ZMP: Separation of Variables

1 Literature

Reviews: [1].

Classical integrability, Hamiltonian systems, Levi-Civita theorem: [2–4]. Also selected chapters of [5]

Beginnings of quantum integrability by Sklyanin: [6,7].

Paradigmatic example applications of SoV:

- Sklyanin works on the Gaudin model (also precursory of geometric Langlands) [1,6,8].
- Gaudin–Pasqier: Periodic Toda chain [9].
- Derkachov–Korchemsky–Manashov: $\mathrm{SL}(2,\mathbb{R})$ XXX model [10].

The latter two use a Feynman-diagrammatic calculus for Q-operators and SoV kernel based on the star-triangle relations, which is central for many applications.

Higher rank: [11–14].

Quantum lattice models: [15].

More mathematics-related literature: Hitchin systems [16], Hilbert schemes [17]. Relation to Quantum Spectral Curves and Geometric Langlands: [18–20].

Optional algebraic component: How to obtain Q-operators from represenation theory? From the physics side: [21]. Relevant mathematical works: [22,23].

Optional component: Relation to $\mathcal{N} = 2$, $d = 4$ SUSY QFT: Recent works by Jeong, Nekrasov, Lee [24,25], also [26].

QSC in $\mathcal{N} = 4$ SYM: [27] and in the fishnet limit: [28]. Review: [29].

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