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Description of Selection Efficiencies in Searches for Heavy Particles with Finite Lifetime

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Abstract

Searches for production of new, heavy particles that decay into an invisible plus a detectable particle depend on the lifetime of the heavy particle. Depending on the lifetime, searches are performed for one or more particles plus missing energy, tracks with large impact parameters, decay vertices ("kinks" or displaced photons), or stable heavy particles with anomalous energy loss.

In order to derive limits for the production of these particles it is necessary to parametrize the selection efficiency as function of the particle's mass and lifetime and the centre–of–mass energy. This note presents fit functions which describe these efficiencies. It will be shown that these functions depend on the detector geometry and the kinematics of the decay only. They allow a better description of the efficiencies than simple interpolation and drastically reduce the amount of Monte Carlo data sets necessary for these type of analyses which are scanning a wide range of lifetimes for many data sets with different center–of–mass energies at LEP.

1 Introduction

Strategies for searches for new, heavy particles depend on the lifetime of the particle searched for. Short lived particles that decay close (within millimetres) to the main vertex are searched for e. g. by looking for the decay particles plus missing energy [1]. Searches for particles which decay before the sensitive region of the tracking devices, e. g. the main tracking chamber of OPAL starts ≈30 cm around the interaction point, have to find tracks with large impact parameters. Longer–lived particles may decay within the jet chamber and can be identified by their decay vertices, which are visible as kinks in the decay tracks or by detecting photons which show a different timing than photons originated from the primary interaction point [2] [3]. Very long–lived charged particles decay outside the detector and are identified by their anomalous $\mathrm{d}E/\mathrm{d}x$ values [4].

These search strategies are complementary, and all searches should be combined to get production cross section limits independent as a function of the particle's lifetime. Therefore values of the selection efficiency of each individual analysis and the combination of all are needed as a function of the particle's mass and lifetime. In order to avoid generating and processing excessive amounts of Monte Carlo samples and to arrive at good description of the efficiencies over the whole mass and lifetime range it is necessary to have a descriptive function which calculates the efficiencies.

This note presents such a fit function. First, the general principle of the function is described. Discussing its application for the individual analyses, it is then shown that this very simple fit function with only three free parameters gives an adequate description of the selection efficiency as a function of particle mass, lifetime, and the energy of the decaying particle. The fit functions are presented for the searches for long–lived, and promptly decaying particles and particles having a large impact parameter or a visible decay vertex. Finally, the fit functions are combined into a function which describes the efficiency of the combined search, taking into account the overlap between the individual analyses. The examples are taking from the search for new particles in the framework of gauge–mediated supersymmetry breaking models [5].

2 General Principle of the Fit Function

The general ansatz of the function uses the following assumptions.

- The heavy particle decay is descibed by $\exp(-\frac{t}{\tau})$;
- The detector to measure the decay is a sphere with an minimum length l_{start} and a maximum length l_{stop}.

Furthermore it is assumed that the detection efficiency of the detector as well as the efficiency of the analysis is constant within the bounds l_{start} and l_{stop}. Initial state radiation is neglected, which is a good assumption for particles near the kinematic limit.

Assume a particle of mass m_1 produced in an $\mathrm{e}^+\mathrm{e}^-$ collision at a centre–of–mass energy $\sqrt{s}$, it has a lifetime τ and decays into two particles with mass m_2 and m_3. Then for this particle, the proper decay time t_{prop} is distributed according to

$$\mathcal{P}\left(t_{\mathrm{prop}}\right) = \frac{1}{\tau}\exp(-\frac{t_{\mathrm{prop}}}{\tau}).$$

The observed decay time in the laboratory, t_{lab}, for a fixed γ_1 is given by $t_{\mathrm{lab}} = \gamma_1 t_{\mathrm{prop}}$, and distributed according to

$$\mathcal{P}\left(t_{\mathrm{lab}}\right) = \frac{1}{\gamma_1\tau}\exp(-\frac{t_{\mathrm{lab}}}{\gamma_1\tau}).$$

The decay length l_{dec} in the laboratory is given by $l_{\mathrm{dec}} = c\beta_1\gamma_1 t_{\mathrm{prop}} = c\beta_1 t_{\mathrm{lab}}$, and is distributed according to

$$\mathcal{P}\left(l_{\mathrm{dec}}\right) = \frac{1}{c\beta_1\gamma_1\tau}\exp(-\frac{l_{\mathrm{dec}}}{c\beta_1\gamma_1\tau}).$$

Assuming that the decay is identified with a constant probability ϵ_0 if the decay occurs at a decay length $l_{\text{start}} \le l_{\text{dec}} \le l_{\text{stop}}$, then the overall probability to observe the decay of the particle is given by

$$\epsilon_{\text{single}} = \int\limits_{l_{\text{start}}}^{l_{\text{stop}}} \mathcal{P}\left(l_{\text{dec}}\right) \mathrm{d}l_{\text{dec}} = \epsilon_0 \left[\exp\left(-\frac{l_{\text{start}}}{c\beta_1\gamma_1\tau}\right) - \exp\left(-\frac{l_{\text{stop}}}{c\beta_1\gamma_1\tau}\right)\right].$$

In case the particle is pair–produced, we can calculate $\beta_1\gamma_1$:

$$\beta_1\gamma_1 = \frac{|p|}{m_1} = \sqrt{\frac{s}{4m_1^2} - 1},$$

where $|p|$ is the momentum of the particles in the laboratory frame. This yields

$$\epsilon_{\text{single}}\left(m_1,\,\tau\right) = \epsilon_0 \left[\exp\left(-\frac{l_{\text{start}}}{l_{\text{mean}}\left(m,\,\tau\right)}\right) - \exp\left(-\frac{l_{\text{stop}}}{l_{\text{mean}}\left(m,\,\tau\right)}\right)\right].$$

with $l_{\text{mean}}\left(m,\,\tau\right)$ defined as

$$l_{\text{mean}}\left(m,\,\tau\right) = c\tau\sqrt{\frac{s}{4m_1^2} - 1}.$$

In another case relevant for the search for new, heavy stable particles, the decay particles are not pair–produced but are also decay products of a pair–produced particle. Here, $\beta_1\gamma_1$ has to be calculated. This can be done by calculating the kinematics of a two–body decay of a particle A with mass m_{A} into the particle 1 (m_1) and particle B (m_{B}). In the restframe of the primary particle A one gets the relations

$$E_1 = \frac{m_{\text{A}}^2 - m_{\text{B}}^2 + m_1^2}{2m_{\text{A}}}$$

and

$$|p_1| = \frac{(m_{\text{A}}^2 - (m_1 + m_{\text{B}})^2)(m_{\text{A}}^2 - (m_1 - m_{\text{B}})^2)}{2m_{\text{A}}}$$

for one of the decay products. To get its energy and momentum in the lab frame one has to calculate the boosted momentum $p_1^{\star 2} = {p_{\parallel\,1}^{\star}}^2 + {p_{\perp\,1}}^2$ with

$$p_{\parallel\,1}^{\star} = \gamma_{\text{A}} p_{\parallel\,1} + \gamma_{\text{A}}\beta_{\text{A}} E_1 = \gamma_{\text{A}} |p_1| \cos\theta + \gamma_{\text{A}}\beta_{\text{A}} E_1$$

and

$$p_{\perp\,1}^{\star} = p_{\perp\,1} = |p_1| \sin\theta.$$

Here θ is the angle between primary particle A and the secondary particle 1. To get the average momentum the average decay angle has to be chosen. In case of an isotropic decay this angle is $\pi/2$. $\beta_{\text{A}}\gamma_{\text{A}}$ are given by

$$\beta_{\text{A}}\gamma_{\text{A}} = \frac{|p_{\text{A}}|}{m_{\text{A}}} = \sqrt{\frac{s}{4m_{\text{A}}^2} - 1}.$$

