# Integrability and the spectrum of two-dimensional fishnet CFT

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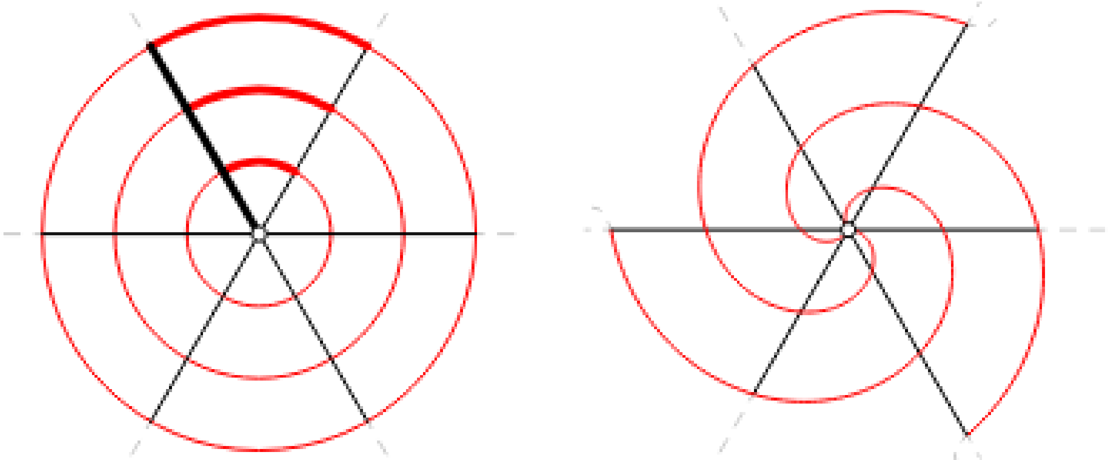

4D biscalar fishnet theory

$$\mathcal{L}_\phi = \frac{N_c}{2}\mathrm{Tr}\left(\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + 2\xi^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2\right)$$

SU(N) matrix-valued fields

Non-hermitian

- complex energies

coupling constant

- No gauge fields
- No SUSY
- Non-unitary

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So why care?

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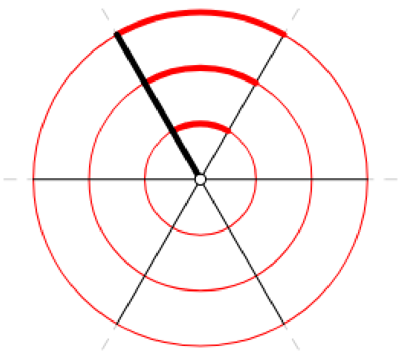
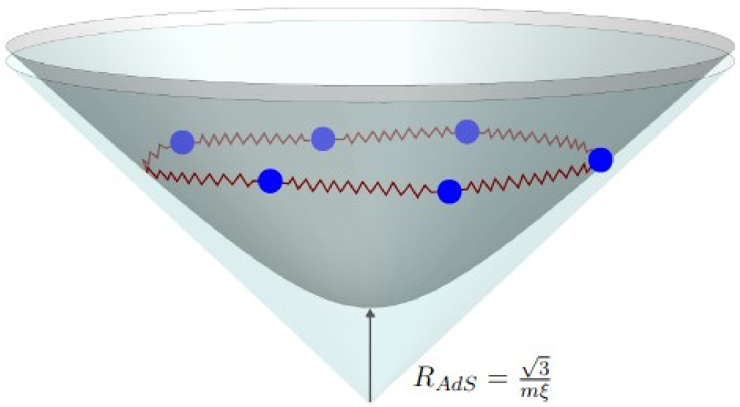
## So why care?

