Today

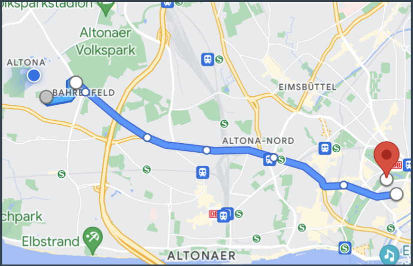
Evidences/Applications of relativistic kinematics:

- GPS
- Waves, and the Doppler effect
- Muon decay
- Twin paradox

The Global Positioning System (GPS)

A Real-life Application of Special Relativity

Nowadays, ubiquitous in life…

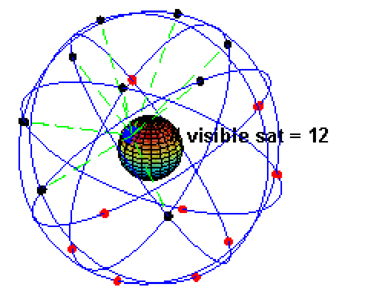


Based on three segments:

- Space segment
- Control segment
- User segment

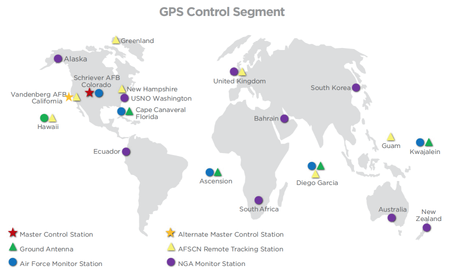
Space Segment

- 31 satellites
  (27 always active)
- Every position on Earth visible by ≥ 4 satellites
- Equipped with atomic clock
- Altitude: 20200 km
- Period: ~ 12 h
- Speed: ~ 4 km/s



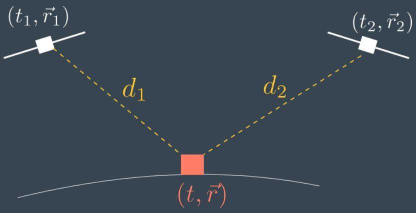
Control Segment

- On-ground stations
- Equipped with high-precision atomic clocks
- Tracks satellites at all times
- Resyncs clocks
- Updates orbits



User Segment

- Small receiver
- Computes position from satellite signal

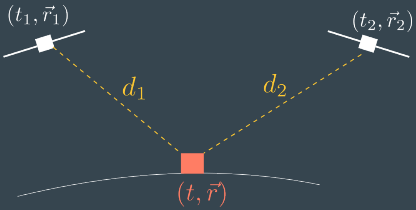


$$d_i = |\vec{r} - \vec{r}_i| = c(t - t_i)$$

- 4 equation, 4 variables
- More satellites, more precision

Possible errors

- Satellite drift
- Signal noise
- Relativity!



Relativistic errors $\Delta t_{\text{sat}} = \gamma \Delta t_{\text{obs}}$

- $c = 300\,000$ km/s
- $1\,\mu$s $= 10^{-6}$s inaccuracy $\Rightarrow$ 300 m error
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- $v_{\text{sat}} = 4\,\text{km/s}$. $v_{\text{sat}} \ll c$ but

$\gamma \sim 1 + 10^{-10}$ $\sim 8\,\mu\text{s/day} \Leftrightarrow 2\text{km/day}$ error!

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Clocks need to be resynchronised constantly! NB: there are also General Relativity effects to be dealt with

Waves and Electromagnetism

Wave mechanics

Wave: a “disturbance” moving through space while maintaining its shape.

Examples:

- Wave in water
- Sound
- Light

…

The Wave Equation

$$\Box = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Any quantity $\vec{u}(t, \vec{x})$ that satisfies:

$$\frac{1}{c^2}\frac{\partial^2 \vec{u}}{\partial t^2} = \Box\,\vec{u}$$

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c : speed of propagation

$c_{\text{air}} \sim 340$ m/s

$c_{\text{water}} \sim 1400$ m/s

$c_{\text{wood}} \sim 4000$ m/s

$c_{\text{diamond}} \sim 12000$ m/s

...

Solutions

A particular solution is the sinusoidal wave:

$$\vec{u}_\lambda(t,\vec{x}) = \vec{A}\sin(\frac{2\pi}{\lambda}(\vec{n}\cdot\vec{x} - ct) + \varphi)$$

λ: Wavelength (*Wellenlänge*)

$\vec{A}$: Amplitude (Auslenkung)

$\vec{n}$: direction of movement ($|\vec{n}| = 1$)

φ: the phase

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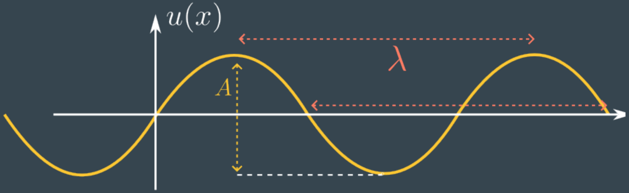

Superposition

General solution via superposition*:

$$u(x,t) = \sum_{\lambda} u_{\lambda}(t, \vec{x})$$

Visualisation: `https://ophysics.com/w3.html`

Electromagnatic waves

Shaking Maxwell's equations (in vacuum):

$$\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \Box \vec{E} \qquad \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = \Box \vec{B}$$

- Electromagnetic radiation propagates as waves
- Gives sinusoidal wave characterised by λ
- For EM waves, energy given by:

$$\mathcal{E} = \frac{hc}{\lambda} \qquad h \simeq 6.26\,\mathrm{J}\cdot\mathrm{s}$$

Electromagnatic waves

Visible light only a tiny fraction of electromagnetic spectrum

$\lambda \in \{380\,\text{nm}, 750\,\text{nm}\}$

What about other types of EM waves?