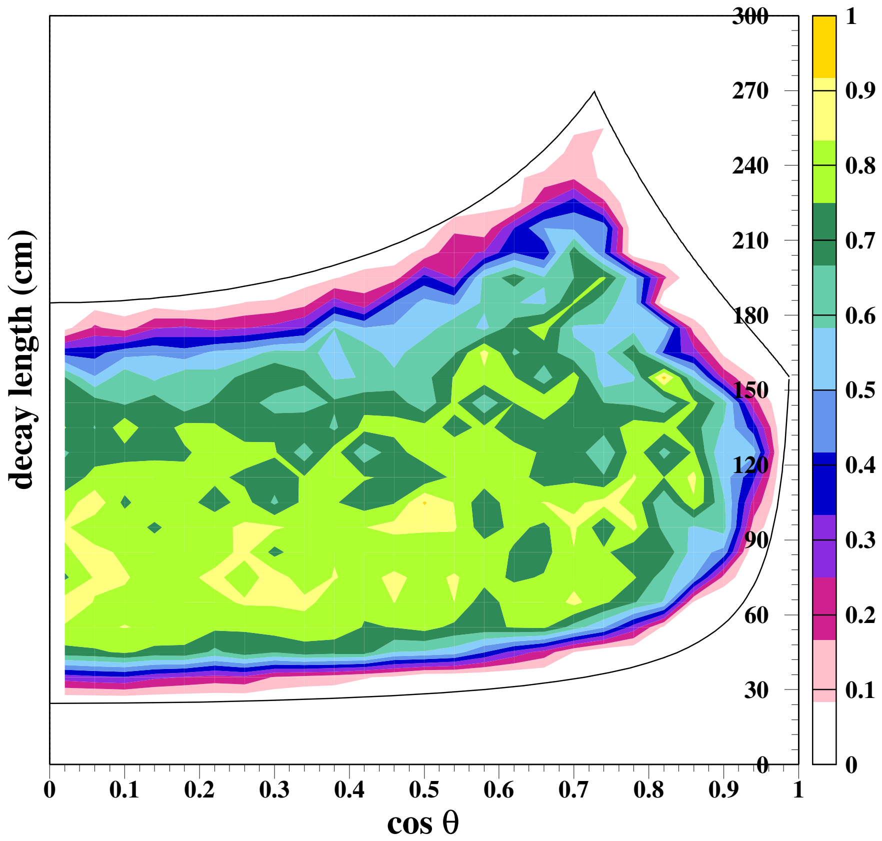

Figure 1: Kink finding effiency for single particles as function of $\cos\theta$, where θ is the production angle of the particle, and the true decay length l_{dec}. The black lines indicate the walls of the OPAL Jet Chamber. This example shows the obtained efficiencies for $\tilde{\mu}$ search with the medium lifetime "kink" analysis for $\sqrt{s} = 189\,\text{GeV}$ Monte Carlo and demonstrates a flat distribution of the efficiency in the detector.

$\beta_1\gamma_1$ can now be calculated to

$$\beta_1\gamma_1 = \frac{|p_1|}{m_1} = \frac{\sqrt{{p^{\star}_{\parallel 1}}^2 + {p_{\perp 1}}^2}}{m_1}.$$

Hence for this case $l_{\text{mean}}\,(m,\,\tau)$ is defined defined as

$$l_{\text{mean}}\,(m,\,\tau) = c\tau\frac{\sqrt{{p^{\star}_{\parallel 1}}^2 + {p_{\perp 1}}^2}}{m_1}.$$

Depending on whether the anlalysis accepts events where one or both of the particles have to decay in the detector, the event selection efficiency ϵ_{event} is given by

$$\epsilon_{\text{event}}\,(m,\,\tau) = 1 - [1 - \epsilon_{\text{single}}\,(m,\,\tau)]^2$$

in case of one decay or

$$\epsilon_{\text{event}}\,(m,\,\tau) = \epsilon^2_{\text{single}}\,(m,\,\tau).$$

in case of two decays are required to be detected.

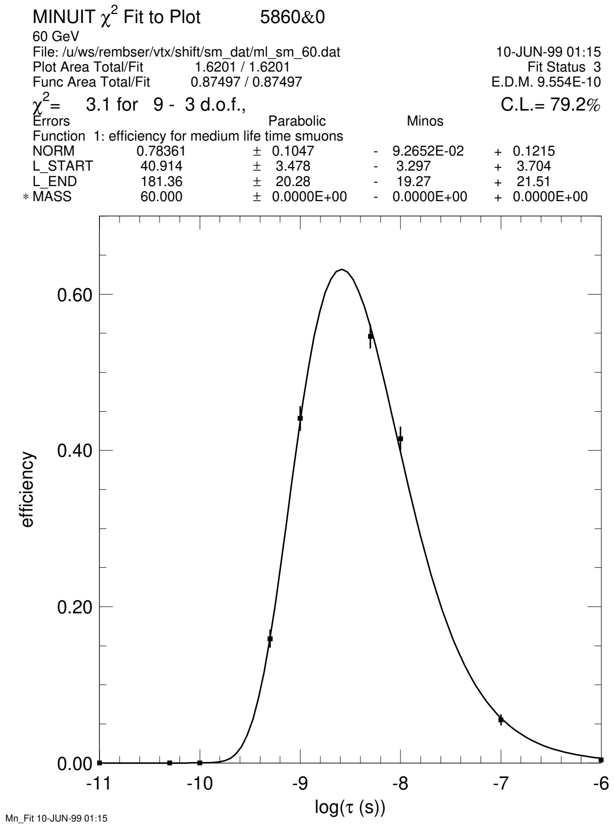

Figure 2: Example for the result of the fit for the smuon analysis ($m_{\tilde{\mu}} = 60\,\mathrm{GeV}$).

This formula describes the selection efficiency of the search analysis over a wide range of m and τ with only three parameters, ϵ_0, l_{start} and l_{stop}. Input variables are the mass of the decaying particle as well as the center–of–mass energy. These parameters can be fitted to the Monte Carlo efficiencies, and should be considered as effective parameters.

Let us take a closer look on the function and interpret the parameters. Obviously the function assumes a spherical detector with a constant efficiency over the range of $0 \leq |\cos\theta| \leq 1$. As this is not true for HEP detectors, the parameter ϵ_0 is an average efficiency over all $\cos\theta$. If the angular distribution of the produced particles changes, ϵ_0 changes, too. The parameters l_{start} and l_{stop} give radii where the constant efficiency ϵ_0 starts and ends. E. g. for the kink search analysis where both tracks, the primary and the charged decay particle, have to be detected, l_{start} and l_{stop} should be of the order of the inner and outer radius of the OPAL Central Jet Chamber. Figure 1 shows the kink finding effiency derived from Monte Carlo simulation for single particles as function of $\cos\theta$, where θ is the production angle of the particle, and the true decay length l_{dec}. As one can see, in a large region the kink finding efficiency is relatively constant at a value around 0.7. This justifies the assumption of a constant ϵ_0. The kink finding efficiency drops for decay lengths of less than about 40 cm or larger than about 200 cm, which are approximately the inner and outer radius of CJ. For values of $\cos\theta$ larger than about 0.5, the minimum and maximum decay lengths at which kinks can be found rise, as expected. For values of $\cos\theta$ larger than about 0.7, the maximum decay length drops again, because the particles escape through the end walls of CJ. A proper description of the kink finding efficiency would take this kink finding efficiency distribution and fold it with the $\cos\theta$ and l_{dec} distribution of the produced particles.

However, the values ϵ_0, l_{start} and l_{stop} must be considered effective values which have to be determined from a fit, and should be expected to vary slightly with the production process; they depend on the $\cos\theta$ distribution of the particles, so they are expected to be different e. g. for selectrons which have a more peaked spectrum in the forward and backward direction due to the presence of t–channel diagrams and for smuons where s–channel diagrams only do contribute.

Figure 2 shows as an example the result of the fits for the smuon analysis. For different masses, the efficiency as function of τ was fitted seperately. The fitted values of ϵ_0, l_{start} and l_{stop} are found to be compatible within the errors for all masses.

3 Extension of the Fit Function for Special Cases

In this section corrections to the fit function are presented which are necessary to describe exceptional kinematic cases of the decay process. It is also presented how to transform the parameters l_{start} and l_{stop} into other parameters which allow to perform a simultaneous fit over all masses and lifetimes. It is shown that in cases where the analysis method does not depend on the particle mass, no extension of the number of parameters is required.

3.1 Description of Efficiencies near the Kinematic Limit and for Low Δm

Close to the kinematic limit where the mass of the primary particle gets close to the energy available for its production (E_{prod}), the fit function might need modifications. This is true for searches which need to detect the primary particle itself (e. g. the "kink" search and the search for heavy charged particles). In this extreme case, not enough energy is left in the particle production to give it a sufficient momentum to reach the detecting chambers. Hence the efficiency ϵ_0 has to drop to zero. The decrease in energy is visible at around $m > E_{prod} - 2\,\text{GeV}$. Monte Carlo studies have shown that it is sufficient to multiply the plateau efficiency ϵ_0 by a Fermi–function

$$\frac{\epsilon_0}{1 + \exp(a[m - m_{\text{thresh}}])} \ .$$

to describe the decrease of efficiency. m_{thresh} and a are free parameters that have to be fitted. [1]

[1]It might be more applicable to use the velocity β instead of the mass m of the particle.