- Conformal
- Feynman diagrams immensely simple
  - one diagram each loop order
- Double scaling limit of planar N=4 SYM
  → inherits integrability
  + exact spectrum from Quantum Spectral Curve

wheel graphs



Exact holographic dual

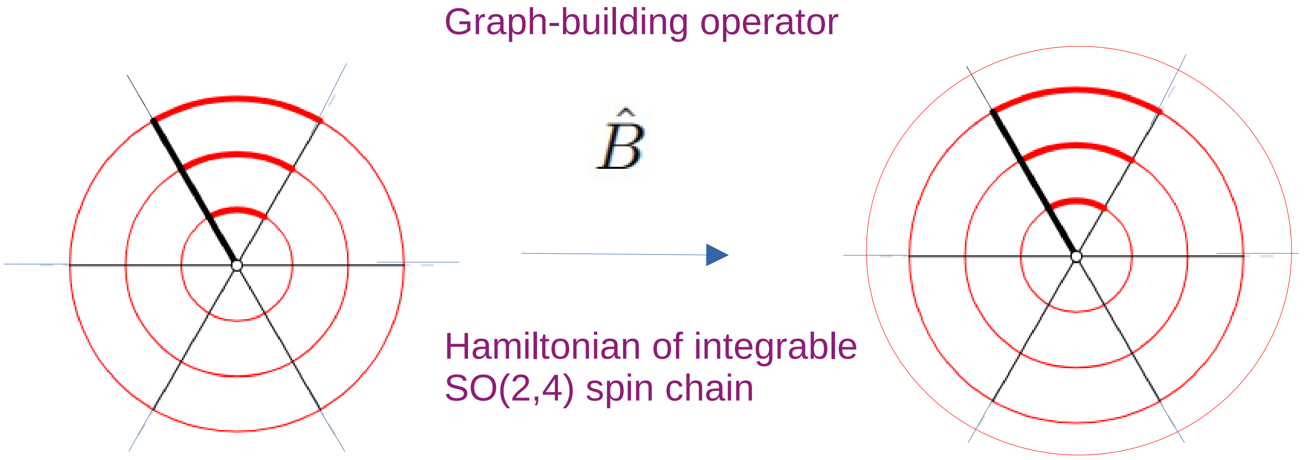


Integrability of fishnet theory

at level of Feynman diagrams
(point-split 2-pt function)

Exact point-split correlators

$$\hat{B}\Psi(x) = \Psi(x)$$



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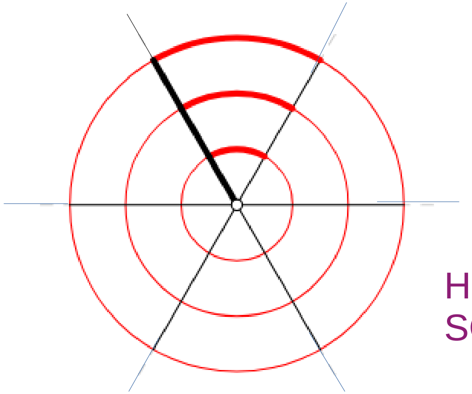
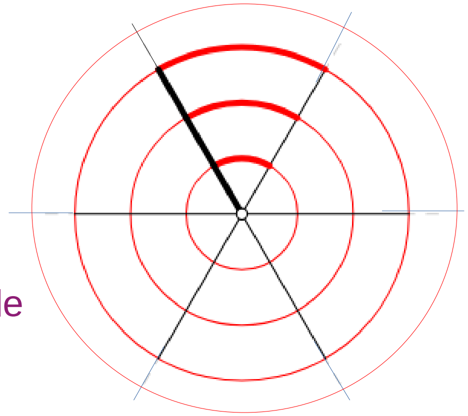


Graph-building operator

$$\hat{B}$$

Hamiltonian of integrable
SO(2,4) spin chain



Fishnet theory admits extension to any D

$$\mathcal{L} = N\,\mathrm{tr}\left[\phi_1^\dagger(-\partial_\mu\partial^\mu)^{\frac{D}{4}}\phi_1 + \phi_2^\dagger(-\partial_\mu\partial^\mu)^{\frac{D}{4}}\phi_2 + (4\pi)^{\frac{D}{2}}\,\xi^2\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\right]$$

- generally non-local + non-unitary
- Conformal + integrable (same Feynman diagram structure) – SO(1,D+1) spin chain structure
- No “parent” theory like N=4 SYM to import exact techniques like Quantum Spectral Curve from

