Introduction to Integrability: Syllabus

1 Schedule of Lectures and Exercise Classes

L: Lectures, **E:** Exercises

April	
L1	17.4.2019
L2	23.4.2019
L3	24.4.2019

May	
E1	07.5.2019
L4	08.5.2019
L5	15.5.2019
E2	21.5.2019
L6	22.5.2019
L7	29.5.2019

June	
E3	04.6.2019
L8	05.6.2019
L9	18.6.2019
L10	19.6.2019
E4	25.6.2019
L11	26.6.2019

July	
L12	02.7.2019
L13	03.7.2019
E5	09.7.2019
L14	10.7.2019

2 Syllabus

0. **Prelude:** What is integrablity? Some history. Integrability versus solvability. Where does integrability appear? Some integrable models.

1. **Integrable Classical Mechanics:** Hamiltonian mechanics. Integrals of motion. Liouville integrability (phase space structure, quadrature, compact level sets). Comparison of classes (chaos, integrability, super-integrability).

2. **Structures of Classical Integrability:** Lax pair. Classical R-matrix. Spectral parameter. Spectral curve, dynamical divisor, reconstruction.

3. **Integrable Field Theory:** Classical field theory. Korteveg–de Vries equation: Solitons, factorized scattering. Integrability structures. Lax monodromy and Lax scattering. Inverse scattering method (auxiliary linear problem, scattering data, inverse scattering transformation, GLM equation). Spectral curves (Heisenberg magnet, Riemann surface of monodromy matrix, quasi-momentum, periods and moduli, finite-gap construction).

4. **Integrable Spin Chains:** Heisenberg spin chain (boundary conditions, symmetry). Spectrum of the closed chain (direct diagonalization, Bethe equations). Coordinate Bethe ansatz (magnon states, scattering factor, factorized scattering, solution of the infinite chain). Bethe equations (periodicity for closed chains, rapidities). Heisenberg XXX model with higher spin. Bethe ansatz for higher-rank algebras (scattering matrix, nested Bethe ansatz).

5. **Long Spin Chains:** Magnon spectrum (finite M at large L). Ferromagnetic continuum (quasi-momentum, spectral curve).

6. **Quantum Integrability:** Quantum integrability in $(1+1)$-dimensional relativistic field theory (conserved charges of different Lorentz spins implies conservation of individual momenta, factorized scattering and the quantum

Yang–Baxter equation (qYBE)). R-matrix formalism (action on tensor products, qYBE, monodromy matrix, transfer matrix, conserved local charges). Quantum inverse scattering method (algebraic Bethe ansatz, RTT algebra, analytic Bethe ansatz, Baxter equation). Classification of R-matrices.

7. **AdS/CFT Integrability:** Gauge theory. $\mathcal{N} = 4$ super Yang–Mills theory. AdS/CFT duality. Planar limit. λ–N_c diagram. Weak/strong coupling. Spectrum of scaling dimensions. Appearance of integrability in dilatation operator and string sigma model.