# Bin Center Corrections, Acceptance, and All That

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June 7, 1995

## 1 Introduction

The differential $ep$ cross section $d^2\sigma_{ep}^{obs}$ for the photoproduction of a vector meson can be expressed as

$$\begin{aligned} d^2\sigma_{ep}^{obs} &= [A^T(y, Q^2) \cdot \mathcal{F}^T(y, Q^2) \cdot \sigma_{\gamma^* p}^T(y, Q^2) + \\ &\quad A^L(y, Q^2) \cdot \mathcal{F}^L(y, Q^2) \cdot \sigma_{\gamma^* p}^L(y, Q^2)]\, dy\, dQ^2 \qquad (1) \\ &= A^T(y, Q^2) \cdot \mathcal{F}^T(y, Q^2) \cdot \sigma_{\gamma p}^T(y, Q^2) \cdot \\ &\quad [1 + \alpha(y, Q^2)\epsilon(y, Q^2)R(Q^2)]\, dy\, dQ^2, \qquad (2) \end{aligned}$$

where $A^T(y, Q^2)$ ($A^L(y, Q^2)$) is the $y$- and $Q^2$-dependent acceptance (including detector acceptance, trigger and reconstruction efficiency) for vector mesons produced by transversely (longitudinally) polarized photons, $\mathcal{F}^T(y, Q^2)$ ($\mathcal{F}^L(y, Q^2)$) is the flux of transverse (longitudinal) photons, $\sigma_{\gamma^* p}^T(y, Q^2)$ ($\sigma_{\gamma^* p}^L(y, Q^2)$) is the $\gamma^* p$ cross section for vector meson production off transverse (longitudinal) photons, and the related quantities $\alpha$, $\epsilon$, and $R$ are defined as

$$\alpha(y, Q^2) = \frac{A^L(y, Q^2)}{A^T(y, Q^2)} \qquad (3)$$

$$\epsilon(y, Q^2) = \frac{\mathcal{F}^L(y, Q^2)}{\mathcal{F}^T(y, Q^2)} \qquad (4)$$

$$R(Q^2) = \frac{\sigma_{\gamma^* p}^L(y, Q^2)}{\sigma_{\gamma^* p}^T(y, Q^2)}. \qquad (5)$$

The measurement is necessarily made in bins of finite size, therefore the $ep$ cross section inside a bin $\sigma_{ep}^{obs,bin}$ is given by

$$\sigma_{ep}^{obs,bin} = \int\limits_{y,\, Q^2} d^2\sigma_{ep}^{obs}\, dy\, dQ^2. \qquad (6)$$

Usually $y$ (and in the DIS case, $Q^2$) are measured so well that smearing corrections have not to be taken into account.

Trivially, this can be rewritten as

$$\begin{aligned}
\sigma_{ep}^{obs,bin} &= \sigma_{\gamma^* p}^T(y_0, Q_0^2)[1+\epsilon(y_0, Q_0^2)R(Q_0^2)] \cdot \int\limits_{y,Q^2} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2 \cdot \\
&\quad \frac{\int_{y,Q^2} \frac{\sigma_{\gamma^* p}^T(y,Q^2)}{\sigma_{\gamma^* p}^T(y_0,Q_0^2)} \frac{1+\epsilon(y,Q^2)R(Q_0^2)}{1+\epsilon(y_0,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2}{\int_{y,Q^2} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2} \cdot \qquad (7) \\
&\quad \frac{\int_{y,Q^2} A^T(y, Q^2) \frac{\sigma_{\gamma^* p}^T(y,Q^2)}{\sigma_{\gamma^* p}^T(y_0,Q_0^2)} \frac{1+\alpha(y,Q^2)\epsilon(y,Q^2)R(Q_0^2)}{1+\epsilon(y_0,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2}{\int_{y,Q^2} \frac{\sigma_{\gamma^* p}^T(y,Q^2)}{\sigma_{\gamma^* p}^T(y_0,Q_0^2)} \frac{1+\epsilon(y,Q^2)R(Q_0^2)}{1+\epsilon(y_0,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2} \\
&= \sigma_{\gamma^* p}^T(y_0, Q_0^2)[1+\epsilon(y_0, Q_0^2)R(Q_0^2)] \cdot \Gamma^T \cdot B(y_0, Q_0^2) \cdot A, \qquad (8)
\end{aligned}$$

