QFT I exercises - sheet 7

To be handed in monday 16th of December @ start of lecture (12:00).

Fun with spinors

a Compute for $k^2 = 0$ and in 4 space-time dimensions

$$\mathrm{tr}\left(\not{p}_1 \gamma^\mu \not{k} \gamma^\nu \not{p}_2 \gamma_\nu \not{k} \gamma_\mu\right)$$

Hint: $\not{k}\not{k} = k^2 = 0$

b Show

$$\begin{aligned}\mathrm{tr}\left[(\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu)(\not{p} + m)(\gamma_\nu \not{k} \gamma_\mu + 2\gamma_\mu p_\nu)(\not{p}' + m)\right] \\ = 32((p' \cdot k)(p \cdot k) - m^2(p' \cdot k - 2p \cdot k + p \cdot p') + 2m^4)\end{aligned}$$

c Show in the chiral representation that the operator $C := \gamma^0 \gamma^2$ satisfies

$$C^2 = a \quad \text{and} \quad C\gamma^\mu C = b(\gamma^\mu)^T$$

and compute a and b.

d Show $\mathrm{tr}\left(\gamma^{\mu_1}\gamma^{\mu_2}\ldots\gamma^{\mu_n}\right) = c\,\mathrm{tr}\left(\gamma^{\mu_n}\ldots\gamma^{\mu_2}\gamma^{\mu_1}\right)$ and compute c. Hint: insert 'CC' between all gamma matrices.

Scattering $e^-\mu^- \to e^-\mu^-$ in Quantum Electrodynamics

Consider

$$e^-(p_1)\mu^-(p_2) \to e^-(p_1')\mu^-(p_2')$$

in the approximation $m_e = 0$

a Using Feynman graphs, show $\frac{1}{4}\sum_{\text{spins}} |M|^2$ equals

$$\frac{8e^4}{((p_1 - p_1')^2)^2}\left((p_1 \cdot p_2)(p_1' \cdot p_2') + (p_1 \cdot p_2')(p_1' \cdot p_2) - m_\mu^2 (p_1 \cdot p_1')\right)$$

to first order in perturbation theory.

b Show that in the center of mass frame this evaluates to

$$\frac{2e^4}{k^2(1 - \cos(\theta))^2}\left((E + k)^2 + (E + k\cos(\theta))^2 - m_\mu^2(1 - \cos(\theta))\right)$$

for natural center of mass coordinates E, k and θ. (hint: first make a sketch)

Mandelstam variables

For a general scattering process with two incoming massive particles with momenta p_1 and p_2 and outgoing massive particles with momenta p_3 and p_4 one usually defines so-called Mandelstam variables

$$s := (p_1 + p_2)^2 \qquad t := (p_3 - p_1)^2 \qquad u := (p_4 - p_1)^2 \tag{1}$$

Note these momenta obey momentum conservation,

$$p_1 + p_2 - p_3 - p_4 = 0 \tag{2}$$

a Show that $s + t + u = \sum_{i=1}^4 m_i^2$ holds.

b Show that all possible inner products between momenta, i.e. $p_i \cdot p_j$ can be expressed in the Mandelstam invariants and the four masses m_i

c Express the result for $\frac{1}{4}\sum_{\text{spins}} |M|^2$ in item a of the previous exercise in terms of Mandelstam invariants in the high energy limit $m_\mu^2 = 0$.

d Express $\frac{1}{4}\sum_{\text{spins}} |M|^2$ for the $e^+e^- \to \mu^-\mu^+$ scattering process obtained in class in terms of Mandelstam invariants as well.

e Which formal operation relates the results for c and d?