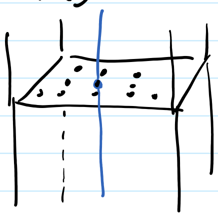
Generalised symmetries and Line Operator RG flows

Plan

(i) Introduction & Motivation
(ii) Generalised symm dictionary
(iii) 1-form symm. in unitary gauge th. (+ matter)
(iv) Line ops RG flows

(i) ① Org. principles ←
② selection rules ←

$d+1$ dim



bulk theory CFT

DCFT

UV: short distance

$\mathbb{R} \times SO(d-1)$

Line operator $\overset{IR}{\rightsquigarrow}$ Conformal line ops

$SL(2,\mathbb{R}) \times SO(d-1)$ DCFTs

$\mathbb{1} \times L = L$

$\langle O_b \rangle = \frac{1}{r_\perp}$

① in the IR $L \to$ trivial line $\mathbb{1}$

$\langle O \rangle = \frac{a(\Lambda)}{r_\perp} \to 0$

② $L \to \tilde{L}$

(ii)	Standard	Generalised
Conserv. current	$d * j^{(1)} = 0$ 1+0 form	$d * j^{(p+1)} = 0$ p+1
Charged ops	Loc ops $O(x)$ 0 dim	Extended ops $\mathcal{V}_q$ p dim
Ward idn.	$d * \langle j(x)\, O(x_1) \cdots O(x_n) \rangle = \sum q_a T^a \delta(x - x_i) \langle \cdots \rangle$	$d * \langle j^{(p+1)}(r)\, \mathcal{V}(r_1) \cdots \mathcal{V}(r_n) \rangle = \sum q_a T^a \delta^{(d-p-1)} \wedge \langle \cdots \rangle$
Charge op	$U_g(M^{d-1}) = \exp(ig \cdot \int * j)$ U_g top $U_g(M^{d-1})\, O(x)\, U_g^{-1} = R(g) \cdot O(x)$	$U_g(M^{d-p-1}) = \exp(ig \int * j^{(p+1)})$ U_g top $U_g(M^{d-1-p})\, \mathcal{V}(r)\, U_g(M^{d-1-p}) = R(g)\, \mathcal{V}(r)$

$$U_g \circ U_{g'} = \sum_{g'' \in C} U_{g''}$$

$[Q, \mathcal{O}] = R(g)\mathcal{O}$

$Q\mathcal{O}Q^{-1} - \mathcal{O} = R(g)\,\mathcal{O}\,Q^{-1}$

(3) 1-form symmetries

4d

Charge ops (top) S^2

U(1) Maxwell (+ matter)

$$\mathcal{L} = F \wedge * F$$

$$dF = 0 \qquad d*F = 0$$

$$U_g^{(e)} = \exp(ig\int_{S^2} *F) \qquad U_g^{(m)} = \exp(ig\int_{S^2} F)$$

$g \in U(1)$

$$U_g^{(e)} \mathcal{L}_{(e,m)} = \exp(ige)\,\mathcal{L}_{(e,m)} \qquad \Gamma^{(1)} = U_e(1) \times U_m(1)$$

$$U_g^{(m)} \mathcal{L}_{(e,m)} = \exp(igm)\,\mathcal{L}_{(e,m)}$$

$U(1) = \Gamma^{(1)} \longrightarrow G \subset \Gamma^{(1)}$ preserves the matter

charge q scalar field $\qquad e^{igq/2\pi} = 1$

$gq \in \mathbb{Z}$

$g \in \mathbb{Z}_q$

$\Gamma^{(1)} = U(1) \rightarrow \mathbb{Z}_q$

$SU(N)$ gauge th (pure)

$g \in Z(G)$

$\Gamma_e^{(1)} = \mathbb{Z}_N \qquad \Gamma_m^{(1)} = \phi \qquad g A g^{-1} = A$

$\Gamma_e^{(1)} = \mathbb{Z}_N$

$\boxed{G} \qquad \Gamma^{(1)}_e = Z(G)$

$SU(2) \qquad \mathbb{Z}_2 \qquad su(2) \qquad \Gamma^{(1)(e)}_{SU(2)} = \mathbb{Z}_2$

$SO(3) \qquad e \qquad \Gamma^{(1)(e)}_{SO(3)} = \phi$

$SU(2)$ $\quad \mathbb{Z}_2$ $\quad$ $[illegible]$ $\quad SU(2)$ $\quad \mathbb{Z}_2$

