Interpolating families of integrable AdS_3 backgrounds

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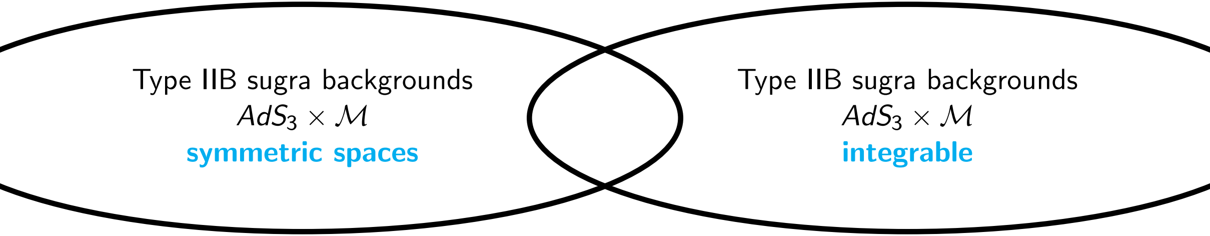
Abstract: We construct families of integrable deformations that interpolate between the $AdS_3 \times S^3 \times S^3 \times S^1$ and either $AdS_3 \times S^3 \times S^2 \times T^2$ or $AdS_3 \times S^2 \times S^2 \times T^3$. They preserve half of the supersymmetry of the original background, namely one copy of the $\mathfrak{d}(2,1;\alpha)$ algebra. From this it follows a similar integrable interpolation between $AdS_3 \times S^3 \times T^4$ and $AdS_3 \times S^2 \times T^5$, which also preserves half of the supersymmetry, namely a copy of the $\mathfrak{psu}(1,1|2)$ algebra. In all cases, the interpolating backgrounds are constructed by using TsT transformations, which makes it easy to implement them in the integrability formalism in the full quantum theory. To illustrate this point, we discuss the lightcone gauge fixing of the models and compute their pp-wave Hamiltonian.

Type IIB sugra backgrounds

$AdS_3 \times \mathcal{M}$

symmetric spaces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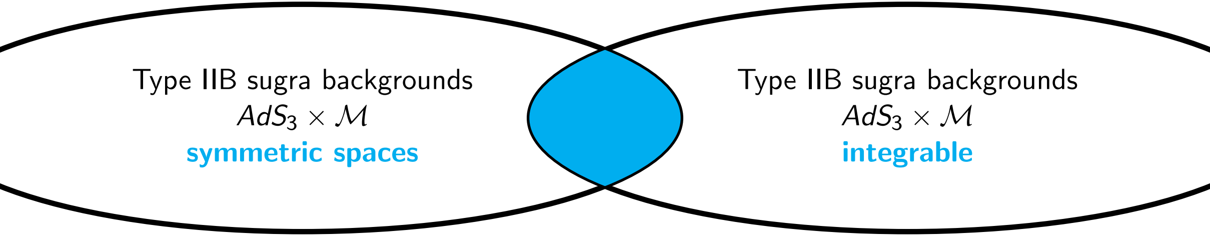

Case	Geometry	SUSY	Manifest symmetries
1.	$AdS_3 \times S^3 \times S^3 \times S^1$	16	$D(2,1;\alpha)^2 \times U(1)$
2.	$AdS_3 \times S^3 \times S^2 \times T^2$	8	$D(2,1;\alpha) \times SU(1,1) \times SU(2) \times U(1)^2$
3.	$AdS_3 \times S^2 \times S^2 \times T^3$	8	$D(2,1;\alpha) \times SU(1,1) \times U(1)^3$
4.	$AdS_3 \times S^3 \times T^4$	16	$PSU(1,1\|2)^2 \times U(1)^4$
5.	$AdS_3 \times S^2 \times T^5$	8	$PSU(1,1\|2) \times SU(2) \times U(1)^5$

- Construct integrable type IIB backgrounds that interpolate between the different cases
  - Preserve the AdS_3 geometry ($\rightarrow$ application to AdS_3/CFT$_2$)
  - Integrable (Lax integrable, factorised scattering)
  - Preserve 8 supersymmetries (coming from one $D(2,1;\alpha)$ copy)

Case	Geometry	SUSY	Manifest symmetries
1.	$AdS_3 \times S^3 \times S^3 \times S^1$	16	$D(2,1;\alpha)_L \times D(2,1;\alpha)_R \times U(1)$

$$ds^2 = R^2 ds^2(AdS_3) + R_1^2 ds^2(S_1^3) + R_2^2 ds^2(S_2^3) + ds^2(S^1)$$
$$H_3 = 2q \left(R^2\, \Omega(AdS_3) + R_1^2\, \Omega(S_1^3) + R_2^2\, \Omega(S_2^3)\right)$$
$$F_3 = 2\hat{q} \left(R^2\, \Omega(AdS_3) + R_1^2\, \Omega(S_1^3) + R_2^2\, \Omega(S_2^3)\right)$$

$$R_1 = \frac{R}{\sqrt{\alpha}}\,, \quad R_2 = \frac{R}{\sqrt{1-\alpha}}\,, \qquad 0 < \alpha < 1\,, \qquad q^2 + \hat{q}^2 = 1\,,$$

