



- Neglects incoming momentum on Higgs leg, but fine for m_h << $m_{H,A,H^\pm}$
- Similar to approach of effective-potential calculations of Higgs mass or trilinear Higgs coupling (→ see Parts 4 and 5)

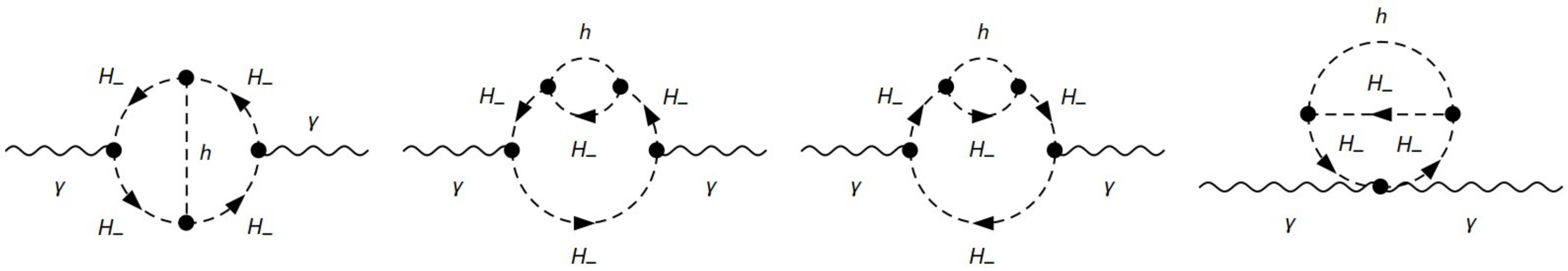
Computing two-loop BSM corrections to $\Gamma(h \to \gamma\gamma)$

- **All known SM contributions**:
  - QCD up to 3L [Djouadi '08] (+ refs. therein)
  - EW SM-like to full 2L [Degrassi, Maltoni '05], [Actis et al. '09]

- Our new calculation: **leading two-loop BSM contributions**
  - genuine, dominant, 2L contributions involving inert scalars
  - purely scalar and fermion-scalar contributions to (1L)^2 terms from external-leg and VEV renormalisation

$$C_{h\gamma\gamma}^{(2),\,\text{IDM}} = C_{h\gamma\gamma}^{\mathcal{O}(\lambda_3^2)} + C_{h\gamma\gamma}^{\mathcal{O}((\lambda_4+\lambda_5)^2)} + C_{h\gamma\gamma}^{\mathcal{O}((\lambda_4-\lambda_5)^2)} + C_{h\gamma\gamma}^{\mathcal{O}(\lambda_2)} + C_{h\gamma\gamma}^{\text{ext.-leg.+VEV}}$$

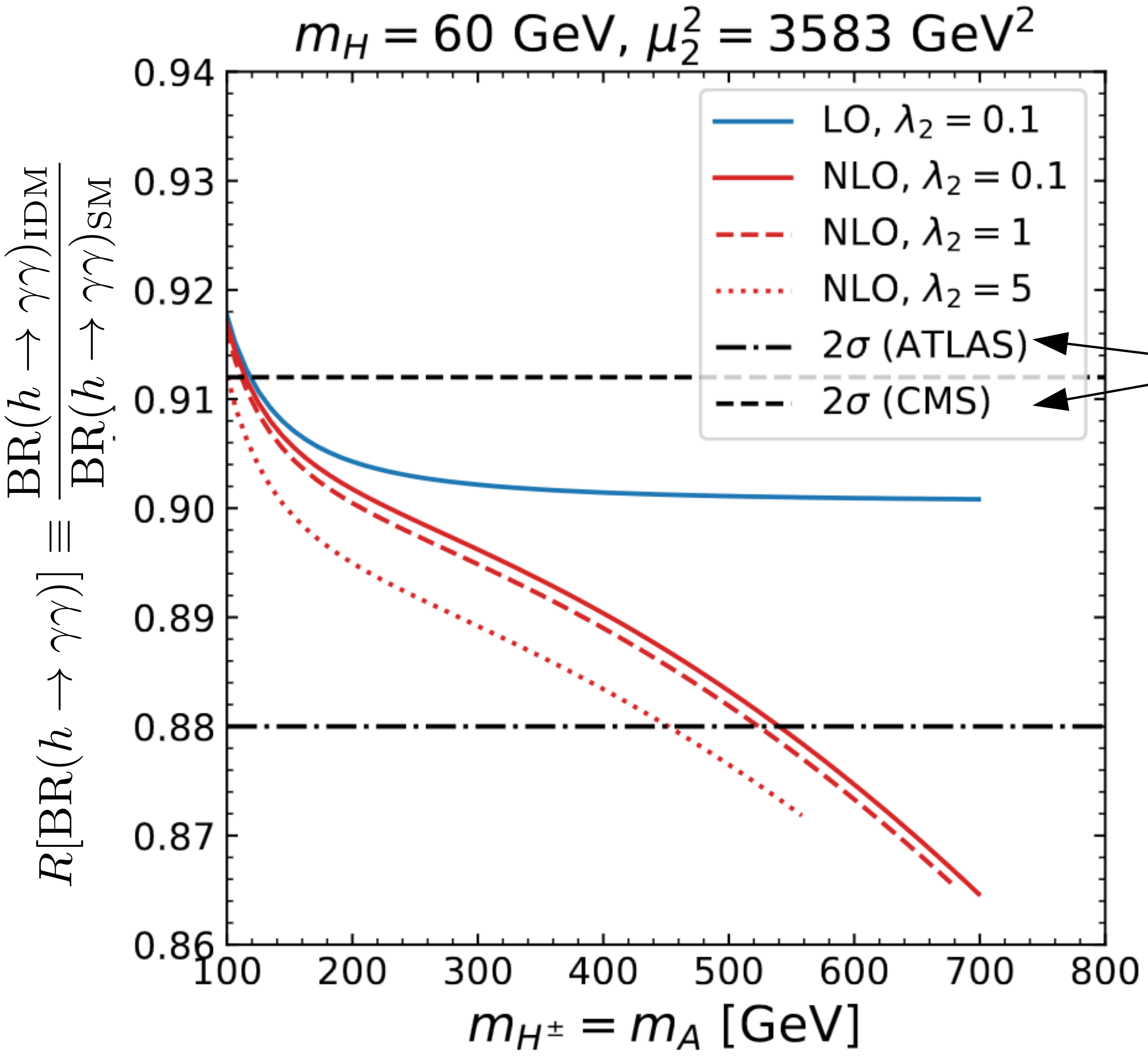
Example:
$O(\lambda_3^2)$ diagrams



- Photon self-energy diagrams generated with `FeynArts`, computed with `FeynCalc` and `Tarcer`, reduced to (limits of) integrals known analytically; then derivative w.r.t. h taken

Results for the Higgs resonance scenario

[Aiko, JB, Kanemura '23]



HL-LHC expected limits

Higgs boson and CP measurements

Higgs boson and CP measurements

- Additional sources of CP **needed for baryogenesis** (c.f. Sakharov conditions)
  → could CP be broken in the Higgs sector ?

- CP violation in **Higgs-gauge** (hVV) **couplings**:
  - Already very constrained via VBF and VH Higgs-production and h→4ℓ decay
    see e.g. [ATLAS,CMS: 2002.05315, 2104.12152, 2109.13808, 2202.06923, 2205.05120]
  - Can only appear at loop level → typically small for most BSM models

- How about **CP violation in Yukawa interactions**?
  - Possible from tree level
  - Numerous ongoing collider investigations at LHC (+ EDMs, c.f. later)
  - Conveniently parametrised in terms of a **Higgs characterisation model** with modified Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = -\frac{1}{\sqrt{2}} \sum_f y_f^{\text{SM}} \bar{f} \left(c_f + i\gamma_5 \tilde{c}_f \right) f h$$

$c_f, \tilde{c}_f$ *: CP-even and CP-odd coupling modifiers*

- NB: in an extended Higgs sector (e.g. 2HDM), there could also be *CP violation in scalar couplings*!

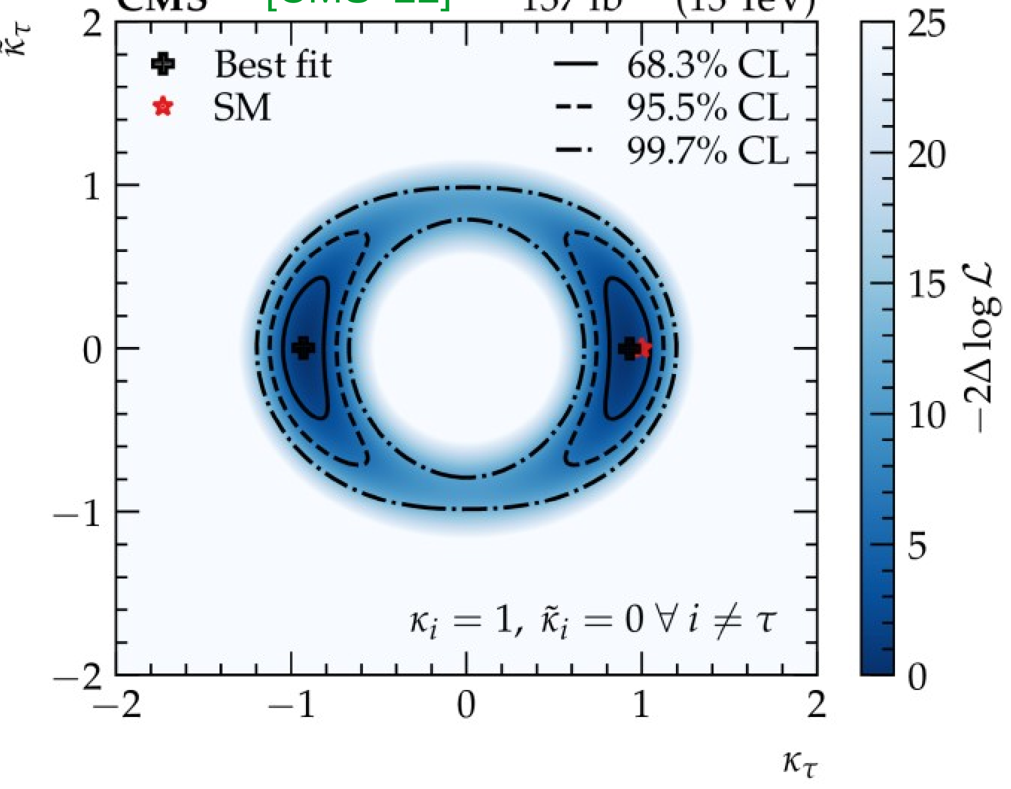
Constraining CP violation in the tau-Yukawa coupling

- A **CP-odd observable** to probe CP structure of tau Yukawa: **Decay angle in h→ττ**

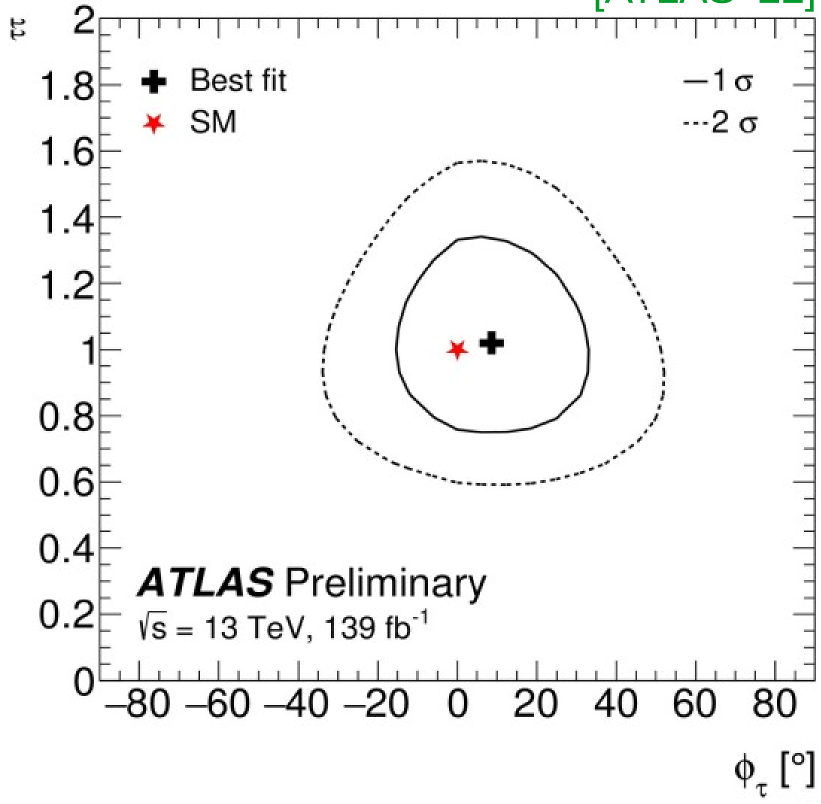
  → sensitive to CP phase in tau Yukawa

- Using angular correlation between decay products in h →ττ decays, ATLAS and CMS have obtained the following experimental constraints:

[CMS '22]



[ATLAS '22]



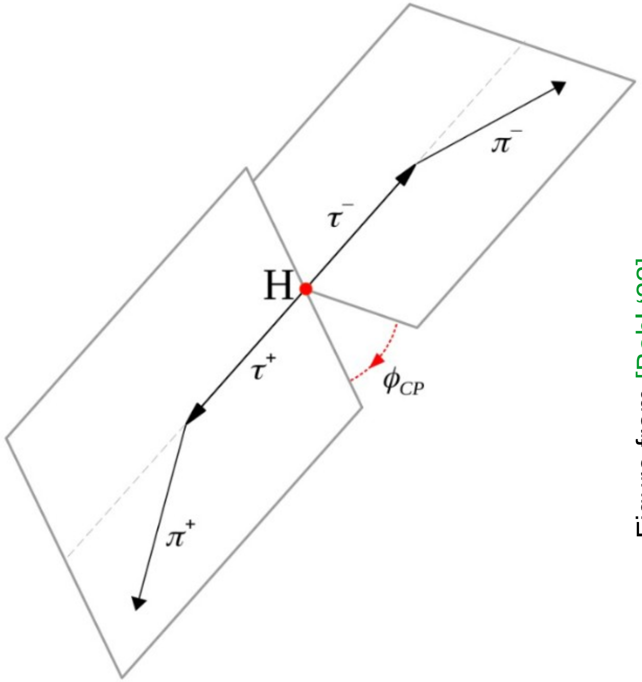


Figure from [Bahl '23]

Constraining CP violation in the tau-Yukawa coupling

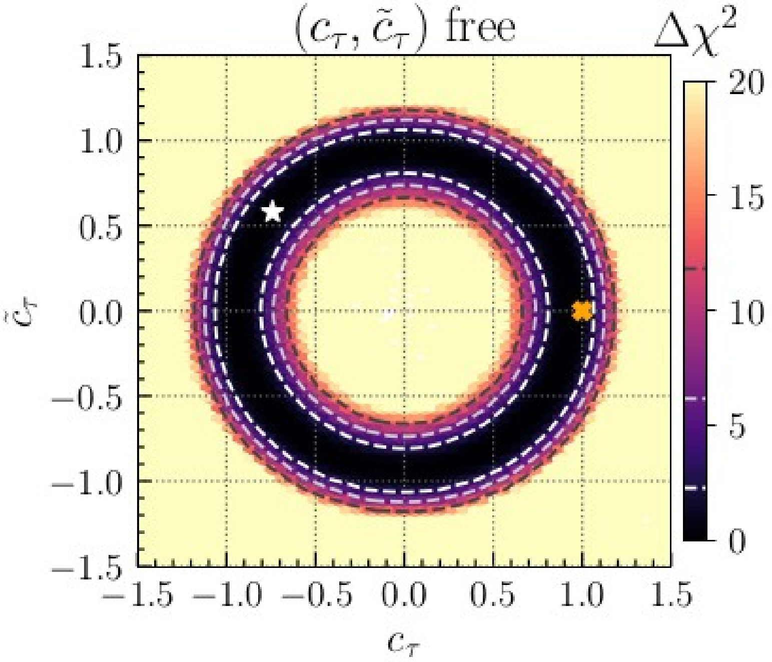
➢ Include CMS results on CP phase to global fit in Higgs/tau characterisation model

