High Energy Physics - Theory

[Submitted on 11 Mar 2024]

Holography and Regge Phases with $U(1)$ Charge

Giulia Fardelli, A. Liam Fitzpatrick, Wei Li

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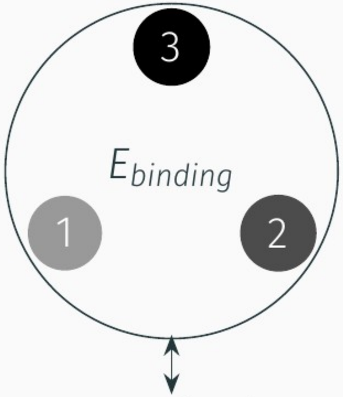
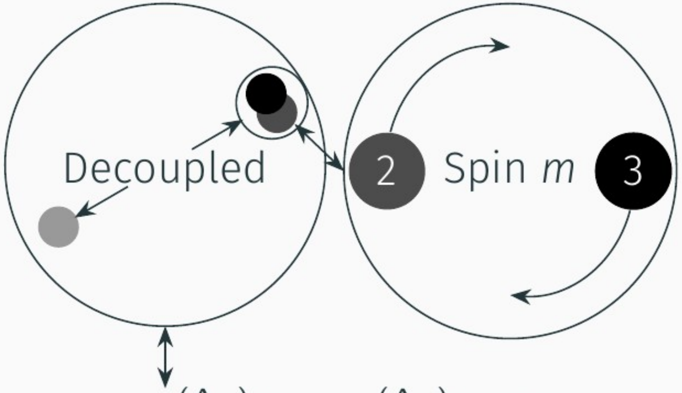
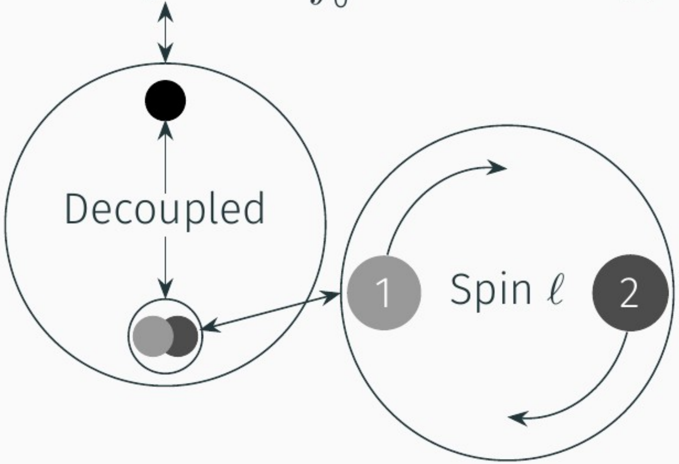
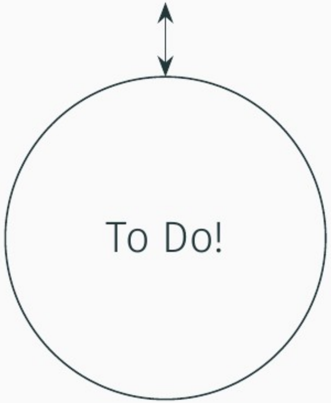
Perturbative Interactions between Three Bodies



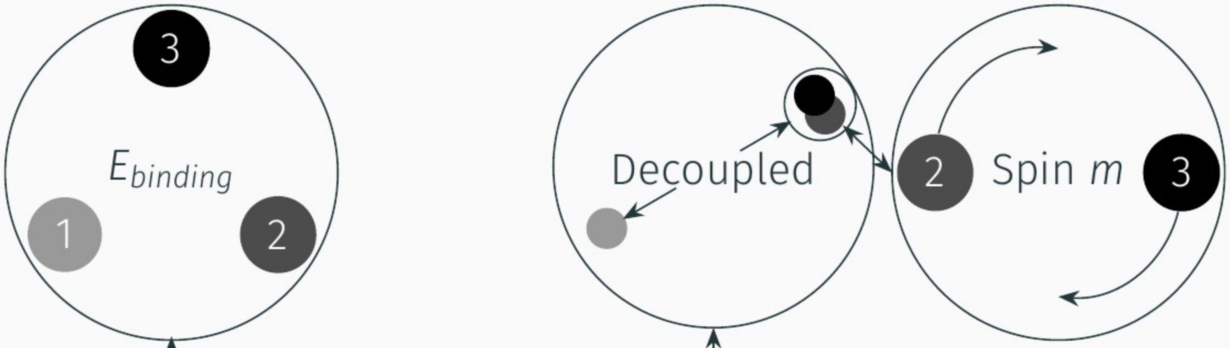


$$\frac{\gamma_{k\ell}(J,\kappa)}{\gamma_0} = \frac{\delta(k-\ell)}{\ell^{2h_\star}} + \int_0^\infty \mathrm{d}m \, \frac{W_{km}^{(\Delta_\phi)}(J,\kappa)\, W_{m\ell}^{(\Delta_\phi)}(J,\kappa)}{m^{2h_\star}} + O(J^{-2h_\star}).$$

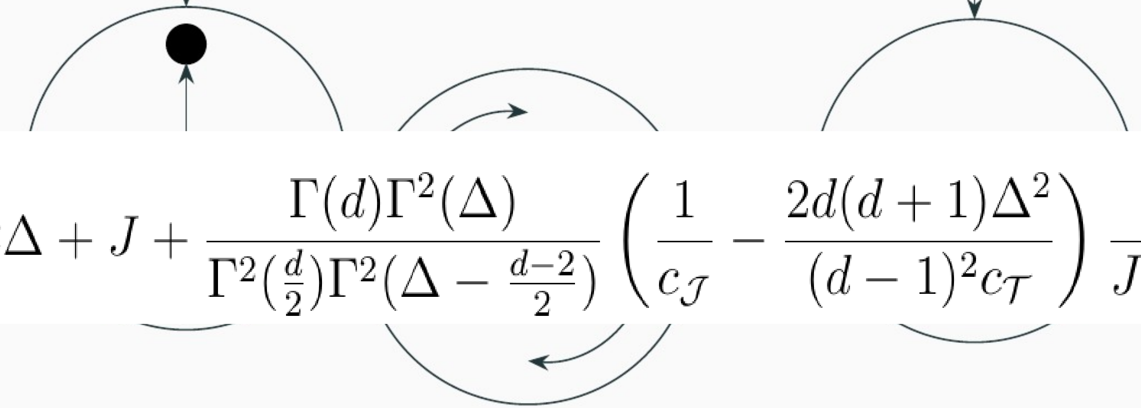




Perturbative Interactions between Three Bodies



$$\frac{\gamma_{k\ell}(J,\kappa)}{\gamma_0} = \frac{\delta(k-\ell)}{\ell^{2h_\star}} + \int_0^\infty dm \frac{W^{(\Delta_\phi)}_{km}(J,\kappa) W^{(\Delta_\phi)}_{m\ell}(J,\kappa)}{m^{2h_\star}} + O(J^{-2h_\star}).$$



$$\Delta_{[\Phi,\Phi]_J} = 2\Delta + J + \frac{\Gamma(d)\Gamma^2(\Delta)}{\Gamma^2(\frac{d}{2})\Gamma^2(\Delta - \frac{d-2}{2})} \left(\frac{1}{c_{\mathcal{J}}} - \frac{2d(d+1)\Delta^2}{(d-1)^2 c_{\mathcal{T}}} \right) \frac{1}{J^{d-2}} + \dots.$$

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Comments: 88 pages, many nice figures
Subjects: **High Energy Physics - Theory (hep-th)**
Cite as: arXiv:2403.07079 **[hep-th]**
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many nice figures

$$S = \frac{1}{\kappa^2} \int d^{d+1}x \sqrt{-g} \left(\frac{R - \Lambda}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (D^\mu \phi)^\dagger (D_\mu \phi) - m^2 \phi^\dagger \phi \right),$$

where we have defined the covariant derivative

$$D_\mu = \nabla_\mu - i g_{\mathrm{U}(1)} A_\mu .$$

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where we have defined the covariant derivative

$$D_\mu = \nabla_\mu - ig_{\mathrm{U}(1)}A_\mu .$$

such that

$$S_{\text{eff}} = \int d^{d+1}x\sqrt{-\widetilde{g}}\Big(-\nabla^\mu\phi^\dagger\nabla_\mu\phi - m^2\phi^\dagger\phi - V_{\text{eff}}[\phi,\phi^\dagger]\Big)$$

$$V_{\text{eff}}[\phi,\phi^\dagger] = -\frac{1}{2}\kappa\, A^\mu[\phi,\phi^\dagger]\mathcal{J}_\mu[\phi,\phi^\dagger] - \frac{1}{4}\kappa\, h^{\mu\nu}[\phi,\phi^\dagger]\mathcal{T}_{\mu\nu}[\phi,\phi^\dagger]\,,$$

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$$\gamma^{(d,Q)}(J) = \int d^dx\sqrt{-\tilde{g}}\;{}_J\langle\Psi|\,V_{\mathrm{eff}}[\phi,\phi^\dagger]\,|\Psi\rangle_J \equiv {}_J\langle\Psi|\,\mathcal{V}_{\mathrm{eff}}[\phi,\phi^\dagger]\,|\Psi\rangle_J\,.$$

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$$\gamma^{(d,Q)}(J) = \int d^dx\sqrt{-\widetilde{g}}\; {}_J\langle\Psi|\, V_{\mathrm{eff}}[\phi,\phi^\dagger]\,|\Psi\rangle_J \equiv {}_J\langle\Psi|\, \mathcal{V}_{\mathrm{eff}}[\phi,\phi^\dagger]\,|\Psi\rangle_J \, .$$

$$\langle\ell_2\ell_4|\, V_{\mathrm{eff}}[\phi,\phi^\dagger]\,|\ell_1\ell_3\rangle = -2\Bigg(\frac{(\kappa g_{\mathrm{U(1)}})^2}{2}\langle\ell_2\ell_4|A[\phi,\phi^\dagger]\cdot\mathcal{J}[\phi,\phi^\dagger]|\ell_1\ell_3\rangle + \frac{\kappa^2}{4}\langle\ell_2\ell_4|h[\phi,\phi^\dagger]\cdot\mathcal{T}[\phi,\phi^\dagger]|\ell_1\ell_3\rangle + \ell_2\leftrightarrow\ell_4\Bigg),$$

$$S = \frac{1}{\kappa^2}\int d^{d+1}x\sqrt{-g}\left(\frac{R-\Lambda}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - (D^\mu\phi)^\dagger(D_\mu\phi) - m^2\phi^\dagger\phi\right),$$

where we have defined the covariant derivative

$$\gamma^{(d,Q)}(J) = \int d^dx\sqrt{-\widetilde{g}}\;{}_J\langle\Psi|\,V_{\text{eff}}[\phi,\phi^\dagger]\,|\Psi\rangle_J \equiv {}_J\langle\Psi|\,\mathcal{V}_{\text{eff}}[\phi,\phi^\dagger]\,|\Psi\rangle_J\,.$$

$$D_\mu = \nabla_\mu - i g_{\mathrm{U}(1)} A_\mu.$$

many nice figures

$$\langle\ell_2\ell_4|\,V_{\text{eff}}[\phi,\phi^\dagger]\,|\ell_1\ell_3\rangle = -2\left(\frac{(\kappa g_{\mathrm{U}(1)})^2}{2}\langle\ell_2\ell_4|A[\phi,\phi^\dagger]\cdot\mathcal{J}[\phi,\phi^\dagger]|\ell_1\ell_3\rangle\right.$$

$$\left.+\frac{\kappa^2}{4}\langle\ell_2\ell_4|h[\phi,\phi^\dagger]\cdot\mathcal{T}[\phi,\phi^\dagger]|\ell_1\ell_3\rangle + \ell_2\leftrightarrow\ell_4\right),$$