A consequence of another kinematic effect is especially important for the "kink" and the large impact parameter search. For both analyses it is essential to detect one of the decay products which has to show either a visible decay vertex or a large impact parameter. If the difference in mass $\Delta m = m_1 - m_2$ of the primary particle (m_1) and the visible secondary particle (m_2) gets very small, not enough transverse momentum for the secondary particle w.r.t. the flight direction of the primary particle is left. Hence analyses sensitive to the angles between primary and secondary particles lose sensitivity. This effect starts giving an impact on the efficiency ϵ_0 at mass differences $\Delta m < 1\,\mathrm{GeV}$. As for the effect mentioned before it is sufficient to multiply the plateau efficiency ϵ_0 by a Fermi–function

$$\frac{\epsilon_0}{1 + \exp(b[\Delta m - \Delta m_{\mathrm{thresh}}])}$$

to describe the decrease of efficiency. Again, $\Delta m_{\mathrm{thresh}}$ and b are parameters that have to be fitted.

Describing efficiencies for searches for pair–produced particles with finite lifetime in practice only one of these corrections is needed. In other scenarios e. g. the pair production of neutralinos which decay into a slepton having finite lifetime and a lepton, both corrections have to be applied.

3.2 Calculation of l_{start} for the Analyses Sensitive to Short Lifetimes

It is expected that the parameter l_{start} can be calculated as a function of the cut value of the analysis on the impact parameter d_0 of a track. Assuming a symmetric spherical detector one gets for the 2–dimensional projection of the impact parameter (Figure 3) as a function of l_{start}:

$$d_0 = l_{\mathrm{start}} \sin\alpha$$

where θ is the average decay angle between primary (E_1, m_1, p_1) and secondary track (E_2, m_2, p_2). If the average angle θ in the rest frame of the decaying particle is known, the decay angle $\theta^\star$ in the lab frame can be calculated by using

$$\tan\theta^\star = \frac{p_{\perp 2}}{p^\star_{\parallel 2}}$$

with

$$p^\star_{\parallel 2} = \gamma_1 p_{\parallel 2} + \gamma_1\beta_1 E_1 = \gamma_1 |p_2|\cos\theta + \gamma_1\beta_1 E_2 = \frac{E_1(m_1^2 - m_2^2)\cos\theta - |p_1|(m_1^2 + m_2^2)}{2m_1^2}$$

and

$$p^\star_{\perp 2} = p_{\perp 2} = |p_2|\sin\theta.$$

Therefore one yields for the average decay angle $\theta^\star$ in the lab frame as a function of the average decay angle θ in the rest frame

$$\theta^\star = \arctan\frac{m_1\sin\theta}{E_1\cos\theta + \sqrt{E_1^2 - m_1^2}}$$

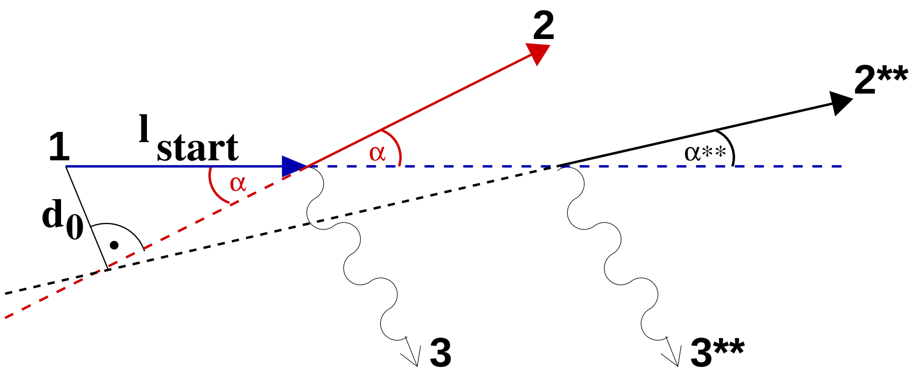

Figure 3: Knowing the average decay angle, the parameter l_{start} can be transformed to a quantity describing the minimum impact parameter d_0 which is required by the analyses. The scetch of the two–body decay of a particle 1 into a detectable particle 2 and an invisible particle 3 demonstrates the change in l_{start} with a change of the angle α.

Assuming an isotropic decay of the primary particle the average decay angle in the rest frame is $\pi/2$. Depending on the mass of the primary particle and its energy (e. g. $E_1 = \sqrt{s}/2$ for pair–produced particles), the average angle can be calculated and the parameter l_{start} is replaced by $d_0 = l_{\text{start}} \sin\theta$. The same transformation is done for the parameter l_{stop}, as if the momentum of the track is greater than a few GeV hence curvature in the magnetic field of the detector can be neglected.

This transformation of the fit parameters is not only cosmetic but allows us to obtain flat distributions of the fit parameters for all masses. This allows a two–dimensional description of the efficiency. If no transformation would be applied, one gets a decrease of the parameter l_{start} and l_{stop} with increasing particle mass. This can be interpreted in the following way. Figure 3 shows that all decays into a detectable particle 2 having an angle less than α will not be detected as the impact parameter cut is not satisfied. Within the cone with angle α is a certain percentage of decays depending on the momentum of the particle. For higher momenta, the same amount of decays is in a cone with opening angle $\alpha^{\star\star}$. The fit always assumes a constant efficiency ϵ_0 which should be the same for all masses. If it is so, the amount of decays within the cone defined by α and $\alpha^{\star\star}$ has to be the same. But this requieres an increase of l_{start} and l_{stop} with an increase of the momentum of the secondary particle or – in other words – an increase of l_{start} and l_{stop} with decreasing mass of the primary particle.

However, the transformed values l_{start} and l_{stop} must be considered as effective values which have to be determined from a fit.

3.3 Efficiency Fit with Reduced Number of Parameters for the Stable Particle Search

The long lifetime search looks for heavy particles which do not decay within the detector and are identified e. g. by their anomalous ionization. This effectively imposes a minimum decay length for the particles but no restriction on the maximum flight length is needed. Therefore the efficiency that a single particle is identified can be described with two parameters only, ϵ_0 and l_{start} and are estimated to be

$$\epsilon_{\text{single}} = \int\limits_{l_{\text{start}}}^{\infty} \mathcal{P}\left(l_{\text{dec}}\right) \mathrm{d}l_{\text{dec}} = \epsilon_0\left(m\right) \exp\left(-\frac{l_{\text{start}}}{c\beta\gamma\tau}\right) = \epsilon_0\left(m\right) \exp\left(-\frac{l_{\text{start}}}{l_{\text{mean}}\left(m,\,\tau\right)}\right).$$

If both particles are required to be seen, both have decay lengths above l_{start}, and so the event selection efficiency is given by

$$\epsilon_{\text{event}}\left(m,\,\tau\right) = \epsilon^2_{\text{single}}\left(m,\,\tau\right).$$

3.4 Efficiency Fit with Reduced Number of Parameters for Particles with Very Short Lifetime

The zero lifetime search looks for particles originating from the interaction point. This effectively imposes a maximum decay length for the particles but no restriction on the minimum flight length is needed. Therefore the efficiency that a single particle is identified can be described with two parameters only, ϵ_0 and l_{stop} and are estimated to be

$$\epsilon_{\text{single}} = \int\limits_{0}^{l_{\text{stop}}} \mathcal{P}\left(l_{\text{dec}}\right) \mathrm{d}l_{\text{dec}} = \epsilon_0\left(m\right)[1-\exp\left(-\frac{l_{\text{stop}}}{c\beta\gamma\tau}\right)] = \epsilon_0\left(m\right)[1-\exp\left(-\frac{l_{\text{stop}}}{l_{\text{mean}}\left(m,\,\tau\right)}\right)].$$

If both particles are required to be seen, both have decay lengths above l_{start}, and so the event selection efficiency is given by

$$\epsilon_{\text{event}}\left(m,\,\tau\right) = \epsilon^2_{\text{single}}\left(m,\,\tau\right).$$

3.5 Transforming Fit Parameters for Analyses requiring Timing Cuts

So far, the fit function uses spacial information only and gives interpretable parameters for analyses which perform cuts on the geometry of the topology searched for. This is not true for searches for displaced photons where timing cuts on the decay photons are used. In this case, it is more sensible to use a minimum time which the photon needs to travel from its origin to the timing detectors t_{min} instead of the parameter l_{start} to achieve a flat distribution of the fit parameter for all masses . This minimum time corresponds to the timing cut which distinguishs non–pointing photons from pointing ones. Nevertheless it is recomended to keep the parameter l_{stop} as the outer detector volume is a fixed geometrical quantity. The minimum time t_{min}, the decay angle between primary and secondary particle is set to be 0, can be calculated the following way.