[Bahl, Fuchs, Heinemeyer, Katzy, Menen, Peters, Saimpert, Weiglein '22]

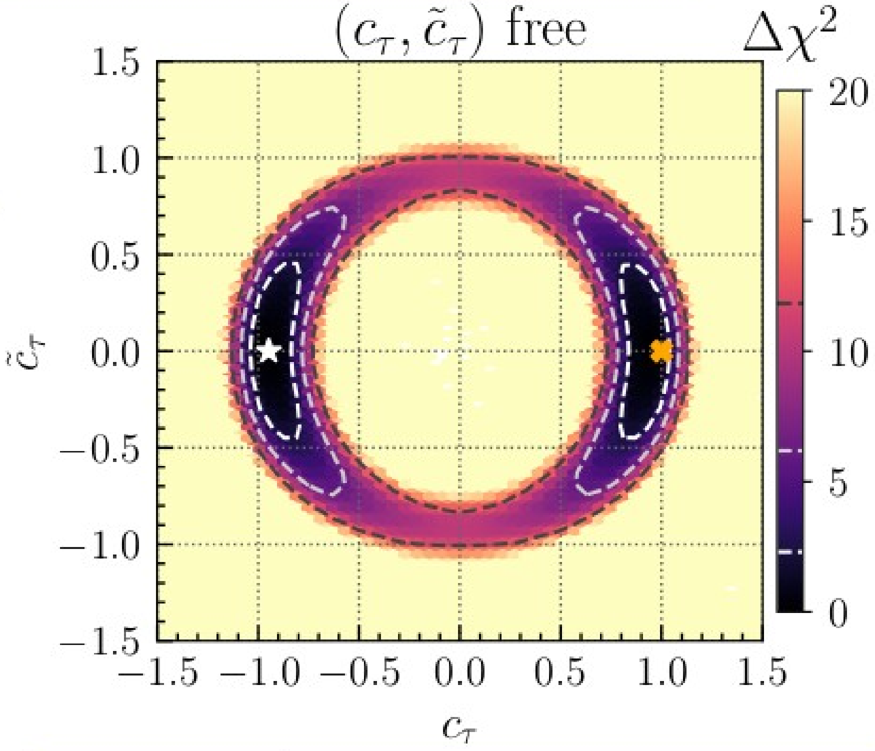
$$\mathcal{L}_{\text{Yuk}} = -\sum_f \frac{y_f}{\sqrt{2}} \bar{f}\left(c_f + i\gamma_5 \tilde{c}_f\right) f h,$$

can also be analyzed in EFT

Global fit using **HiggsSignals** + recent analyses



CMS 2110.04836
h→ ττ CPV analysis



$c_\tau \simeq \pm 1$ almost degenerate minima of $\Delta\chi^2$

Ring-structure from upper/lower bound on BR

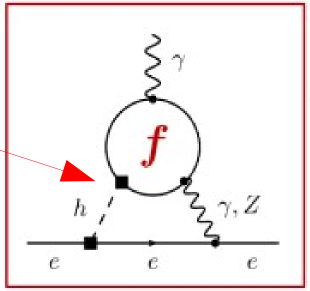
CMS analysis excludes large $\tilde{c}_\tau$

Slide elements from H. Bahl and G. Weiglein

Constraining CP violation in the tau-Yukawa coupling

- **Electron electric dipole moment (EDM)** also gives strong limit on CP violation

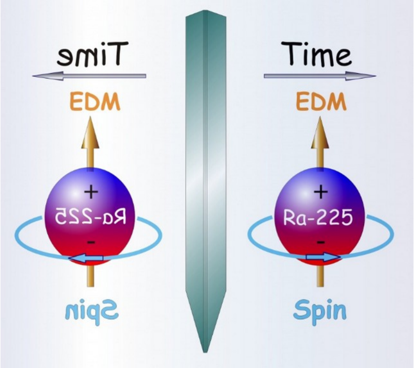
CPV phase from tau Yukawa enters Barr-Zee diagram



ACME [Nature '18]:

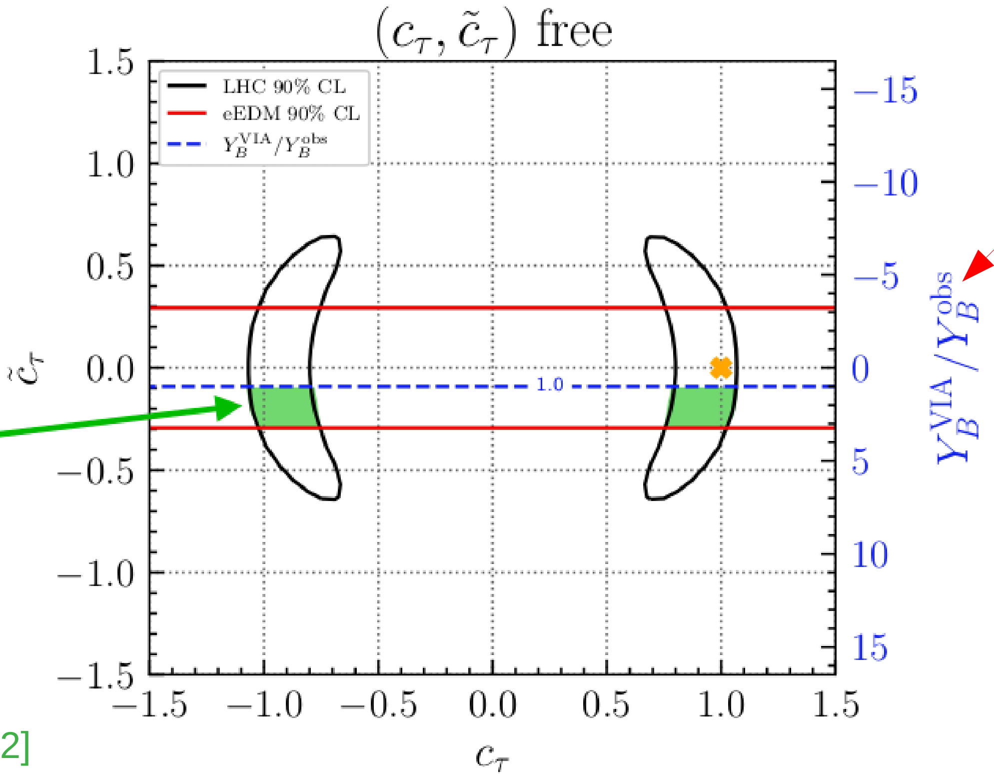
$$d_e \leq 1.1 \times 10^{-29}\, e\,\mathrm{cm} \text{ at } 90\%\ \mathrm{CL}$$

Using [Panico, Pomarol, Riembau '18], [Brod, Haisch, Zupan '13], [Brod, Stamou '18],...



- How do constraints from electron EDM and from LHC h→ττ angular distributions compare?

Electron electric dipole moment $d_e \propto \tilde{c}_f$



Allowed by LHC, EDM constraints and baryogenesis!

Prediction for baryon asymmetry BAU

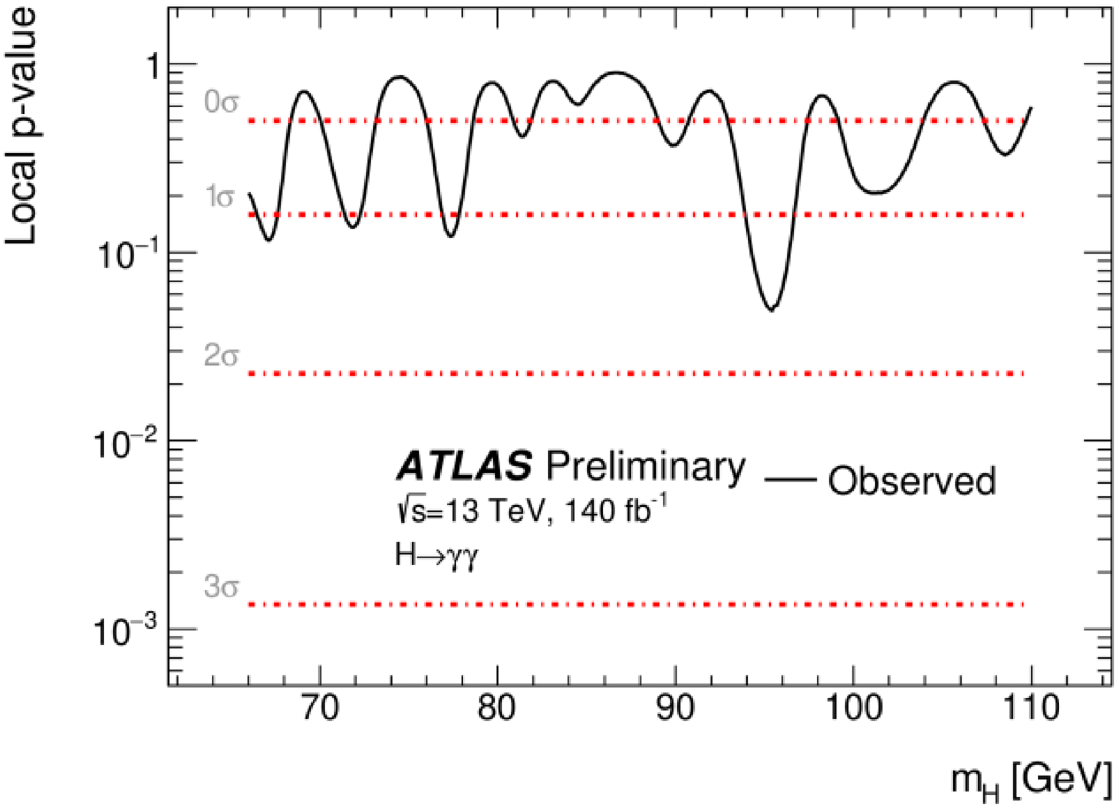
- CP violation in tau-Yukawa (alone) could suffice to **explain matter-antimatter asymmetry** of Universe + this can be **probed in complementary ways by colliders and EDMs**

[Bahl, Fuchs, Heinemeyer, Katzy, Menen, Peters, Saimpert, Weiglein '22]

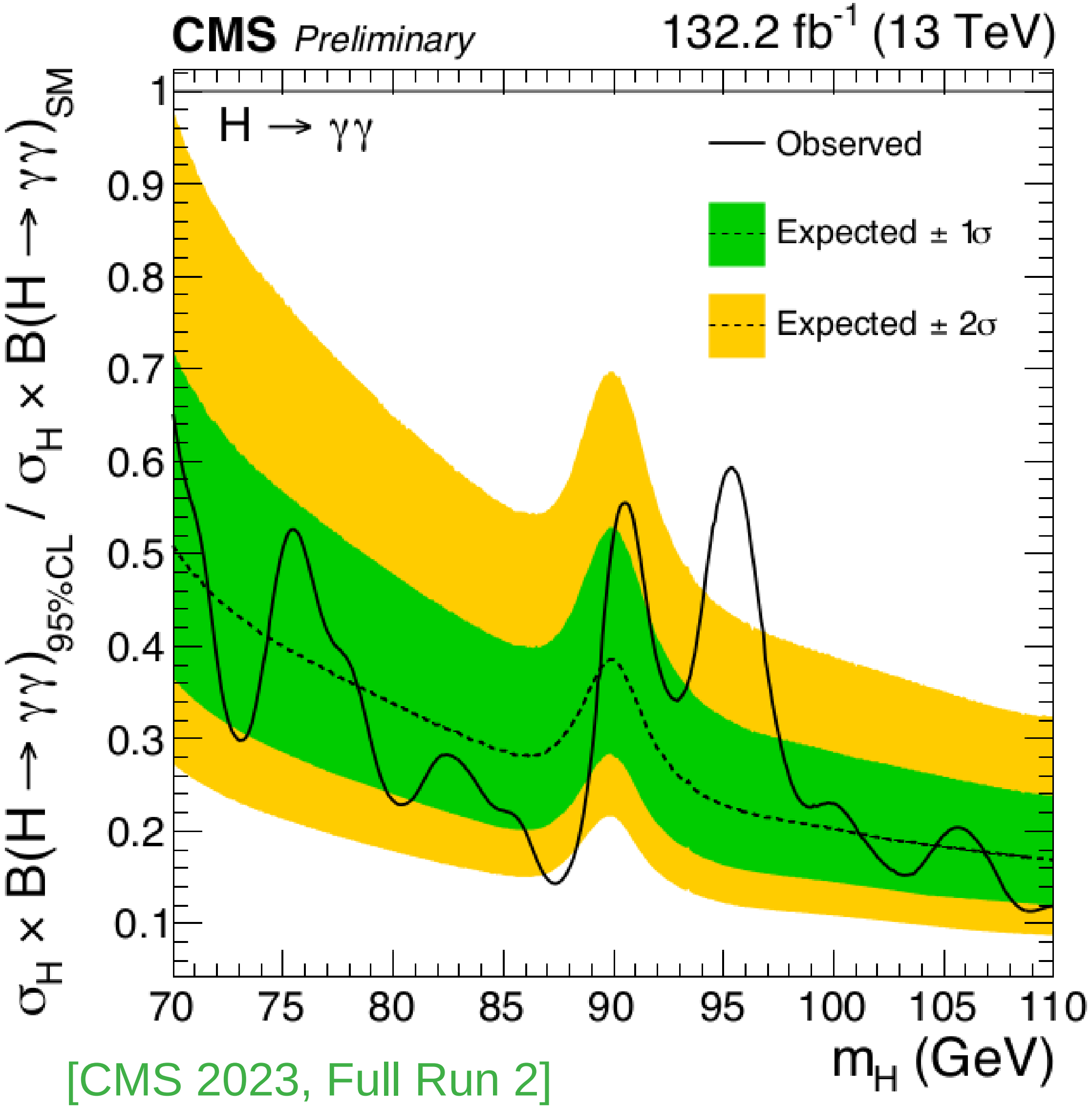
Slide elements from H. Bahl and G. Weiglein

Intermezzo:
additional Higgs bosons at the LHC?