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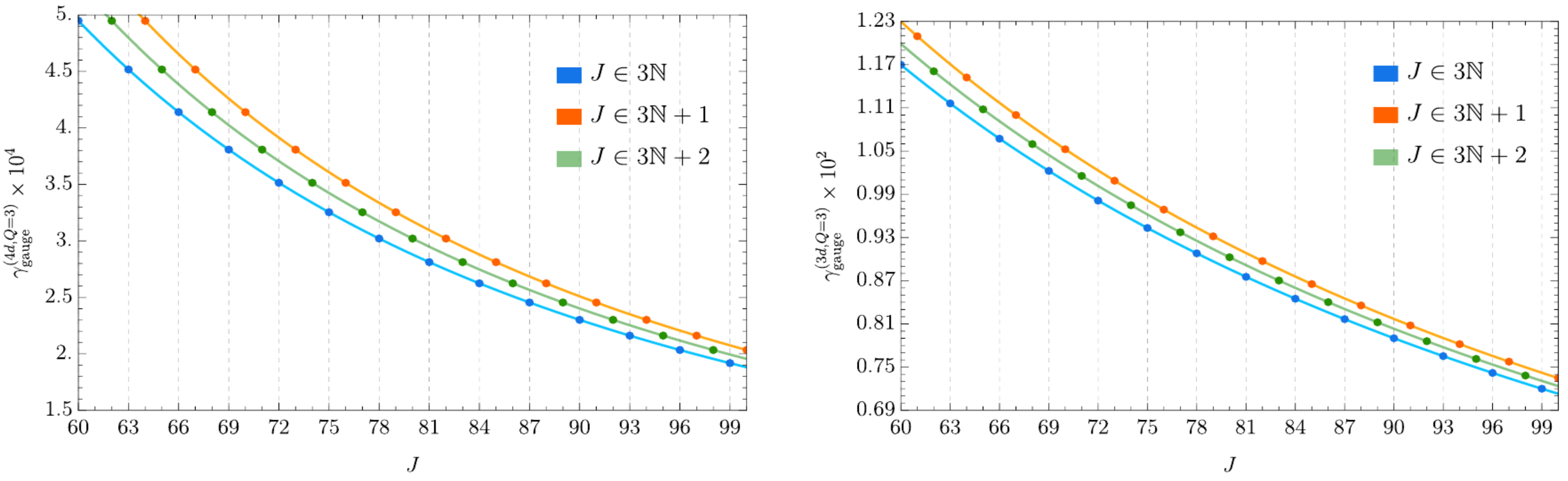
Only Coulomb Force

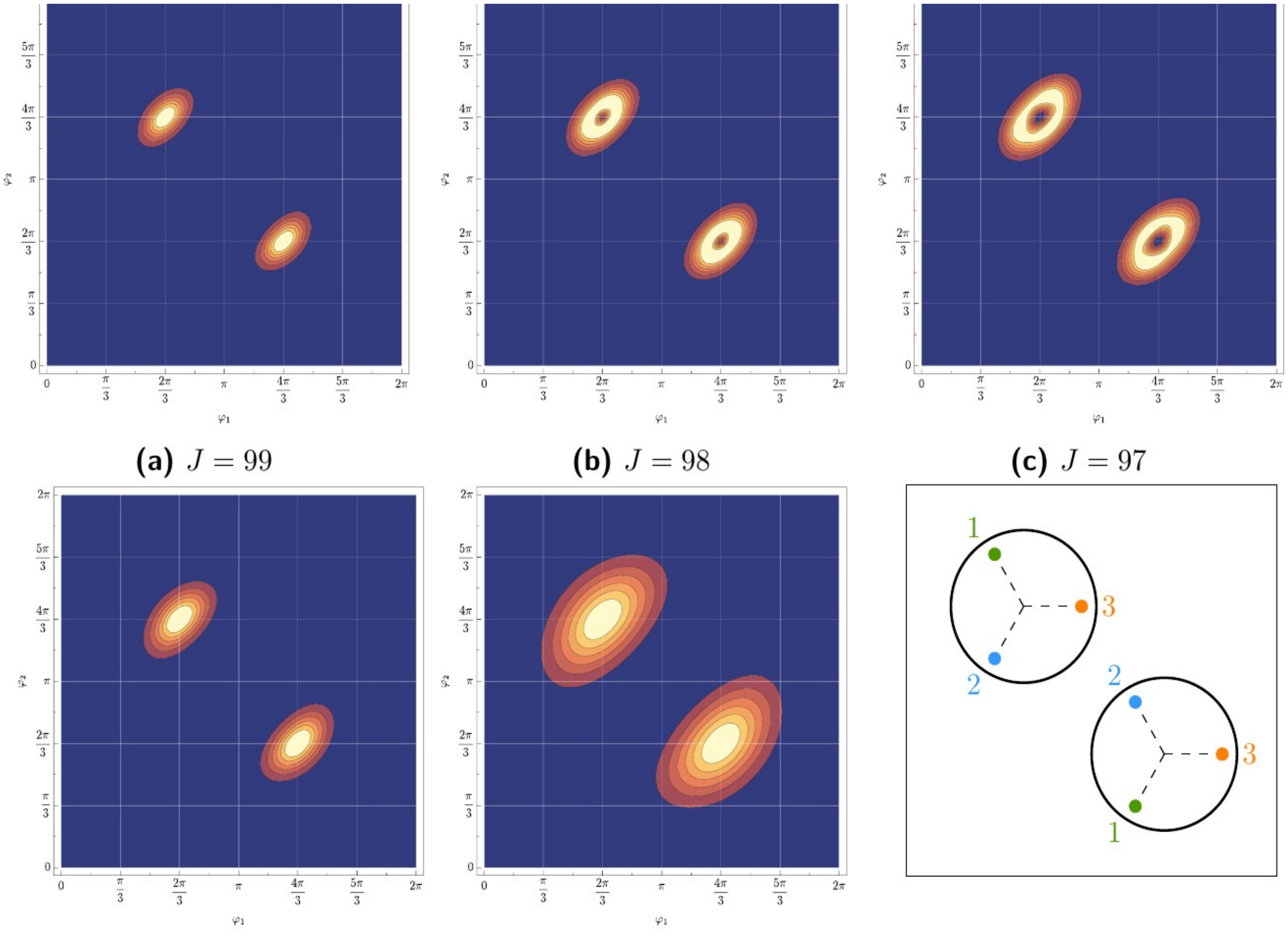


(a) $Q = 3\ \Delta = 4$ in $4d$

(b) $Q = 3\ \Delta = \frac{5}{2}$ in $3d$

Figure 3. Anomalous dimensions corresponding to the lowest eigenvalue at each spin for

$|\Psi_{\min}(\varphi_1, \varphi_2)|$ for the 3-particle state with $\Delta = \frac{5}{2}$ in $3d$

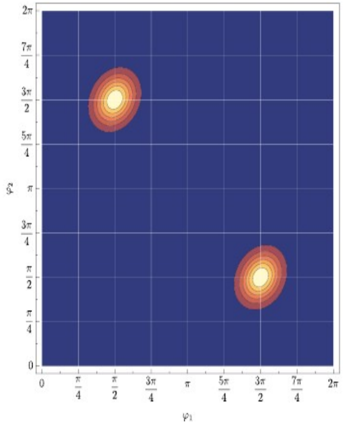


(a) $J = 99$ (b) $J = 98$ (c) $J = 97$

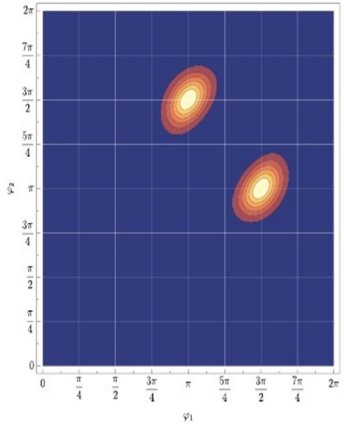
(d) $J = 57$ (e) $J = 18$ (f) Cartoon version

(Third particle is at angular position 0 on the equator)

The same pictures for four partons:

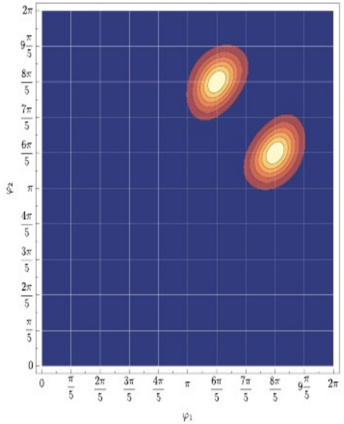


(a) $\varphi_3 = \pi$

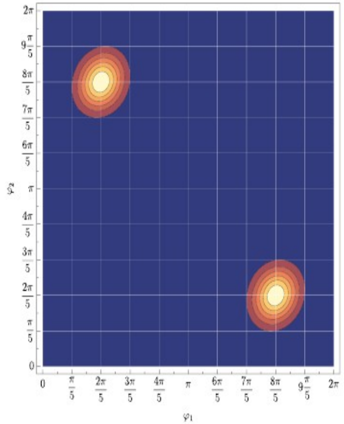


(b) $\varphi_3 = \frac{\pi}{2}$

Figure 7. $Q = 4\ |\Psi_{\rm min}(\varphi_1, \varphi_2, \varphi_3, 0)|$ for $\Delta = \frac{5}{2}$ in $3d$ and $J = 60$.



(a) $(\varphi_3, \varphi_4) = \left(\frac{2\pi}{5}, \frac{4\pi}{5}\right)$



(b) $(\varphi_3, \varphi_4) = \left(\frac{6\pi}{5}, \frac{4\pi}{5}\right)$

Figure 8. $Q = 5\ |\Psi_{\rm min}(\varphi_1, \varphi_2, \varphi_3, \varphi_4, 0)|$ for $\Delta = \frac{5}{2}$ in $3d$ and $J = 50$.