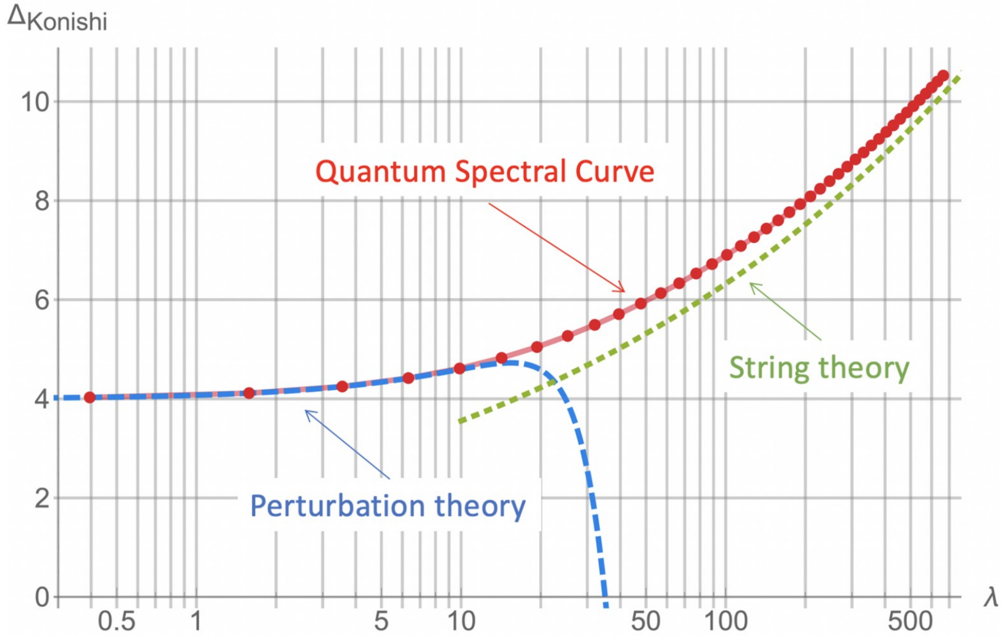
# Main Motivation

Quantum Spectral Curve

Exact solution of spectral problem of planar N=4 SYM

[Gromov, Kazakov, Leurent, Volin]

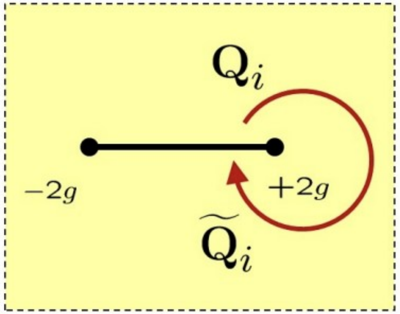
Interpolates between weakly-coupled gauge theory and string theory at strong coupling




## What is it?

- collection of functions Q(u) of a single complex variable u
- relations fixed by symmetry – psu(2,2|4)
- model-dependent analytic properties

Branch points related to ‘t Hooft coupling




$$g = \frac{\sqrt{\lambda}}{4\pi}$$

Q-functions encode conformal dimension

$$Q(u) \sim u^{\Delta}$$

## A Quantum Spectral Curve based approach to structure constants?

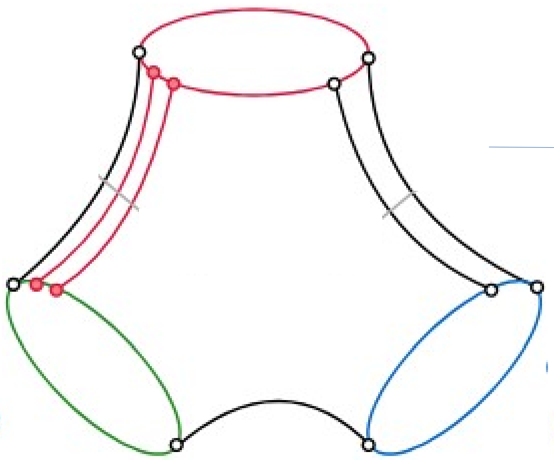
- any operator, any size (hexagons need large operators)
- strong coupling
- should naturally incorporate qualitative features like analyticity in spin (spin discrete in hexagons)

On-going program

[Basso, Cavaglià, Bargheer, Bercini, Caron-Huot, Coronado, Ekhammar, Georgoudis, Giombi, Gromov, Homrich, Jiang, Julius, Komatsu, Kostov, Lai, Levkovich-Maslyuk, Preti, PR, Serban, Sokolova, Sueiro, Trinh, Vieira, Volin, Zahraee ...]