Possible hints of a Higgs boson at 95 GeV – diphoton channel



[ATLAS 2023, Full Run 2]

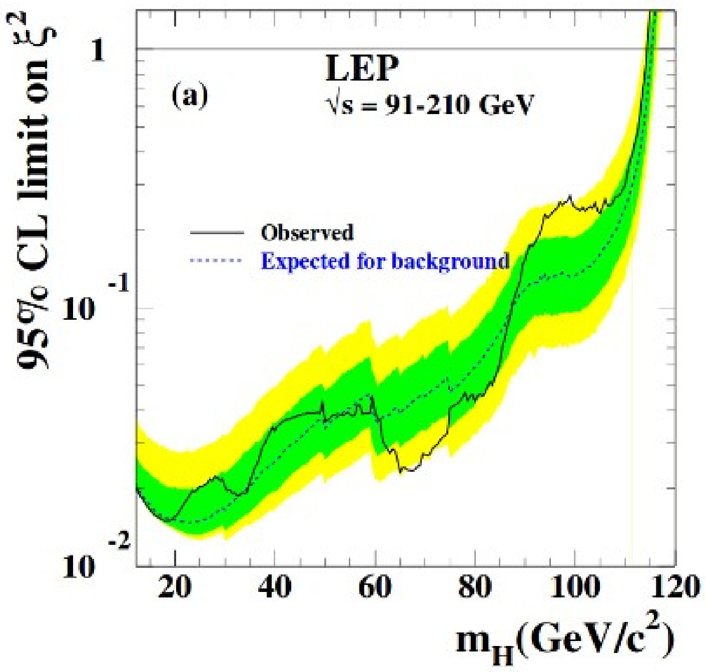


[CMS 2023, Full Run 2]

1.7 σ local excess at 95.4 GeV

2.9 σ local excess at 95.4 GeV

Possible hints of a Higgs boson at 95 GeV – several channels



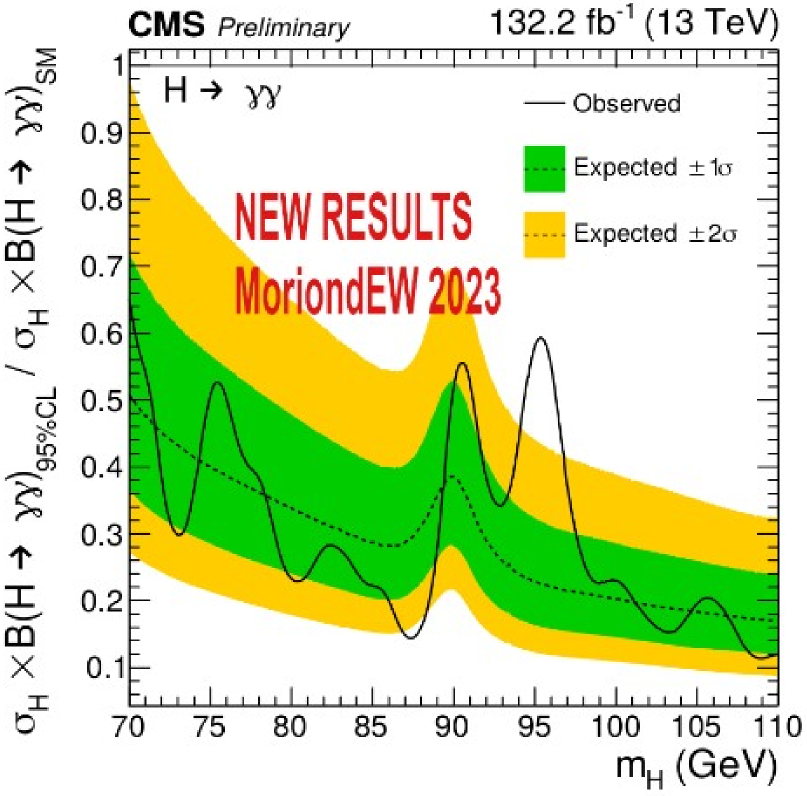
[LEP: hep-ex/0306033]

Local significance: 2.3σ

Signal strength:

$$\mu_{bb}\left(e^+e^- \to Zh \to Zb\bar{b}\right) = 0.117 \pm 0.057$$

[Cao, Guo, He, Wu, Zhang: 1612.08522]

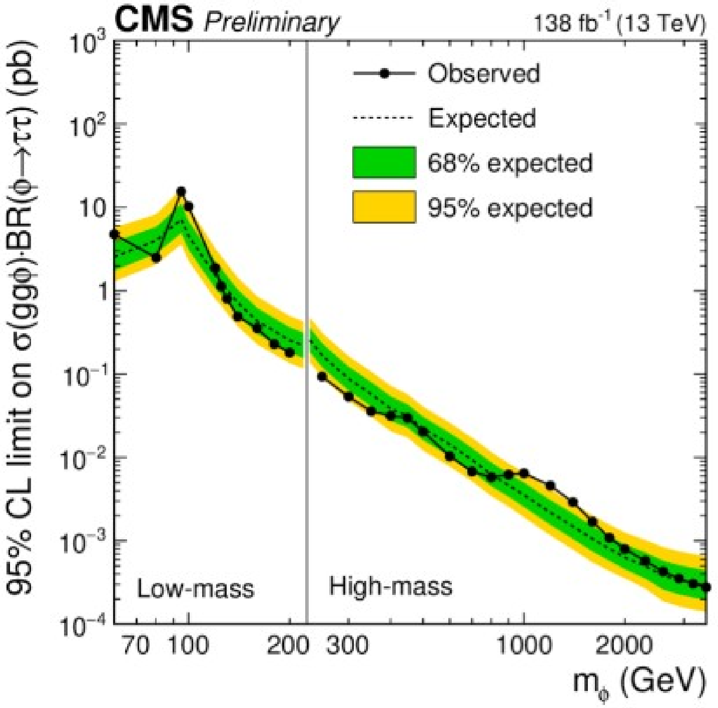


[CMS HIG-20-002]

Local significance: 2.9σ

Signal strength:

$$\mu_{\gamma\gamma}\left(gg \to h \to \gamma\gamma\right) \approx 0.33^{+0.19}_{-0.12}$$



[CMS: 2208.02717]

Local significance: 3.1σ

Signal strength:

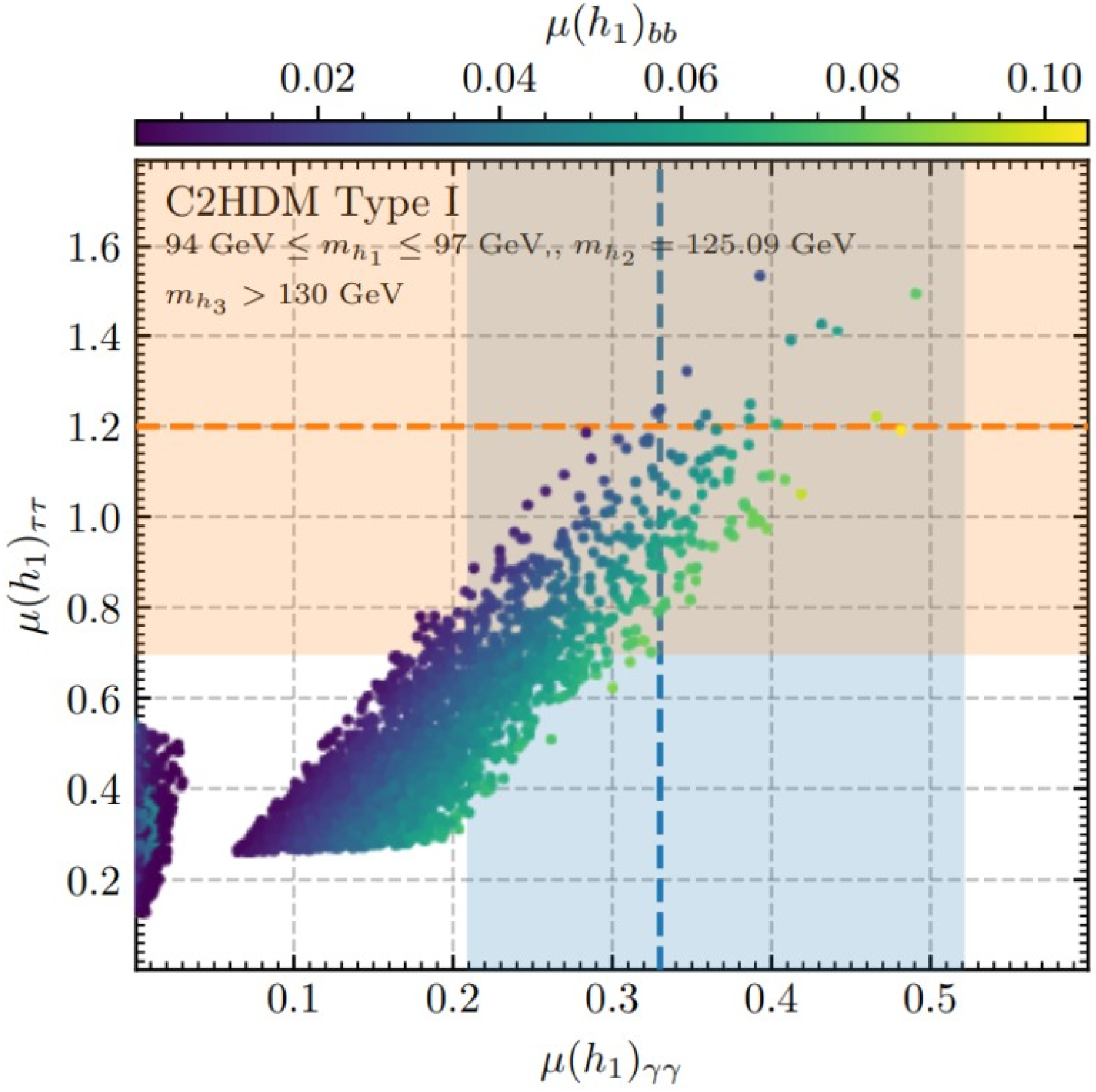
$$\mu_{\tau\tau}\left(gg \to h \to \tau^+\tau^-\right) = 1.2 \pm 0.5$$

[CMS: 2208.02717]

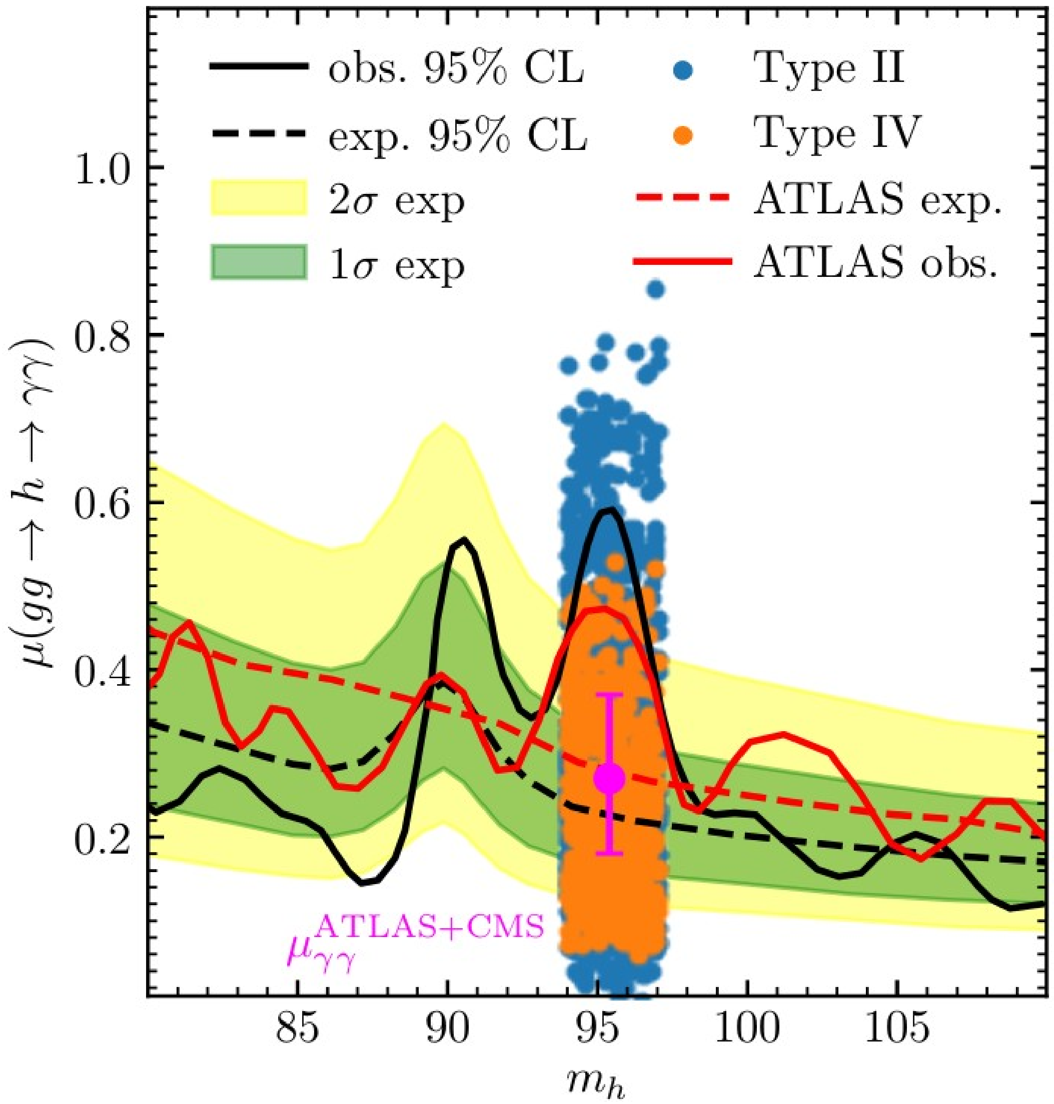
Slide from T. Biekötter

Possible hints of a Higgs boson at 95 GeV – interpretations

Example 1: in a **Two-Higgs-Doublet Model** with complex parameters (**C2HDM**) [Azevedo, Biekötter, Ferreira '23]

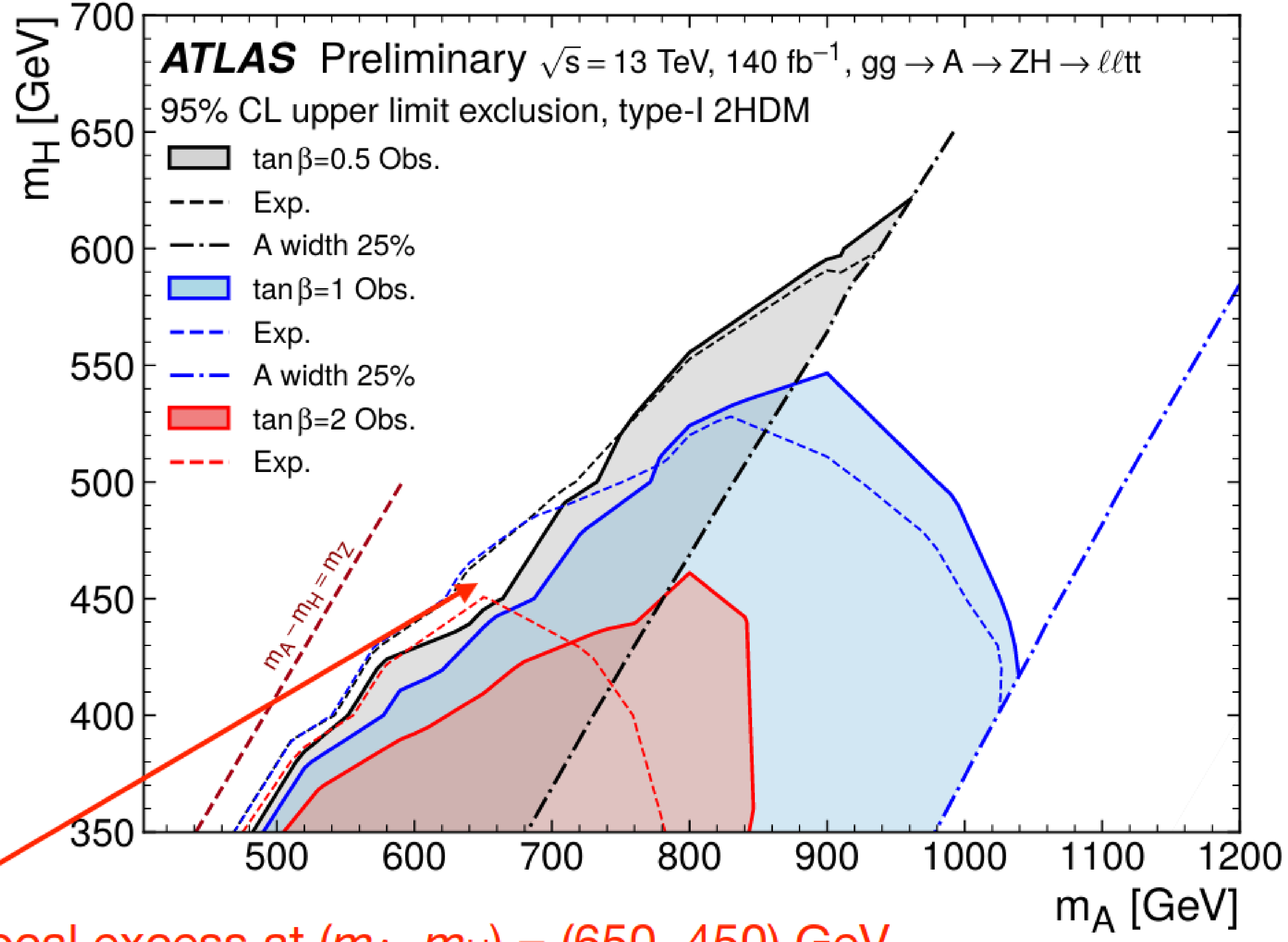


Example 2: **in a Two-Higgs-Doublet Model extended by a complex singlet (S2HDM)** [Biekötter, Heinemeyer, Weiglein '23]



Hints for two Higgs bosons at 450 and 650 GeV?