Excited states and radial resolution

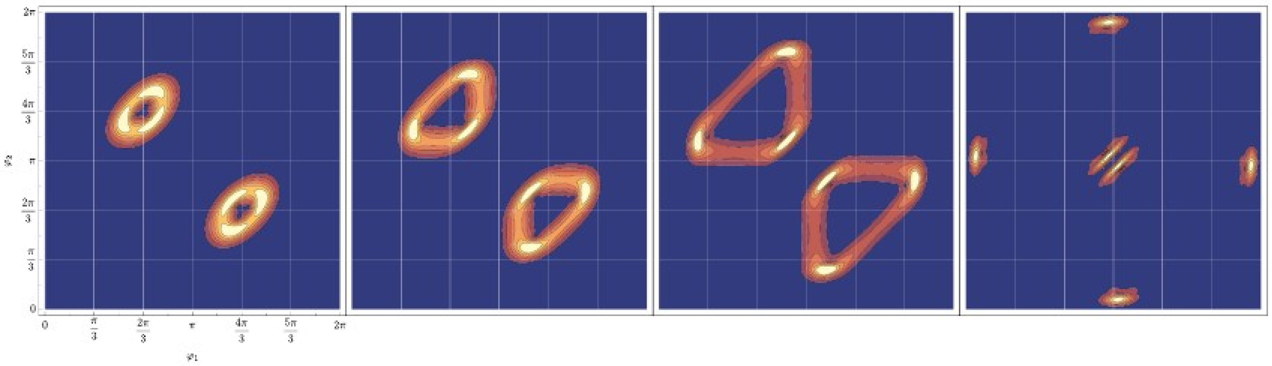
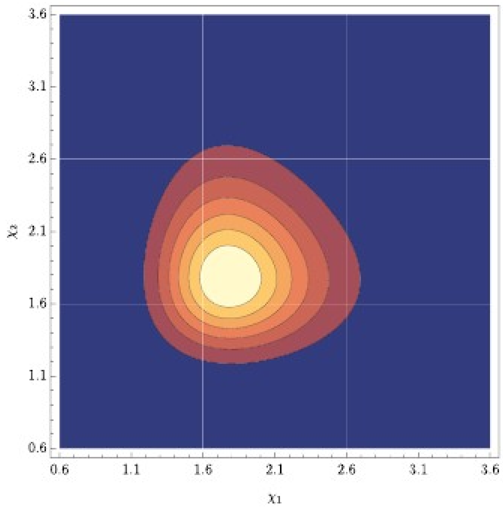
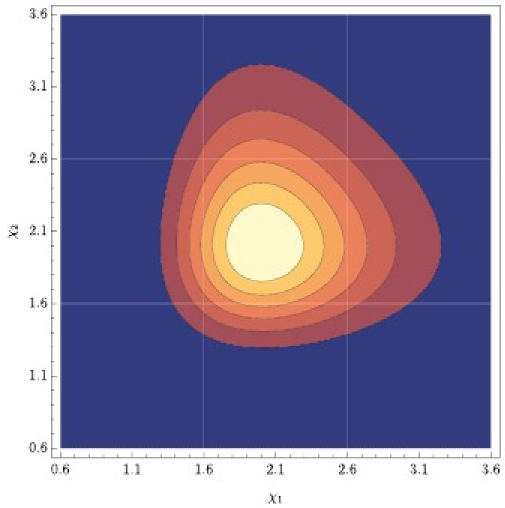


Figure 9. $|\Psi_r(\varphi_1, \varphi_2)|$ for the 3-particle state with $\Delta = \frac{5}{2}$ in $3d$ at $J = 99$ for various eigenvalues. Recall that r labels different primaries at a fixed spin , $r = 1, \cdots, \mathcal{N}(3, 99) = 17$. We plot, from left to right, $r = 2, 4, 8, 16$.



(a) $\Delta = 4$ and $\chi_3 \sim 1.78$



(b) $\Delta = \frac{5}{2}$ and $\chi_3 \sim 2$

Only Gravitational Interaction

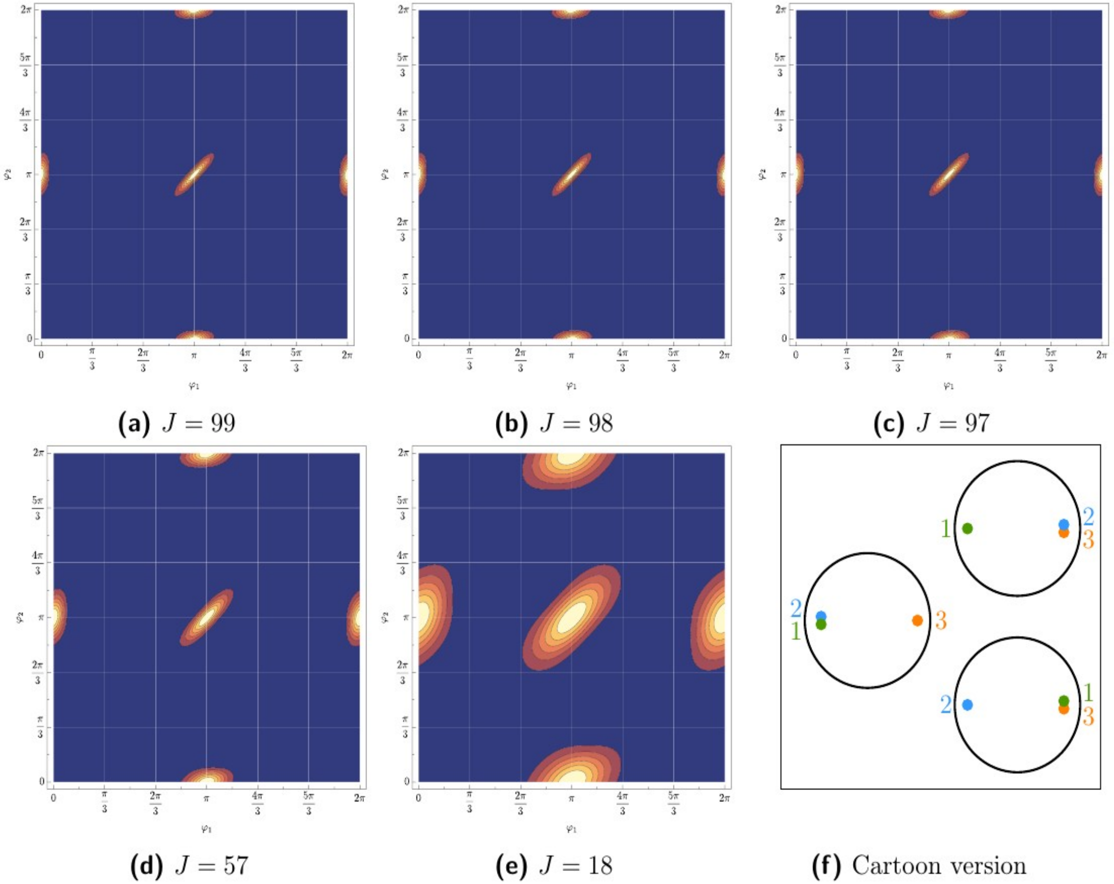
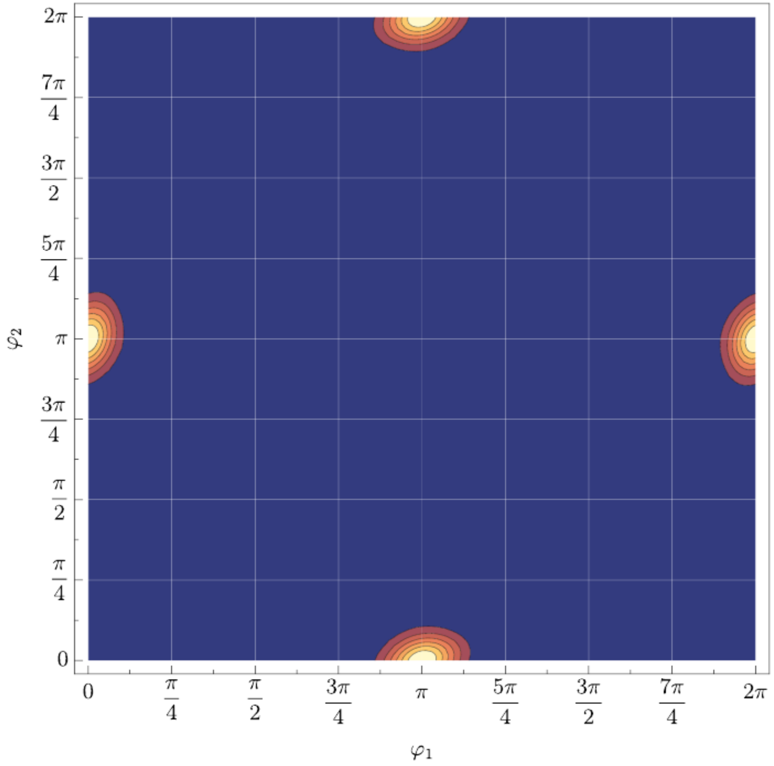


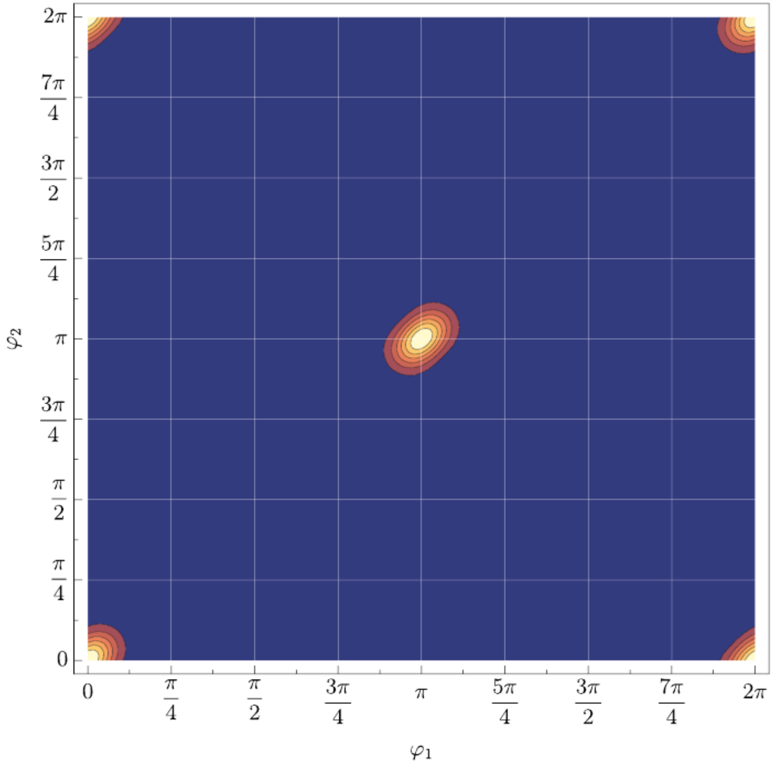
Figure 16. $|\Psi_{\rm min}(\varphi_1, \varphi_2)|$ for the 3-particle state with $\Delta = \frac{5}{2}$ in $3d$ for different values

(Third particle is at angular position 0 on the equator)

