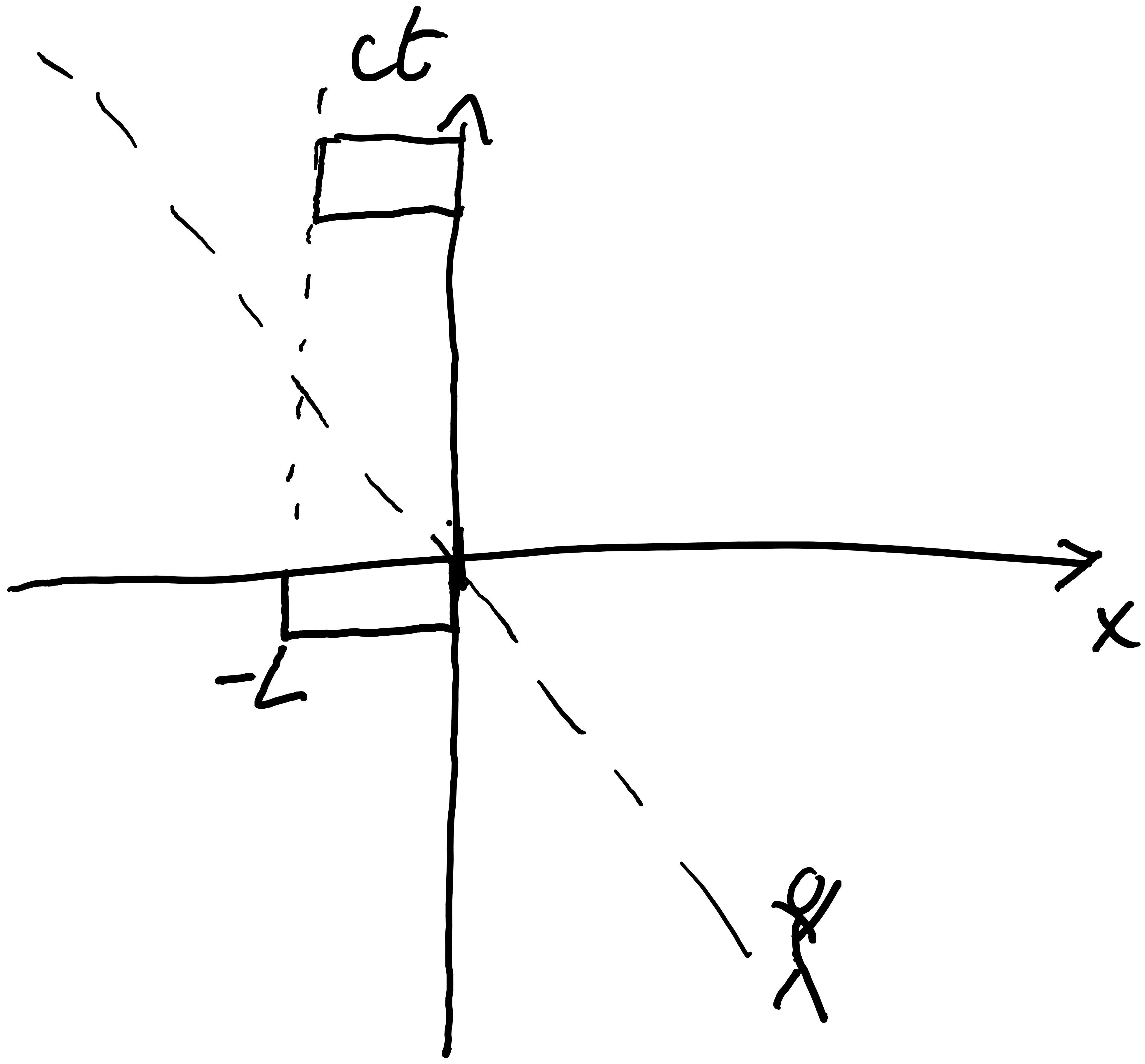
GR 4

We start w/ a Minkowski diagram



For O, the pedestrian approaches the car w/ velocity $-v$, sees the car at

front: $(ct_1, x_1) = (0, 0)$

rear: $(ct_2, x_2) = (ct_2, -L)$ $\quad t_2 = \frac{L}{v}$

For $\tilde{O}$: $\tilde{L} = |\tilde{x}_2 - \tilde{x}_1| = v|\tilde{t}_2 - \tilde{t}_1|$