[ATLAS '23]



2.85 σ local excess at (m_A, m_H) = (650, 450) GeV

Part 4:
The Higgs boson mass,
a new precision observable

Measurements of the Higgs boson mass

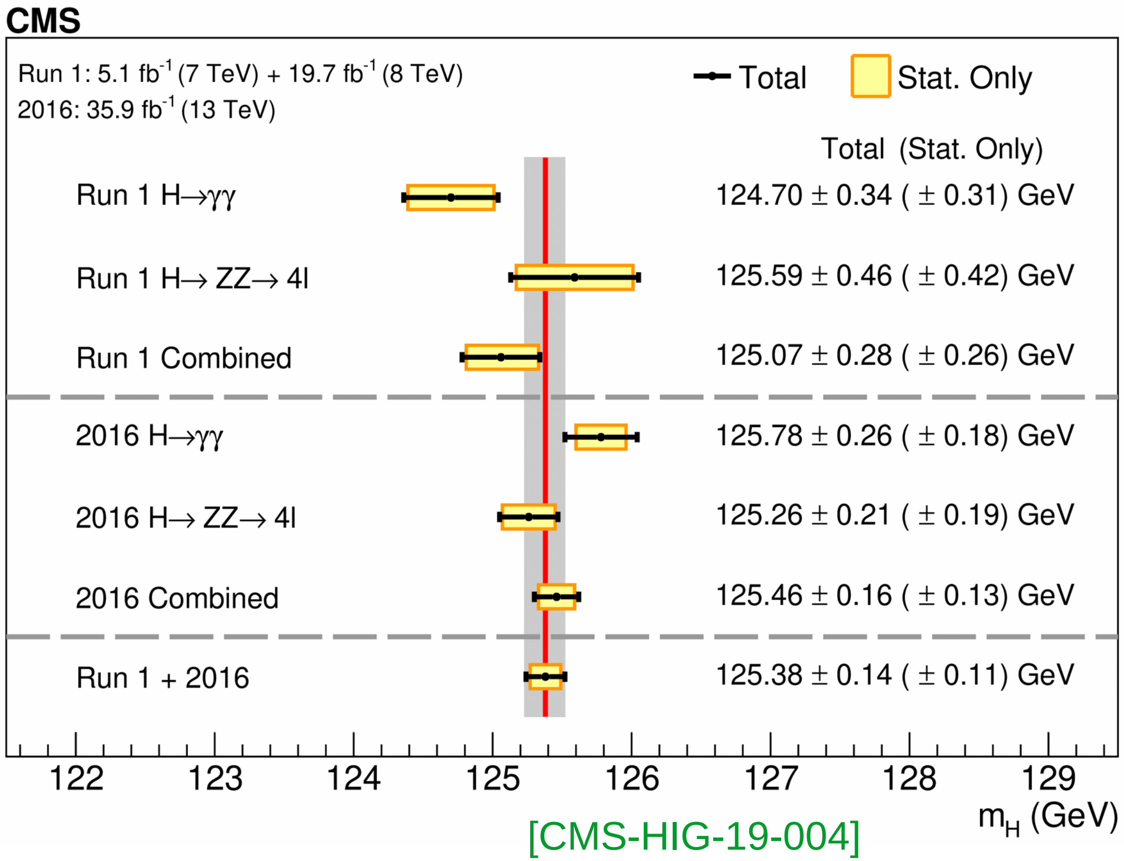
- **Higgs mass** already measured at sub-permille level! → new **precision observable**!

M_h = 125.09 ± 0.21(stat.) ± 0.11(syst.) GeV

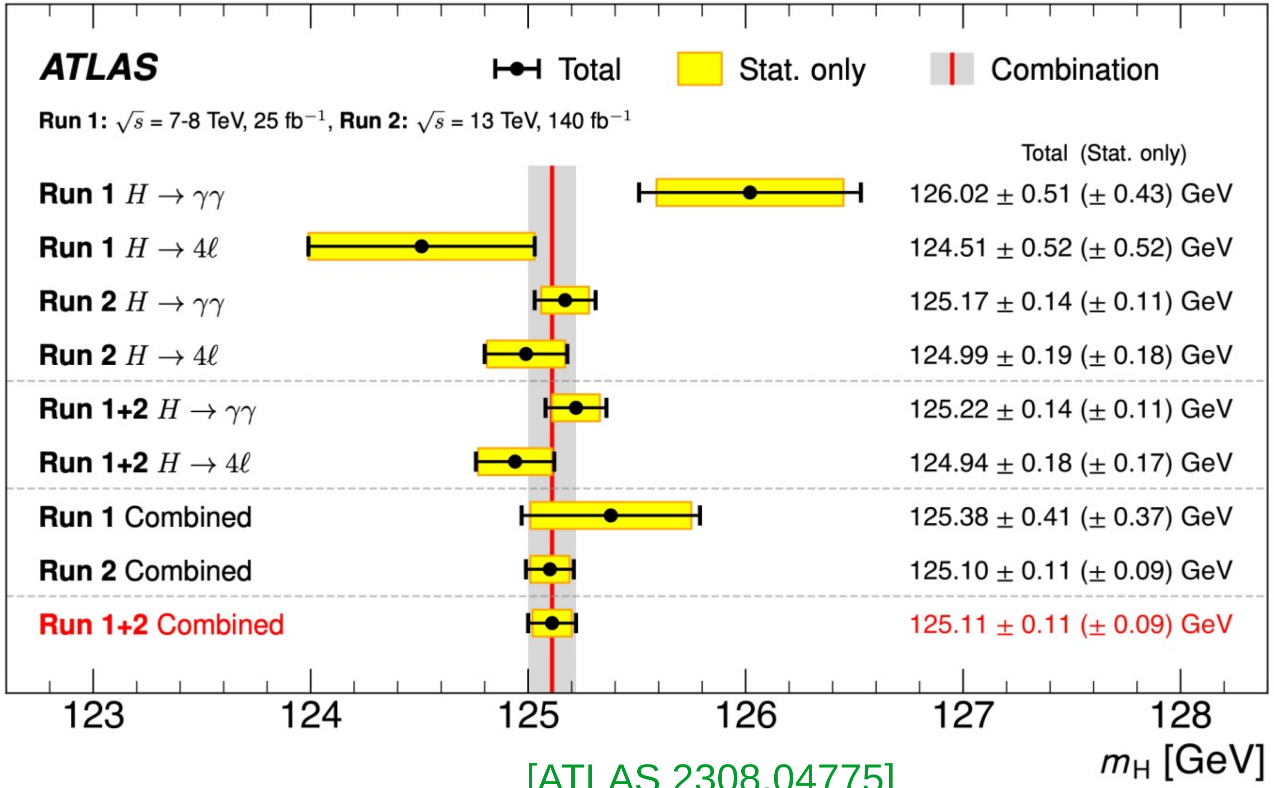
[ATLAS & CMS Run 1 combined, Moriond 2015]

M_h = 125.11 ± 0.11(stat.) ± 0.09(syst.) GeV

[ATLAS 2308.04775 from Run1+Run2 in h→γγ and h→4l channels]



[CMS-HIG-19-004]



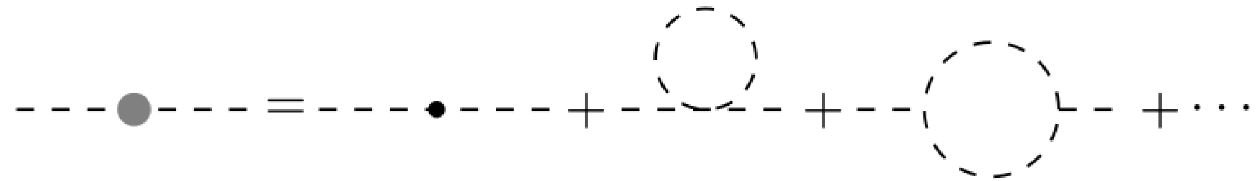
[ATLAS 2308.04775]

Radiative corrections to the Higgs boson mass

*NB: other possible approach → **EFT** (more later)*

- **Feynman-diagrammatic** calculations, *i.e.* solve for M_h^2

$$\underbrace{M_h^2}_{\text{pole mass}} = \underbrace{(m_h^2)^{\text{tree}}}_{\text{tree-level mass}} + \underbrace{\hat{\Sigma}_{hh}(p^2 = M_h^2)}_{\text{self-energy}}$$

$\leftrightarrow$ 

$\hat{\Sigma}_{hh}(p^2)$ computed order-by-order in perturbation theory as Feynman diagrams.
Difficulty: momentum dependence of self-energy diagrams not always known at two loops and higher + long numerical calculations

- **effective potential** approximation $V_{\text{eff}} = V^{(0)} + \Delta V$
  where ΔV are quantum corrections, computed as
  - **one loop**: supertrace formula
  - **two loops and beyond**: 1PI vacuum bubble diagrams

$$\left.\frac{\partial^2 V_{\text{eff}}}{\partial h^2}\right|_{\text{min}} \leftrightarrow \hat{\Sigma}_{hh}(0)$$

$\Rightarrow$ much simpler/faster calculations, but with lower accuracy

- **tadpole equation(s)** $\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{\text{min}} = 0$ are needed to properly relate all couplings

Two interpretations of Higgs mass calculations

- Higgs mass M_h is computed as a function of Lagrangian parameters, in particular quartic Higgs coupling λ

$$M_h = M_h(\lambda, \cdots)$$

- **Case 1: λ is a free parameter of the theory**
  e.g. in SM and many extensions (SSM, 2HDM, etc.)
  → one cannot predict M_h
  → but one can use the equation $M_h(\lambda, \cdots) = 125.09\ \mathrm{GeV}$ to **extract λ** and study the **high-scale behaviour of the theory**

- **Case 2: λ is predicted by the theory**
  e.g: - in SUSY, λ is related to other couplings (EW gauge couplings + eventually SUSY scalar couplings)
  - in (classical) scale invariant models, λ=0 at the scale at which the symmetry is imposed
  - but also the case in a non-SUSY extension of the SM taken as low-energy limit of a UV-model in which λ is predicted (*more on this later*)
  → **M_h can be predicted** as a function of the model parameters

$$(m_h^2)_{\text{tree}} \leq M_Z^2 c_{2\beta}^2, \qquad M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \ldots$$

  → Comparing computed and measured values of M_h → **constrain allowed BSM parameter space**

Case 1:
Extracting scalar quartic couplings from physical spectra (masses & mixing angles)

Extracting a scalar quartic coupling from M_h

In the absence of mixing

Higgs pole mass M_h related to Lagrangian coupling λ by

$$\underbrace{M_h^2}_{\text{pole mass}} = \underbrace{2\lambda v^2}_{\text{tree-level}} + \underbrace{\sum_\ell \frac{1}{(16\pi^2)^\ell}\delta^{(\ell)} m_h^2(\lambda)}_{\text{loop corrections}}$$

$\rightarrow$ highly non-linear equation in λ...

but can be inverted analytically with a perturbative expansion of λ as

$$\lambda = \lambda^{(0)} + \frac{1}{16\pi^2}\delta^{(1)}\lambda + \frac{1}{(16\pi^2)^2}\delta^{(2)}\lambda + \cdots$$

$\rightarrow$ one obtains

$$\lambda^{(0)} = \frac{M_h^2}{2v^2}$$

$$\delta^{(1)}\lambda = -\frac{1}{2v^2}\delta^{(1)} m_h^2 \,\Big|_{\lambda=\lambda^{(0)}}$$

$$\delta^{(2)}\lambda = -\frac{1}{2v^2}\left[\delta^{(1)}\lambda \frac{\partial}{\partial\lambda}\delta^{(1)} m_h^2 + \delta^{(2)} m_h^2\right]_{\lambda=\lambda^{(0)}}$$

Extracting the SM quartic Higgs coupling from M_h

- Renormalisation-group running of SM quartic Higgs coupling λ determines the **fate of the EW vacuum**
  → **stable / metastable / unstable**

- **High-precision calculation of $M_h(\lambda)$ to extract λ is required for a reliable study of vacuum stability**

- State-of-the-art in the SM = Full 2L (+leading 3L and 4L) diagrammatic calculation of m_h and extraction of λ
  [Degrassi et al. '12], [Buttazzo et al. '13], [Martin '13], [Martin '14], [Martin, Robertson '14, '19], [Kniehl et al. '15, '16], [Alam, Martin '22]

  → High-precision study of the stability of EW vacuum in SM since [Degrassi et al. '12], [Buttazzo et al. '13] (see also [Chigusa, Moroi, Shoji '18])

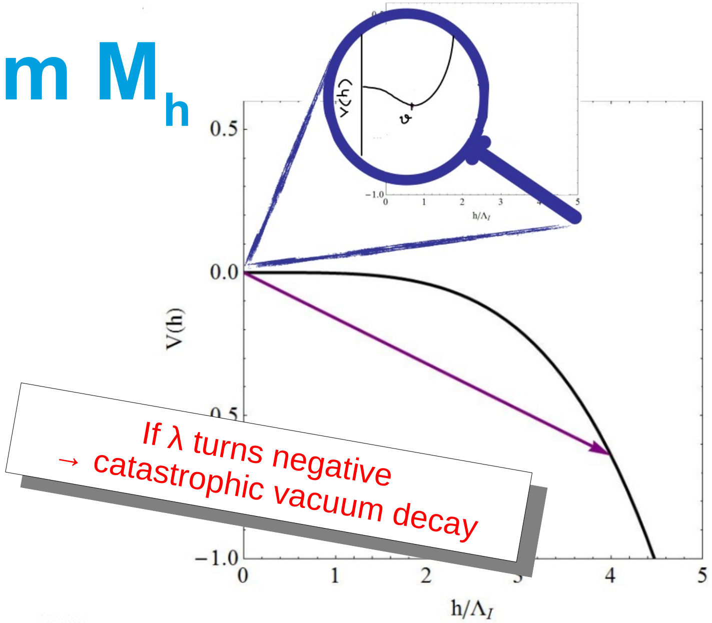
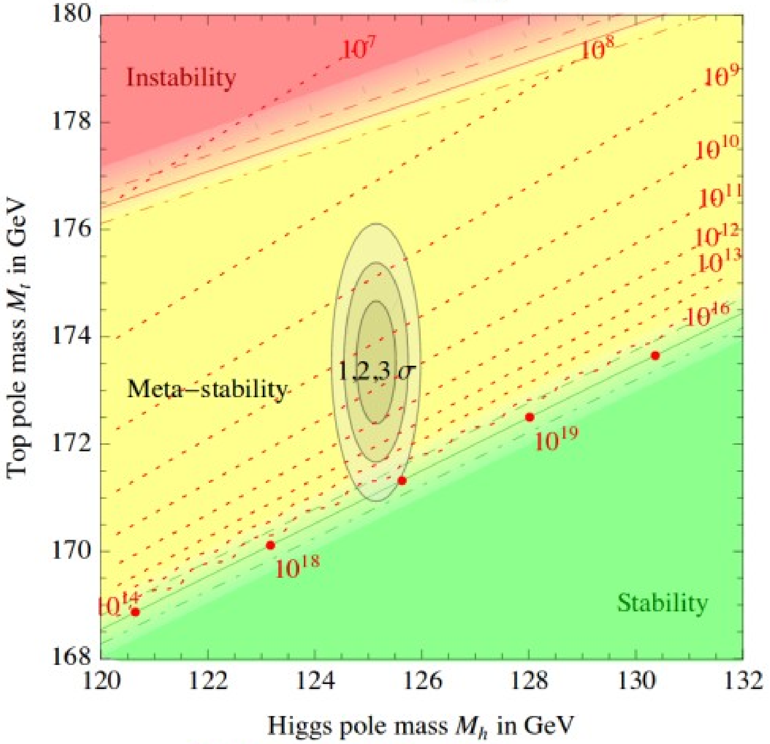


Figure from [Grojean '23]



taken from [Buttazzo et al. '13]

Extracting a scalar quartic coupling from M_h

- *A BSM example without scalar mixing*: the $\mathbf{Z_2}$**SSM**

$\mathbb{Z}_2$SSM $\equiv$ SM + real singlet S + $\mathbb{Z}_2$ symmetry under which $S \to -S$
$\Rightarrow$ no mixing between S and h

$$\boxed{V^{(0)} = \mu^2|\mathbf{H}|^2 + \tfrac{1}{2}M_S^2 S^2 + \tfrac{1}{2}\lambda|\mathbf{H}|^4 + \tfrac{1}{2}\lambda_{SH}S^2|\mathbf{H}|^2 + \tfrac{1}{2}\lambda_S S^4}$$