Four Partons

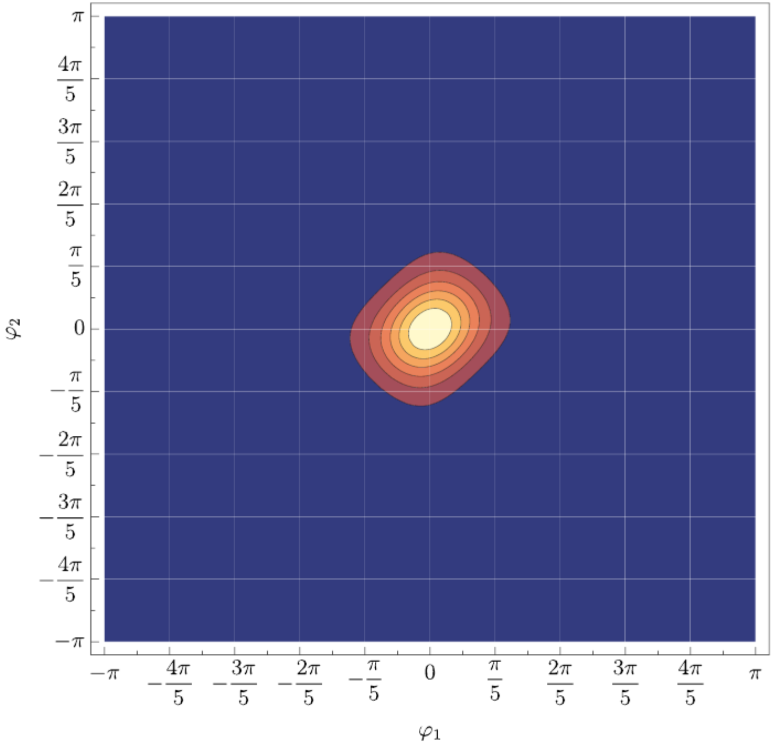


(a) $Q = 4\ \Delta = 4$ in $4d$ and $\varphi_3 = 0$

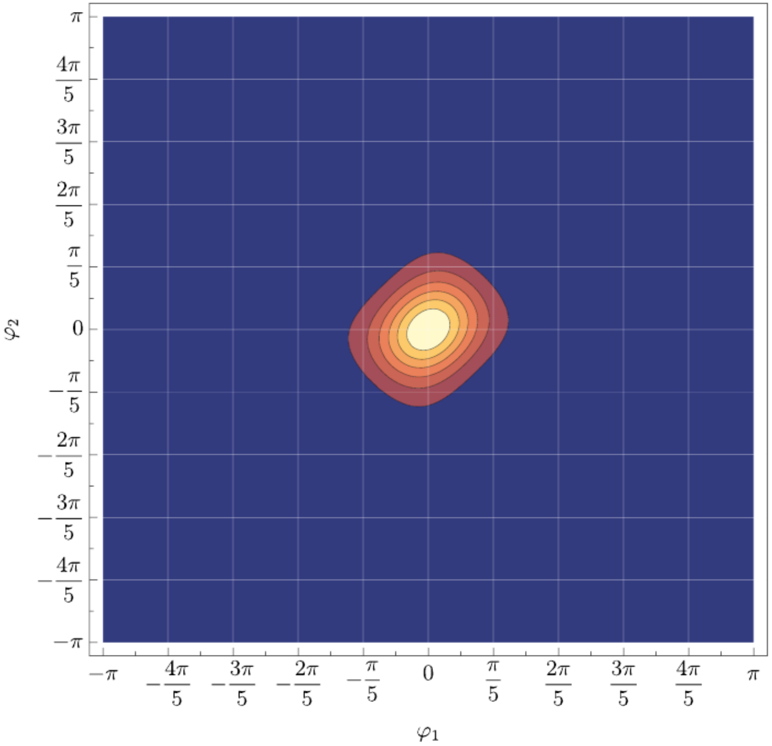


(b) $Q = 4\ \Delta = 4$ in $4d$ and $\varphi_3 = \pi$

Five Partons

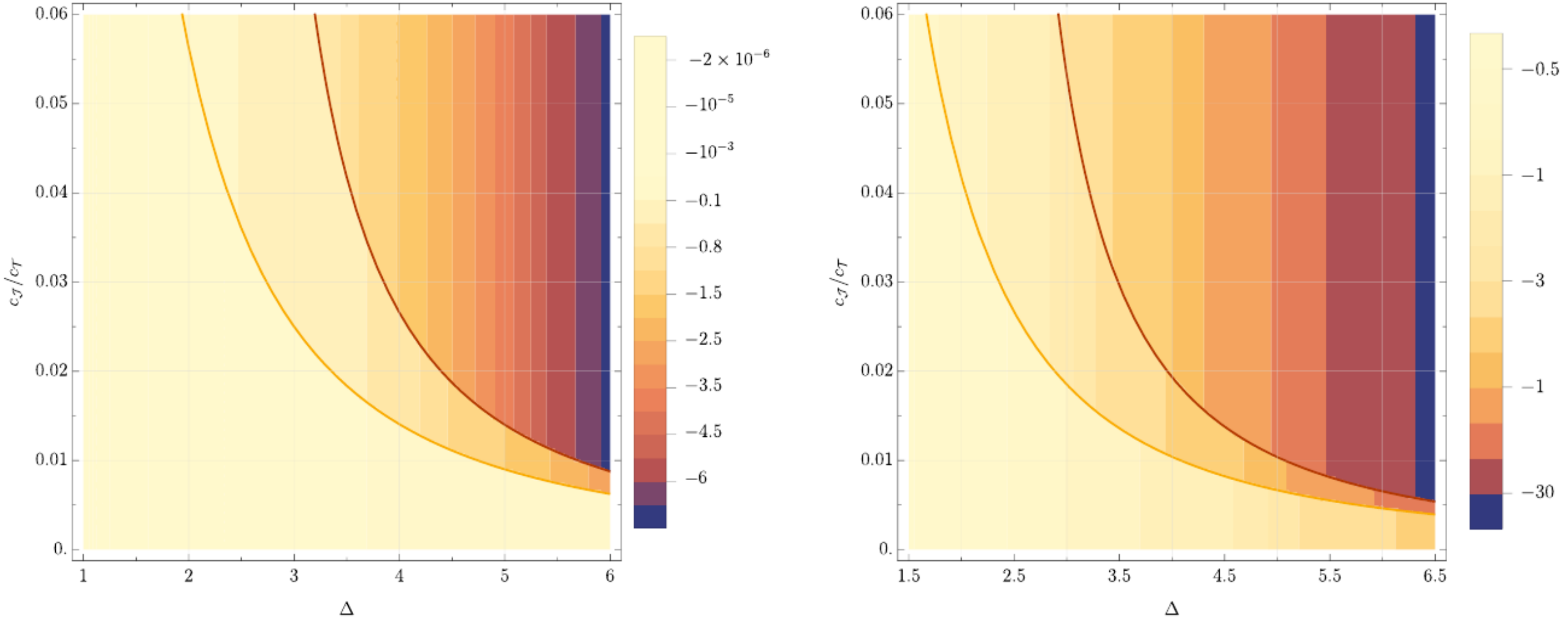


(a) $Q = 5\ \Delta = 4$ in $4d$ and $J = 55$



(b) $Q = 5\ \Delta = \frac{5}{2}$ in $3d$ and $J = 50$

Gauge + Gravity = Pretty Phase Diagrams!



(a) $4d$ **(b)** $3d$

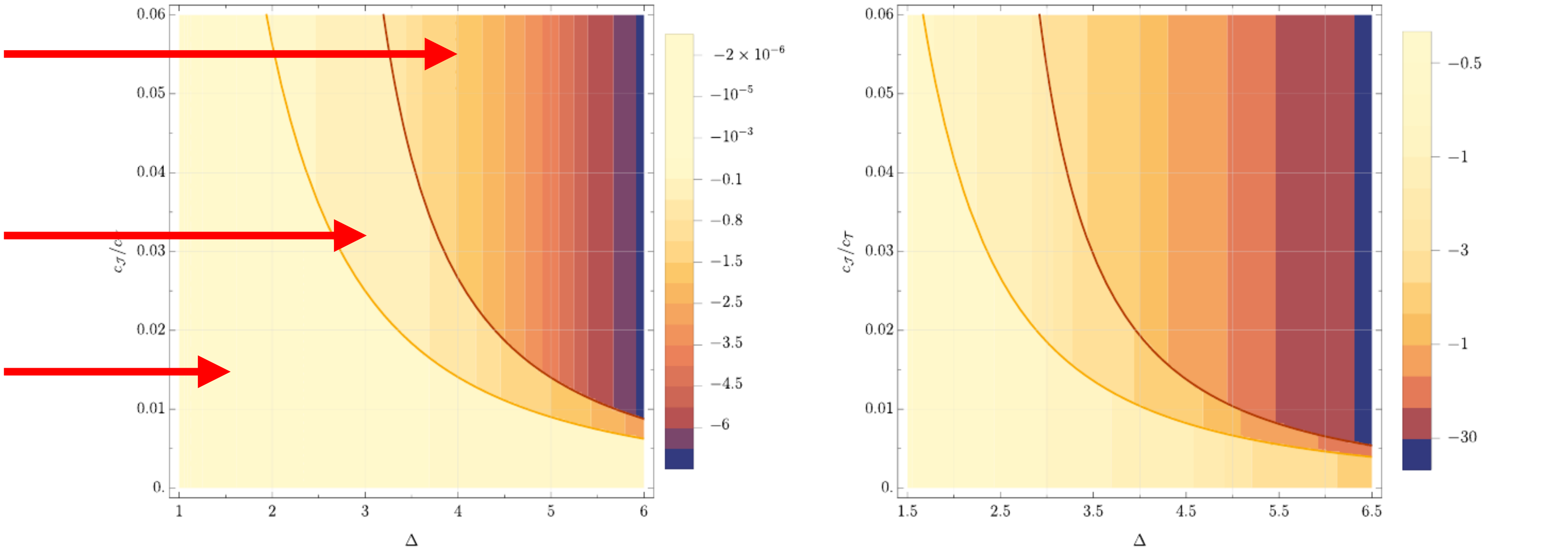
Figure 26. $\frac{\partial\gamma^{(d,Q)}}{\partial(c_{\mathcal{J}}/c_{\mathcal{T}})}$ as a function of Δ and $(c_{\mathcal{J}}/c_{\mathcal{T}})$ at fixed $J = 100$ and $Q = 3$ in four and three dimensions. The lines correspond to a function $(c_{\mathcal{J}}/c_{\mathcal{T}}) = \beta(\Delta)$ such that the system passes from the repulsive $\to$ attractive phase (orange line) and $a_0 = \gamma^{(d,Q=2)}(c_{\mathcal{J}}/c_{\mathcal{T}})|_{\ell=2} \to a_0 = \gamma^{(d,Q=2)}(c_{\mathcal{J}}/c_{\mathcal{T}})|_{\ell=0}$ (red line).

Gauge + Gravity = Pretty Phase Diagrams!