The delay time t_{min} between production of the primary particle 1 and the detection of the decay particle 2 is given by the time t_1 the primary particle needs to travel to the decay point at $l = l_1$ plus the time t_2 the photon needs to reach the detecting chambers at $l = l_2$:

$$t_{min} = t_1 + t_2 = \frac{l_1}{c\beta_1} + \frac{(l_2 - l_1)}{c} = \frac{l_1}{c}(\frac{1}{\beta_1} - 1) + \frac{l_2}{c}.$$

l_2 corresponds to l_{stop} which should be kept as a fit parameter. l_{start} which is equivalent to l_1 has to be replaced by t_{min} and can be extracted from the formula above to

$$l_{\text{start}} = l_1 = \frac{c\beta_1 t_{min} - \beta_1 l_{\text{stop}}}{\beta_1 - 1}$$

The transformed fit parameter t_{min} corresponds to the value of the timing cut used for the search analysis.

4 Application of the Fit Function to OPAL Analyses

In the following sections applications of the fit function to OPAL analyses are described. Examples given here are taken from the search for pair–produced sleptons and neutralinos in the framework of GMSB. The description of the individual cases serves as a reference for OPAL publications.

4.1 Efficiency of the Large Impact Parameter Search

The large impact parameter search for pair produced smuons with finite lifetime requires two tracks with large impact parameters. As the particles are pair–produced the energy of a particle is set to $E_{\tilde{\mu}} = \sqrt{s}/2$. The fit function used has the form:

$$\epsilon_{\text{event}}(m, \tau) = \epsilon^2_{\text{single}}(m, \tau)$$

with

$$\epsilon_{\text{single}}(m_1, \tau) = \epsilon_0 \left[\exp\left(-\frac{l^\star_{\text{start}}}{l_{\text{mean}}(m, \tau)} \right) - \exp\left(-\frac{l^\star_{\text{stop}}}{l_{\text{mean}}(m, \tau)} \right) \right]$$

where $l_{\text{mean}}(m, \tau)$ is given by

$$l_{\text{mean}}(m, \tau) = c\tau \sqrt{\frac{s}{4m_1^2} - 1}.$$

$l^\star_{\text{start}}$ and $l^\star_{\text{stop}}$ are calculated by

$$l^\star_{\text{start}} = \frac{l_{\text{start}}}{\sin\theta^\star} \quad \text{and} \quad l^\star_{\text{stop}} = \frac{l_{\text{stop}}}{\sin\theta^\star}.$$

The average decay angle $\theta^\star$ for the isotropic decay of a particle with mass m is given by

$$\theta^\star = \arctan \frac{2m_1 \sin\theta}{\sqrt{s}\cos\theta + \sqrt{s - 4m_1^2}}; \quad \theta = \pi/2.$$

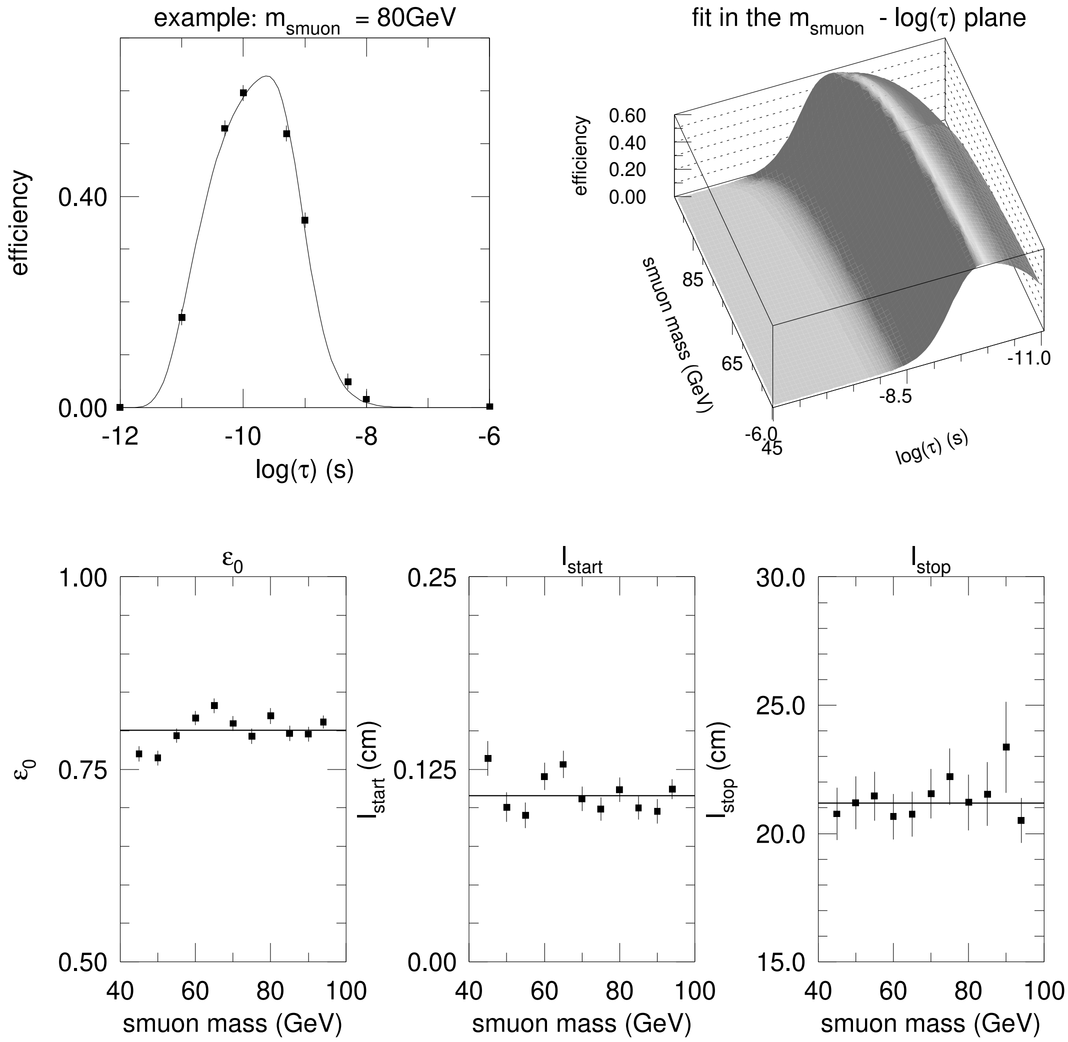

Figure 4: Description of the efficiency of the large impact parameter search for smuons with the OPAL detector at $\sqrt{s} = 189\,\mathrm{GeV}$. The upper left plot gives an example of the fit for $m_{\tilde{\mu}} = 80\,\mathrm{GeV}$. The upper right plot shows the description of the efficiency in the plane of the smuon mass and the lifetime of the smuon. The description was achieved by a simultaneous fit to all Monte Carlo points. The lower plots show the fit parameters ϵ_0, l_{start} and l_{stop} for the one dimensional fits (dots) and the two dimensional fit (line).

Figure 4 shows the result of the fit function applied to to single mass points of the $\tilde{\mu}$ as well as the simultaneous fit to all masses. The parameters ϵ_0, l_{start} and l_{stop} are found to be compatible within the errors for all masses. For the two dimensional fit the parameters were found to be

- $\epsilon_0 = 0.798 \pm 0.003$. This value is equivalent to the average of all ϵ_0 of the single mass fits.
- $l_{\mathrm{start}} = (0.109 \pm 0.003)\,\mathrm{cm}$. It describes an effective impact parameter cut. The value is compatible with the actual impact parameter cut applied in the analysis of 0.05 cm. The difference between fitted and actual values are due to the asymmetric cut in the analysis, where the impact parameter in the r-ϕ plane was set to 0.05 cm and in z to be 20 cm.
- $l_{\mathrm{stop}} = (21.4 \pm 0.3)\,\mathrm{cm}$. This corresponds to the distance between the interaction point and the inner walls of the Jet Chamber and is compatible with the maximum value of the impact parameter which is accepted by the analysis.

As the efficiency can be well described by the three parameters only and other input parameters are just $m_{\tilde{\mu}}$, its lifetime τ and the center–of–mass energy $\sqrt{s}$, efficiencies for other center–of–mass energies can be calculated by the variation of $\sqrt{s}$ for the same analysis.