Extract λ from m_h at two loops

[JB, Goodsell, Krauss, Opferkuch, Staub '17]

$$\delta^{(1)}\lambda = \delta^{(1)}_{\text{SM}}\lambda - \frac{1}{2}\lambda_{SH}^2\,\overline{\log}\,m_S^2$$

$$\begin{aligned}\delta^{(2)}\lambda \simeq \delta^{(2)}_{\text{SM}}\lambda &+ \frac{9}{4v^2}\lambda_{SH}\lambda m_S^2(\overline{\log}\,m_S^2 - 1)\\ &+ \lambda_{SH}^3\left[1 - 2\,\overline{\log}\,m_S^2 + \overline{\log}^2\,m_S^2\right]\\ &+ \frac{1}{4}\lambda_{SH}^2\lambda\Big[-18 - 6\,\overline{\log}^2\,m_S^2\\ &\qquad + (36\,\overline{\log}\,m_h^2 - 12)\,\overline{\log}\,m_S^2\Big]\\ &+ 3\lambda_{SH}^2\lambda_S\left[-1 + \overline{\log}\,m_S^2 + \overline{\log}^2\,m_S^2\right] + \mathcal{O}(v^2/m_S^2)\end{aligned}$$

with $\overline{\log}x \equiv \log x/Q^2$
& Q: renormalisation scale

Extracting scalar quartic couplings from physical spectra

In presence of mixing

- Several relations between loop-corrected masses m_ϕ^2 and parameters/couplings λ_i

$$m_\phi^2 = \text{eig}[(\mathcal{M}^2)^{\text{loop}}(\{\lambda_i\})]_\phi$$

ϕ: list of fields

i: list of couplings

$\Rightarrow$ very difficult to invert analytically

- Instead proceed by numerical iterations: compute $m_\phi^2(\{\lambda_i, \cdots\})$ at given loop-order, for **varying Lagrangian parameters**, until results correspond to desired **physical spectrum**

- Numerical calculations can now be performed up to two-loop order with `SARAH`/`SPheno` *(more later)* + can be done with scanning tools like `BSMArt` [Goodsell, Joury '23]

[Staub '08, '15], [Porod '03], [Staub, Porod '11]

Extracting scalar quartic couplings from physical spectra

- *An example BSM model with scalar mixing*: the **Two-Higgs-Doublet Model (2HDM)**

$$
\begin{aligned}
V^{(0)} =\ & m_{11}^2 \Phi_1^\dagger \cdot \Phi_1 + m_{22}^2 \Phi_2^\dagger \cdot \Phi_2 + m_{12}^2 \left[\Phi_1^\dagger \cdot \Phi_2 + \Phi_2^\dagger \cdot \Phi_1\right] \\
& + \lambda_1 \left(\Phi_1^\dagger \cdot \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \cdot \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \cdot \Phi_1\right)\left(\Phi_2^\dagger \cdot \Phi_2\right) \\
& + \lambda_4 \left(\Phi_1^\dagger \cdot \Phi_2\right)\left(\Phi_2^\dagger \cdot \Phi_1\right) + \frac{1}{2}\lambda_5 \left[\left(\Phi_1^\dagger \cdot \Phi_2\right)^2 + \left(\Phi_2^\dagger \cdot \Phi_1\right)^2\right]
\end{aligned}
$$

$$
\tan\beta = \frac{v_2}{v_1}
$$

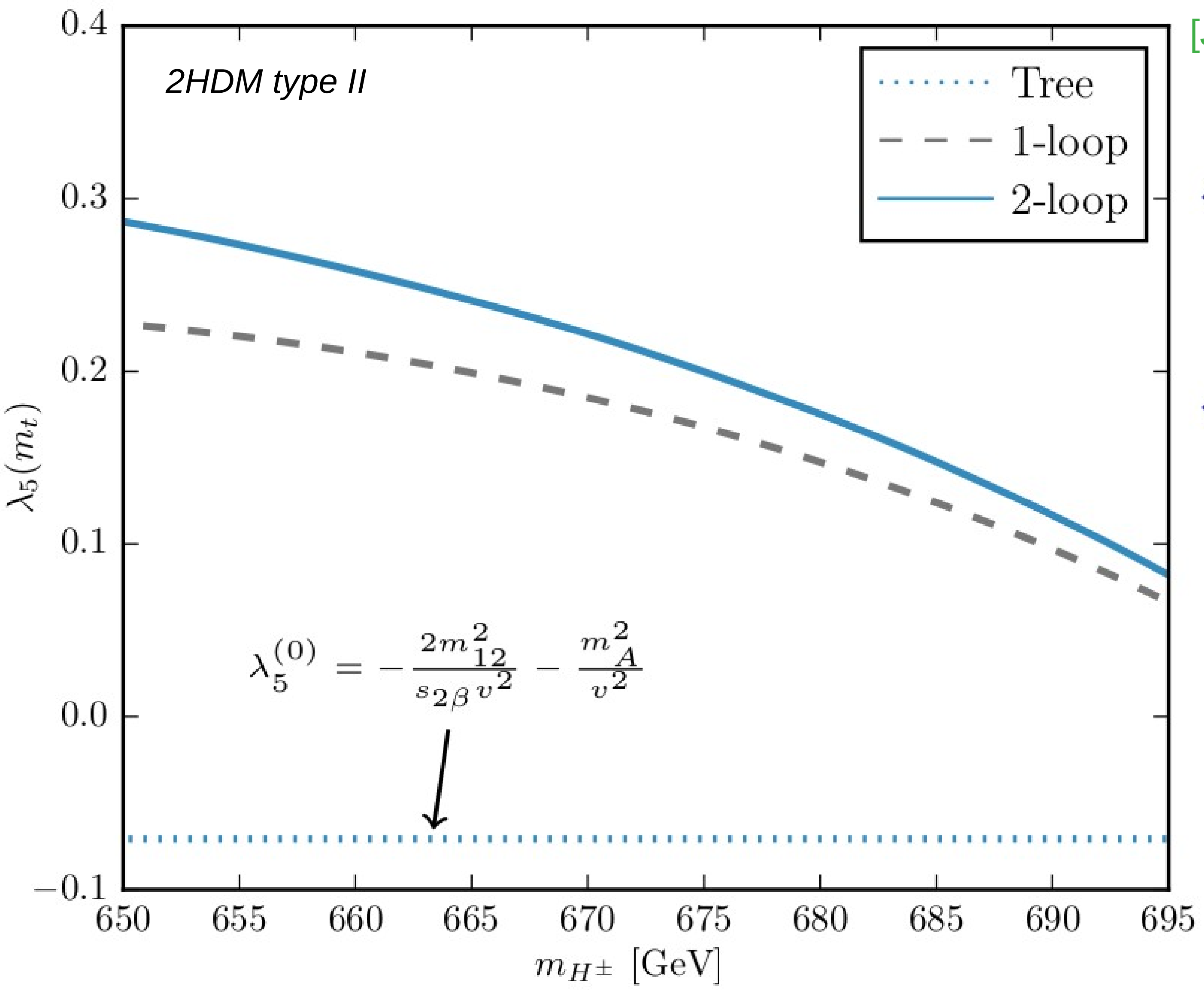
Numerical iterations with SARAH/SPheno [Staub '08, '15], [Porod '03], [Staub, Porod '11]

At N-**loop level** ($N = 0, 1, 2$), an iterative extraction can be done

$$
\left.\begin{array}{c}
\text{Take } m_{12}^2 \text{ and } \tan\beta \text{ as } \overline{\text{MS}} \text{ inputs} \\
\& \\
m_{11}^2 \text{ and } m_{22}^2 \text{ found by solving the } N\text{-}\textbf{loop} \text{ tadpole equations} \\
\& \\
\text{Fix Higgs spectrum} - m_h^2,\ m_H^2,\ m_A^2,\ m_{H^\pm}^2,\ \tan\alpha\ - \\
\text{at } N\text{-}\textbf{loop order}
\end{array}\right\} \Rightarrow \begin{array}{c} \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \\ \textbf{at } N \textbf{ loops!} \end{array}
$$

Extracting scalar quartic couplings from physical spectra

[JB, Goodsell, Krauss, Opferkuch, Staub ‘17]



- ◇ Take $\tan\beta = 1.4$;
  $m_{12}^2 = -(500\text{ GeV})^2$
- ◇ Fix $m_h = 125$ GeV;
  $m_H = 750$ GeV;
  $m_A = 730$ GeV;
  $m_{H^\pm} \in [650, 695]$ GeV
  $\tan\alpha = -0.71$
  at desired order

→ **UV behaviour of 2HDM can be drastically changed by loop-level extraction of quartic couplings!**

Case 2:
SUSY Higgs mass calculations

SUSY Higgs mass calculations – fixed-order calculation

- SUSY models contain extended scalar sectors → **physical masses** found as solutions for p^2 of equation

$$\det\left[p^2\delta_{ij} - (\mathcal{M}^2_{\text{tree}})_{ij} - \Delta\mathcal{M}^2_{ij}(p^2)\right] = 0$$

- At tree level, $m_h \leq M_Z$, however, since early 1990's ([Okada, Yamaguchi, Yanagida '90], [Ellis, Ridolfi, Zwirner '90], [Haber, Hempfling '90]) it has been known that loop corrections can raise m_h to 125 GeV

$$M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M^2_{\text{SUSY}}}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

tanβ: ratio of Higgs VEVs
X_t: stop mixing parameter
M_{SUSY}: SUSY-breaking scale
$\widehat{X}_t \equiv X_t/M_{\text{SUSY}}$

- Since then, huge efforts to improve precision of SUSY Higgs mass calculations
  → summarised in recent report of "Precision SUSY Higgs Mass Calculation Initiative KUTS"
  [Slavich, Heinemeyer (eds.) et al 2012.15629]
  → for the MSSM, state-of-the-art is now almost full 2L in effective-potential approximation, + leading 2L momentum-dependent effect + leading 3L corrections
  → for the NMSSM and beyond (e.g. Dirac gaugino models), leading 2L corrections

  + reliable estimates of theoretical uncertainties (from missing higher orders & parametric uncertainties) → 1-3 GeV depending on point

- However, experimental searches now put lower bounds on stop (scalar partner of top quarks) masses beyond 1 TeV → fixed-order calculations start to suffer from **large logs**

SUSY Higgs mass calculations – the problem of large logs

- Scale of New Physics M_{NP} driven higher by experimental searches

$\Rightarrow$ in fixed-order calculations, **large logarithmic terms** $\propto \log \frac{M_{\text{NP}}}{m_{\text{EW}}}$ can spoil the accuracy, or even the validity, of the perturbative expansion, *e.g.*

$$\mathcal{O} = \underbrace{\alpha^0 a_0}_{\text{tree-level}} + \underbrace{\alpha(b_1 L + a_1)}_{\text{one-loop}} + \underbrace{\alpha^2(c_2 L^2 + b_2 L + a_2)}_{\text{two-loop}} + \cdots$$

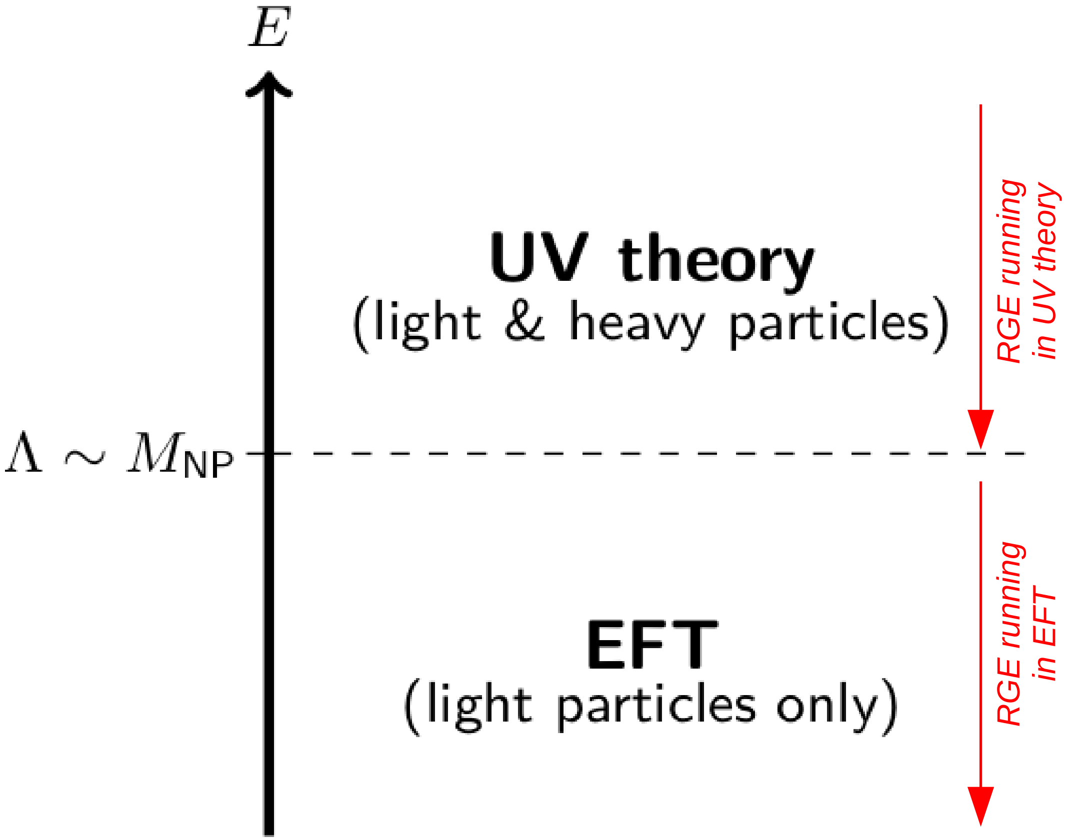
$\alpha \equiv (g/4\pi)^2$, $L \equiv \log M_{\text{NP}}/m_{\text{EW}}$, $a_i, b_i, c_i \in \mathbb{C}$.
Loss of perturbativity if

$$\alpha L \gtrsim 1 \ \cdots$$

The perturbative expansion must be reorganised $\rightarrow$ **EFT calculation**

Intermezzo: an EFT primer

- **Integrate out heavy fields** at some scale $\Lambda \sim M_{\text{NP}}$ and work in a low energy EFT below Λ
- Couplings in the EFT receive **threshold corrections** at the matching scale Λ



Coupling of the UV theory $\tilde{g}$

$\Downarrow$

Match effective actions computed in UV th. and EFT, at ℓ loops:

$$\tilde{\Gamma}^{\text{UV}}(Q=\Lambda) = \Gamma^{\text{EFT}}(Q=\Lambda)$$

$\Downarrow$

ℓ-loop threshold correction to coupling of the EFT g

$$g = \tilde{g} + \Delta g$$

- Use **RGEs** to run the couplings from the high input scale, to the low scale ($\ll M_{\text{NP}}$) at which the calculation is performed $\Rightarrow$ **large logs are resummed!**

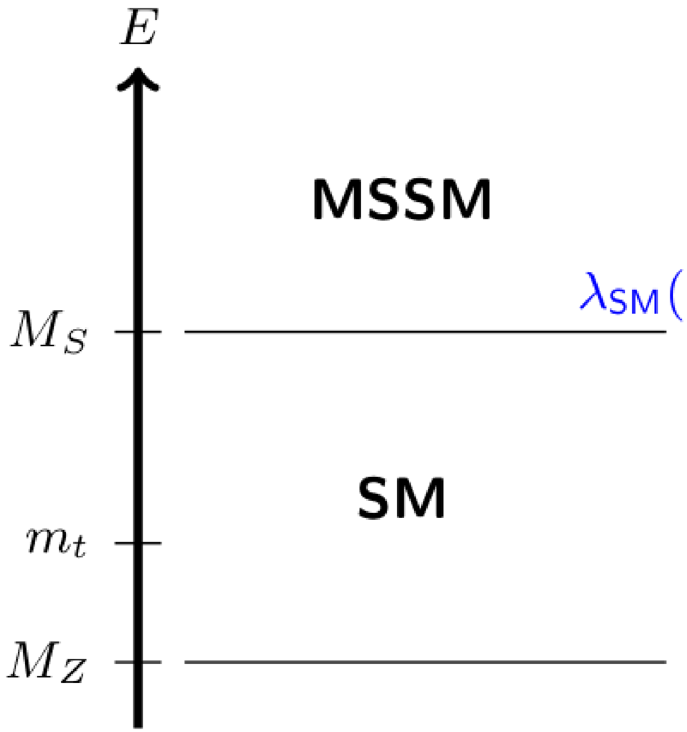
SUSY Higgs mass calculations – EFT approach

- *Simplest example:* **UV theory** $\rightarrow$ **MSSM**, and **EFT** $\rightarrow$ **SM**

see $e.g$ [Bernal, Djouadi, Slavich '07], [Draper, Lee, Wagner '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15], [Bagnaschi, Pardo Vega, Slavich '17], [Athron et al. '17], [Harlander, Klappert, Ochoa Franco, Voigt '18], etc.