where

$$\Gamma^T = \int\limits_{y,Q^2} \mathcal{F}^T(y, Q^2)dy\,dQ^2 \qquad (9)$$

is the integrated transverse photon flux for the bin considered,

$$B(y_0, Q_0^2) = \frac{\int_{y,Q^2} \frac{\sigma_{\gamma^* p}^T(y,Q^2)}{\sigma_{\gamma^* p}^T(y_0,Q_0^2)} \frac{1+\epsilon(y,Q^2)R(Q_0^2)}{1+\epsilon(y_0,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2}{\int_{y,Q^2} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2} \cdot \qquad (10)$$

is the bin–center correction for the bin–center given by $y_0$ and $Q_o^2$, and

$$A = \frac{\int_{y,Q^2} A^T(y, Q^2) \frac{\sigma_{\gamma^* p}^T(y,Q^2)}{\sigma_{\gamma^* p}^T(y_0,Q_0^2)} \frac{1+\alpha(y,Q^2)\epsilon(y,Q^2)R(Q_0^2)}{1+\epsilon(y_0,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2}{\int_{y,Q^2} \frac{\sigma_{\gamma^* p}^T(y,Q^2)}{\sigma_{\gamma^* p}^T(y_0,Q_0^2)} \frac{1+\epsilon(y,Q^2)R(Q_0^2)}{1+\epsilon(y_0,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y, Q^2)\,dy\,dQ^2} \qquad (11)$$

is the bin–averaged acceptance.

## 2 The Bin–Center Correction

$B(y_0,\, Q_0^2)$ is defined as

$$
\begin{aligned}
B(y_0,\, Q_0^2) &= \frac{\int\limits_{y,\,Q^2} \frac{\sigma^T_{\gamma^* p}(y,\,Q^2)}{\sigma^T_{\gamma^* p}(y_0,\,Q_0^2)} \frac{1+\epsilon(y,\,Q^2)R(Q_0^2)}{1+\epsilon(y_0,\,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y,\,Q^2)\,dy\,dQ^2}{\int\limits_{y,\,Q^2} \mathcal{F}^T(y,\,Q^2)\,dy\,dQ^2} && (12)\\
&= \frac{\text{bin–averaged cross section}}{\text{cross section at bin center}}. && (13)
\end{aligned}
$$

Obviously it is always possible for a given parametrization of $\sigma^T_{\gamma^* p}(y,\, Q^2)$ and $R(Q_0^2)$ to find a set of values for $y_0$ and $Q_0^2$ inside the bin where $B(y_0,\, Q_0^2) = 1$. This may however not be the best choice, since generally $B$ will depend on the chosen parametrization of $\sigma^T_{\gamma^* p}(y,\, Q^2)$ and $R(Q_0^2)$. The point where $B(y_0,\, Q_0^2) = 1$ corresponds to a value of $y_0$ and $Q_0^2$ where the average cross section is equal to the cross section at this particular point. A different parametrization of the cross section though may change the value of $B$ considerably, say by 10 %, so that these 10 % have to be included into the systematical error, whereas for a different choice of $y_0$ and $Q_0^2$ the variation of $B$ for different parametrizations may be much smaller.

Note that the value of $B(y_0,\, Q_0^2)$ depends on the choice of the bin–center, but not on the acceptance (i.e. detector effects) in the bin. It may therefore be computed efficiently to an arbitrary precision using fast numerical integration programs.

## 3 The Bin–Averaged Acceptance

$A$ is defined as

$$
\begin{aligned}
A &= \frac{\int\limits_{y,\,Q^2} A^T(y,\,Q^2) \frac{\sigma^T_{\gamma^* p}(y,\,Q^2)}{\sigma^T_{\gamma^* p}(y_0,\,Q_0^2)} \frac{1+\alpha(y,\,Q^2)\epsilon(y,\,Q^2)R(Q_0^2)}{1+\epsilon(y_0,\,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y,\,Q^2)\,dy\,dQ^2}{\int\limits_{y,\,Q^2} \frac{\sigma^T_{\gamma^* p}(y,\,Q^2)}{\sigma^T_{\gamma^* p}(y_0,\,Q_0^2)} \frac{1+\epsilon(y,\,Q^2)R(Q_0^2)}{1+\epsilon(y_0,\,Q_0^2)R(Q_0^2)} \mathcal{F}^T(y,\,Q^2)\,dy\,dQ^2}\\
&= \frac{\text{Number of events triggered and reconstructed}}{\text{Number of events that occured}}. && (14)
\end{aligned}
$$

$A$ is usualy determined from Monte Carlo, which limits the accuracy with which $A$ is known (typically one has about 10 times more Monte Carlo statistics than data statistics, so that the statistical error on $A$ is about 30 % of the statistical error from the data).