4.2 Efficiency of the “Kink” Search

The efficiency function for the “kink” search will be discussed for two examples. One example is the description of the efficiency for pair–produced smuons (see previous section). The other example is the efficiency function for the search for neutralino pair production where each neutralino decays into a slepton with finite lifetime plus a lepton of the same flavour.

4.2.1 Efficiency for Displaced Vertices in $\tilde{\mu}$ Pair Production

The “kink” search for pair produced smuons with finite lifetime requires at least one identified kink in the central tracking chamber. The particles are pair–produced so the energy of a particle is set to $E_{\tilde{\mu}} = \sqrt{s}/2$. The fit function used has the form:

$$\epsilon_{\mathrm{event}}\,(m,\,\tau) = 1 - \left[1 - \epsilon_{\mathrm{single}}\,(m,\,\tau)\right]^2$$

with

$$\epsilon_{\mathrm{single}}\,(m_1,\,\tau) = \epsilon_0^{\star}\left[\exp\left(-\frac{l_{\mathrm{start}}}{l_{\mathrm{mean}}\,(m,\,\tau)}\right) - \exp\left(-\frac{l_{\mathrm{stop}}}{l_{\mathrm{mean}}\,(m,\,\tau)}\right)\right]$$

with

$$l_{\mathrm{mean}}\,(m,\,\tau) = c\tau\sqrt{\frac{s}{4m_1^2} - 1}.$$

To describe the efficiency drop close to the kinematic limit $\epsilon_0^\star$ has the form

$$\epsilon_0^\star = \frac{\epsilon_0}{1+\exp(a[m - m_{\mathrm{thresh}}])}\ .$$

Figure 5 gives examples of the fit function for the "kink" analysis as well as the parameters calculated by the fit. To study the behaviour close to the kinematic limit simulated events for masses of the $\tilde{\mu}$ with $m_{\tilde{\mu}} = 91, 92, 93\,\mathrm{GeV}$ were analysed. Parameters obtained from the single mass fits are found to be compatible within the errors for all masses if the additional corrections introduced by the fermi function are applied. Hence a simultaneous fit for all masses is possible. Results for the overall fit are:

- $\epsilon_0 = 0.759 \pm 0.022$ with $m_{\mathrm{thresh}} = (92.87 \pm 0.03)\,\mathrm{GeV}$ and the slope of the fermi function a is found to be $a = (0.42 \pm 0.04)\,\mathrm{GeV}$.
- $l_{\mathrm{start}} = (41.29 \pm 0.79)\,\mathrm{cm}$ which approximately corresponds to the inner radius of the OPAL Central Jet Chamber plus an additional 10 cm which are needed to detect the primary track of the kink.
- $l_{\mathrm{stop}} = (191.73 \pm 5.15)\,\mathrm{cm}$ which corresponds to the outer radius of the OPAL Central Jet Chamber minus an additional 10 cm which are needed to detect the secondary track of the kink.

As seen for the efficiency of the large impact parameter search, efficiencies for the $\tilde{\mu}$ search with visible decays in the Jet Chamber at different center–of–mass energies can be calculated by adjusting $\sqrt{s}$ to the actual value of the energy.

4.2.2 Efficiency for Displaced Vertices in $\tilde{\chi}^0$ Pair Production

The fit function was used here to describe the efficiency of the search for pair–produced neutralinos; $\mathrm{e^+e^-} \to \tilde{\chi}^0\tilde{\chi}^0$ where each neutralino decays via $\tilde{\chi}^0 \to \tilde{\ell}\ell$ ($\tilde{\ell} \to \ell\tilde{\mathrm{G}}$). Monte Carlo samples simulating this process containe smuons and selectrons.

The fit function used for the parametrization is

$$\epsilon_{\mathrm{event}}\,(m,\,\tau) = 1 - [1 - \epsilon_{\mathrm{single}}\,(m,\,\tau)]^2$$

with

$$\epsilon_{\mathrm{single}}\,(m_1,\,\tau) = \epsilon_0^\star \left[\exp\left(-\frac{l_{\mathrm{start}}}{l_{\mathrm{mean}}\,(m,\,\tau)}\right) - \exp\left(-\frac{l_{\mathrm{stop}}}{l_{\mathrm{mean}}\,(m,\,\tau)}\right)\right]$$

with

$$l_{\mathrm{mean}}\,(m,\,\tau) = c\tau\frac{\sqrt{{p^\star_{\parallel\,\tilde{\ell}}}^2 + {p_{\perp\,\tilde{\ell}}}^2}}{m_{\tilde{\ell}}}.$$

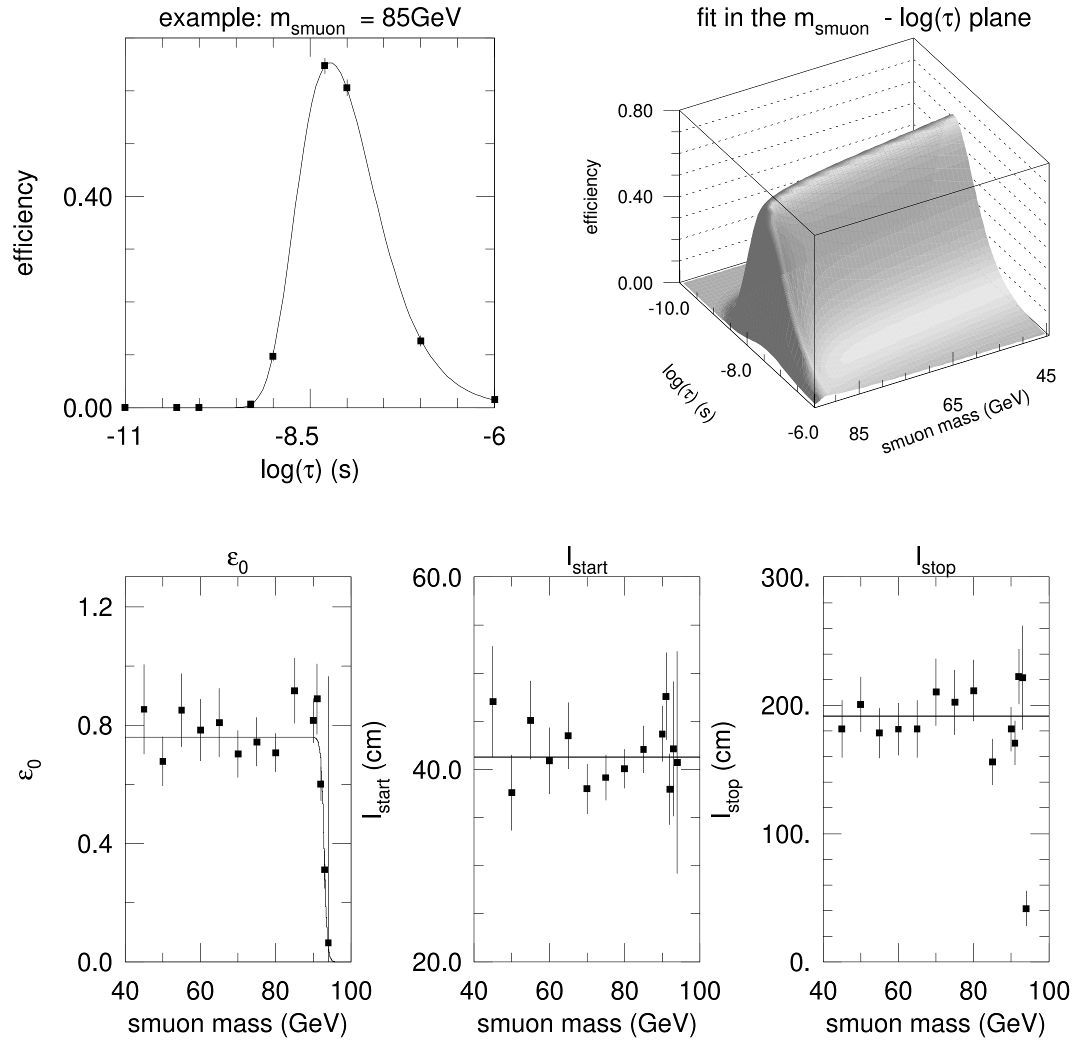

Figure 5: Description of the efficiency of the displaced vertex search for smuons with the OPAL detector at $\sqrt{s} = 189\,\text{GeV}$. The upper left plot gives an example of the fit for $m_{\tilde{\mu}} = 85\,\text{GeV}$. The upper right plot shows the description of the efficiency in the plane of the smuon mass and its lifetime. The description was achieved by a simultaneous fit to all Monte Carlo points. The lower plots show the fit parameters ϵ_0, l_{start} and l_{stop} for the one dimensional fits (dots) and the two dimensional fit (line). Parameter ϵ_0 is extended by a fermi function to describe the behaviour close to the kinematic limit.

