Do null defects dream of conformal symmetry?

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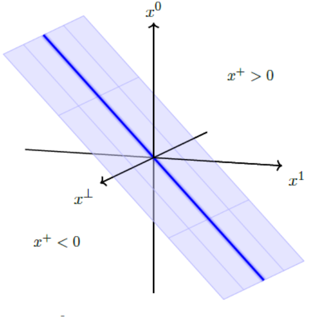
Abstract: We initiate the study of null line defects in Lorentzian conformal field theories in various dimensions. We show that null lines geometrically preserve a larger set of conformal isometries than their timelike and spacelike counterparts, explain a connection to non-relativistic systems, and constrain correlation functions using conformal Ward identities. We argue that having conformal symmetry, and especially maximal conformal symmetry, is extremely constraining – nearly trivializing systems. We consider the (3+1)d scalar pinning field and null Wilson line examples in depth, compare their results to ultraboosted limits of timelike and spacelike systems, and argue that shockwave-type solutions are generic. A number of physical consistency conditions compel us to consider defect correlators as distributions on a restricted subspace of Schwartz test functions. Consequently, we provide a resolution to the longstanding problem of ultraboosted limits of gauge potentials in classical electromagnetism. We briefly analyze semi-infinite sources for the scalar in $(4-\epsilon)$-dimensions, consider solutions on the Lorentzian cylinder, and introduce the "perfect null polygon" which emerges for compatibility between Gauss' law and ultraboosted limits.

Motivation

- Defects in QFT: impurities, phase transitions, dualities.
- Traditionally studied in Euclidean, but Lorentzian aspects are important (causality, real-time dynamics).
- Null line defects: intrinsically Lorentzian, correspond to impurities or charges moving at the speed of light.



Goal: systematic study of null defects in Lorentzian CFTs.

Main results

- Symmetry of null lines is larger than timelike or spacelike

$$\mathfrak{n}_d := (\mathfrak{sl}(2,\mathbb{R}) \times \mathbb{R} \times \mathfrak{so}(d-2)) \ltimes \mathfrak{h}_{d-2}.$$

If full $\mathfrak{n}_d$ is preserved, 1pt functions vanish and 2pt functions are characterized by discontinuities along shockwave planes.

- Examples: pinning defect in $d = 3+1$ free scalar theory, null Wilson line in pure Maxwell theory,...

$$F_{+i} = \frac{g}{2\pi} \frac{x_i}{|x_\perp|^2} \delta\left(x^+\right) \, .$$

Correlators must be distributions (general feature).

- In $d = 1+1$, unitary irreps of $\mathfrak{n}_2$ give abstract description of potential defect local operators and Hilbert space.

Symmetry of null lines in CFT

$$\mathfrak{n}_d := (\mathfrak{sl}(2,\mathbb{R}) \times \mathbb{R} \times \mathfrak{so}(d-2)) \ltimes \mathfrak{h}_{d-2}.$$

- Transverse transformations

$$K_-, \quad K_i, \quad M_{-i}, \quad M_{ij}, \quad \bar{J}_0 \equiv -\frac{1}{2}\left(D + \frac{1}{F^2} M_{+-}\right)$$

- Parallel transformations

$$P_-, \quad K_+, \quad J_0 \equiv -\frac{1}{2}\left(D - \frac{1}{F^2} M_{+-}\right)$$

- $\mathfrak{h}_{d-2}$ is the Heisenberg algebra $[K_i, M_{-j}] = i\delta_{ij} K_-$.
- Larger symmetry than e.g. timelike line,

$$\mathfrak{sl}(2,\mathbb{R}) \times \times \mathfrak{so}(d-1)$$

- $\mathfrak{n}_d$ is Schrodinger algebra + $\bar{J}_0$. Time $\sim$ lightcone direction, space $\sim$ transverse directions. Connection to non-relativistic CFTs with $z = 2$?

Why larger symmetry?

- Null directions are fixed points of more conformal transformations. The entire lightcone is invariant under special conformal transformations up to rescaling, not just the origin,

$$y \cdot K : x^{\mu} \mapsto \frac{1}{1 - 2(y \cdot x)} x^{\mu}, \quad x \text{ on lightcone} .$$

- Chiral dilatations survive for null defects because x^+ and x^- scale independently along the lightcone

$$\begin{aligned} J_0 : (x^+, x^-, x_\perp) &\to (x^+, \lambda^2 x^-, \lambda x_\perp) \\ \bar{J}_0 : (x^+, x^-, x_\perp) &\to (\lambda^2 x^+, x^-, \lambda x_\perp) \end{aligned}$$

  Only one of these is preserved for timelike/spacelike lines.

Consequences of symmetry

Depending on the amount of preserved symmetry, they find

- Maximal symmetry $\mathfrak{n}_d$,

$$\langle O_\Delta \rangle = 0\,, \quad \langle \mathcal{O}_{\Delta_1}(x_1)\,\mathcal{O}_{\Delta_2}(x_2)\rangle = \delta_{\Delta_1,\Delta_2}\frac{t\,(s_1, s_2)}{|x_1 - x_2|^{2\Delta_1}}.$$

  $t(s_1 \equiv \text{sign}(x_1^+), s_2 \equiv \text{sign}(x_2^+))$ has a jump at $x^+ = 0$.

- Breaking some symmetry (different cases, also non conformal),

$$\langle O \rangle = \frac{c}{|x^+|^\delta} + \delta\left(x^+\right) h\left(|x_\perp|\right)$$

  Correlators are distributions on a space of test functions.

- Vanishing one-point functions $\neq$ trivial defect in Lorentzian.

Example: null Wilson line in Maxwell theory

$$S = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_\mu J^\mu\right), \quad J^\mu = gF^2 v^\mu \delta(x\cdot\bar{v})\delta^2\left(x^\perp\right),$$

where $v^\mu = (1, -u, 0, 0)$. The limit $u = 1$ represents a charged particle moving at speed of light,

$$u = 1: \quad J_1^- = g\delta\left(x^+\right)\delta^2\left(x^\perp\right).$$

They compute

$$F_{+i} = \frac{gF^2}{2\pi}\frac{x_i}{|x_\perp|^2}\delta\left(x^+\right)$$

- Can be reproduced from ultraboosted limit of Lienard-Wiechart solution for $0 < u < 1$ (subtlety with distributions).
- It does not preserve conformal symmetry (breaks K_+).

Other examples

- Pinning defect in $d = 3 + 1$ free scalar theory

$$S = \frac{1}{2} \int d^4x \, (\partial_\mu \phi)^2 + h \int dx^- \phi \left(0, x^-, 0\right) \, .$$

- Semi-infinite line in $d = 4 - \varepsilon$ interacting scalar

$$S = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{\lambda_*}{4!} \phi^4 \right] + h \int_0^\infty dx^- \phi \left(0, x^-, 0\right)$$

- Defects on the Lorentzian cylinder (e.g. two Wilson lines with opposite charges).

In all cases they analyze symmetries, compute 1pt functions and check that it is consistent with Ward identities.

Open problems

- Defect operators and Hilbert space in $d > 2$.
- Notion of defect OPE: no obvious distance between any two points on null surface.
- Interpretation in (Carrollian) holography.
- Connection to Soft Collinear Effective Theory (preserve the same symmetries?).
- Connection to non-relativistic CFTs.