Operators ⟷ Q-functions by QSC

$\mathcal{O}_1 \sim Q_1$



$\mathcal{O}_2 \sim Q_2$ $\mathcal{O}_3 \sim Q_3$

Some functional which eats Q-functions

$$\longrightarrow \mathcal{F}(Q_1, Q_2, Q_3) \longrightarrow C_{123}$$

Separation of Variables

Fishnet CFT is the ideal testing ground to develop this formalism

- 2D especially simple – SL(2) spin chain

- can important Separation of Variables from earlier work [Derkachov, Korchemsky, Manashov]

- No Quantum Spectral Curve – need to develop it!

- Spin chain is so simple we have an exact operatorial construction
  of the model at finite-coupling (don’t have this in N=4 SYM)

- We can develop Quantum Spectral Curve from first principals

# Quantum Spectral Curve for 2D Fishnet

$$\dot{h} - h = S \quad \dot{h} + h = \Delta$$

Conformal symmetry $so(1,3) = sl(2) \oplus sl(2)$

## Undotted sector

Baxter Equation

$$(u + \tfrac{i}{4})^J q(u+i) - t(u)q(u) + (u - \tfrac{i}{4})^J q(u-i) = 0$$

Linearly independent solutions, large-u asymptotics

$$q_1(u) \sim u^h + \ldots, \; q_2(u) \sim u^{1-h} + \ldots$$

Freedom in pole structure $q_j^{\downarrow} \quad q_j^{\uparrow}$

Transition matrix $q_j^{\uparrow} = \Omega_j{}^k(u) q_k^{\downarrow}$

## Dotted sector

$$(u + \tfrac{i}{4})^J \dot{q}(u+i) - \dot{t}(u)\dot{q}(u) + (u - \tfrac{i}{4})^J \dot{q}(u-i) = 0$$

Independent solutions

$$\dot{q}_1(u) \sim u^{\dot{h}} + \ldots, \; \dot{q}_2(u) \sim u^{1-\dot{h}} + \ldots$$

$$\dot{q}_j^{\downarrow} \quad \dot{q}_j^{\uparrow}$$

$$\dot{q}_j^{\uparrow} = \dot{\Omega}_j{}^k(u) \dot{q}_k^{\downarrow}$$

Gluing $\Gamma^{ik}\Omega_k{}^j = \Gamma^{jk}\dot{\Omega}_k{}^i, \quad \Gamma = \begin{pmatrix} 1 & 0 \\ 0 & c \end{pmatrix}$