Attractive spin 0

Attractive spin 2

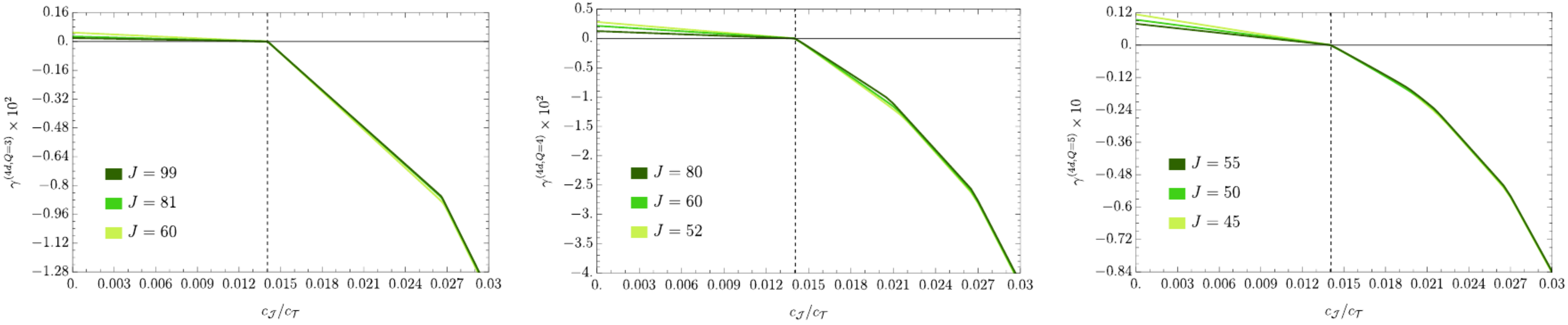
Repulsive



(a) $4d$ **(b)** $3d$

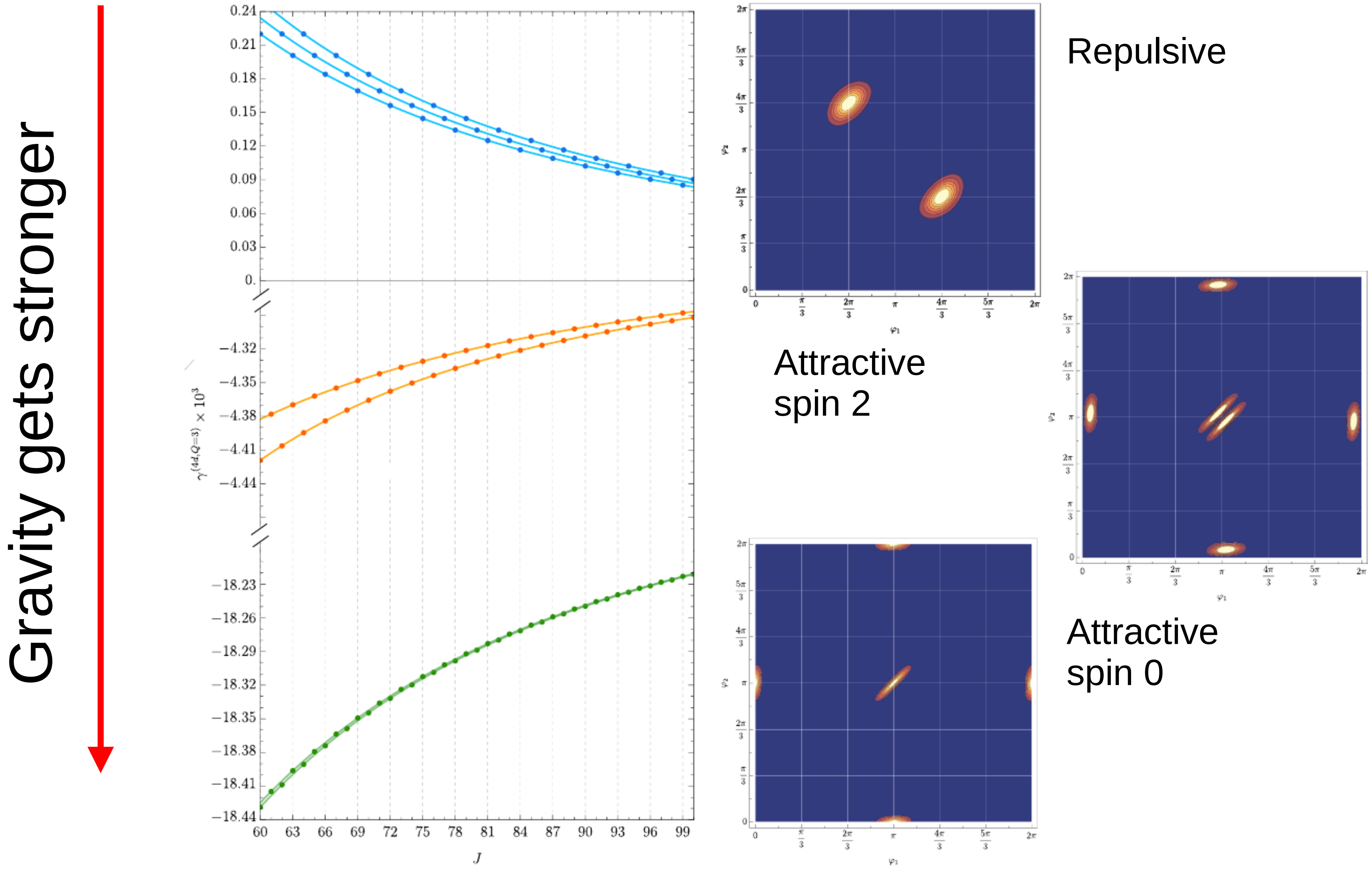
Figure 26. $\frac{\partial \gamma^{(d,Q)}}{\partial (c_{\mathcal{J}}/c_{\mathcal{T}})}$ as a function of Δ and $(c_{\mathcal{J}}/c_{\mathcal{T}})$ at fixed $J = 100$ and $Q = 3$ in four and three dimensions. The lines correspond to a function $(c_{\mathcal{J}}/c_{\mathcal{T}}) = \beta(\Delta)$ such that the system passes from the repulsive $\to$ attractive phase (orange line) and $a_0 = \gamma^{(d,Q=2)}(c_{\mathcal{J}}/c_{\mathcal{T}})|_{\ell=2} \to a_0 = \gamma^{(d,Q=2)}(c_{\mathcal{J}}/c_{\mathcal{T}})|_{\ell=0}$ (red line).

For people who don’t like derivatives:

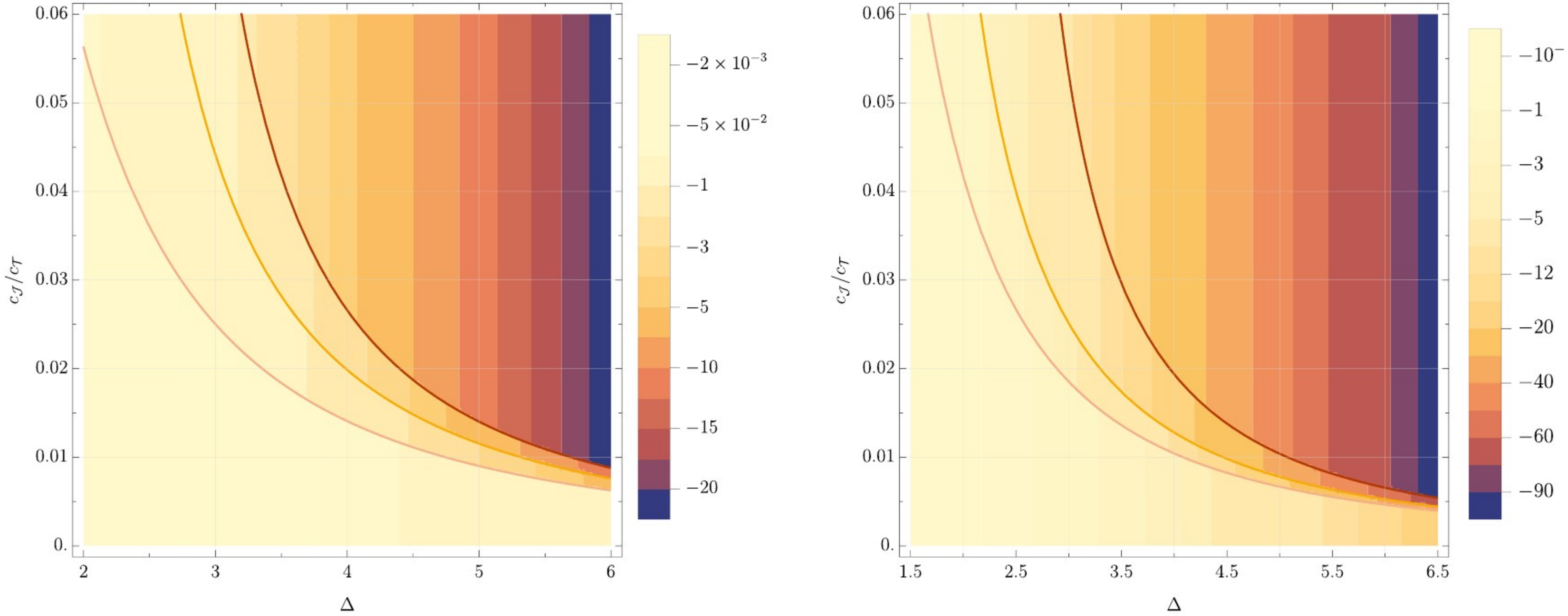


(a) $Q = 3$ **(b)** $Q = 4$ **(c)** $Q = 5$

Figure 27. $4d$ anomalous dimensions for Q-particle states due the photon and graviton exchange combined at $\Delta = 4$ as a function of $c_{\mathcal{J}}/c_{\mathcal{T}}$. At fixed Q, every line corresponds to a different value of the spin. There are Q different slopes corresponding to different configurations of the partons.


Gravity gets stronger
Repulsive
Attractive
spin 2
Attractive
spin 0
$\gamma^{(4d,Q=3)} \times 10^3$
J
φ_1
φ_2

More Partons, more phases.



(a) $4d$ **(b)** $3d$

Figure 30. $\frac{\partial \gamma^{(d,Q)}}{\partial (c_{\mathcal{J}}/c_{\mathcal{T}})}$ as a function of Δ and $(c_{\mathcal{J}}/c_{\mathcal{T}})$ at fixed $J = 100$ and $Q = 4$

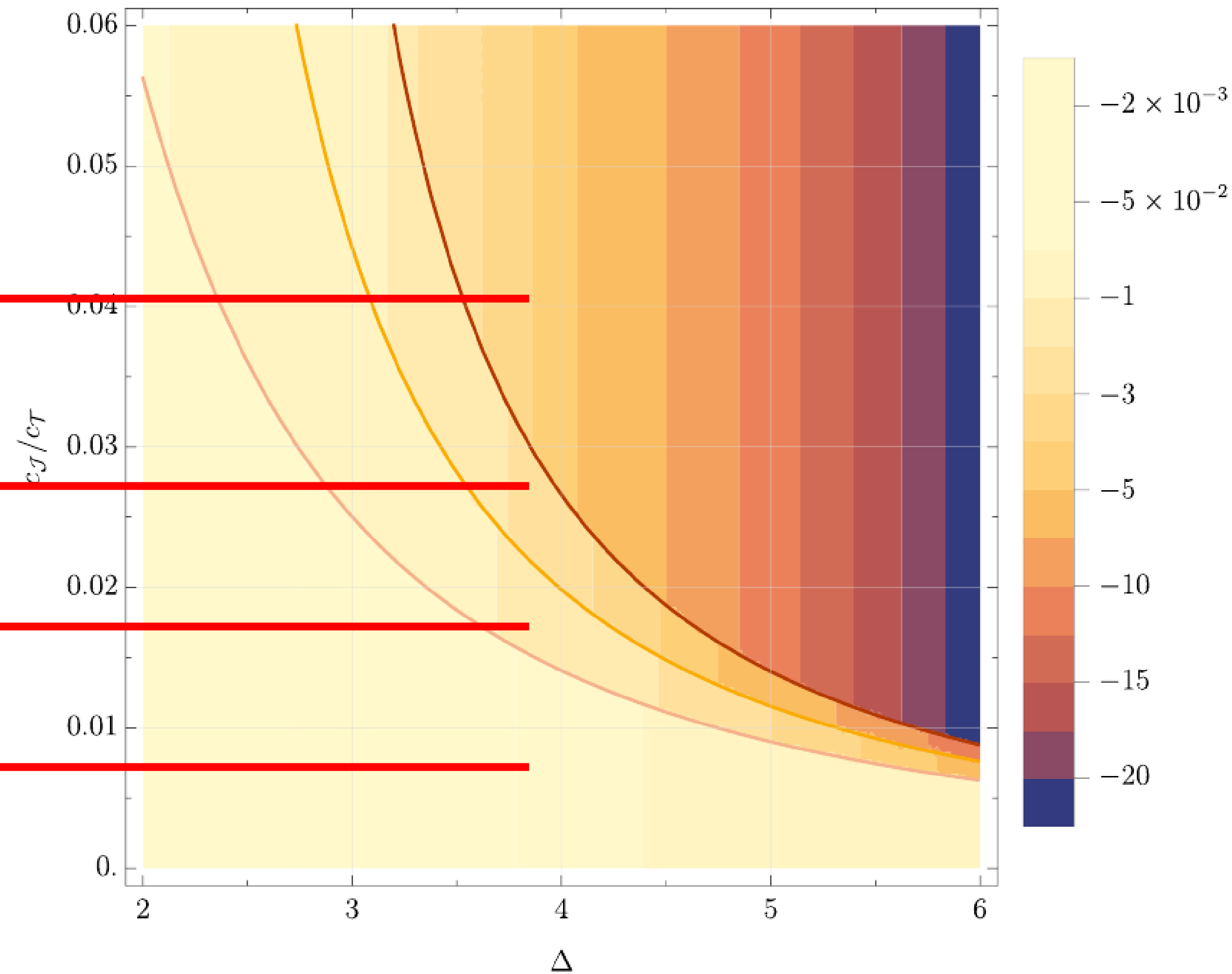
Two blobs of two partons?!