More choices of EFTs also considered, see [KUTS report '20] *and refs. therein*

$e.g.$



MSSM inputs

$\Downarrow$

$$\lambda_{\text{SM}}(M_S) = \frac{1}{4}\left(g^2(M_S) + g'^2(M_S)\right)\cos^2 2\beta + \Delta\lambda^{1\ell}(M_S) + \Delta\lambda^{2\ell}(M_S) + \Delta\lambda^{3\ell}(M_S)$$

$\left[\text{Full } 2\ell \text{ RGEs in MSSM}\right]$

$\Downarrow$

$$M_h^2 = f(\lambda_{\text{SM}}(m_t))$$

fixed-order calculation in the SM

$\left[\text{Full } 2\ell + \text{leading } 3\ell \text{ and } 4\ell \text{ RGE for } \lambda_{\text{SM}} \text{ in SM}\right]$

$\Uparrow$

SM inputs

- In many cases $M_S \gg v \Rightarrow$ effect of higher-dimensional operators $\propto v/M_S$ can be disregarded

NB: $M_S = M_{SUSY}$

SUSY Higgs mass calculations – hybrid approaches

- However, for lower MSUSY, EFT calculations lose accuracy (because of v/M_{SUSY} effects)
- Can one **combine the advantages** of fixed-order (reliable for low M_{SUSY}) and EFT (reliable for high M_{SUSY})?
  → Yes!
- Different approaches

  1) FeynHiggs approach [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein PRL '13]

$$(M_h^2)_{\text{FH hyb.}} = (m_h^2)_{\text{tree}} + \underbrace{\hat{\Sigma}_{hh}^{\text{FO}}(M_h^2)}_{\text{fixed-order}} + \underbrace{\left[\lambda(M_t)v^2\right]_{\text{logs.}}}_{\text{EFT log. resum.}} - \underbrace{\left[\hat{\Sigma}_{hh}^{\text{FO}}(M_h^2)\right]_{\text{logs}}}_{\text{subtraction term}}$$

  2) FlexibleSUSY approach → "*pole mass matching*" [Athron et al '17]

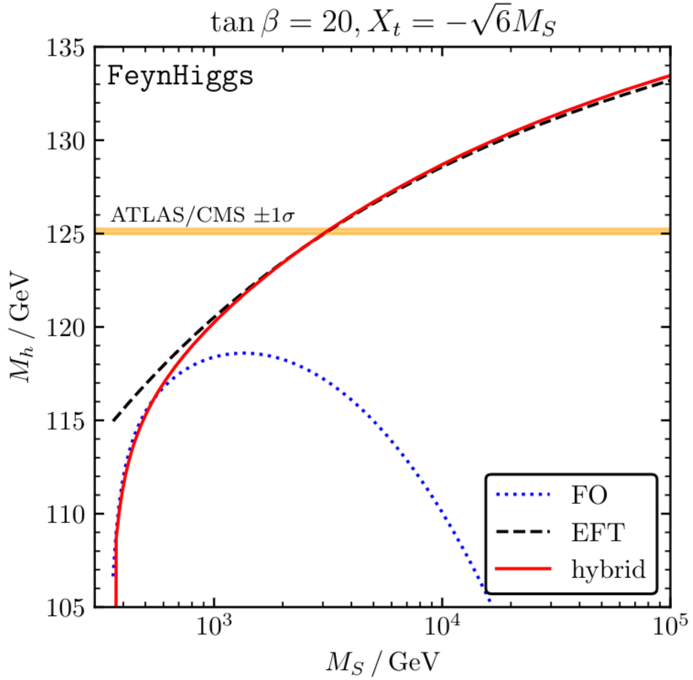
$$\lambda_{\text{SM}}(M_S) = \frac{1}{2\,v^2(M_S)}\left[(M_h^2)_{\text{HET}} - (\Delta M_h^2)_{\text{SM}}\right]$$

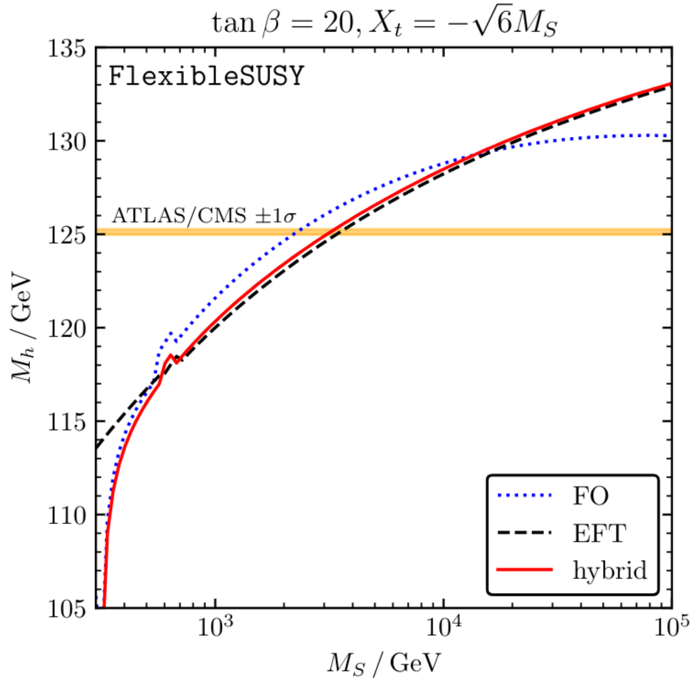
Fixed-order res. in full theory

(also included in SARAH/SPheno)

  3) Aachen group solution
     [Harlander, Klappert, Voigt '19]

$$(M_h^2)_{\text{hyb}} = (M_h^2)_{\text{EFT}} + \Delta_v^{0\ell+1\ell} + \Delta_v^{2\ell}$$





Different types of SUSY Higgs mass calculations – summary

$$M_h^2 \simeq (m_h^2)_{\rm tree} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\rm SUSY}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

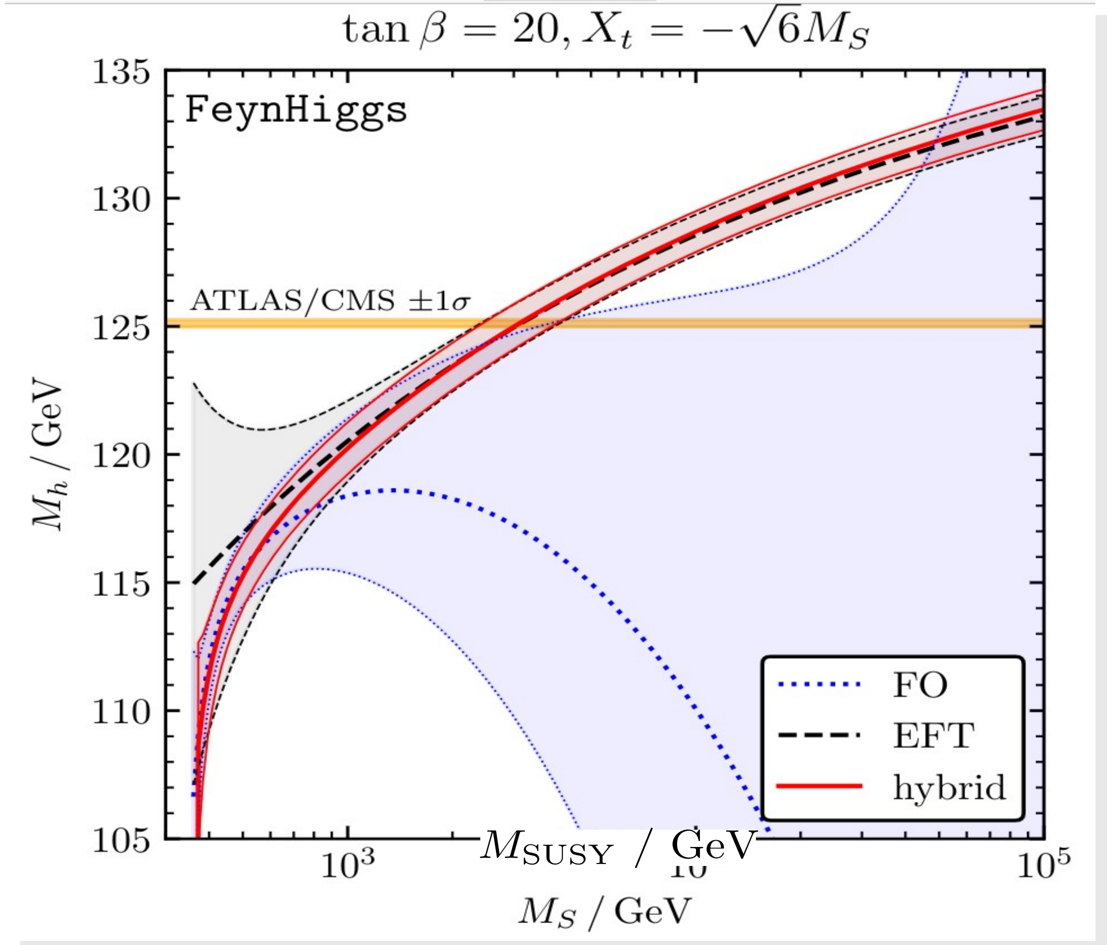
3 types of calculations for M_h:

- **Fixed-order approach**:
  + precise for low SUSY scales
  – but for high scales large logarithms $\log(M_{SUSY}/m_t)$ spoil convergence of perturbative expansion

- **Effective field theory approach**:
  + precise for high SUSY scales (since logarithms are resumed)
  – but for low scales $O(v/M_{SUSY})$ terms are missed if higher-dimensional operators are not included

- **Hybrid approach combing FO and EFT approaches**:
  ++ precise for both low and high SUSY scales.

- Current status in FeynHiggs (*c.f. figure*)
  → FO: full 1L + 2L in gaugeless limit,
  → EFT: full leading-log (LL) + Next-to-LL (NLL) + NNLL + partial N^{3}LL in gaugeless limit



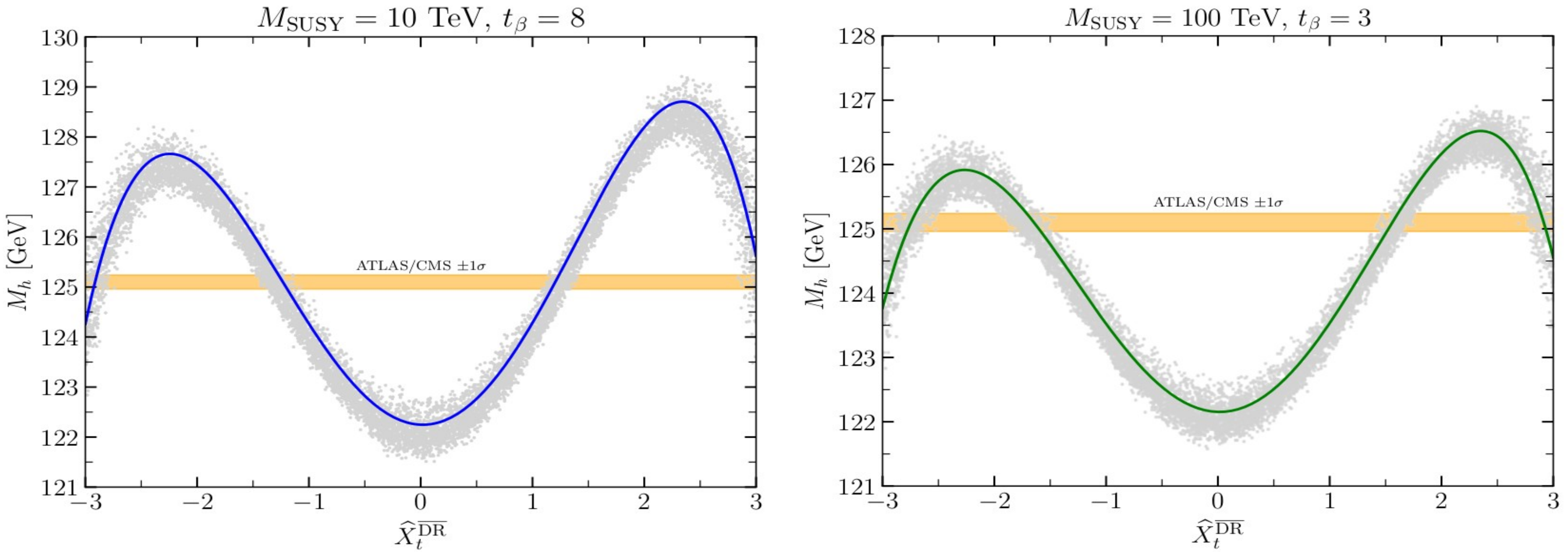
[KUTS report, Slavich, Heinemeyer et al. '20]

Accessing the stop mixing parameter X_t via the Higgs boson mass

- X_t enters prediction of M_h from 1L:

$$M_h^2 \simeq (m_h^2)_{\text{tree}} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

- **Blue/green lines**: common mass scenarios, i.e. all non-SM masses $= M_{\text{SUSY}}$ and $A_{f\neq t} = 0$
- **Grey points**: scan over SUSY parameters (masses and trilinears) between $M_{\text{SUSY}}/2$ and $2\,M_{\text{SUSY}}$



[Bahl, JB, Weiglein '22] *with* `FeynHiggs 2.18.1`

- Significant dependence of M_h on X_t, even for high SUSY scale, at 10 or 100 TeV!
- If stop masses and tanβ known → **X_t can be extracted from M_h**

Automating Higgs mass computations

The motivation for automation

- Interest for non-minimal SUSY and non-SUSY models is growing, driven by experimental results, but in most cases Higgs mass calculations beyond one-loop are still missing
  $\longrightarrow$ huge uncertainties

- Computing corrections from the beginning for every new model would be extremely inefficient and time consuming!

