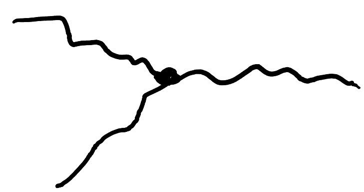
Scattering

phonon = quasi-particle

→ Conservation of Energy / momentum
Impuls

$E_{before} = E_{after}$ $\quad\quad E = \hbar\omega_k$

$\vec{k}_{before} = \vec{k}_{after}$

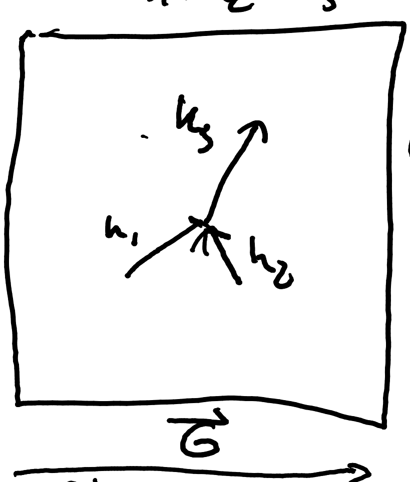


momentum ↔ reciprocal lattice

Normal process

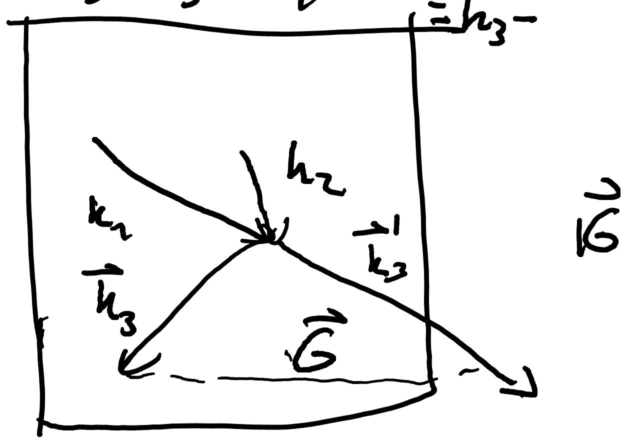
$\vec{k}_1 + \vec{k}_2 = \vec{k}_3$

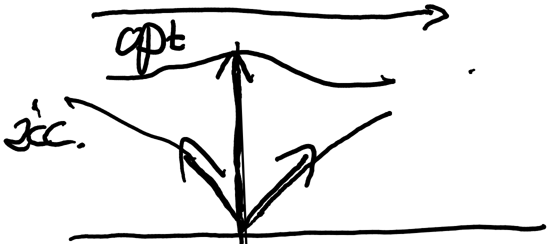
Brillouin zone

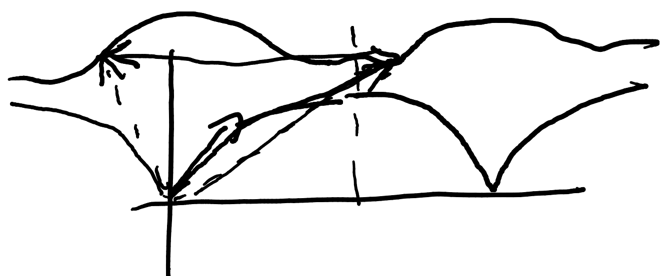


Umklapp process

$\vec{k}_3' = \vec{k}_3 + \vec{G} \;\Big|\; k_1 + k_2 \overset{!}{=} \vec{k}_3' \neq \vec{k}_3$







Umklapp proc. happens if

$|k_{before}| > |\vec{G}|$

Since lattice is periodic, we simply use $\vec{G}$ (lattice vector) to "bring back" the vector inside the Brillouin zone

Since $k \sim \omega \sim E \sim T$

→ Umklapp in phonon-phonon scattering associated w/ high temperature

⟺ important in <u>thermal resistivity</u>

can also have

* electron (e^-) phonon scattering

In solid, many phonons, ... Impossible to describe them all

↳ use statistics

⇝ thermodynamical system

Perfect gas (massive particle

black body (photons)

Solid (phonons)

Need to describe distribution of phonons, photons, particle

⇝ different models

Einstein model

1 isolated has energy

$$U_n = (n + 1/2)\hbar\omega_0$$

single frequency

phonon level

$$U \approx 3N \langle n \rangle \hbar\omega$$

Planck distribution $\langle n \rangle = \dfrac{1}{e^{\hbar\omega/k_BT} - 1}$

$\langle n \rangle = \langle n \rangle(T)$: average level of phonons at temp. T

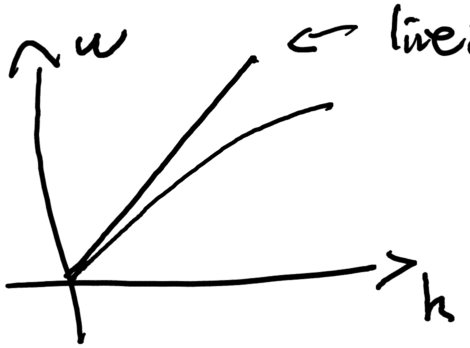
Define $C_v = \dfrac{\partial U}{\partial T}$ (heat capacity)

$T \gg 1$ $\quad C_v \to 3Nk_B$ ← Dulong-Petit ✓ law (experimental)

$T \ll 1$ $\quad C_v \sim e^{T}$ wrong ($C_v \sim T^3$ exper.)

Debye model

Debye : phonons do not all have same ω, but a linear dispersion relation $\omega(k) \sim k$



total energy — density of states — energy of 1 mode

$$U_{Debye} = \int_0^{\omega_D} d\omega \; D(\omega) \; U_n$$

$$U_n = \langle n \rangle \hbar \omega$$

Planck distrib

vol

$$D = \frac{V \omega^2}{2\pi^2 v_s^2}$$

$$C_v = \frac{\partial U}{\partial T}$$

speed of sound

gas cst

cutoff frequency : $\omega_D = \sqrt[3]{\frac{6\pi^2 v_s^3 N_A}{V}}$

Debye

$T \gg 1 \qquad C_V \sim cst$

$T \ll 1 \qquad C_V \sim T^3 \qquad$ good!

Improvement from Einstein model :)