Spin one three parton blob

Spin two three parton blob

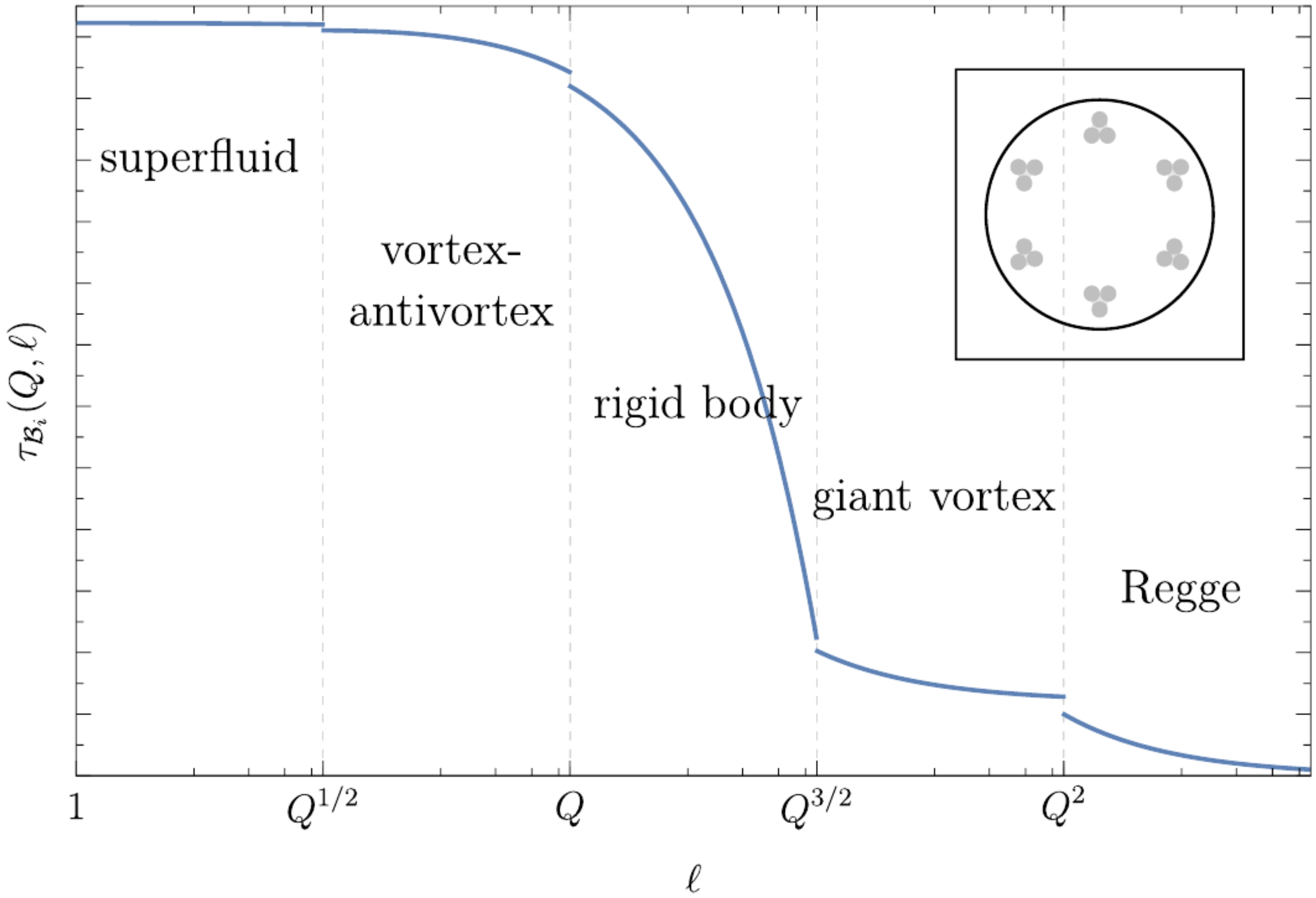
Spin three two parton blobs

Repulsive Phase

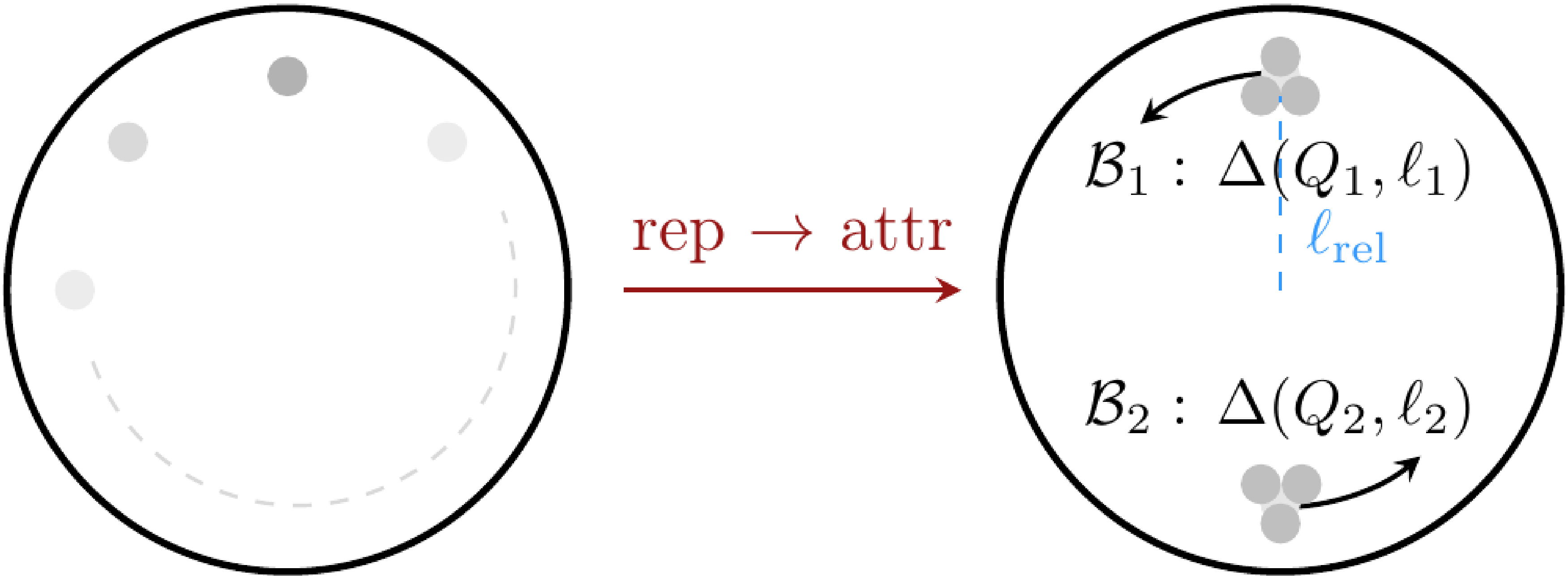


(a) $4d$

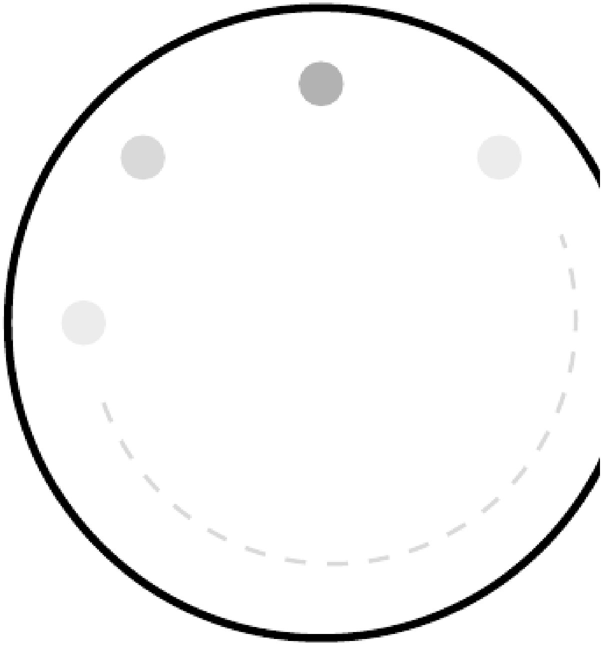
Large Charge EFT

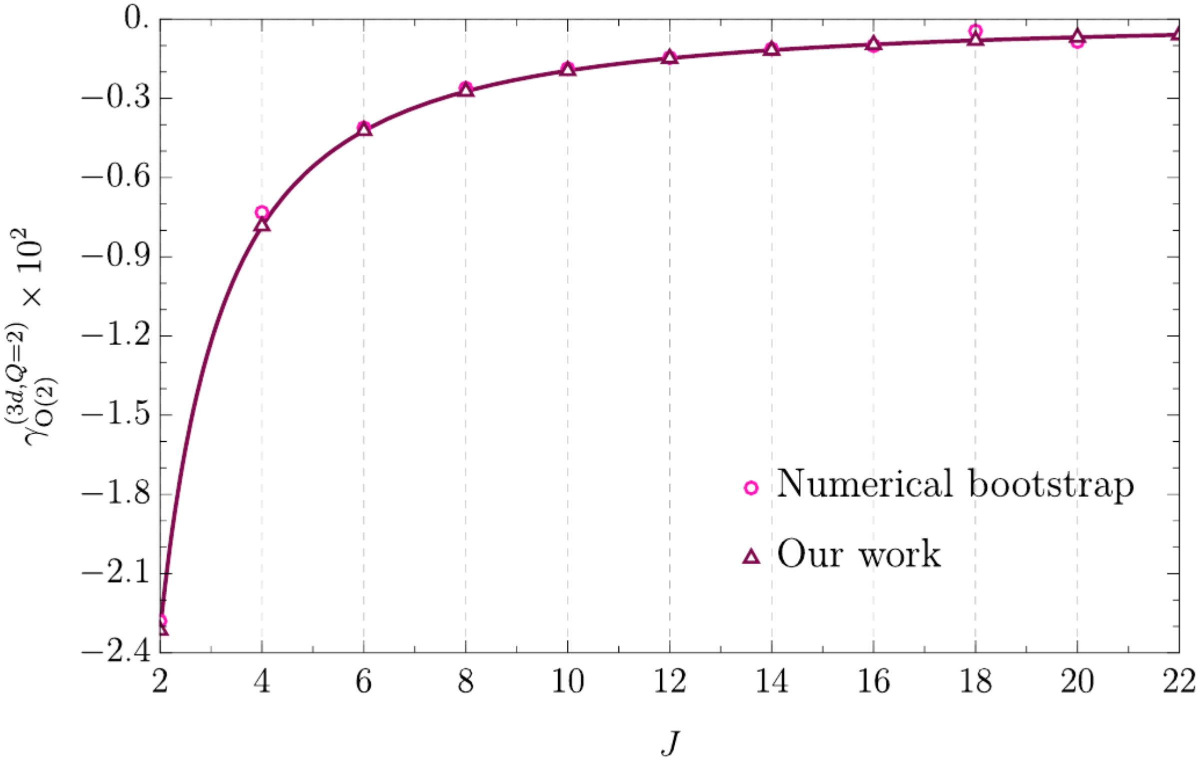


Feeding in non perturbative data and matching known O(2) model results.

