2504.13091

Perturbed symmetric-product orbifold

First order mixing and puzzles for integrability

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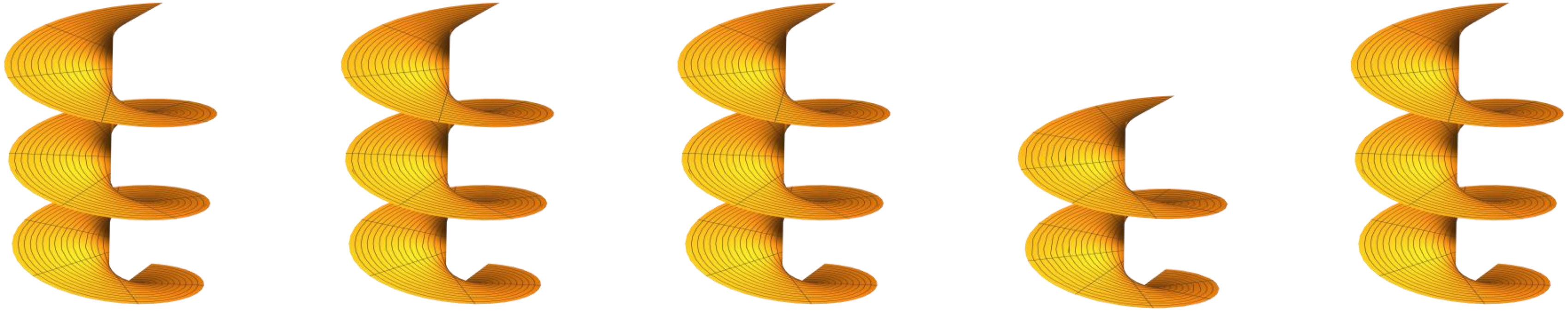


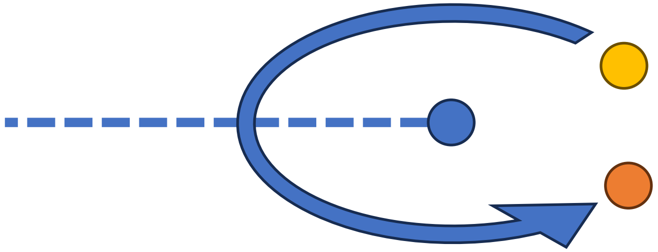
Overview $\mathrm{Sym}_N \mathbb{T}^4$

- Start from free 2d $\mathcal{N}$=(4,4) SUSY theory of 4 bosons and 4 Majorana fermions (central charge c=6)

$$X^{A\dot{A}}(z,\bar{z})\,,\quad \psi^{\alpha A}(z)\,,\quad \tilde{\psi}^{\dot{\alpha}\dot{A}}(\bar{z})$$

- Copy this N times $\phi \to \phi_i$
- Orbifold projection introduces twisted vacua, captured by $\sigma_H\,,\;\; H \in S_N$ e.g.:

$$\phi_1(e^{2\pi i}z)\sigma_{12}(0) = \phi_2(z)\sigma_{12}(0)$$

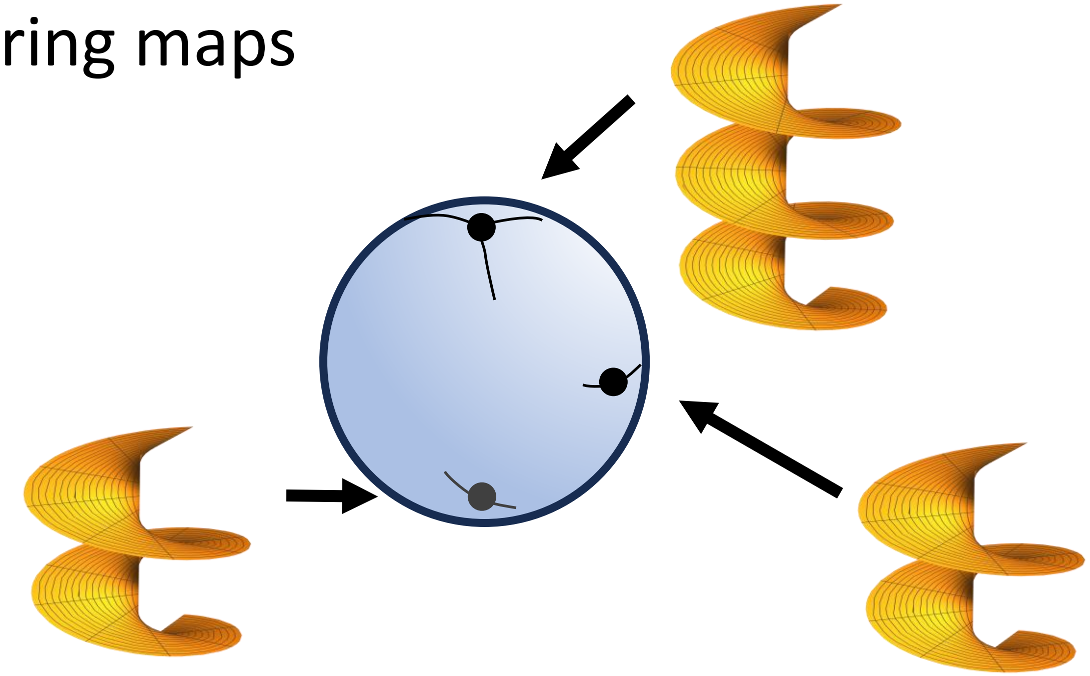


- At large N: Decompose into "single cycle" vacua σ_w of "twist" w

Covering space

- Mode expansion now features fractional modes - **painful**
- Instead consider a covering map $z = \Gamma[t] = t^L$
- We need only one copy of the seed theory on covering space – **nice**
- **Downside:** Need different covering maps
  for each correlator, e.g.:

$$\langle \sigma_2(0)\sigma_2(1)\sigma_3(\infty)\rangle :$$
$$\Gamma[t] = 3t^2 - 2t^3 \,.$$