$p^\star_{\parallel\,\tilde\ell}$ and $p_{\perp\,\tilde\ell}$ are calculated according to

$$p^\star_{\parallel\,\tilde\ell} = \gamma_{\tilde\chi^0} p_{\parallel\,\tilde\ell} + \gamma_{\tilde\chi^0}\beta_{\tilde\chi^0} E_{\tilde\ell} = \gamma_{\tilde\chi^0}|p_{\tilde\ell}|\cos\theta + \gamma_{\tilde\chi^0}\beta_{\tilde\chi^0} E_{\tilde\ell}$$

and

$$p^\star_{\perp\,\tilde\ell} = p_{\perp\,\tilde\ell} = |p_{\tilde\ell}|\sin\theta.$$

Here θ is the average decay angle between the slepton and the neutralino and is set to $\pi/2$ as the decay is isotropic. $E_{\tilde\ell}$ and $|p_{\tilde\ell}|$ are calculated by

$$E_{\tilde\ell} = \frac{m^2_{\tilde\chi^0} - m^2_{\ell} + m^2_{\tilde\ell}}{2m_{\tilde\chi^0}}$$

and

$$|p_{\tilde\ell}| = \frac{(m^2_{\tilde\chi^0} - (m_{\tilde\ell} + m_\ell)^2)\cdot(m^2_{\tilde\chi^0} - (m_{\tilde\ell} - m_\ell)^2)}{2m_{\tilde\chi^0}}.$$

Figure 6 gives an example of the calculated energy of the slepton as a function of the slepton mass and the neutralino mass. Compared are results of the fit where the energy $E_{\tilde\ell}$ is chosen as a free fit parameter and the calculation using the average decay angle θ. Both results show very good agreement and proof confidence in the assumed fit function.

As two effects have an impact on the efficiency $\epsilon^\star_0$ – the drop of efficiency close to the kinematic limit as well as its drop of in the low Δm region – they have to be implemented in the description of ϵ_0. Effects close to the kinematic limit are described as a function of $\beta_{\tilde\ell} = |p_{\tilde\ell}|/E_{\tilde\ell}$

$$\epsilon^\star_0 = \epsilon_0 \cdot \frac{1}{(1+\exp(a[\beta_{\tilde\ell} - \beta_{\tilde\ell\,\mathrm{thresh}}]))} \cdot \frac{1}{(1+\exp(b[\Delta m - \Delta m_{\mathrm{thresh}}]))}\ .$$

For the description the parameters l_{start} and l_{stop} from the fit for pair–produced smuons were used and set to $l_{\mathrm{start}} = 41.29\,\mathrm{cm}$ and $l_{\mathrm{stop}} = 191.73\,\mathrm{cm}$.

Fitting the function given above to the efficiencies found with the Monte Carlo studies for the fixed mass points $m_{\tilde\ell}/m_{\tilde\chi^0}$–plane and fixing the parameters l_{start} and l_{stop} yields a distribution of $\epsilon^\star_0$. This distribution is found to be flat in all regions which are not affected by limitations close to the kinematic limit or in the low Δm regions and is shown in Figure 6 (upper right plot).

The fit parameters are found to be

- $\epsilon_0 = 0.631 \pm 0.005$.
- $\beta_{\tilde\ell\,\mathrm{thresh}} = 0.198 \pm 0.004$ gives the threshold at which velosity of the particle the efficiency decreases. The slope a is calculated to be $a = 3.08\cdot 10^{-2} \pm 0.31\cdot 10^{-2}$.
- $\Delta m_{\mathrm{thresh}}$ was chosen to be 1 GeV as it could not be determined by the fit because of lack of Monte Carlo samples for slepton masses close to the neutralino mass. Hence the value was chosen corresponding to the value of $\Delta m_{\mathrm{thresh}}$ from the fit of the search for pair–produced smuons, and was fitted to be $\Delta m_{\mathrm{thresh}} = (1.0 \pm 0.9)\,\mathrm{GeV}$. The slope b was fitted to be $b = (0.843 \pm 0.167)\,\mathrm{GeV}$.

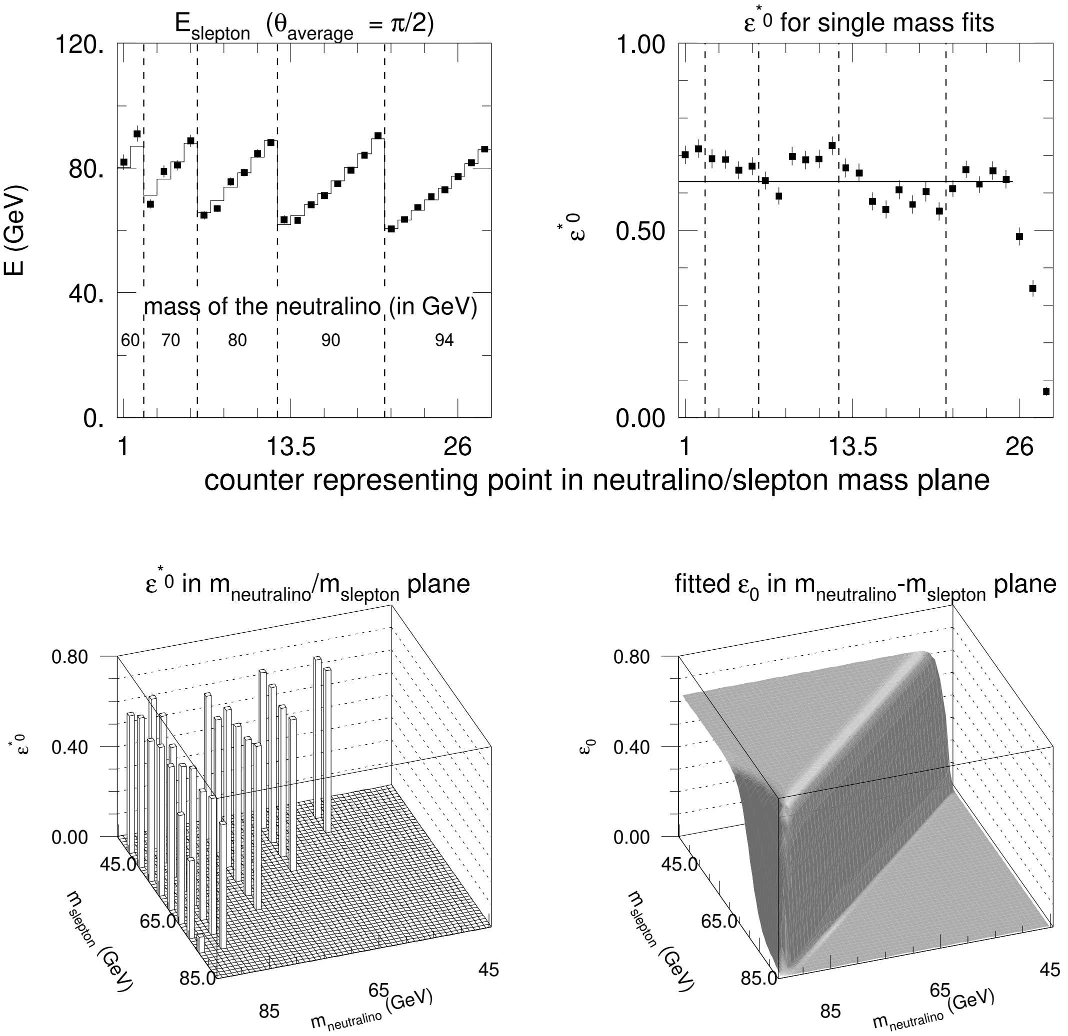

Figure 6: Examples for the efficiency function used for the search for neutralino pair production. The neutralino decays into a slepton with finite lifetime. The upper left plot shows the comparison of the calculated slepton energy (line) with the energy obtained from the fit. The values show very good agreement. For this and the next plot the x–axis is giving a counter representing a point in the $\tilde{\chi}^0/\tilde{\ell}$ mass plane. $m_{\tilde{\chi}^0}$ is indicated, $m_{\tilde{\ell}} = m_{\tilde{\chi}^0} - n \cdot 5\,\mathrm{GeV}$ from the right to the left for every slight in $m_{\tilde{\chi}^0}$. The upper right plot gives the efficiency $\epsilon_0^\star$ for the single masses fit. The line indicates the value for ϵ_0 calculated by the overall fit. The lower left histogram shows their position in the $\tilde{\chi}^0/\tilde{\ell}$ plane. The lower right histogram gives the result of the fit for ϵ_0.

