$2 + 2 = 4$

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DESY Theory J.C.

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by Guido Bonori

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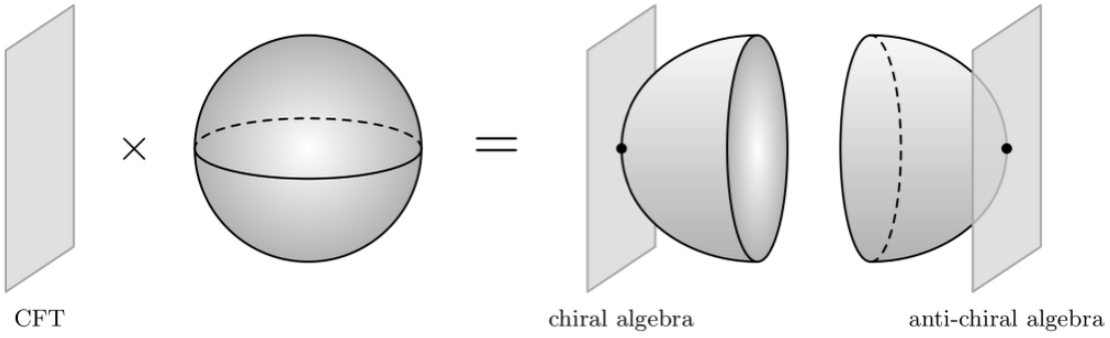
Motivations

- SCFT/VOA correspondence

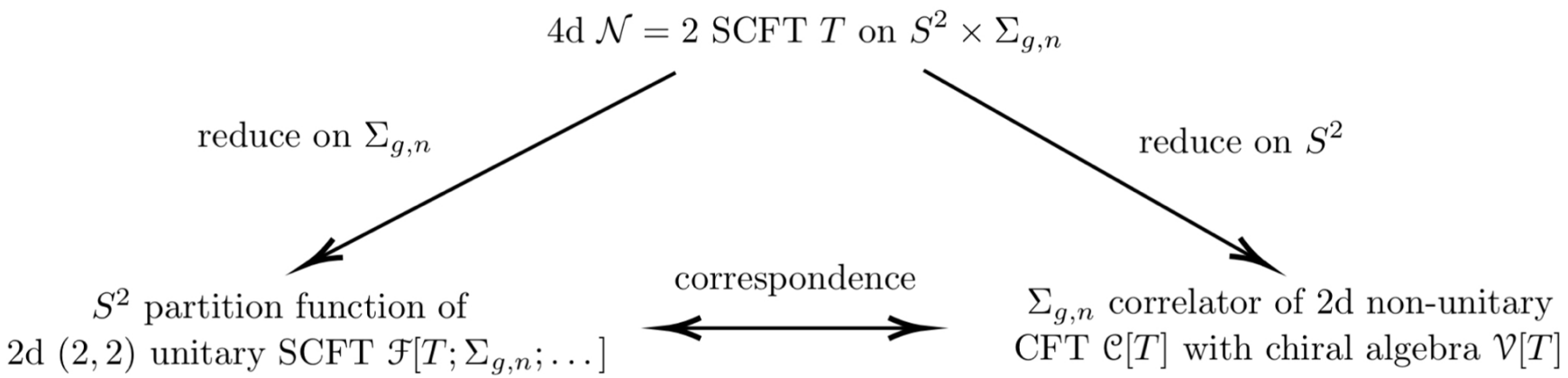
4d $\mathcal{N}=2$ SCFT $\mathcal{T} \longmapsto \mathbb{V}[\mathcal{T}] \approx$ Schur sector of $\mathcal{T}$

- Can we promote $\mathbb{V}[\mathcal{T}]$ to full-fledged 2d CFT ?