First order perturbation theory

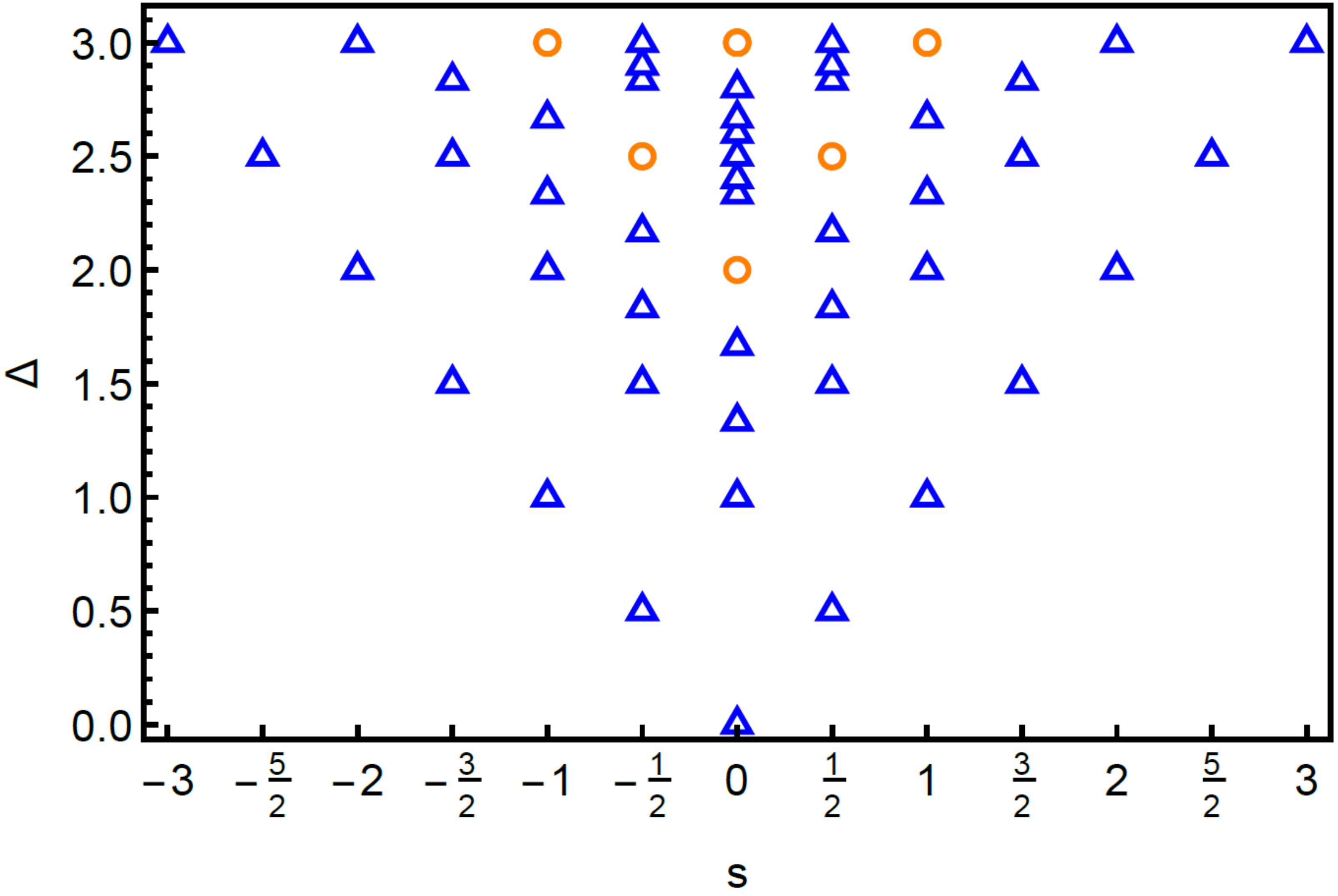
- Symmetric-product orbifold $\leftrightarrow$ IIB string on $AdS_3 \times S^3 \times T^4$ with $k = 1$ unit of NSNS-flux
- Marginal deformation $\mathcal{D}$ $\leftrightarrow$ Turning on RR-flux

$$\mathcal{D} = \frac{1}{2\sqrt{2}} \epsilon_{AB} \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} G^{\alpha A}_{-\frac{1}{2}} G^{\dot{\alpha} B}_{-\frac{1}{2}} \sigma_2^{\beta\dot{\beta}}$$