Electromagnatic waves

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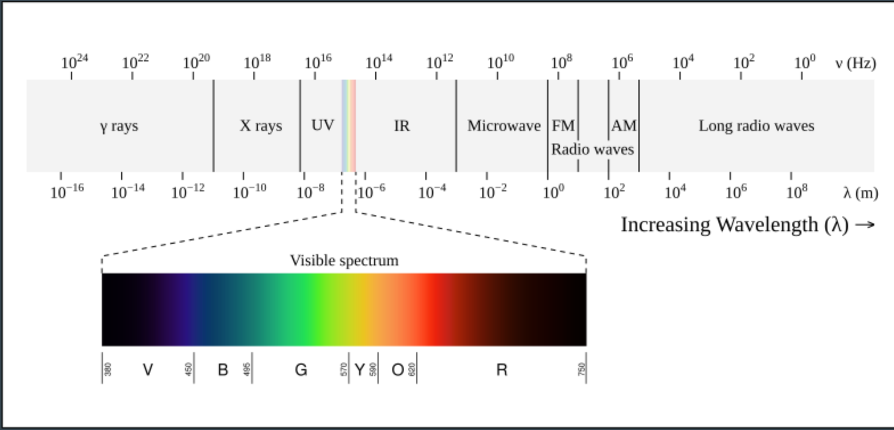


White light

White light: superposition of various wavelengths

Bohr's Model

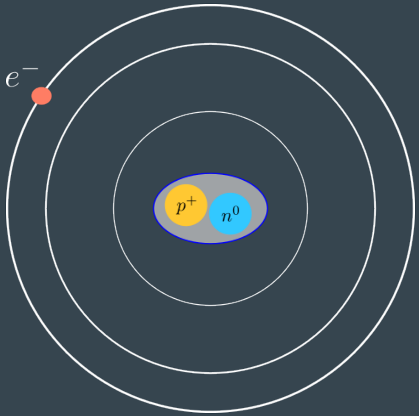
Most common atom in the universe: Hydrogen

- A nucleus ($p^+ + n^0$)
- An electron e^-

Bohr's model:

e^- at fixed/quantised orbit

radius $\Leftrightarrow$ energy $\mathcal{E}$

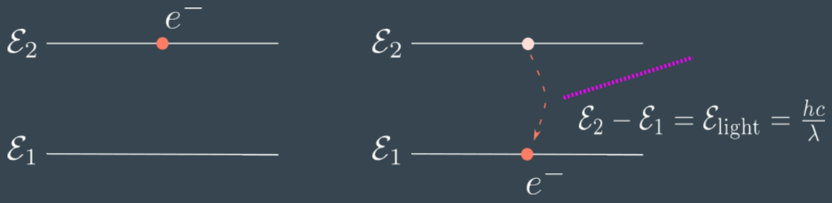


Crude model, but works fairly well...

Spectroscopy

Prediction of Bohr's model: an electron can change orbit

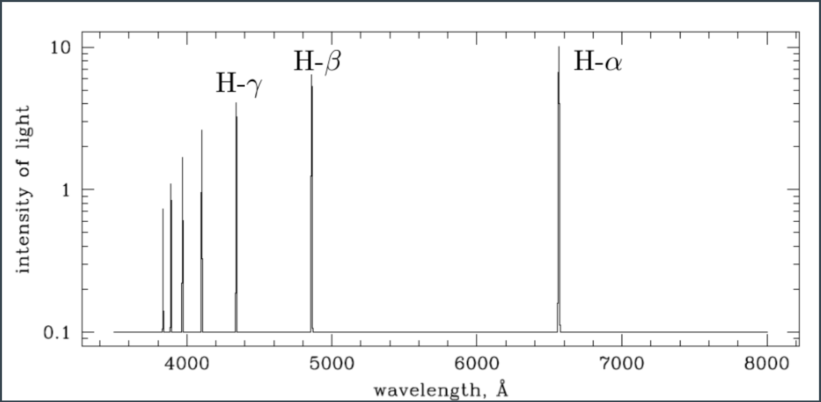
- Absorbing light, go up
- Emitting light, go down



Emission can only happen at precise wavelengths

Spectroscopy

In practice, take a blob of Hydrogen, and count the bits of light at each wavelength for some time



Different material, different peaks. Spectroscopy can be used to identify a substance.

Summary so far

- GPS could not work properly without relativity
- Light is an EM wave (visible or not)
- Characterised by a wavelength
- Spectroscopy: atoms emit light at specific wavelengths

Wave*length*. Affected by Special Relativity?