$\mathcal{T} \longmapsto \mathcal{C}[\mathcal{T}]$ s.t. Chiral Algebra $(\mathcal{C}[\mathcal{T}]) \simeq \mathbb{V}[\mathcal{T}]$

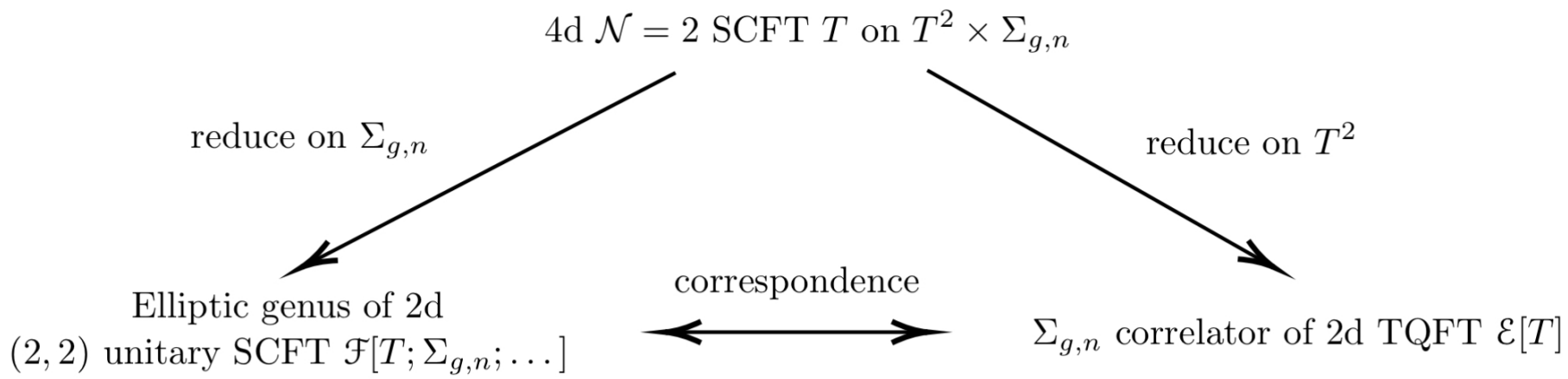


The Correspondence



4d	2d unitary	2d non-unitary
4d $\mathcal{N}=2$ SCFT T	2d $(2,2)$ class $\mathcal{F}$ theory	non-unitary CFT $\mathcal{C}[T]$ with chiral algebra $\mathcal{V}[T]$
$S^2_B\times\Sigma_{g,n}$ partition function	S^2_B partition function	n-point genus g correlator
$\mathbb{R}^2_\epsilon\times\Sigma_{g,n}$ partition function	vortex partition function	chiral conformal block of $\mathcal{V}[T]$
complex structure deformations of $\Sigma_{g,n}$	chiral exactly marginal parameters	complex structure deformations of spacetime
conformal manifold	twisted chiral exactly marginal parameters	—

The Correspondence with the elliptic genus



Building a Background

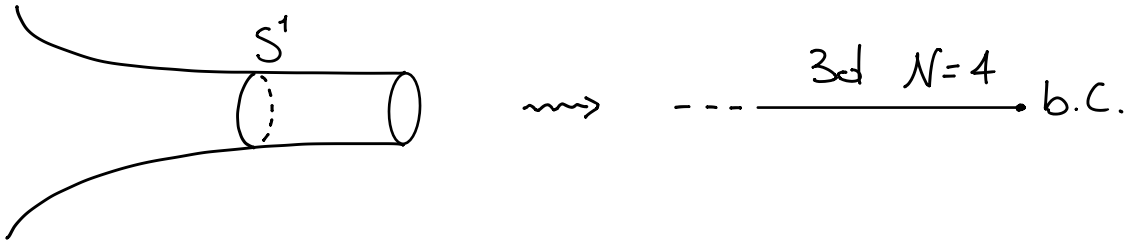
- Compactifications of 4d $\mathcal{N}=2$ SCFTs on $S^2 \times \Sigma$
  - ↳ Construct SUGRA backgrounds that include $SU(2)_R$ twist on Σ and preserve 4 supercharges