- **Idea**:
  Do the **calculation for a general renormalisable theory** and then apply that result to the considered model
  $\longrightarrow$ can be automated, in public tools like `SARAH` [Staub '08-'15] or `FlexibleSUSY` [Athron et al. '14, '17]

Generic calculations of the Higgs boson mass – conventions

Write the most general renormalisable interactions with real scalars ϕ_i, Weyl fermions ψ_I and vector bosons A^a_μ:

$$
\begin{aligned}
\mathcal{L}_S &= -\frac{1}{6}\lambda^{ijk}\phi_i\phi_j\phi_k - \frac{1}{24}\lambda^{ijkl}\phi_i\phi_j\phi_k\phi_l, \\
\mathcal{L}_{SF} &= -\frac{1}{2}y^{IJk}\psi_I\psi_J\phi_k + c.c., \\
\mathcal{L}_{SV} &= -\frac{1}{2}g^{abi}A^a_\mu A^{\mu b}\phi_i - \frac{1}{4}g^{abij}A^a_\mu A^{\mu b}\phi_i\phi_j - g^{aij}A^a_\mu\phi_i\partial^\mu\phi_j\,, \\
\mathcal{L}_{FV} &= g^{aJ}_I\bar{\psi}^I\bar{\sigma}^\mu\psi_J A^a_\mu\,, \\
\mathcal{L}_{\text{gauge}} &= g^{abc}A^a_\mu A^b_\nu\partial^\mu A^{\nu c} - \frac{1}{4}g^{abe}g^{cde}A^{\mu a}A^{\nu b}A^c_\mu A^d_\nu + g^{abc}A^a_\mu\omega^b\partial^\mu\bar{\omega}^c
\end{aligned}
$$

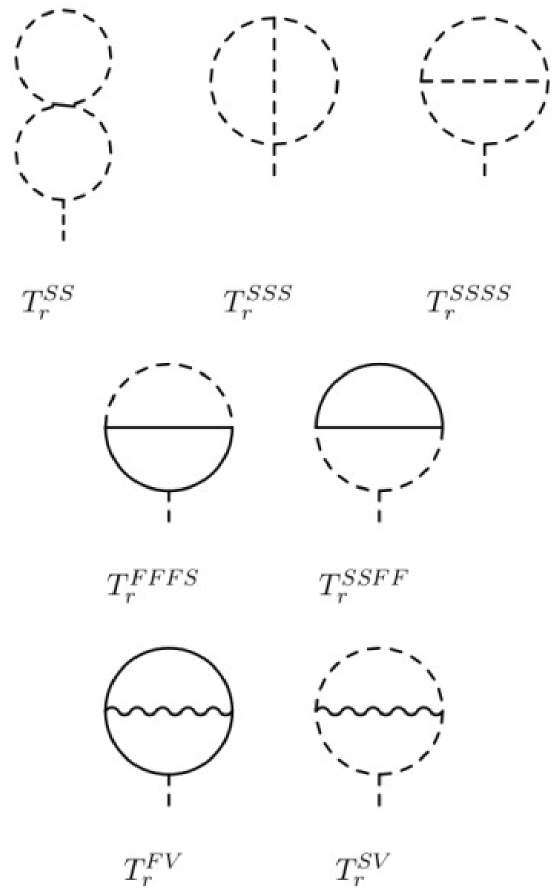
- Here, all fields are defined in mass-diagonal bases
  (some care needed to diagonalise scalar masses)

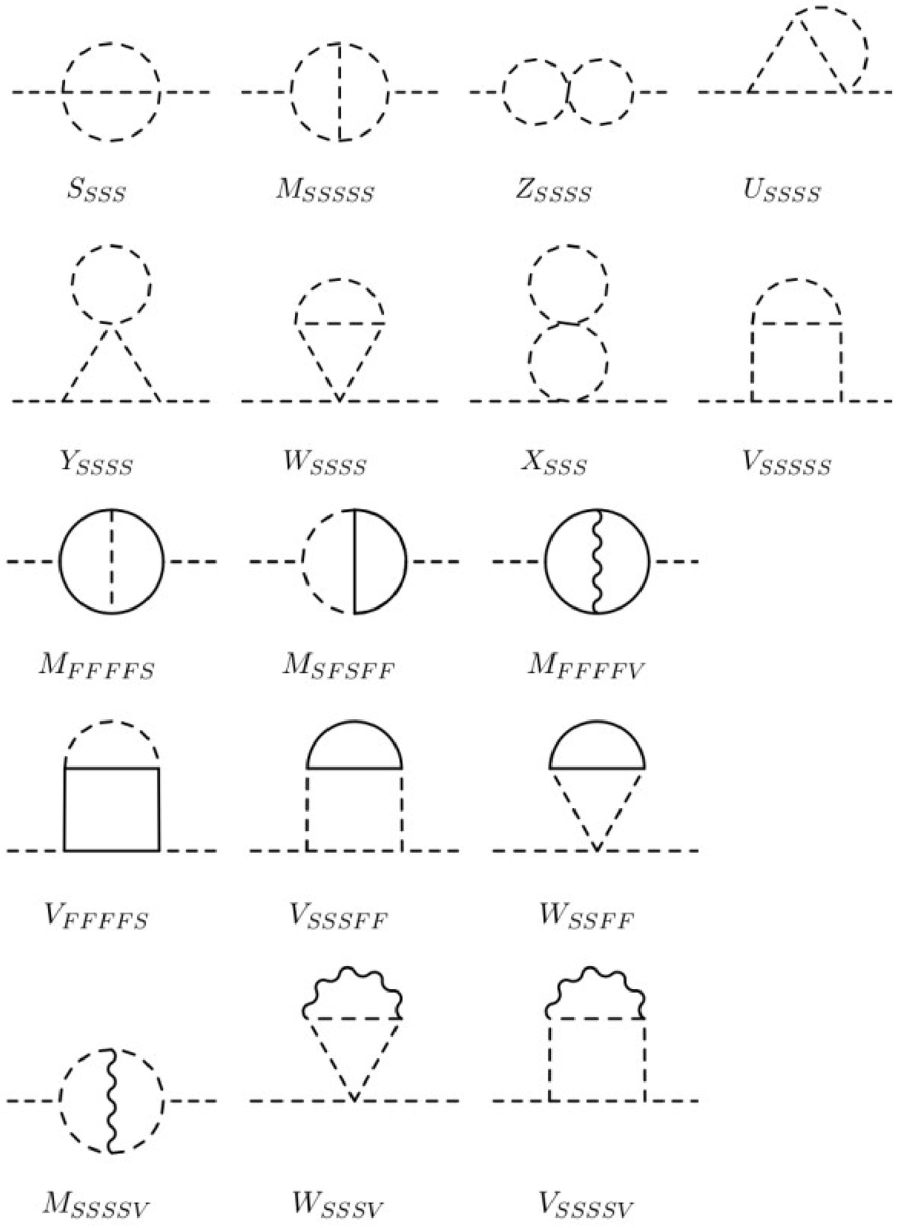
- Interactions between scalars and ghosts turned off by working in Landau gauge

- Parameters usually* renormalised in minimal subtraction schemes ($\overline{\text{MS}}$ or $\overline{\text{DR}}$)
  (: with one notable exception → anyH3 in Part 5)*

Generic calculations of the Higgs boson mass – diagrams

Then we need to compute loop diagrams for V_{eff}, the tadpole equations and the mass diagrams, *e.g.*





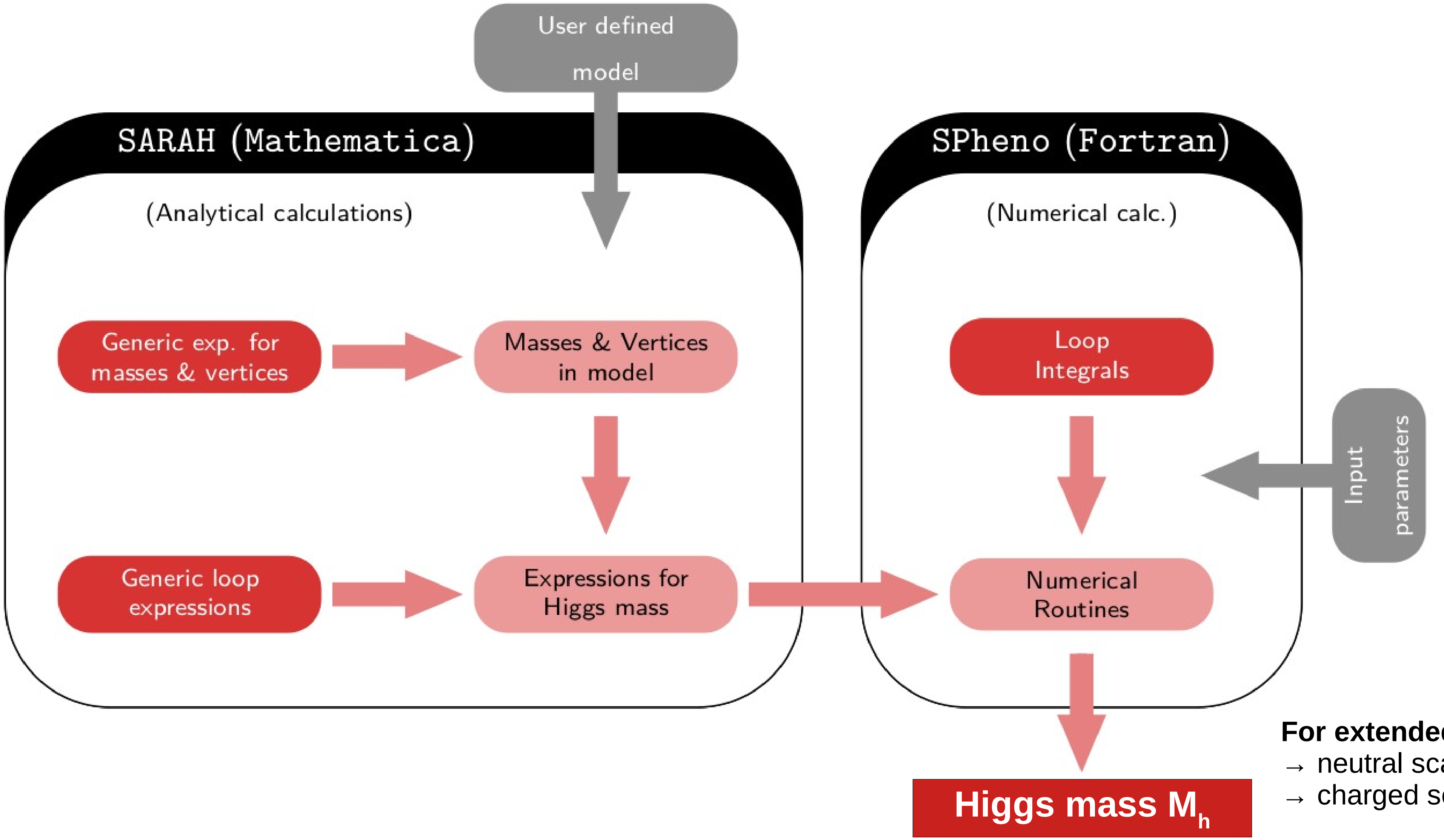
Generic 2L results available for:

- **V_{eff}**:
[Martin '01] (*Landau gauge*),
[Martin, Patel '18] (*general gauge*)
(3L V_{eff} in [Martin '17])

- **Tadpoles**:
[Goodsell, Killian, Staub '15]

- **Self-energies**:
[Martin '03, '05],
[Goodsell, Paßehr '19]

Generic calculations of the Higgs boson mass with SARAH/SPheno



Adapted from a figure by Florian Staub

For extended scalar sectors:
→ neutral scalar masses @ **2L**;
→ charged scalar masses @ **1L**

Many other observables also available! (decays, STU, etc.)

Thank you very much for your attention!

Contact

DESY. Deutsches Elektronen-Synchrotron

www.desy.de

Johannes Braathen
DESY Theory group
Building 2a, Room 208a
johannes.braathen@desy.de

Bonus:
The Goldstone Boson Catastrophe, and its solutions

New results obtained with generic calculations

- **New corrections in MSSM/NMSSM:**
  → effects of non-minimal flavour mixing or R-parity violating operators at 2L in MSSM
  → complete 2L corrections from Yukawas in NMSSM
  → some results only existed in OS scheme (e.g. for the complex MSSM) → now a DR result as well

- **Corrections in models beyond (N)MSSM**: e.g.
  → first two-loop studies in Dirac gaugino models using SARAH [Diessner, Kalinowski, Kotlarski, Stöckinger ’14, ’15] (independent explicit calculation in [JB, Goodsell, Slavich ’16])

- Computations can be done for SUSY models beyond MSSM, or non-SUSY models with almost the same precision as has been reached for the MSSM with explicit calculations

- **However, problems arise when including Goldstones**

First attempt to go beyond the gaugeless limit in the MSSM (at 2L)

[Martin, hep-ph/0211366]

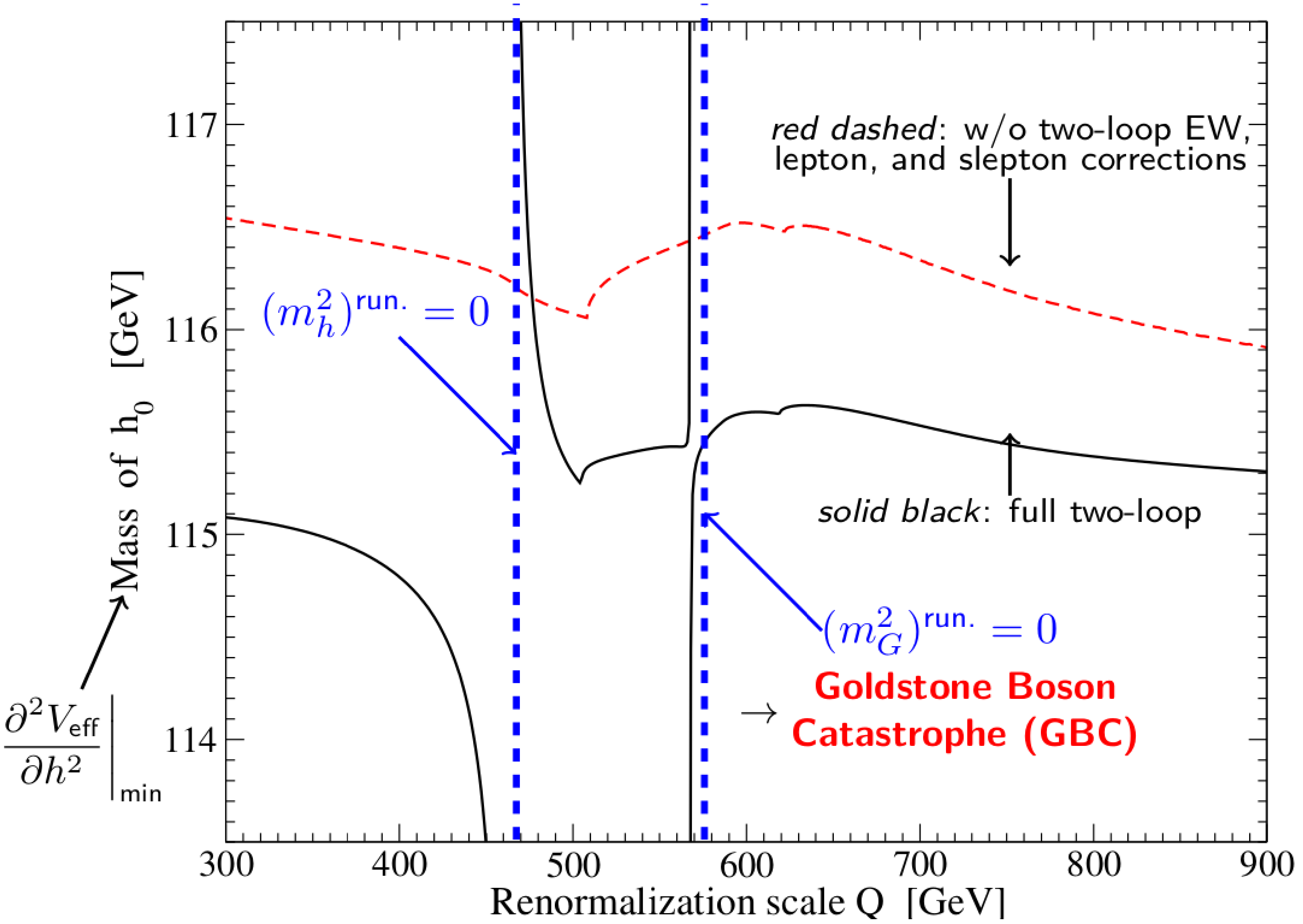


Illustration: the GBC in an Abelian Goldstone model

- 1 complex scalar $\phi = \frac{1}{\sqrt{2}}(v + h + iG)$, no gauge group and only a potential

$$V^{(0)} = \mu^2|\phi|^2 + \lambda|\phi|^4,$$

where $\mu^2 < 0$ and $\lambda > 0$

- Tree-level masses:
  - ▷ Higgs $m_h^2 = \mu^2 + 3\lambda v^2$,
  - ▷ Goldstone $m_G^2 = \mu^2 + \lambda v^2$
- V_{eff} at 2-loop order:

$$V_{\text{eff}} = V^{(0)} + \underbrace{\frac{1}{16\pi^2}\Big[f(m_h^2) + f(m_G^2)\Big]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{1}{(16^2)^2}\Big[\lambda\Big(\frac{3}{4}A(m_G^2)^2 + \frac{1}{2}A(m_G^2)A(m_h^2)\Big) - \lambda^2 v^2 I(m_h^2, m_G^2, m_G^2) + \overbrace{\cdots}^{\text{no Goldstone}}\Big]}_{\text{2-loop}}$$

$$+ \mathcal{O}(\text{3-loop})$$

where $f(x) = \frac{x^2}{4}(\log x/Q^2 - 3/2)$, $A(x) = x(\log x/Q^2 - 1)$ and $I \propto \ominus$

Illustration: the GBC in an Abelian Goldstone model

Tree-level tadpole

$$\left.\frac{\partial V^{(0)}}{\partial h}\right|_{h=0,G=0} = 0 = \mu^2 v + \lambda v^3 = m_G^2 v$$

Loop-corrected tadpole

$$\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{h=0,G=0} = 0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\log \frac{m_G^2}{Q^2}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{m_h^2} A(m_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