We know (ct_i, x_i) $i=1,2$

Go to $\tilde{O}$ frame via Lorentz transf

$$\Lambda_\beta = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} ; \quad \beta = \frac{\text{velocity}}{c} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Here $\tilde{O}$ approaches at velocity $\underline{\underline{-v}}$

$$\begin{pmatrix} \cancel{\gamma} c\tilde{t} \\ \tilde{x} \end{pmatrix} = \gamma \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix} \begin{pmatrix} ct \\ \tilde{x} \end{pmatrix}$$

note: $(0,0) \to (0,0)$

$$\begin{pmatrix} c\tilde{t}_2 \\ \tilde{x}_2 \end{pmatrix} = \gamma \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix} \begin{pmatrix} L\frac{c}{v} \\ -L \end{pmatrix} = \gamma \begin{pmatrix} L\frac{c}{v} - L\frac{v}{c} \\ L - L \end{pmatrix}$$

$$= \gamma \begin{pmatrix} L(+\frac{1}{\beta} - \beta) \\ 0 \end{pmatrix} = \gamma \begin{pmatrix} L/\beta(1-\beta^2) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} c\tilde{t}_2 \\ \tilde{x}_2 \end{pmatrix} = \gamma \begin{pmatrix} \frac{L}{\beta}\underbrace{(1-\beta^2)}_{=1/\gamma^2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{L}{\beta\gamma} \\ 0 \end{pmatrix}$$

$$\tilde{L} = v(\tilde{t}_2 - \tilde{t}_1) = \beta\left(\frac{L}{\beta\gamma} - 0\right) = \frac{L}{\gamma}$$

$\gamma > 1 \Rightarrow \tilde{L} < L \Rightarrow$ Length contraction

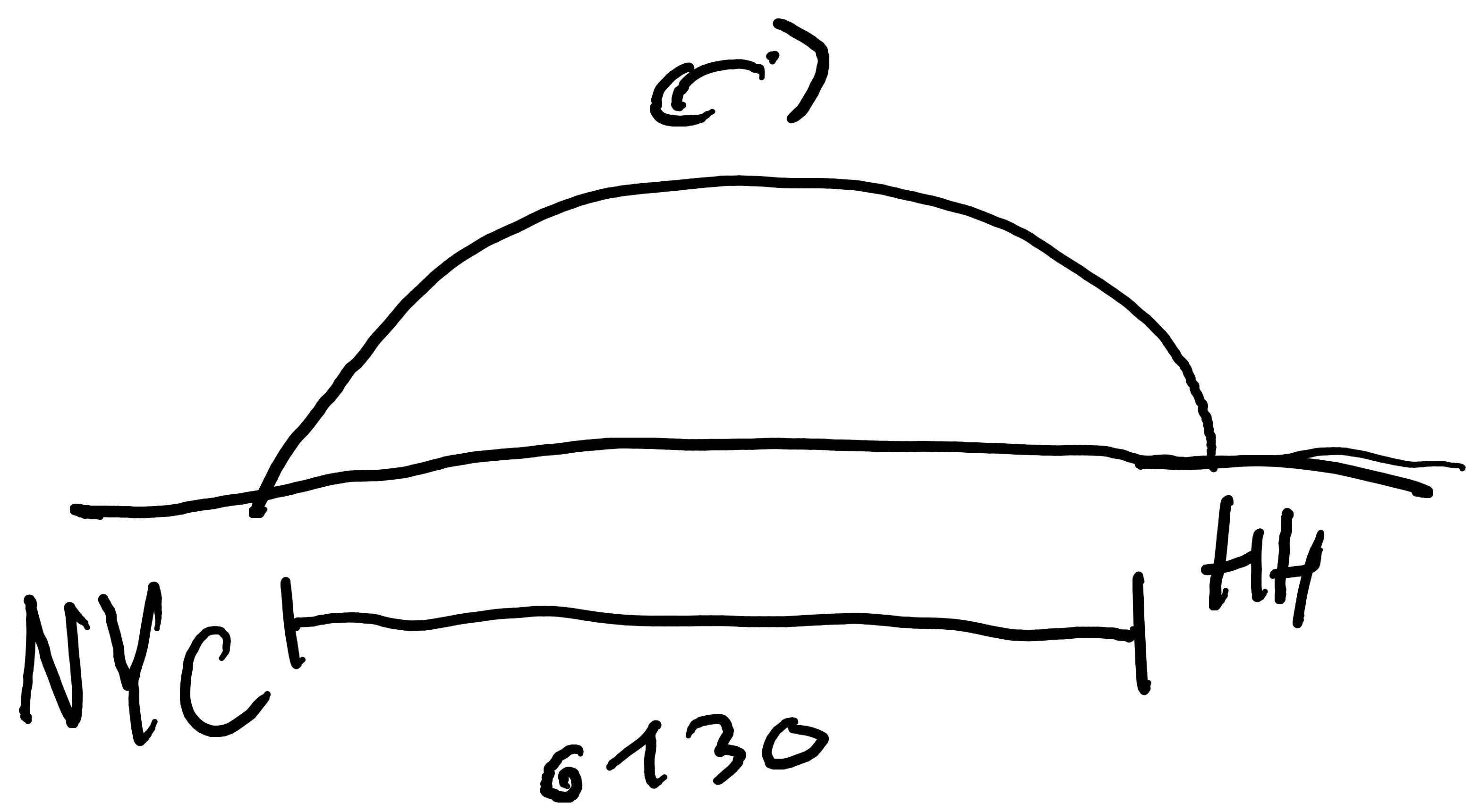
fro time $\Delta t = t_2 - t_1 = \frac{L}{v} \Rightarrow \tilde{\Delta t} = \frac{L}{v\gamma} = \frac{\Delta t}{\gamma}$