- Important choiche to make
  - A-type background $\rightsquigarrow$ $SU(2,1)_A \longrightarrow \mathcal{Z}_{S^2_A}$ { depends on conf. mfd. of T; not " " complex str. mod. on Σ
  - B-type background $\rightsquigarrow$ $SU(2,1)_B \longrightarrow \mathcal{Z}_{S^2_B}$ { not depends on conf. mfd. of T; " " complex str. mod. on Σ

- $\mathcal{Z}$ on $S^2 \times \Sigma \in \mathbb{C}$ $\rightsquigarrow$ CFT $\mathcal{C}$ non unitary!

Unitary Side

- 2d $\mathcal{N}=(2,2)$ theory $\mathcal{F}[T;\Sigma_{g,m}]$
- Punctures $\leftrightsquigarrow$ B.C. of T reduced on a circle



- Anomaly polinomial and central charges

$$I_4 = \chi(2a-c)[C_1^2(+) - C_1^2(-)] \ , \ \ c_+ = c_- = 2\chi(2a-c) + \sum_i \Delta c_i n_i$$

- 2d Operators from 4d BPS spectrum $\rightsquigarrow$ predictions for elliptic genus

Non Unitary Side

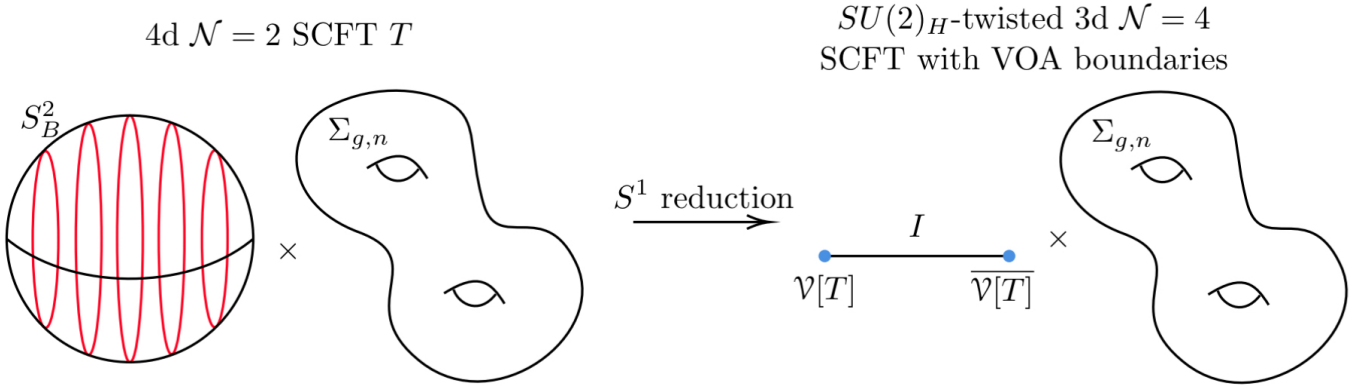
- Reduction of T on S^2

  $\Rightarrow$ CFT $\mathcal{C}[T]$ on Σ by gluing $\mathcal{V}[T], \overline{\mathcal{V}}[T]$

- Punctures $\longrightarrow$ insertion of primary operators

- Construction from gluing hemispheres equivalent to S^2 reduction

  $\rightsquigarrow$ Hemisphere (vortex) part. funct. $\longleftrightarrow$ Conf. blocks



An Example : $T = (A_1, A_2)$

- $T = (A_1, A_2)$ Argyres-Douglas theory
  - no Higgs branch
  - 1 Coulomb branch operator, $\Delta = \frac{6}{5}$
- $C[(A_1, A_2)] = \mathcal{M}(2,5)$ known
- Proposal for concrete lagrangian description of $\mathcal{F}[(A_1, A_2), \Sigma_{0,m}]$

