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Structure Constants from Q-Systems and Separation of Variables

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We introduce a novel method to compute structure constants from Q-functions in the scalar sector of planar $\mathcal{N} = 4$ super Yang–Mills (SYM) and related theories. The method derives from operatorial as well as functional separation of variables, and the structure constants are expressed as determinants of matrices whose entries are integrals over products of Q-functions. In this framework, each operator is twisted by an external angle, mirroring the cusped Maldacena–Wilson loop. The structure constants of local single-trace operators in $\mathcal{N} = 4$ SYM are recovered in the untwisting limit, where we obtain a one-to-one correspondence between our key building blocks and those of the Hexagon formalism. Retaining appropriate twists, our structure constants also perfectly match those of the orbifold points of $\mathcal{N} = 4$ SYM. Our results thus far are valid at leading order in the weak-coupling expansion, but their formulation in terms of Q-functions provides a natural starting point for including loop corrections. Many of the methods we develop in this work apply more generally to the computation of correlation functions in integrable models.

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The Setting

Goal: Solve interacting 4d QFT analytically

Most symmetric setting: $\mathcal{N} = 4$ SYM

Integrability: Quantum Spectral Curve captures spectrum of local (and non-local) operators at finite coupling [1, 2]

Open problem: Extend integrability approach to higher-point correlators

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State of the Art: Expansion in string worldsheet excitations via Hexagon form factors [3]

- Very powerful for large-charge operators
- Applies to n-point correlators of arbitrary operators [4, 5]
- Also applies at subleading orders in $1/N_c$ [6]
- Small/finite charges:
  Re-summations cannot be performed in practice
  Singularities (related to wrapping) are difficult to handle [7]

Program

Want: Correlators expressed in *Q-functions* (associated to operators by QSC)

Natural framework: Separation of Variables (SoV) [8]

- Wave functions factorize into-single particle Q-functions
- Correlators simplify drastically [9, 10]

SoV approach for structure constants (three-point functions):
So far limited to bosonic rank-one sectors $\mathfrak{su}(2)$ and $\mathfrak{sl}(2)$ [11, 12, 13]

At finite coupling: All $\mathfrak{psu}(2,2|4)$ states must be included.
Requires:

- Generalization to higher rank
- Inclusion of fermions/supersymmetry

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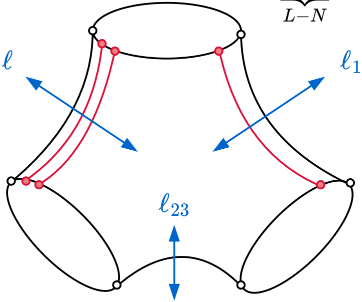
- Generalization to higher rank $\leftarrow$ **This paper!**
- Inclusion of fermions/supersymmetry

Object

$$\mathcal{O}_1 = \widetilde{\mathrm{Tr}}\,(\,\Phi_1 \ldots \Phi_N \underbrace{Z \ldots Z}_{L-N}) + \text{perms}$$



$$\mathcal{O}_2 = \mathrm{Tr}(\overbrace{\mathcal{W}_{\omega,\kappa} \ldots \mathcal{W}_{\omega,\kappa}}^{L_2})$$

$$\mathcal{O}_3 = \mathrm{Tr}(\overbrace{\mathcal{W}_{\kappa} \ldots \mathcal{W}_{\kappa}}^{L_3})$$

$$\mathcal{O}_1 : |\Psi\rangle\,, \qquad \mathcal{O}_2 : \langle \mathcal{W}^{L_2}_{\omega,\kappa}|\,, \qquad \mathcal{O}_3 : \langle \mathcal{W}^{L_3}_{\omega}|\,,$$

$$C_\ell = \frac{\langle \mathcal{W}^{\ell}_{\omega,\kappa} \otimes \mathcal{W}^{L-\ell}_{\omega} | \Psi \rangle}{||\Psi||}\,.$$

Result

$$C_\ell = \mathcal{N}_\ell \times \frac{\mathbb{W}_\omega \, \mathbb{A}_{\ell,\kappa}}{\sqrt{\mathbb{B}}}$$