$\tilde{\Delta t} < \Delta t$

$\tilde{O}$ sees the car pass quicker

$\Rightarrow$ time flows slower at rest $\Rightarrow$ time dilation

Problem 2



$t_{flight} = 7h\,30 = 27000\,s$

$d = 6130\,km$

$$v = \frac{t_{flight}}{d} \simeq 227{,}037\ m/s \sim 9 \cdot 10^{-7} c$$

note $$\frac{\Delta x}{\Delta t} = \frac{\Delta\tilde{x}/\gamma}{\Delta\tilde{t}/\gamma} = \frac{\Delta\tilde{x}}{\Delta\tilde{t}} \checkmark$$

$\mathcal{O}$: Earth frame

$\tilde{\mathcal{O}}$: plane frame

$$\text{diff} = \Delta t - \Delta\tilde{t} = \Delta t\left(1 - \frac{1}{\gamma}\right)$$

$$= 2t_{flight}\left(1 - \frac{1}{\gamma}\right)$$

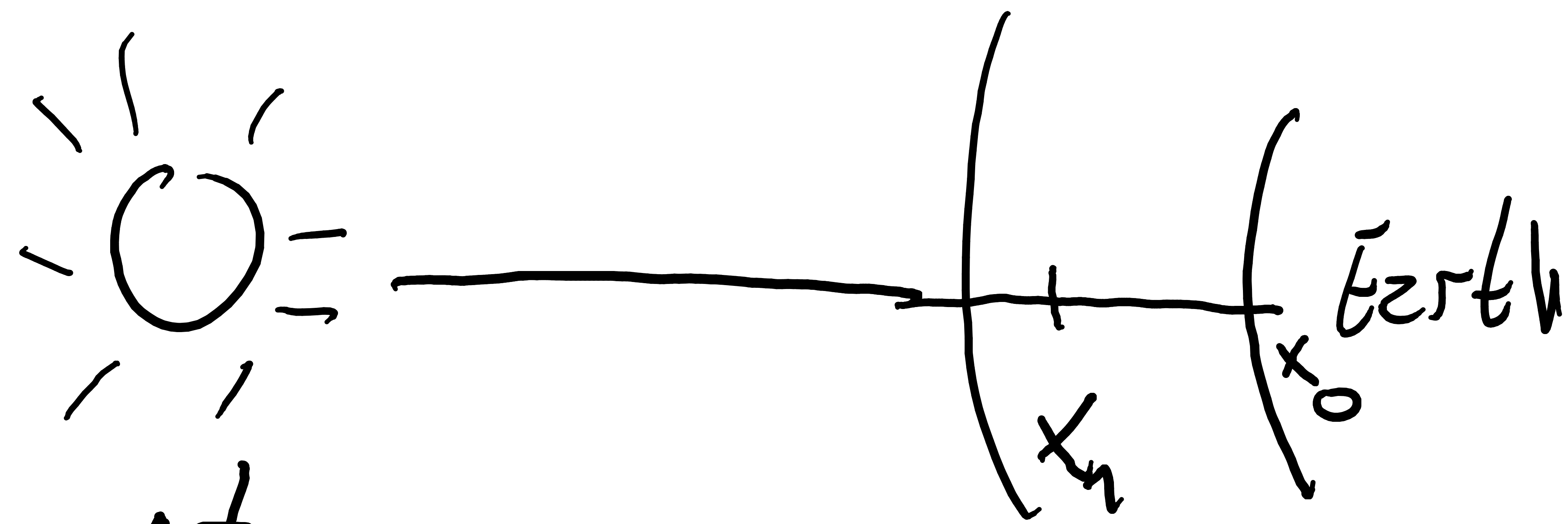
$$\frac{1}{\gamma} = (1-\beta^2)^{1/2} \simeq 1 - \frac{1}{2}\beta^2 \Rightarrow \text{diff} = t_{flight}\,\beta^2$$