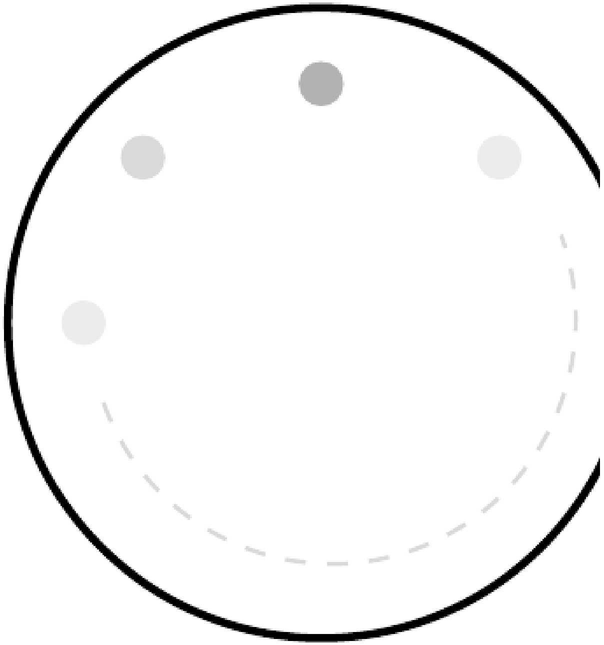


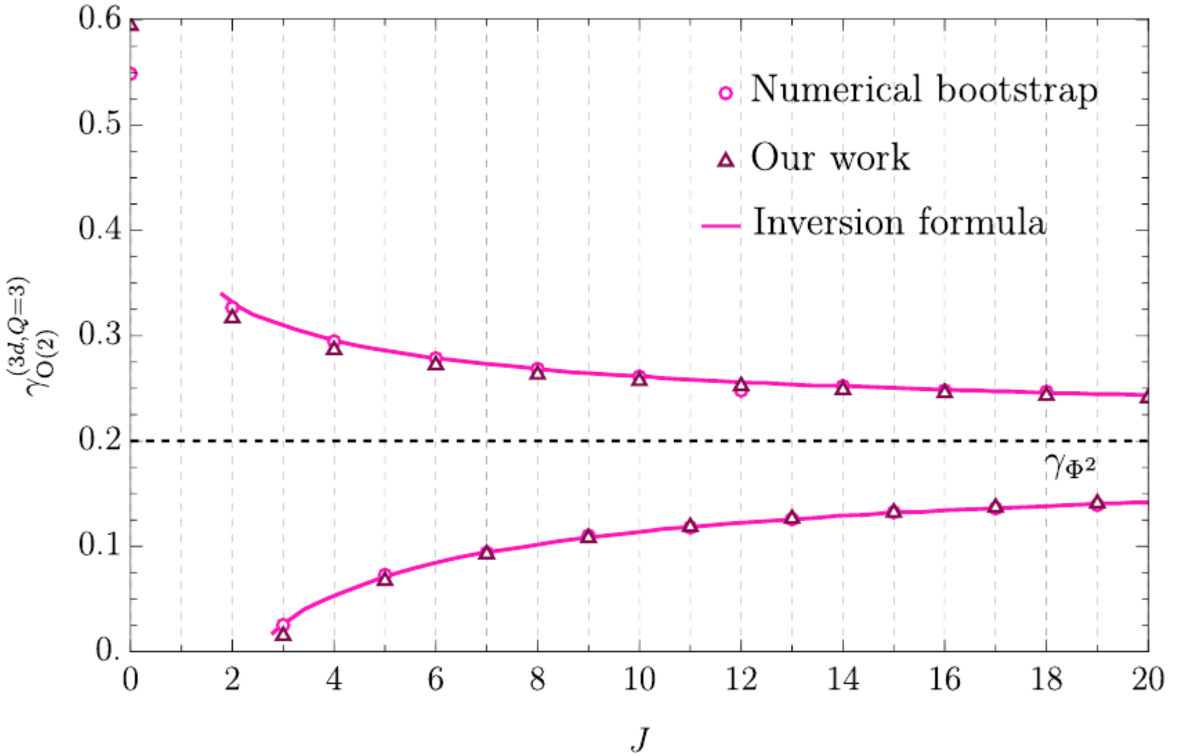
Feeding in non perturbative data and matching known O(2) model results.