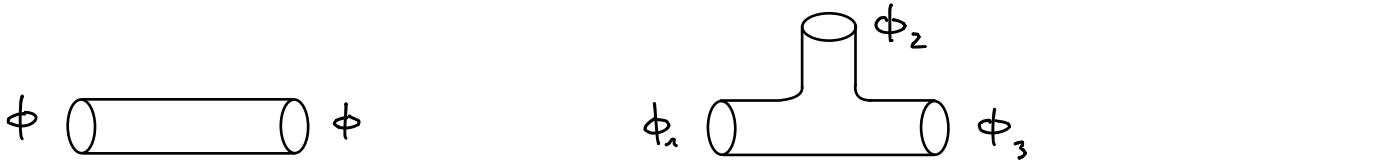
  $\mathcal{F}[(A_1, A_2), \Sigma_{0,4}]$ = GLSM w/ six twisted chirals + twisted U(1) gauge field

  ↳ test with : - anomalies
  - elliptic genus
- punctures $\longleftrightarrow$ operators in $\mathcal{M}(2,5)$

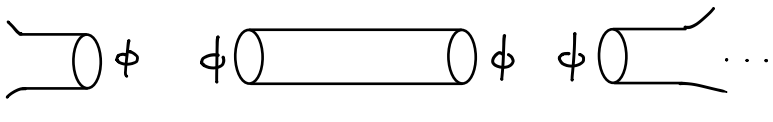
Elliptic Genus and Glouing

- One might expect a glouing prescription similar to Class S
  - ↳ Clear from elliptic genus

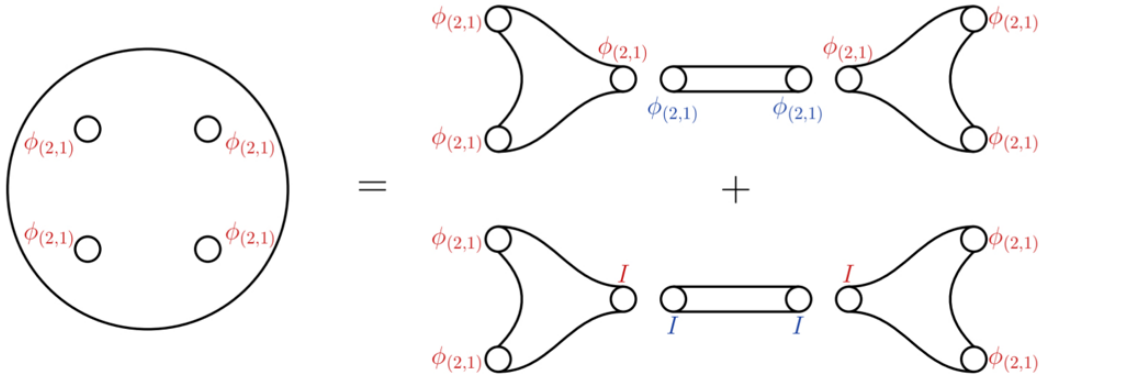
- Elliptic genus of $\mathcal{F}[T,\Sigma]$ can be obtained from building blocks