TsT transformation

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AdS_3	$\times$	S^3_1	$\times$	S^3_2	$\times$	S^1
$SU(1,1)_L \times SU(1,1)_R$	$\times$	$SU(2)_{1,L} \times SU(2)_{1,R}$	$\times$	$SU(2)_{2,L} \times SU(2)_{2,R}$	$\times$	$U(1)$
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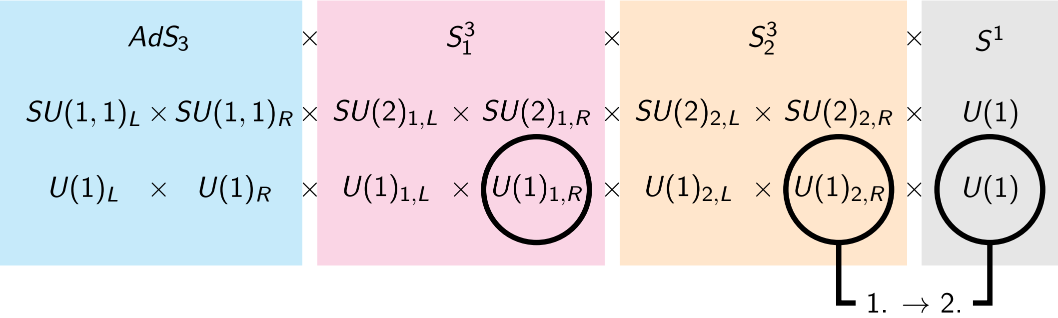

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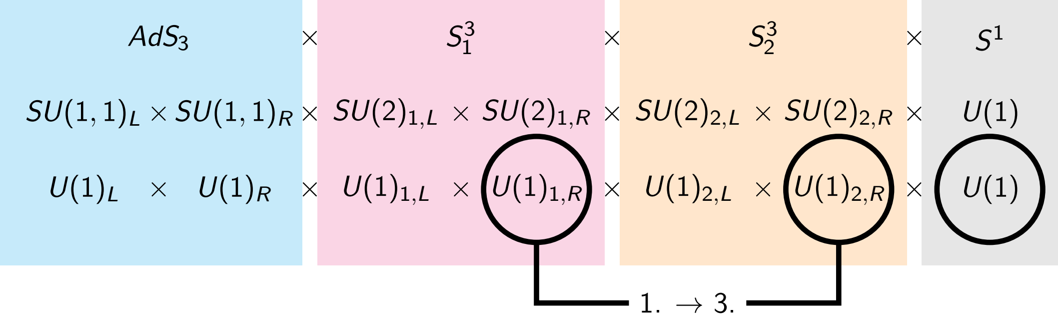

- TsT-deformed background (starting from pure RR):

$$ds^2 = R^2 ds^2(AdS_3) + R_1^2 ds^2(S_1^3) + R_2^2 ds^2(S^3_{2,\Delta}) + dx^2$$
$$H_3 = \frac{1}{2} R_2 \sqrt{1-\Delta}\, dA_2 \wedge dx$$

- "Squashing" of the sphere:

$$ds^2(S^3_{2,\Delta}) = \frac{1}{4}\left(ds^2(S_2^2) + \Delta A_2^2\right) , \qquad s = \frac{2}{R_2}\frac{\sqrt{1-\Delta}}{\sqrt{\Delta}} , \qquad 0 < \Delta \le 1$$

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  - $\Delta \to 1$: undeformed $AdS_3 \times S^3 \times S^3 \times S^1$
  - $\Delta \to 0$ (together with rescaling of coordinates): $AdS_3 \times S^3 \times S^2 \times T^2$

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- Small fluctuations above the geodesics

$$ds^2 = 2dx^+dx^- + \delta_{ij}dx^i dx^j - A_{ij}x^i x^j (dx^+)^2 + 4(1-\alpha)\sqrt{\Delta}\, x^5 dx^6 dx^+$$
$$H_3 = -2(1-\alpha)\sqrt{1-\Delta}\, dx^5 \wedge dx^8 \wedge dx^+$$

where $j = 1, \ldots, 8$ and

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- Fix light-cone gauge

$$x^+ = \tau\,, \qquad p_- = 1\,, \qquad p_+ = -\mathcal{H}\,.$$

$$H = \omega_{1,\pm}\, a^\dagger_{1,\pm} a_{1,\pm} + \omega_{2,\pm}\, a^\dagger_{2,\pm} a_{2,\pm} + \omega_{3,\pm}\, a^\dagger_{3,\pm} a_{3,\pm} + \omega_{4,\pm}\, a^\dagger_{4,\pm} a_{4,\pm} \,.$$

$$\omega_{1,\pm} = \sqrt{p^2 + 1}$$

$$\omega_{4,\pm} = |p|$$

$$\omega_{2,\pm} = \sqrt{p^2 + \alpha^2}$$

$$\omega_{3,\pm} = \sqrt{p^2 + (1-\alpha)^2} \pm (1-\alpha)$$

Outlook

- Integrability for cases 2., 3. and 5. not yet well-understood
- Know that the TsT transformation acts on the exact w.s. S-matrix as a Drinfel'd twist
- Subtelty as the twist involves the light-cone coordinates
- Holographic duals of these TsT-transformed backgrounds?
- Many more integrable deformations: trigonometric and elliptic deformations of S^3 factors.

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