- l_{start} and l_{stop} were chosen as fixed parameters and were set to 41.29 cm and 191.73 cm.

Figure 6 gives the result of the fit in the $m_{\tilde{\chi}^0}/m_{\tilde{\ell}}$ plane for the parameter ϵ_0.

4.3 Efficiency of the Stable Particle Search

The stable particle search looks for pair–produced heavy charged particles which do not decay within the detector and are identified by their anomalous ionization or by comparing the momentum of the particles to the centre–of–mass energy. The analysis requires two tracks from the vertex which are back–to–back and have a minimum number of CJ hits which are used for the $\mathrm{d}E/\mathrm{d}x$ measurement. This cut effectively imposes a minimum decay length for the particles but no restriction on the maximum flight length is given. Therefore the efficiency that a single particle passes the selection cuts can be described with two parameters only, ϵ_0 and l_{start} and are estimated to be

$$\epsilon_{\text{single}} = \int_{l_{\text{start}}}^{\infty} \mathcal{P}\left(l_{\text{dec}}\right) \mathrm{d}l_{\text{dec}} = \epsilon_0\left(m\right) \exp\left(-\frac{l_{\text{start}}}{c\beta\gamma\tau}\right) = \epsilon_0\left(m\right) \exp\left(-\frac{l_{\text{start}}}{l_{\text{mean}}\left(m,\, \tau\right)}\right).$$

Since both particles are required to have a high $\mathrm{d}E/\mathrm{d}x$ value, the event selection demands that both particles have decay lengths above l_{start}, therefore the event selection efficiency is given by

$$\epsilon_{\text{event}}\left(m,\, \tau\right) = \epsilon^2_{\text{single}}\left(m,\, \tau\right).$$

However, for this analysis ϵ_0 and l_{start} depend on the mass of the produced particle, because two different selections are used. One based on $\mathrm{d}E/\mathrm{d}x$, one based on the mass estimate from the particle's momentum. The dependence of two fit parameters ϵ_0 and l_{start} is shown in the lower plots of Figure 7. The effect of the two different analyses is clearly visible: in the region around a particle mass of 60 GeV the specific energy loss of the heavy particle is the same as for standard model particles. Hence kinematic selection cuts are performed for this region. This selection has less efficiency as cuts have to be tight to reduce background which is not present for $\mathrm{d}E/\mathrm{d}x$ cuts. Whereas for the $\mathrm{d}E/\mathrm{d}x$ method only tracks with a certain number of hits are needed which corresponds to a "short" parameter l_{start}, the kinematic analysis requires well measured tracks through the entire volume Central Jet Chamber. The parameter l_{start} gets values which are constistent with the outer radius of the Jet Chamber as seen in the histogram.

But for each individual mass the above ansatz works well, and a global fit can be performed by parametrizing ϵ_0 and l_{start} as function of the particle's mass. For the parametrization superpositions of functions type

$$\epsilon = \frac{\epsilon_0}{1 + \exp(c[\pm m \mp m_{\text{thresh}}])}$$

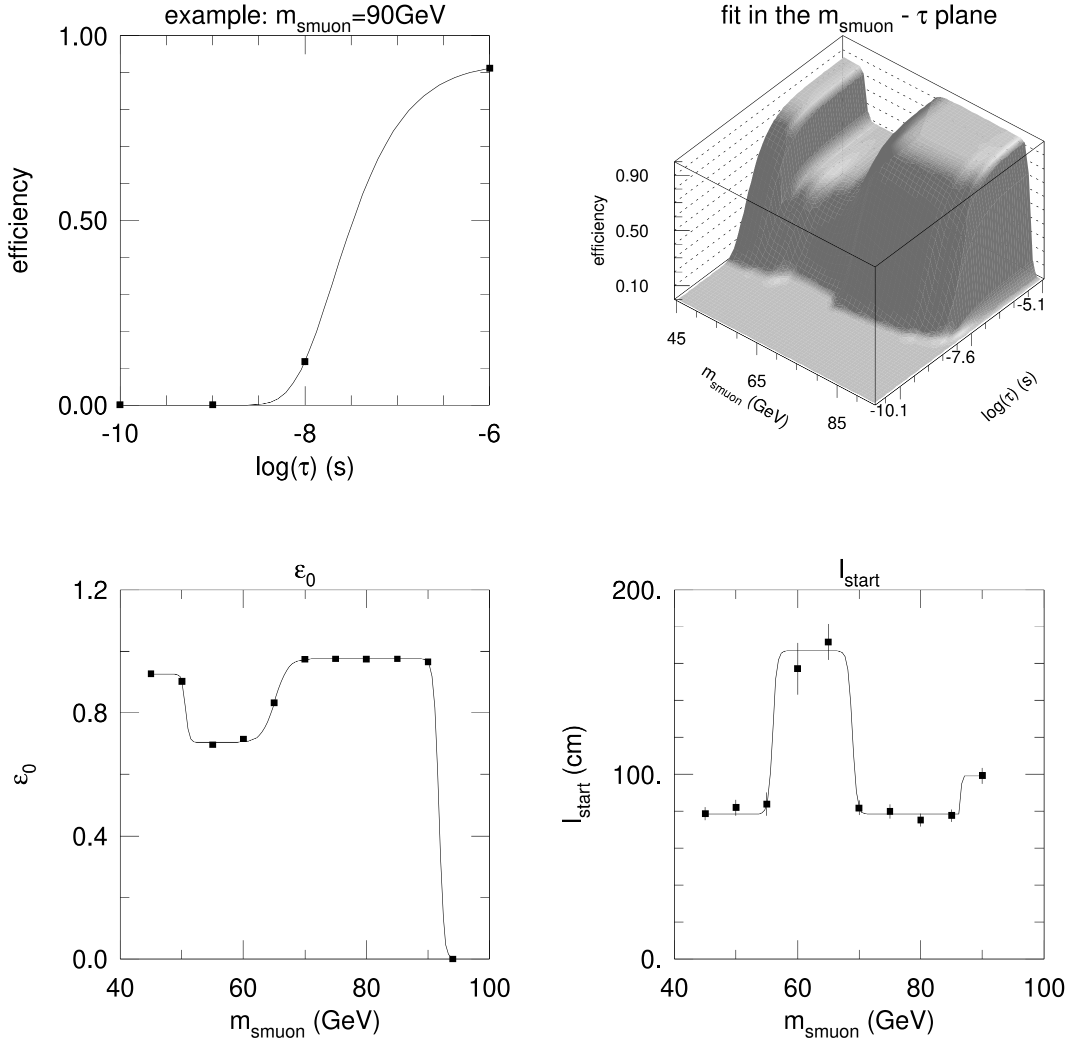

Figure 7: Efficiency function of the search for heavy stable charged particles with the OPAL detector at $\sqrt{s} = 189\,\mathrm{GeV}$. Efficiencies of a sample for pair–produced smuons are shown here. A reduced function with two free parameters only is used. The upper left plot gives an example of the single mass fit, the upper right plot show the fit over the entire $m_{\tilde{\mu}}/\tau_{\tilde{\mu}}$ plane. As the analysis uses two different methodes depending on the $\tilde{\mu}$ mass, the two fit parameters ϵ_0 and l_{start} depend on the $m_{\tilde{\mu}}$ (lower plots). To perform a two–dimensional fit ϵ_0 and l_{start} have to be parametrized as a function of $m_{\tilde{\mu}}$. In the lower plots the parametrization (solid line) is superimposed to the parameters of the single mass fits.

are used. The shape of the parametrized efficiency ϵ is given in Figure7 as well and is superimposed to the fit parameter values. For the parameters ϵ_0 and l_{start} following values were found:

- as seen in Figure7 ϵ_0 is found to be different for three regions of the smuon mass: $m < 50\,\mathrm{GeV}$: $\epsilon_0 = 0.926 \pm 0.012$, $50\,\mathrm{GeV} < m < 65\,\mathrm{GeV}$: $\epsilon_0 = 0.704 \pm 0.018$ and $65\,\mathrm{GeV} < m$: $\epsilon_0 = 0.976 \pm 0.010$.
- values for l_{start} depend on m, too and are found to be $59\,\mathrm{GeV} < m < 66\,\mathrm{GeV}$: $l_{\mathrm{start}} = 166.9 \pm 3.2\,\mathrm{cm}$, $m > 86.5\,\mathrm{GeV}$: $l_{\mathrm{start}} = 99.2 \pm 1.9\,\mathrm{cm}$ and $l_{\mathrm{start}} = 78.4 \pm 0.5\,\mathrm{cm}$ elsewhere.