Correlator of arbitrary states (1 non-BPS, 2 BPS) in the $\mathfrak{su}(4)$ sector

$$\begin{aligned}
\mathbb{B} &= \det_{(\alpha,j,k),(\beta,a,b)} \langle \mathcal{Q}_j^{[a]} u^{\beta-1} \mathcal{Q}^{k[b]} \rangle_{L,\alpha} \,, \\
\mathbb{W}_\omega &= \det_{\alpha,\beta} \langle \omega^{-iu} u^{\beta-1} \mathcal{Q}_{12} \rangle_{L,\alpha} \,, \\
\mathbb{A}_{\ell,\kappa} &= \det_{(\gamma,j),(\delta,a)} \langle \kappa^{-iu} u^{\delta-1} \mathcal{Q}_j^{[a]} \rangle_{\ell,\gamma} \,.
\end{aligned}$$

$$\langle f \rangle_{\ell,\alpha} = \oint \frac{\mathrm{d}u}{(-2\pi)^\alpha} \frac{e^{2\pi u(\alpha-1)}}{(u+i/2)^\ell (u-i/2)^\ell} f(u) \,,$$

Features

Twists: $\mathfrak{su}(4)$ completely broken to $\mathfrak{u}(1)^3 \quad \to \quad$ All degeneracy is lifted

- $\mathcal{N} = 4$ SYM recovered when twists $z_j \to 0$
- $\mathbb{Z}_N$ orbifold structure constants: $z_1 = \mathrm{e}^{2\pi i n_1/N}, z_2 = \mathrm{e}^{2\pi i n_2/N}$ [14]

Left-Right Symmetry: $C_\ell = 0$ unless $(t_+ - t_-)|\Psi\rangle = 0 \quad \to$ manifest!

Derivation:

- Use Functional Separation of Variables (FSoV) [15, 16, 17]
  to build inner products $\langle \Psi_A | \Psi_B \rangle \to \|\Psi\|^2 \propto \det\langle \mathcal{Q}_j^{[a]} u^{\beta-1} \mathcal{Q}^{k[b]} \rangle_{L,\alpha} = \mathbb{B}$
- Similar: $\langle \mathcal{W}_\omega^L | \Psi \rangle \propto \det\langle \omega^{-iu} u^{\beta-1} \mathcal{Q}_{12} \rangle_{L,\alpha} = \mathbb{W}_\omega$
- *Localization:* $\langle \mathcal{W}_{\omega,\kappa}^\ell \otimes \mathcal{W}_\omega^{L-\ell} | = \langle W_\omega^L | P_\kappa(t)$, with $P_\kappa(t)$ polynomial in t's

Match with Hexagons: At the level of building blocks: [3, 18]
Gaudin norms: $\langle \mathbf{u} | \mathbf{u} \rangle \propto \mathbb{B}$ and $\langle \mathbf{v} | \mathbf{w} \rangle \propto \mathbb{W}_{\omega=1}$
Hexagon amplitude: $\mathcal{A}_\ell \propto \mathbb{A}_{\ell,\kappa}$

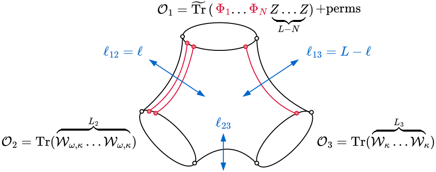
Outlook

This work: General correlator in higher rank $\mathfrak{su}(4)$ sector (one excited state)

Directions:

- Include fermions/supersymmetry to get full $\mathfrak{psu}(2,2|4)$
- Unify with earlier hexagon/Q-function connections at loops [19, 20]
- Promote to loops: Q-functions from QSC
- Analytic continuation in spin, light-ray operators [21]
- Twists/angles z_j, ω, κ play the same role as the angles in the cusped Maldacena–Wilson loop [22, 10]
  $\rightarrow$ Indeed, our result has the same functional form!

Thank you!



$\mathcal{O}_1 = \widetilde{\mathrm{Tr}}\,(\Phi_1 \dots \Phi_N \underbrace{Z \dots Z}_{L-N}) + \text{perms}$
$\ell_{12} = \ell$
$\ell_{13} = L - \ell$
ℓ_{23}
$\mathcal{O}_2 = \mathrm{Tr}(\overbrace{\mathcal{W}_{\omega,\kappa} \dots \mathcal{W}_{\omega,\kappa}}^{L_2})$
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