Coupling

$$Q_+(u) = \Gamma^{ij} q_i^{\downarrow}(u) \dot{q}_j^{\uparrow}(u)$$

$$\xi^{2J} = \frac{Q_+(\frac{3i}{4})}{Q_+(\frac{i}{4})}$$

# Exact Results

$$\Delta = \frac{J}{2} + S + \mathcal{O}(\xi^2)$$

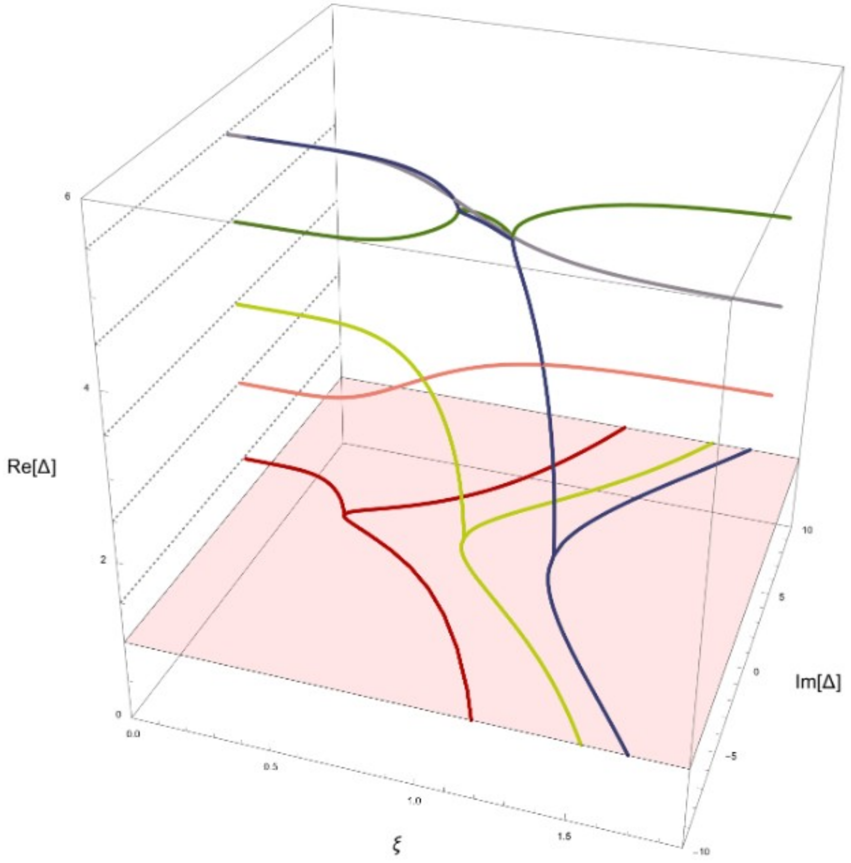



**Figure 3**: We plot the low-lying spectrum for 2D bi-scalar fishnet theory from weak to strong coupling for operators with $J = 3$, $M = 0$, and $S = 0$. As expected, we find collisions among states and complex energy levels.

From high-precision numerics we computed analytic results

Leading wrapping correction

$$\Delta\Big|_{J=3,M=0,S=0} = \frac{3}{2} - \frac{2}{3}\frac{\pi^4}{\Gamma(\frac{3}{4})^8}\xi^6 + \dots$$

$$\gamma_{2J}\Big|_{J=3,M=0,\Delta_0=\frac{3}{2}+|S|} = -\frac{2\sqrt{2}\,\pi^{5/2}}{\Gamma\left(\frac{3}{4}\right)^6}\frac{\Gamma\left(\frac{|S|}{2}+\frac{1}{4}\right)\Gamma\left(\frac{|S|}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{|S|}{2}+\frac{3}{4}\right)\Gamma\left(\frac{|S|}{2}+1\right)}\,.$$

People apparently like these numbers...

# Summary

Formulated Quantum Spectral Curve for 2D Fishnet CFT

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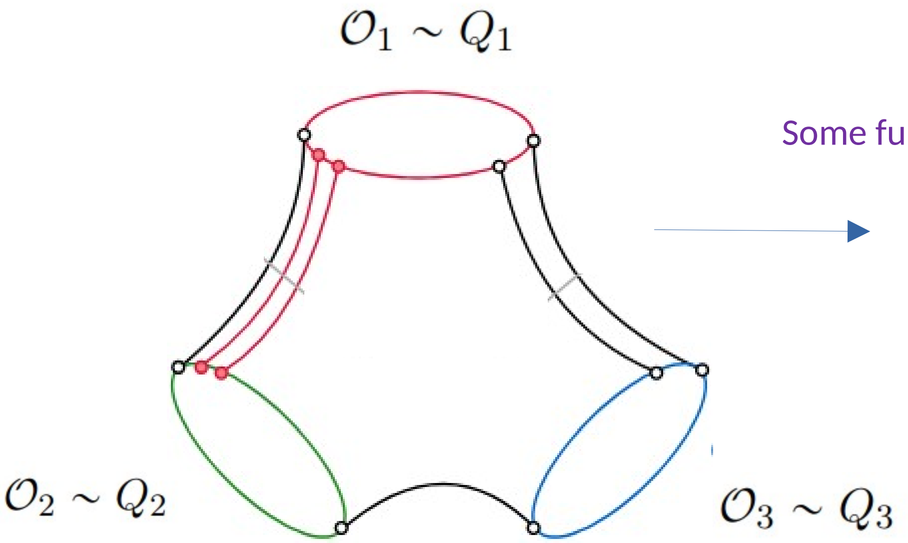
Studied exact spectum

Next step...




Some functional which eats Q-functions

$\rightarrow \mathcal{F}(Q_1, Q_2, Q_3) \rightarrow C_{123}$

In progress right now!