QFT I exercises - sheet 10

To be handed in wednesday 22nd of Januari @ start of lecture (12:00). The exercise marked BONUS is a bonus

Form factor calculation in scalar QED

Let

$$\mathcal{L} = \overline{D_\mu \phi} D^\mu \phi - m^2 \phi \bar{\phi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

be the Lagrangian for 'scalar QED'. This theory describes photons coupled to a complex scalar in four space-time dimensions.

a What should X_μ be such that for $D_\mu = \partial_\mu + X_\mu$ this Lagrangian is invariant under the local gauge transformation $\phi \to e^{i\alpha}\phi(x)$ and $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha$?

b Write down the Feynman rules of this Lagrangian, including external waves. Hint: there is now also a quartic vertex in the theory.

c Draw the Feynman graphs for the scattering process $\phi H^- \to \phi H^-$ to second order in perturbation theory (that is, up to and including one loop), where H is a heavy fermion coupled as in QED. Mark the graphs which dominate in the limit where the fermion is extremely massive.

The fermion H will be taken to be extremely massive for the rest of the exercise. The scalar vertex function in the rest of this exercise is simply the scalar analog of the fermion vertex function in QED

d Graphology:

- Which graph contributes to the scalar vertex function at zero loops?
- Which three graphs contribute to the scalar vertex function at one loop?
- Which graphs contribute to the scalar self-energy at tree level and which at one loop?
- Which graphs contribute to the photon self-energy at tree level and which at one loop?
- Which three graphs contribute in principle to the soft bremsstrahlungs part of the calculation relevant to obtain a one loop cross section for $\phi H^- \to \phi H^-$?

e Which of these last three graphs does *not* contribute to the leading infrared divergence of the cross section for $\phi H^- \to \phi H^-$? Hint: show that the integral over the unobserved small momentum is less divergent than for the fermion case when the photon energy $|k| \to 0$ for contributions to the cross-section which involve this graph. (you don't have to do the integral over unobserved photon momentum explicitly)

f Derive the soft bremsstrahlung contribution to the cross-section from the sum of the other two graphs and show it is structurally the tree level result times an integral over the unobserved photon momentum. Compare this result to the fermion case in Peskin and Schroeder. (you don't have to do the integral over unobserved photon momentum)

g Show the scalar vertex function must be proportional to $(p + p')^\mu$ to all loop orders, where μ is the Lorentz index on the photon leg. Hint: a certain not-yet proven identity should be useful.

h Argue the scalar vertex function at $q^2 = 0$ measures the charge of the scalar field.

j Compute the scalar vertex function in dimensional regularization (leave the Feynman parameter integrals)

k Write down a precise additional term in the Lagrangian whose only effect in perturbation theory is to make the scalar vertex function at $q^2 = 0$ equal to its tree level value to the one loop order. (leave the Feynman parameter integrals)

l Show Pauli-Villars regularisation as defined in Peskin and Schroeder does *not* cancel all ultra-violet divergences for the self-energy of the scalar field at one loop. Hint: the more powers of loop momentum in the numerator, the more divergent the graph...

m Show one of the graphs of the scalar field self-energy at one loop is zero in dimensional regularisation. Hint: multiply its integrand by $\frac{l^2-\alpha^2}{l^2-\alpha^2}$ and follow the usual rules for computation of loop graphs. BONUS: show the volume of space, $\int d^D l 1$, vanishes in dimensional regularisation.

n Compute the self-energy of the scalar field at one loop in dimensional regularisation (leave the Feynman parameter integral).

o Compare the one-loop correction to 'Z'-factor of the scalar field to the coefficient of the term found under k. Are they the same, just as in QED?