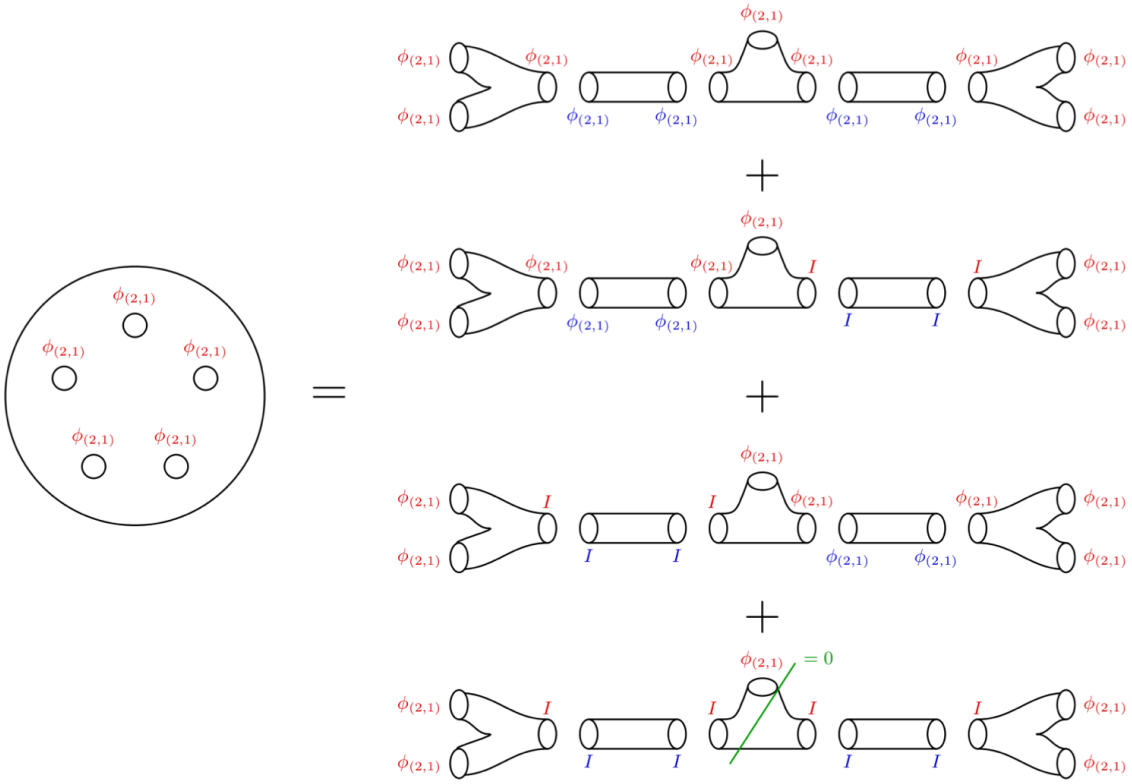
propagator of $\mathcal{E}[T]$ structure constants of $\mathcal{E}[T]$

- For (A_1, A_2) C_{III}, $C_{\phi_{(2,1)}II}$, $C_{\phi_{(2,1)}\phi_{(2,1)}I}$, $C_{\phi_{(2,1)}\phi_{(2,1)}\phi_{(2,1)}}$ T_{II}, $T_{\phi_{(2,1)}\phi_{(2,1)}}$
- Glouing $= \sum_{\phi}$ 