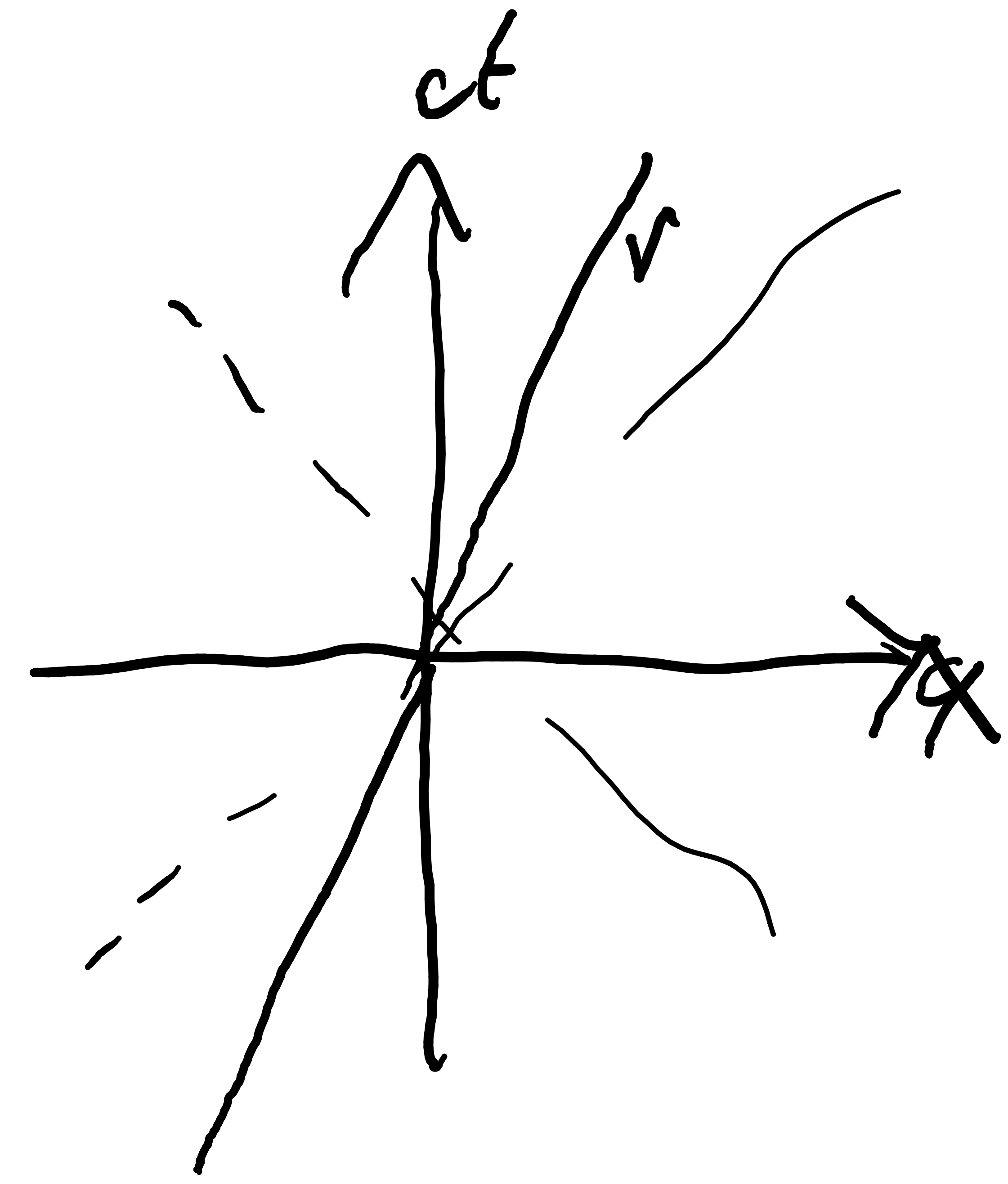
$$\sim 15\ ns$$

Problem 3

$m_\mu \approx 200\, m_e$

but decays





rest

$\Delta \tau = \frac{\Delta t}{\gamma}$

$x_1 - x_0 = 10\text{ km}$

$r_0 = 200$ muons$/sm^2$

$r_1 = 336$ muons$/sm^2$

at rest

exponential decay: $N(t) = N_0 e^{-\lambda \tau}$

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda \tau_{1/2}} \Rightarrow \ln\left(\tfrac{1}{2}\right) = -\lambda t_{1/2}$$

$$\lambda = \ln 2 / t_{1/2}$$

$$\frac{r_0}{r_1} = e^{-\tau \lambda} \Rightarrow \tau = \frac{1}{\lambda} \ln \frac{r_1}{r_2}$$

$$v = \frac{h}{\Delta t} \overset{\tau = \gamma \Delta t}{=} \frac{h}{\gamma \tau} \qquad \frac{h}{\tau} = \gamma v$$

$$\Rightarrow \frac{v}{c} \sim 0.99$$