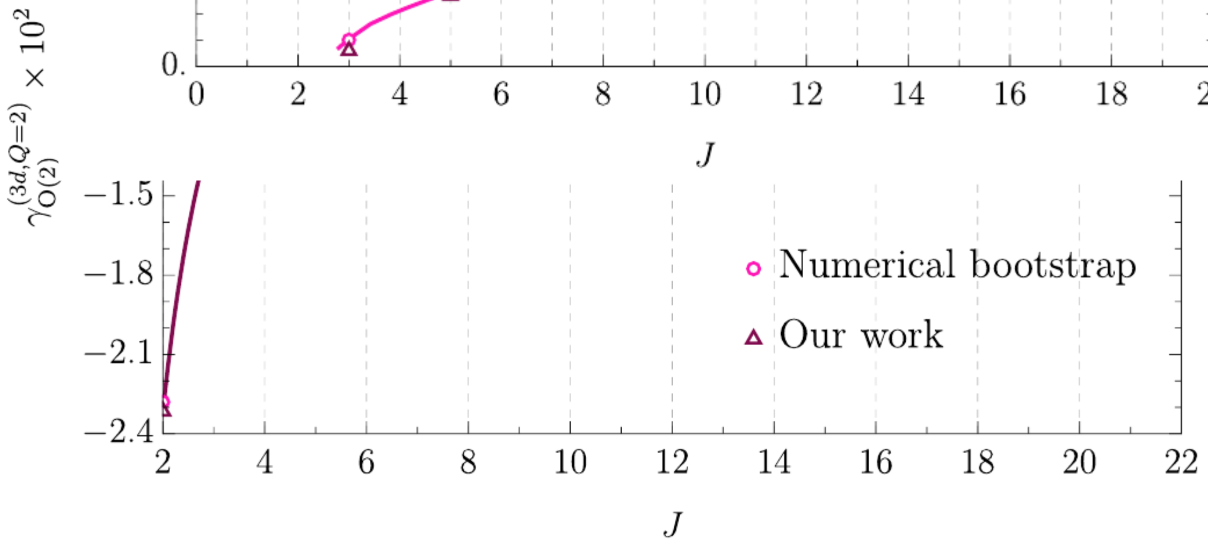


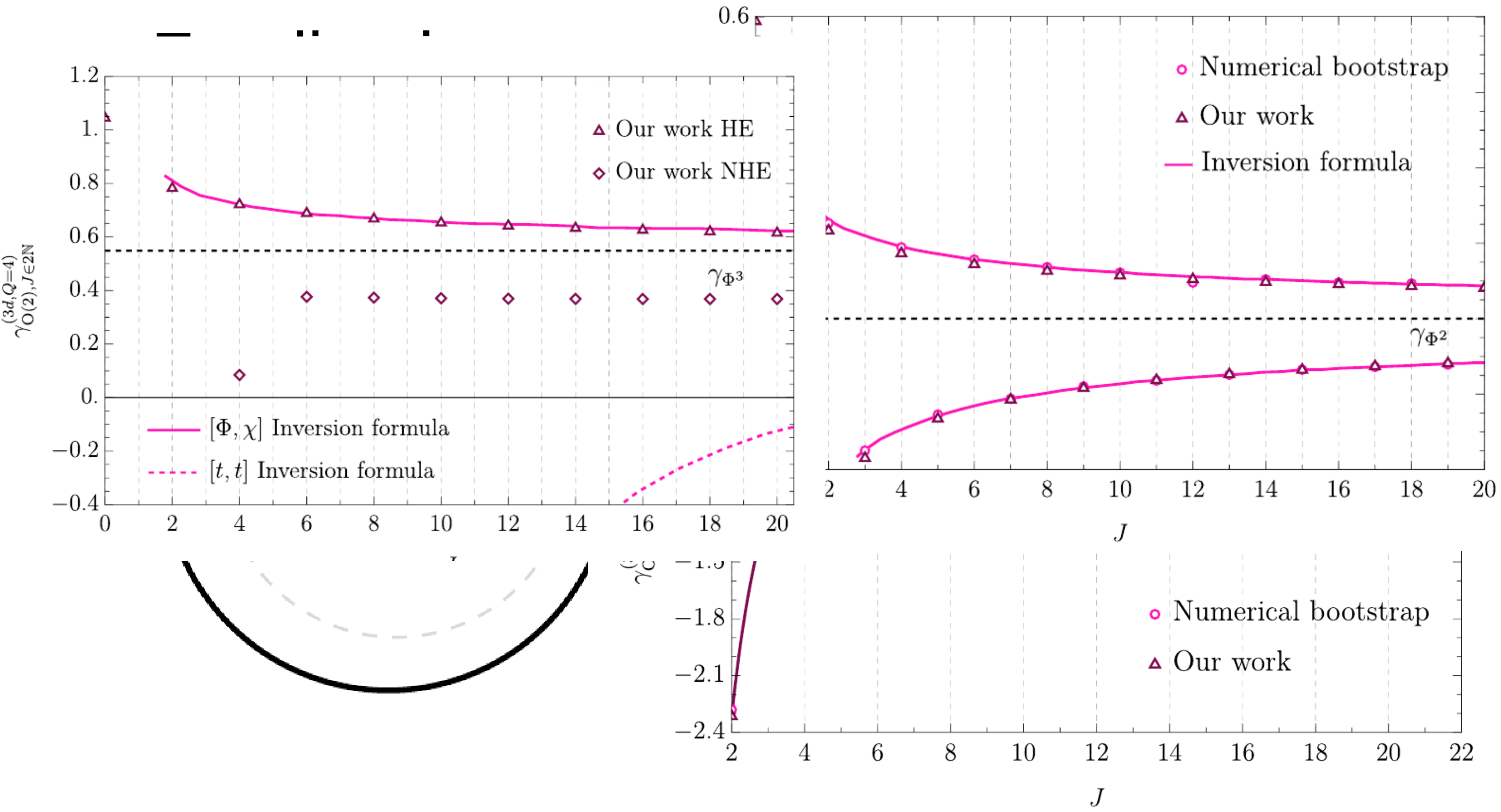


Feeding in non p
matching known

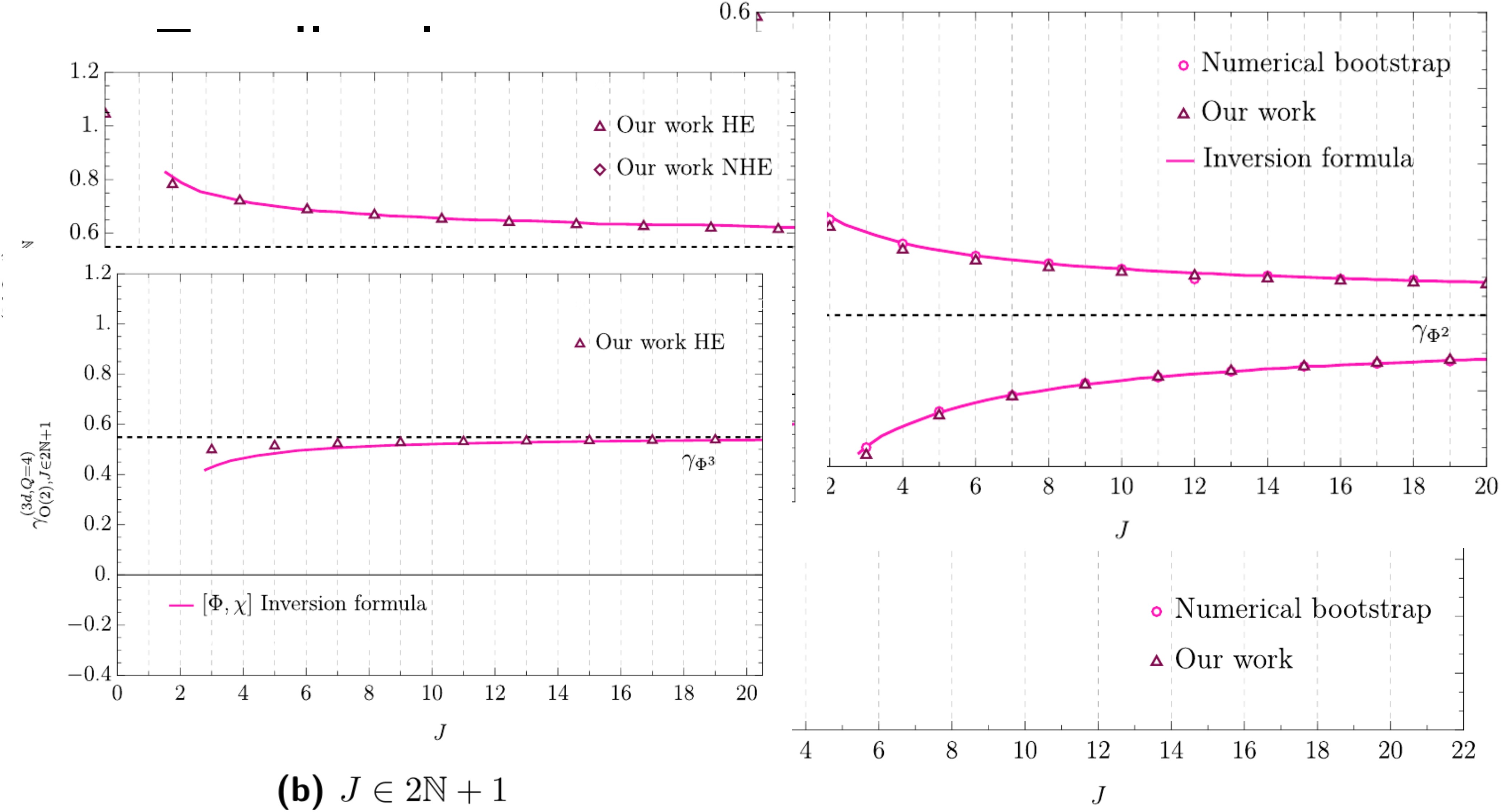




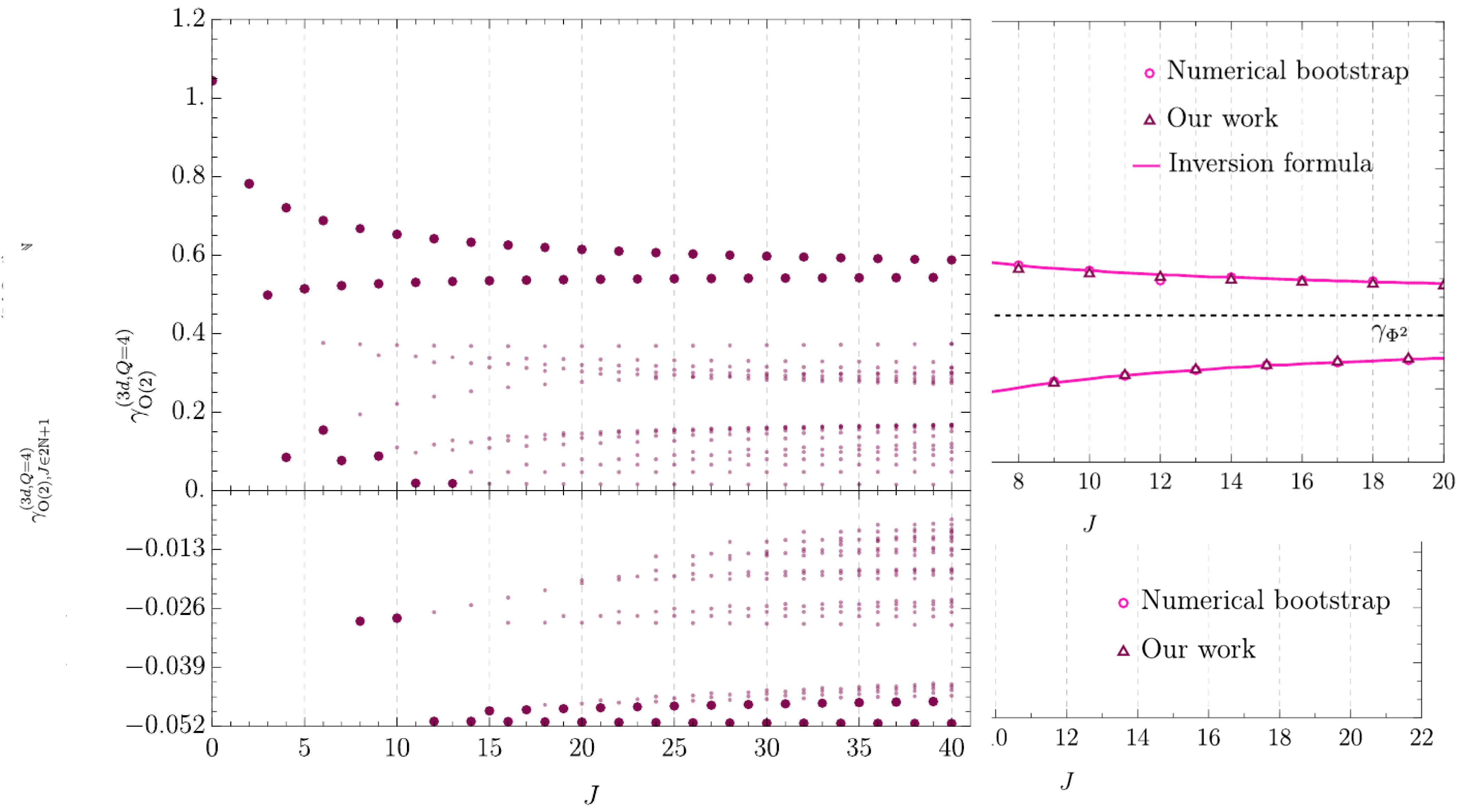




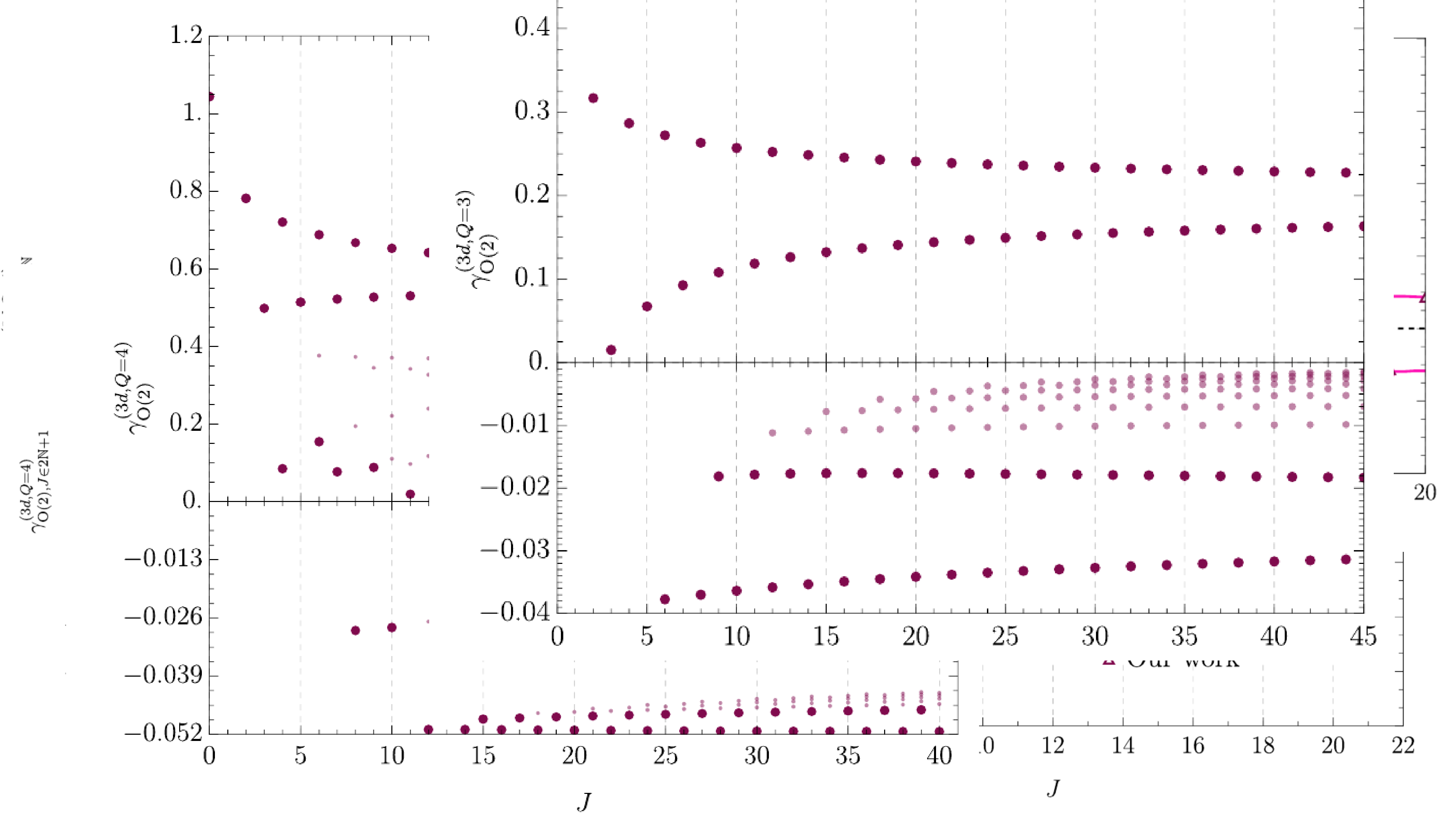
$\gamma^{(3d,Q=4)}_{O(2),J\in 2\mathbb{N}}$
Our work HE
Our work NHE
γ_{Φ^3}
$[\Phi,\chi]$ Inversion formula
$[t,t]$ Inversion formula
Numerical bootstrap
Our work
Inversion formula
γ_{Φ^2}
J
Numerical bootstrap
Our work
J



(b) $J \in 2\mathbb{N}+1$



Numerical bootstrap
Our work
Inversion formula
γ_{Φ^2}
$\gamma^{(3d,Q=4)}_{O(2)}$
$\gamma^{(3d,Q=4)}_{O(2),J\in 2N+1}$
J
Numerical bootstrap
Our work



$\gamma^{(3d,Q=4)}_{\mathrm{O}(2),J\in 2\mathbb{N}+1}$
$\gamma^{(3d,Q=4)}_{\mathrm{O}(2)}$
$\gamma^{(3d,Q=3)}_{\mathrm{O}(2)}$
J
Our work