Nevertheless, $A$ also depends on the cross section parametrization. For example, for $J/\psi$ photoproduction $W$ is strongly correlated with the polar angle of the observed tracks, and therefore $A$ varies considerably with $W$ in regions where the track angles become large ($> 150°$) or small ($< 30°$, when forward tracks are not used). Therefore, the error $\delta A$ of $A$ contains a contribution from the cross section parametrization, which is highly correlated to the error $\delta B$ of the bin–center correction $B$. This contribution is hard to quantify if it is smaller than the statistical error on $A$, since the change of $A$ due to a different parametrization of the cross section can then not be determined reliably. In this case it is probably reasonable to treat $A$ and $B$ as uncorrelated quantities in the evaluation of systematic errors.

If however Monte Carlo ststistics is so large that the statistical error of $A$ is small compared to the variation of $A$ due to different parametrizations of $\sigma^T_{\gamma^* p}(y, Q^2)$ and $R(Q_0^2)$, then $A$ and $B$ are highly correlated, and to be conservative, $\delta A$ and $\delta B$ should be added linearly instead of quadratically.

## 4 Calculation of the Cross Section

Obviously what one measures is

$$\sigma_{ep}^{obs,bin} = \frac{N^{obs,bin}}{\int \mathcal{L}\, dt}, \tag{15}$$

where $N^{obs,bin}$ is the number of events observed in the bin (after background subtraction), and $\int \mathcal{L}\, dt$ is the integrated luminosity. The acceptance–corrected $ep$ cross section is then given by

$$\sigma_{ep}^{bin} = \frac{N^{obs,bin}}{A \int \mathcal{L}\, dt}. \tag{16}$$

The value of

$$\sigma^T_{\gamma^* p}(y_0, Q_0^2)[1 + \epsilon(y_0, Q_0^2) R(Q_0^2)] = \sigma^T_{\gamma^* p}(y_0, Q_0^2) + \epsilon(y_0, Q_0^2)\sigma^L_{\gamma^* p}(y_0, Q_0^2)$$

is

$$\sigma^T_{\gamma^* p}(y_0, Q_0^2)[1+\epsilon(y_0, Q_0^2) R(Q_0^2)] = \frac{\sigma_{ep}^{bin}}{B(y_0, Q_0^2)\,\Gamma_T} = \frac{N^{obs,bin}}{A\, B(y_0, Q_0^2)\,\Gamma_T \int \mathcal{L}\, dt}. \tag{17}$$

If one wants to quote as final result of the measurement $\sigma^T_{\gamma^* p}(y_0, Q_0^2)[1+\epsilon(y_0, Q_0^2)R(Q_0^2)]$, $\sigma^T_{\gamma^* p}(y_0, Q_0^2)$, or $\sigma^T_{\gamma^* p}(y_0, Q_0^2) + \sigma^L_{\gamma^* p}(y_0, Q_0^2)$ depends on the actual situation.

For the low–$Q^2$ photoproduction of vector mesons, $\sigma_{\gamma^* p}$ is usually qoted at $Q_0^2 = 0$, where $\sigma^L_{\gamma^* p}$ vanishes anyway. Also in this case longitudinal photons contribute little to the total observed cross section, so hat the error introduced due to an *à priori* unknown value of $R$ is small compared to other errors.

In the high–$Q^2$ region one may want to quote $\sigma^T_{\gamma^* p}(y_0, Q_0^2)+\sigma^L_{\gamma^* p}(y_0, Q_0^2)$ rather than $\sigma^T_{\gamma^* p}(y_0, Q_0^2)[1+\epsilon(y_0, Q_0^2)R(Q_0^2)]$, as one does in the case of the measurement of the structure function $F_2$. In this case one has

$$
\begin{aligned}
&\sigma^T_{\gamma^* p}(y_0, Q_0^2) + \sigma^L_{\gamma^* p}(y_0, Q_0^2) \\
&= \frac{1+R(y_0, Q_0^2)}{1+\epsilon(y_0, Q_0^2)R(y_0, Q_0^2)}\sigma^T_{\gamma^* p}(y_0, Q_0^2)[1+\epsilon(y_0, Q_0^2)R(Q_0^2)] \quad (18) \\
&= \frac{1+R((y_0, Q_0^2)}{1+\epsilon(y_0, Q_0^2)R(y_0, Q_0^2)} \frac{N^{obs,bin}}{A\,B(y_0, Q_0^2)\,\Gamma_T \int \mathcal{L}\,dt}. \quad (19)
\end{aligned}
$$

At HERA-energies, $\epsilon$ is usually very nearly 1, so that even a broad variation of possible parametrizations of $R$ will introduce a tolerable error.