Illustration: the GBC in an Abelian Goldstone model

Tree-level tadpole equation

$$\left.\frac{\partial V^{(0)}}{\partial h}\right|_{h=0,G=0} = 0 = \mu^2 v + \lambda v^3 = m_G^2 v$$

Loop-corrected tadpole equation

$$\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{h=0,G=0} = 0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\Big[3A(m_h^2) + A(m_G^2)\Big]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\overbrace{\log\frac{m_G^2}{Q^2}}^{\text{GBC!}}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{m_h^2} A(m_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2\to 0}}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

The Goldstone Boson Catastrophe

- The occurence and severity of the GBC comes from 2 common choices, made for reasons of simplicity:
  - working in the Landau gauge, in order to decouple the ghosts
    $\Rightarrow$ Goldstones are treated as actual massless bosons *i.e.* $(m_G^2)^{\text{OS}} = 0$
  - using running ($\overline{\text{MS}}/\overline{\text{DR}}'$) masses for the particles in the loops

$$(m_G^2)^{\text{run.}} = (m_G^2)^{\text{OS}} - \Pi_{GG}(p^2 = (m_G^2)^{\text{OS}}) = -\Pi_{GG}(p^2 = 0),$$

  where Π_{GG} is the Goldstone self-energy $\Rightarrow$ dependence on renorm. scale Q

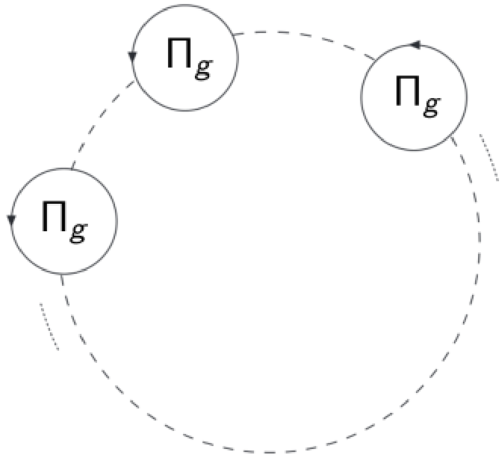
- V_{eff}, tadpoles, and self-energies contain $\log$s of $(m_G^2)^{\text{run.}}$, via loop functions
- Under RG flow, $(m_G^2)^{\text{run.}}$ may
  - $\rightarrow$ become 0 $\Rightarrow$ IR divergences
  - $\rightarrow$ change sign $\Rightarrow$ unphysical imaginary parts

$\equiv$ **Goldstone Boson Catastrophe**

NB: working in another gauge would require long and painful calculations, and would **not** necessarily evade the GBC for all values of Q!
(previous example had no gauge group, but still encountered the GBC...)

First solutions to the Goldstone Boson Catastrophe

- By hand: drop the imaginary part of V_{eff} or adjust Q
- Some work-arounds for automated codes (SARAH) $\to$ different ways of changing the Goldstone contributions to avoid the problem, but **physically wrong**! (especially wrong when Goldstone contributions to scalar masses are large $\to$ non-SUSY models)
- First theoretical solution: resum the Goldstone contributions to V_{eff}
  SM [Ellias-Miro, Espinosa, Konstandin '14], [Martin '14]; **MSSM**: [Kumar, Martin '16];
  generic theories: [JB, Goodsell '16]



[Adapted from arXiv:1406.2652]

- Power counting $\to$ most divergent contribution to V_{eff} at ℓ-loop $=$ ring of $\ell-1$ Goldstone propagators and $\ell-1$ insertions of 1PI subdiagrams Π_g involving **only** heavy particles
- Π_g obtained from Π_{GG}, Goldstone self-energy, by removing "soft" Goldstone terms $\to$ can be done with *method of regions* [Espinosa, Konstandin '17]
- Resumming Goldstone rings $\Rightarrow$ ℓ-loop resummed effective potential, free of leading Goldstone boson catastrophe

- However extending the resummation to generic theories is technically difficult, and doesn't solve the GBC for self-energies *(more on this in a few slides)*

Setting the Goldstone boson "on-shell"

[JB, Goodsell arXiv:1609.06977]

Adopt an on-shell scheme for the Goldstone(s)

- Replace $(m_G^2)^{\text{run.}}$ by $(m_G^2)^{\text{OS}}(=0)$ and $\Pi_{GG}(0)$

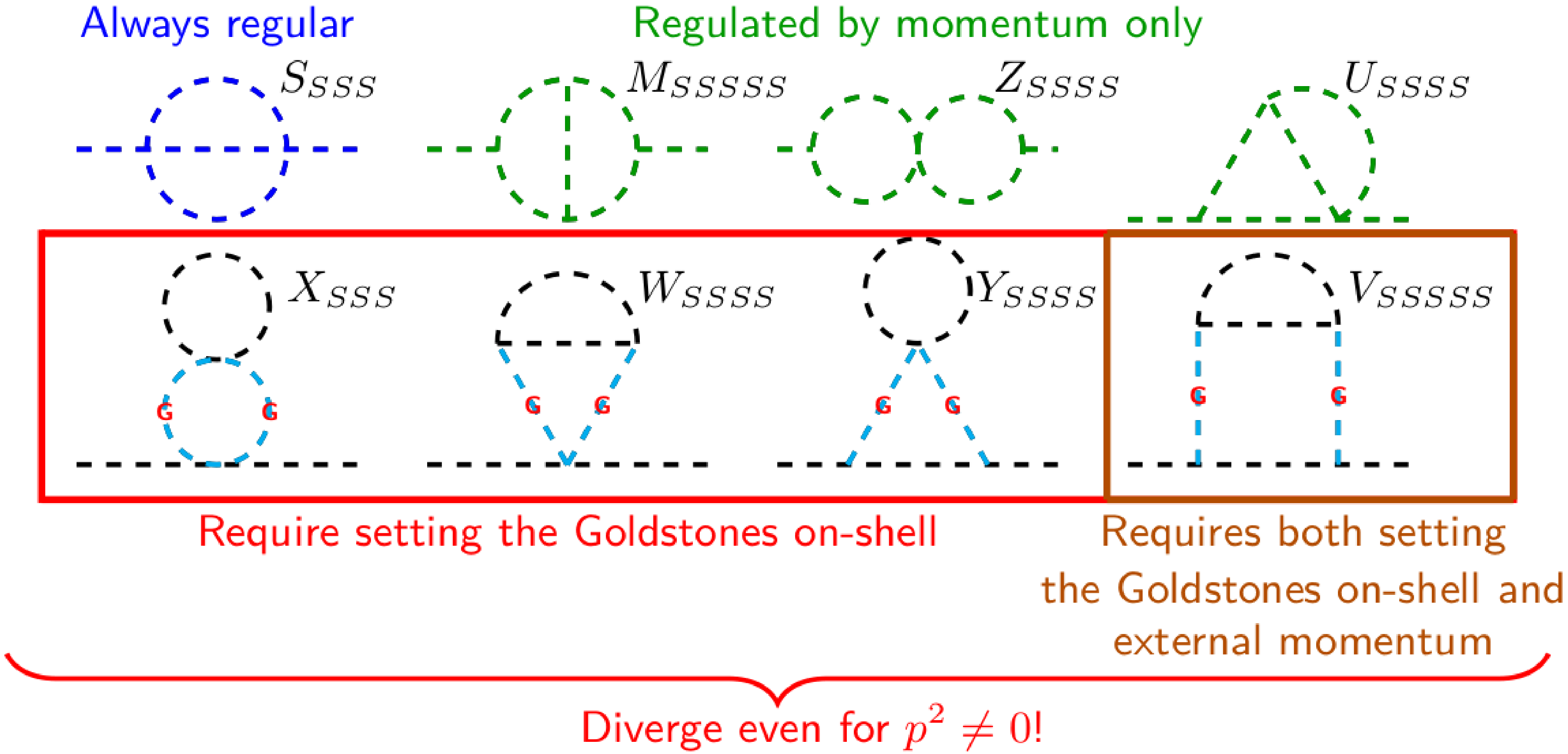
$$\underbrace{(m_G^2)^{\text{run.}}}_{\text{running mass}} = \underbrace{(m_G^2)^{\text{OS}}}_{\text{on-shell mass}} - \underbrace{\Pi_{GG}((m_G^2)^{\text{OS}})}_{\text{self-energy}} \underset{\text{(in Landau gauge)}}{=} -\Pi_{GG}(0)$$

- This can be done **directly** in the tadpole equations or self-energy diagrams!

Next: show how this prescription cures the IR divergences, with the example of purely scalar self-energy diagrams
(same idea for other self-energy diags. and tadpole diags.)

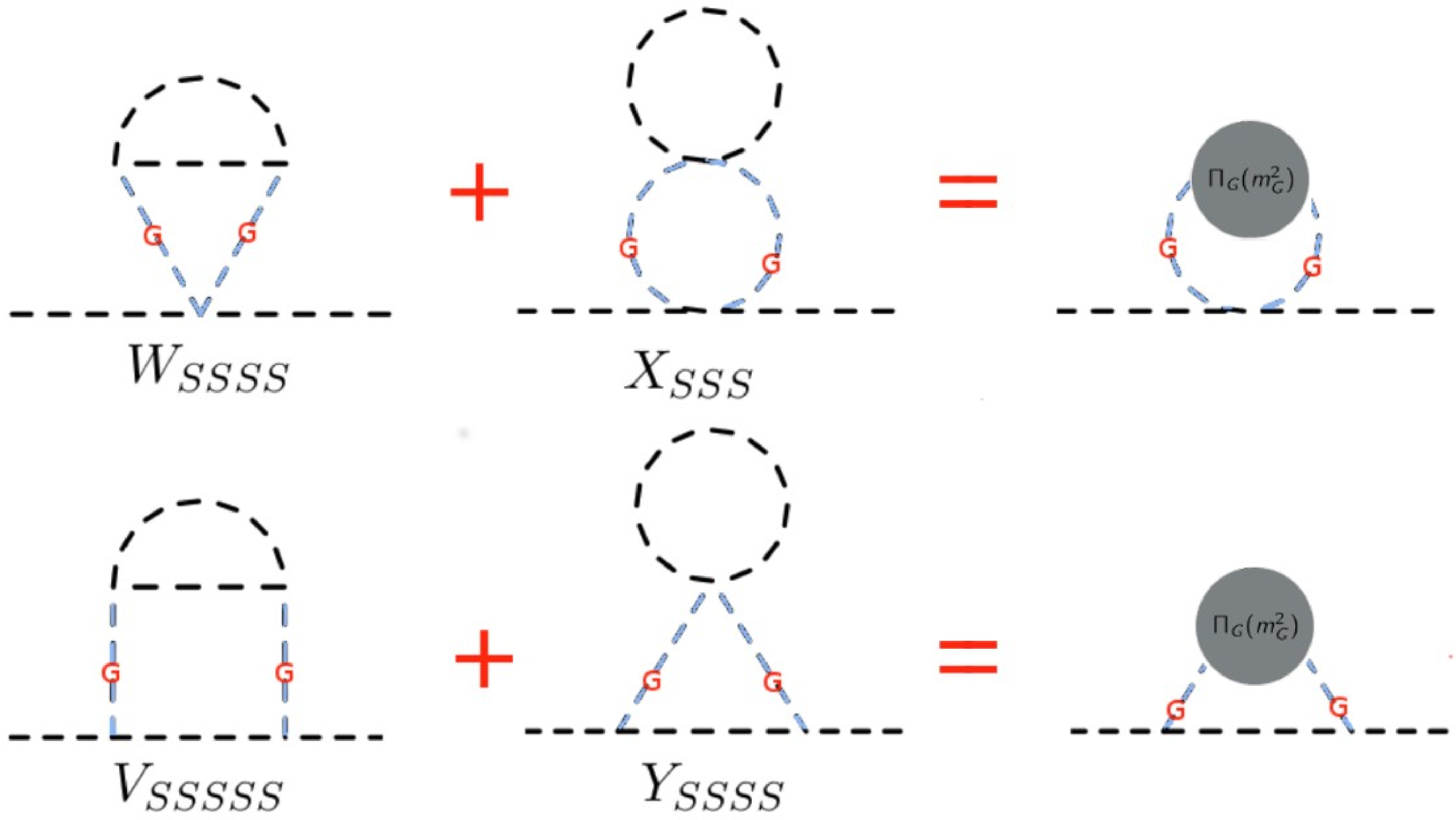
Setting the Goldstone boson "on-shell"

- Earlier literature: inclusion of momentum cures all the IR divergences
- We found
  $\Rightarrow$ true at 1-loop order
  $\Rightarrow$ at 2-loop, $\exists$ diagrams that still diverge for $m_G^2 \to 0$ **even with external momentum included**



Setting the Goldstone boson "on-shell"

Rewrite the divergent two-loop mass diagrams as one-loop diagrams with insertion of $\Pi_G(m_G^2)$

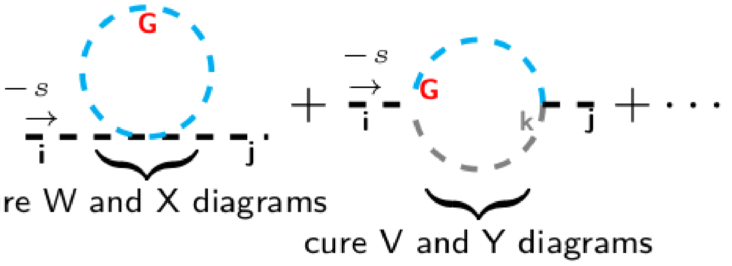


Setting the Goldstone boson “on-shell”

Setting the Goldstone(s) on-shell in mass diagrams

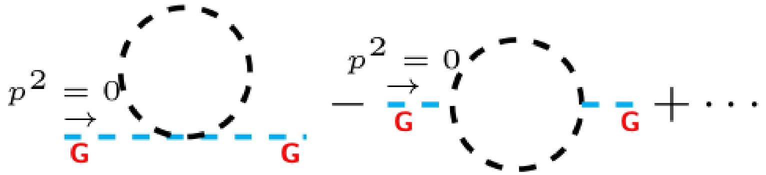
1. Goldstone contributions to the 1-loop scalar self-energy

$$\Pi^{(1)}_{ij}(s = \text{-}p^2) =$$



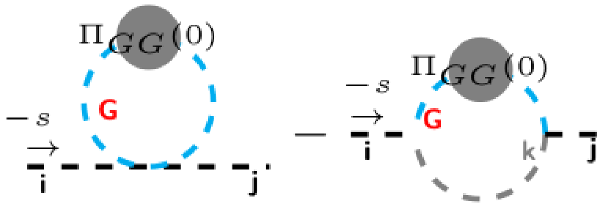
2. Shifting the Goldstone mass to on-shell scheme gives

$$(m_G^2)^{\text{run.}} = -$$



$\rightarrow$ 2-loop shift to the mass diagrams

$$\delta\Pi^{(1)}_{ij}(s) = -$$



✓ cancels the divergence in the V, X, Y, W mass diagrams !

A generalised effective potential approximation

- Solving the GBC has required introducing external momentum $s = -p^2$ in diagrams with pseudo-scalars
- Two-loop diagrams with $p \neq 0$ in general found by solving diff. eqs., implemented e.g. in `TSIL` [Martin, Robertson '05] $\Rightarrow$ very costly numerically!
- For the diagrams where the IR divergence from the Goldstones is cured by external momentum (M, Z, U and V topologies) we need to include the part of the momentum dependence that does the cancellation

$\Rightarrow$ Keep only terms divergent when $s \to 0$ or constant, neglect terms of $\mathcal{O}(s)$ and higher:

$$\Pi^{(2)}_{ij}(s) = \underbrace{\frac{\overline{\log}(-s)}{s}\Pi^{(2)}_{-1\,l,ij} + \frac{1}{s}\Pi^{(2)}_{-1,ij} + \Pi^{(2)}_{l^2,ij}\,\overline{\log}^2(-s) + \Pi^{(2)}_{l,ij}\,\overline{\log}(-s)}_{\substack{\text{div. when } s = -p^2 \to 0; \\ \text{not included in calculation with } V_{\text{eff}}}} + \Pi^{(2)}_{0,ij} + \cancel{\sum_{k=1}^{\infty} \Pi^{(2)}_{k,ij}\frac{s^k}{k!}}$$