4.4 Efficiency for the Combination of Searches

With a parametrization for each individual analysis at hand one can proceed to parametrize the combination of all analyses.

The combination of the fit functions for the individual analyses is done by superposition. Due to the overlap between them, the combined selection efficiency is smaller than the sum of the individual selection efficiencies, the combined selection efficiency for the Monte Carlo is determined by selecting events which are chosen by at least one of the analyses. To incorporate the overlap into the fit, the parametrizations of the different analyses and correction terms are used in the following way.

$$\begin{aligned}
\epsilon_{\mathrm{comb}}(m,\,\tau) = {} & \epsilon_1(m,\,\tau) + \epsilon_2(m,\,\tau) + \epsilon_3(m,\,\tau) + \epsilon_4(m,\,\tau) \\
& -|r_{12}|\,\epsilon_1(m,\,\tau)\,\epsilon_2(m,\,\tau) - |r_{13}|\,\epsilon_1(m,\,\tau)\,\epsilon_3(m,\,\tau) \\
& -|r_{14}|\,\epsilon_1(m,\,\tau)\,\epsilon_4(m,\,\tau) - |r_{23}|\,\epsilon_2(m,\,\tau)\,\epsilon_3(m,\,\tau) \\
& -|r_{24}|\,\epsilon_2(m,\,\tau)\,\epsilon_4(m,\,\tau) - |r_{34}|\,\epsilon_3(m,\,\tau)\,\epsilon_4(m,\,\tau)
\end{aligned}$$

Here, $\epsilon_1(m,\,\tau)$, $\epsilon_2(m,\,\tau)$, $\epsilon_3(m,\,\tau)$ and $\epsilon_4(m,\,\tau)$ are the selection efficiencies for the zero lifetime particle search[2] , the large impact parameter search, the kink search, and the long lived particle search, respectively, parametrized as described before. $\epsilon_{\mathrm{comb}}(m,\,\tau)$ is the efficiency for the combined analysis. The parameters r_{nm} describe the correlation of the selection probabilities of the different analyses, and are expected to be 0 if no correlation is present. Studies on the Monte Carlo samples showed that no event is selected by more than two analyses so correlations among three and more analyses are assumed to be 0. In the fit following overlaps were found to be negligable, $r_{13} = r_{14} = r_{24} = 0$.

Correlations were seen for the following analyses.

- $r_{12} = 1.075 \pm 0.014$ represents the correlation between the zero lifetime particle search and the large impact parameter search. As standard OPAL

[2]The analysis used for the search for short lived partcles is the OPAL search for acoplanar leptons [1]. The analysis is based on a mass–dependent likelyhood selection and the selection efficiency function is obtained in a simillar way like described for the heavy stable particles. Contrary to the function for the heavy stable particles the parameter l_{start} is set to 0 and l_{stop} is calculated as particles with no lifetime are originated in the interaction point.

cuts on the impact parameter are around 2 cm the correlation is expected to be non zero.

- $r_{23} = 0.352 \pm 0.054$ represents the correlation between the large impact parameter search and the displaced vetex search. As secondary tracks of the "kink" have large impact parameters, a non–zero correlation is expected especially due to decays at the start of the Central track chamber.
- $r_{34} = 0.952 \pm 0.024$ shows the correlation between the displaced vertex search and the search for heavy stable charged particles. As the sensitive region of both analyses overlap and a primary "kink" track of a heavy particle shows high ionization losses this correlation is expected to be non-zero.

An example of the combined fit is given in Figure 8

5 Conclusions

It has been shown that a function to describe selection efficiencies for heavy particles with finite lifetime can be found with a minimal set of a few free parameters. These parameters represent an effective overall efficiency of the analysis and two parameters which describe the inner and outer boarder of the sensitive region of the detectors used for the analysis. The other variables used for the fit function can be calculated from the kinematics of the decay process. If the analysis does not change as a function of the particle mass the selection efficiency can be fitted directly to the entire mass–lifetime phase space. Otherwise a parametrization of the mass dependence has to be applied. The selection efficiency function is not a specific function for the OPAL detector but can be applied for other HEP detectors as well.

The selection efficiency function allows a better description than simple interpolation and can drastically reduce the amount of Monte Carlo data sets which one would need to scan the wide range of particle masses, lifetimes and LEP energies.

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[3] OPAL Collaboration, G. Abbiendi et al. (2000): Search for Anomalous Photonic Events with Missing Energy in e+e- Collisions at $\sqrt{s} = 189$ GeV. — *To be submitted to Eur. Phys. J.*

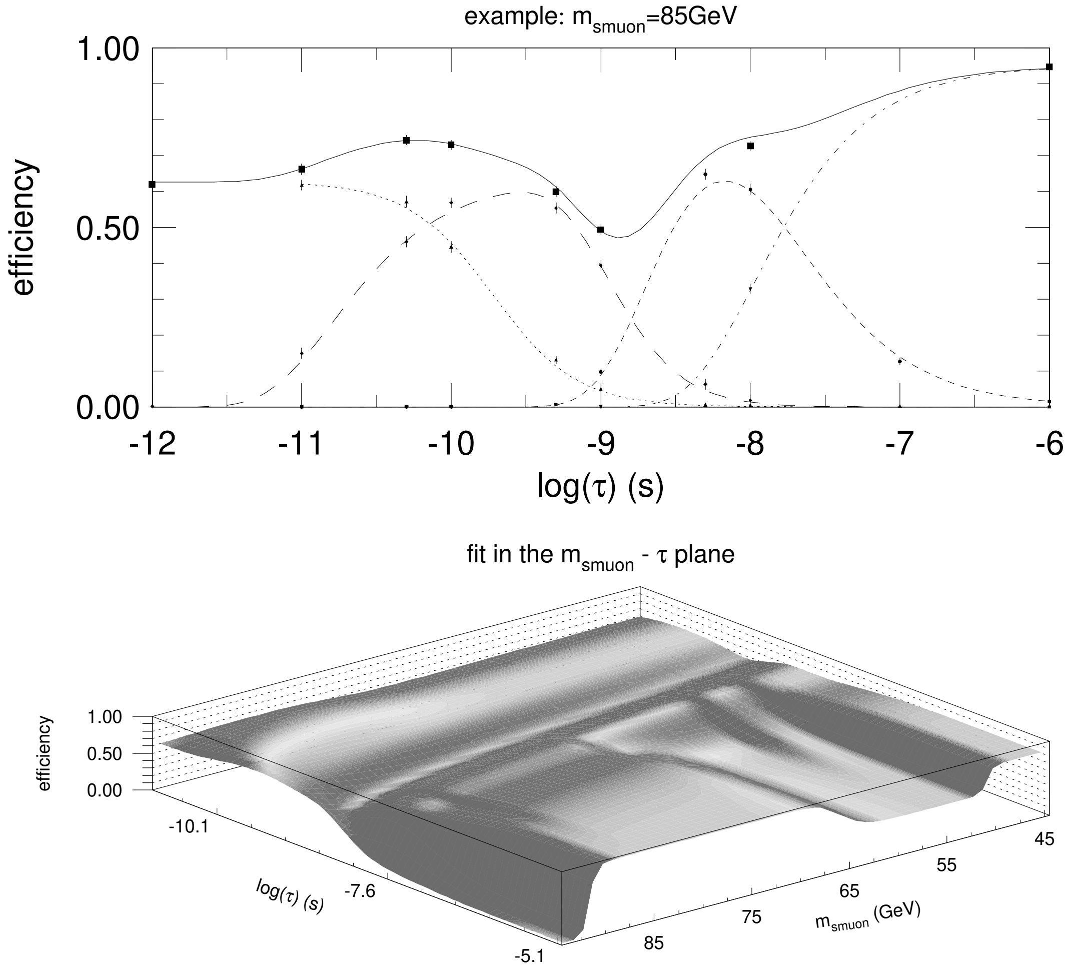

Figure 8: Example of the combined selection efficiency for the search for pair–produced smuons with the OPAL detector at $\sqrt{s} = 189\,\text{GeV}$. The upper plot shows an example of the combined function for a smuon mass of 85 GeV (solid line). The analyses which the combinated fit describes are the search for acoplanar leptons (dotted line), the large impact parameter search (long dashed line), the “kink” search (short dashed line) and the search for heavy stable charged particles (dashed–dotted line). The function in the entire phase–space is shown in the lower plot.

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[5] S. Ambrosanio, G. D. Kribbs and S. P. Martin (1997): Signals for Gauge–Mediated Supersymmetry Breaking Models at the CERN LEP2 Collider. — *Phys. Rev.* ***56****, 1761–1777.*