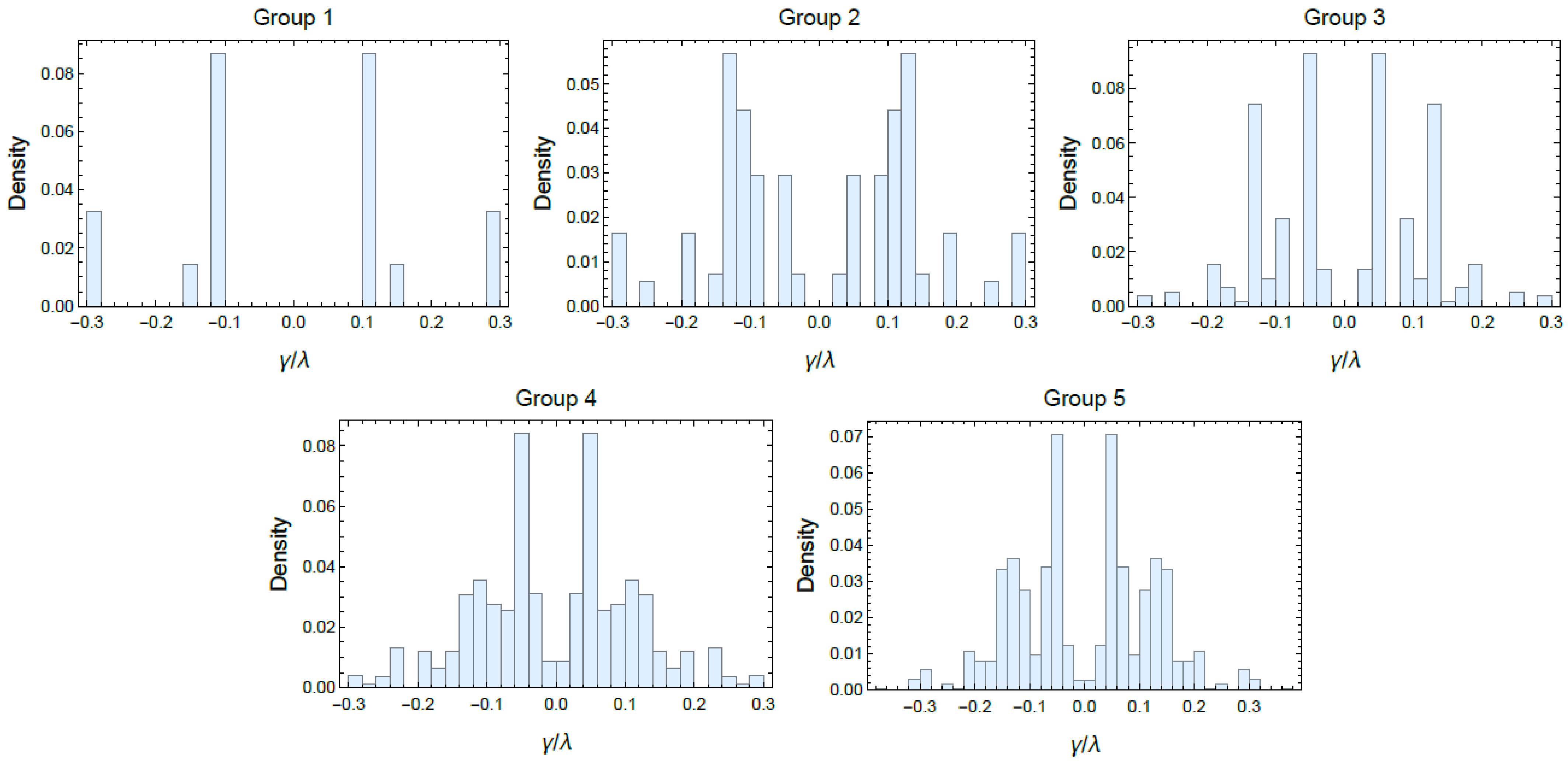
First order mixing matrix elements take the form

$$\langle \phi_{-\frac{n}{w}} \dots \sigma_w | \mathcal{D} | \phi_{-\frac{n}{w\pm 1}} \dots \sigma_{w\pm 1} \rangle$$

Lightest states



Results



Statistics

group	Δ	# states	# deformed states	fraction
1	2	276	74	26.8%
2	$\frac{5}{2}$	4×1090	4×464	42.6%
3	3	4×2368	4×1210	51.1%
4	3	4×6467	4×3828	59.2%
5	3	2×8280	2×4342	52.4%
All	≤ 3	248778	30766	12.4%

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Sparse

Big

Abundant

Large twist

- Checked that the dominant contributions to the mixing matrix go as $\frac{1}{w}$
- This is reassuring, because the integrability prediction is

$$E(p) = \sqrt{p^2 + 4\lambda^2 \sin^2(\pi p)}$$

→ There should be now O(λ) terms, at least asymptotically

- For small twist maybe due to wrapping? **Puzzle for integrability**

- **Causing trouble on the side:** Gaberdiel et al. 23' claimed to have found $\{S_1, S_2\} \left|\text{phys.}\right\rangle \neq 0$ **only** at large w. We cannot confirm this.

Thank you !!!