Nice Pictures

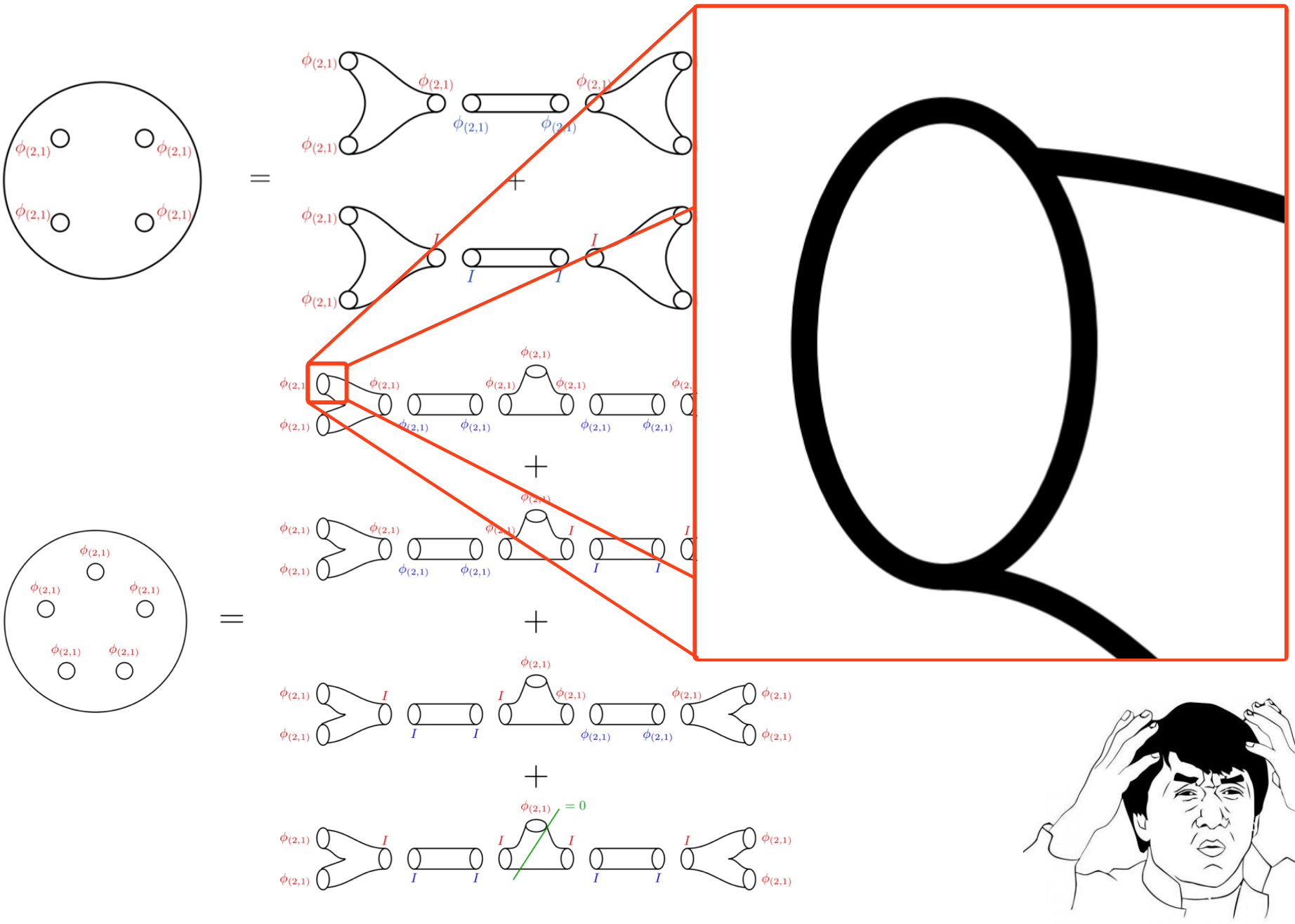


$$\mathcal{F}\left[(A_1, A_2), \Sigma_{0,4}\right]$$



$$\mathcal{F}\left[(A_1, A_2), \Sigma_{0,5}\right]$$

Nice Pictures (?)



Outlook

- Gluing picture in $\mathcal{F}[T, \Sigma]$ setup ?
- Identify $\mathcal{E}[T]$
- A-type correspondence from $\mathcal{Z}_{S^2_B}$
- More VOAs
- $6 = 4 + 2 = 2 + 2 + 2$

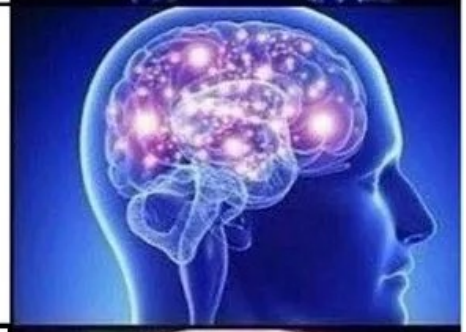
  $\Sigma_1 \times \Sigma_2 \times S^2$
- Interplay with holography

The End

Thank you

6=4+2

4=2+2



6=2+2+2

11=5+2+2+2

∞=1+1+1+ …