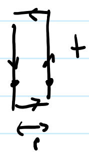
$SO(3)$ $\quad e$ $\quad P^{(1)(e)}_{SO(3)} = \phi$

$$SU(N)/\mathbb{Z}_k \quad ? \quad = P^{(1)}/\mathbb{Z}_k$$

+ add matter $\quad \mathbb{Z}_k \to \mathbb{Z}_k/\mathbb{Z}_q \quad q$

$$SU(3) \times SU(2) \times U(1)/\mathbb{Z}_3 \times \mathbb{Z}_2 \quad \text{(Tong)}$$

1-form symmetry



$\langle W \rangle = 0$ preserved & gapped

$\langle W \rangle \neq 0$ S.B. & gapless

$$\langle W \rangle \sim e^{-V(r)t} \;\nearrow e^{-A} \to 0 \qquad V(r) \gtrsim r$$
$$\qquad\qquad\qquad \searrow e^{-P} \neq 0 \qquad V(r) < r$$

$\langle W \rangle = 0$ preserved

$\langle W \rangle \neq 0$

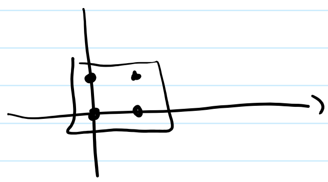
SSB of p-form symm

spin p excitation (Massless)

1 form S.B. $\rightsquigarrow$ photon

$SU(2)$ $\quad D_{(e,m)}$

$$e, m \in Z(G) \times Z(G) = \mathbb{Z}_2 \times \mathbb{Z}_2$$



$(0,0)$

$(1,0)$

$SO(3)_+$	$SO(3)_-$
$(0,1)$	$(1,1)$
$(0,0)$	$(0,0)$

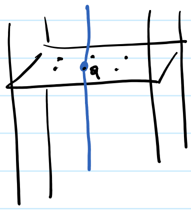
$SO(3)$ $\quad p^{(e)} = 0$

$\qquad\quad p^{(m)} = \mathbb{Z}_2$

$$W = \mathrm{Tr}_{\text{Vect}} \exp\left(A + n^I E_I\right)$$

$r \bmod 2 = 0$

(massless) Scalar QED (2+1) dim



CFT in the bulk

ϕ (= $q = 1$)

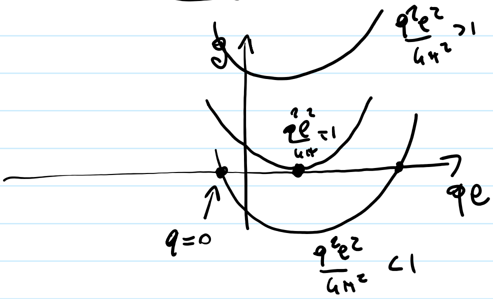
$$S = \int (F)^2 + |\partial\phi|^2 + |\phi|^4 + q\int dt (A_t + g\,\phi^\dagger\phi)$$

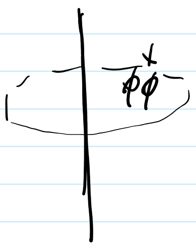
$E(r) \sim \frac{q^2}{r^2}$ "$\hat{\phi}^\dagger\hat{\phi}$"

$$\Delta_{\hat\phi^\dagger\hat\phi} = 1 + \sqrt{1 - \underbrace{\frac{q^2e^2}{4\pi^2}}}$$

$q=0$ $\Delta_{\phi^\dagger\phi} = 2$

$g \to \infty$ $e^{\Lambda/\sqrt{1-\cdots}}$





IR

scre... line

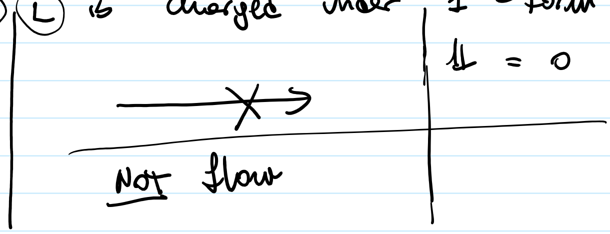
$q \sim 67$

137 $\to$ 6

$q \sim 7$ $\Lambda \sim 10^{-2}$ m

$\Gamma^{(1)} = U(1) \to e$

O ≠ L is charged under 1-form symm



no d-2 form symmetries

SU(2) G.T. $\mathbb{Z}_2$ $q\int (A_t dt + \cancel{g\Phi^\dagger\Phi} + n^I \Phi_I)$

there are just odd lines in the IR (at s.c.)

$\mathcal{N}=4$ SYM $SU(2) \overset{S}{\leftrightarrow} SO(3)$

't Hooft at w.c.