QFT I exercises - sheet 5

Time-reversal reverses time

The spin-flipped $u^{-s}(p)$ is given by

$$u^{-s}(p) = \frac{1}{N} \begin{pmatrix} (p \cdot \sigma + m)(-i\sigma^2 \xi^s) \\ (p \cdot \bar{\sigma} + m)(-i\sigma^2 \xi^s) \end{pmatrix}$$

As verified last week in an exercise, s is the spin in the z-direction in the rest-frame.

a Verify in the rest frame that the spinor $u^{-s}(p)$ indeed has it's spin flipped compared to u^s given in last week exercise.

b Show:

$$u^{-s}(\tilde{p}) = -\gamma_1\gamma_3 \left(u^s(p)\right)^* , \qquad \text{with } \tilde{p} = (p_0, -\vec{p})$$

The anti-unitary operator T which implements time-reversal acts on operators as

$$T a^s_{\vec{p}} T = \eta a^{-s}_{-\vec{p}} \qquad T b^s_{\vec{p}} T = \eta b^{-s}_{-\vec{p}}$$

Here a and b are the operators which appear in the expansion of the Dirac spinor $\psi(x)$ in the Heisenberg picture and η is some phase.

c Show that

$$T\psi(t, \vec{x})T = \eta\gamma_1\gamma_3\psi(-t, \vec{x})$$

holds and argue that $T^2 = 1$ leads to $\eta = \pm\mathrm{i}$

d Compute

$$T\left[\bar{\psi}\psi(t, \vec{x})\right]T, \qquad T\left[\bar{\psi}\gamma^5\psi(t, \vec{x})\right]T, \qquad T\left[\bar{\psi}\gamma^\mu\psi(t, \vec{x})\right]T$$

My first Feynman graphs

a Draw all Feynman graphs for the amplitude $\langle\Omega|T\{\phi(x)\phi(y)\}|\Omega\rangle$ to order λ^2 using the interaction $H_I(t) = \frac{\lambda}{4!}\int d^4z\phi(z)^4$.

b Draw all Feynman graphs for the amplitude $\langle\Omega|T\{\phi(x)\phi(y)\}|\Omega\rangle$ to order λ^2 using the interaction $H_I(t) = \frac{\lambda}{3!}\int d^4z\phi(z)^3$. Argue that only λ^{even} terms contribute.

c Draw all Feynman graphs for the amplitude $\langle\Omega|T\{\phi(x)\phi(y)\}|\Omega\rangle$ to order λ^2 using the interaction $H_I(t) = \frac{\lambda}{5!}\int d^4z\phi(z)^5$.

d Give for each diagram of a) the associated expression in terms of Feynman-propagators, including the symmetry factor.

e Show that the order λ^1 term of a) is proportional to the integral $\int d^D p \frac{\mathrm{i}}{p^2 - m^2}$

f By dimensional analysis compute the dependence on m of this integral as a function of D. What happens in the limit $m \to 0$?

g Argue that *if* the integral were defined, the outcome for $m = 0$ would be 0 by dimensional analysis.

Suppose we look at $H_I(t) = \int d^4 z \frac{\kappa}{3!} \phi(x)^3$ and are only interested in *counting* graphs which are connected and contain no loops. Let a graph of this type contain m external lines, d vertices and l lines (both internal and external)

h Show $3d + m - 2l = 0$ follows from the connectedness prescription

i Show $d - m + 2 = 0$ follows from the requirement there are no loops

j Show the number of these graphs is $(2m - 5)!!$. Hint: consider adding one external line to all graphs with one external line less.

k Compare this to $m!$ for large m and argue that computing Feynman graphs for large numbers of external lines is not feasible.

The graph counting part of this exercise comes from appendix a) of R. Kleiss and H. Kuijf, Nucl. Phys. B 312, 616, 1989, who do the counting including an additional ϕ^4-type interaction. Read the appendix for more information.