Broken global symmetry and defect conformal manifolds

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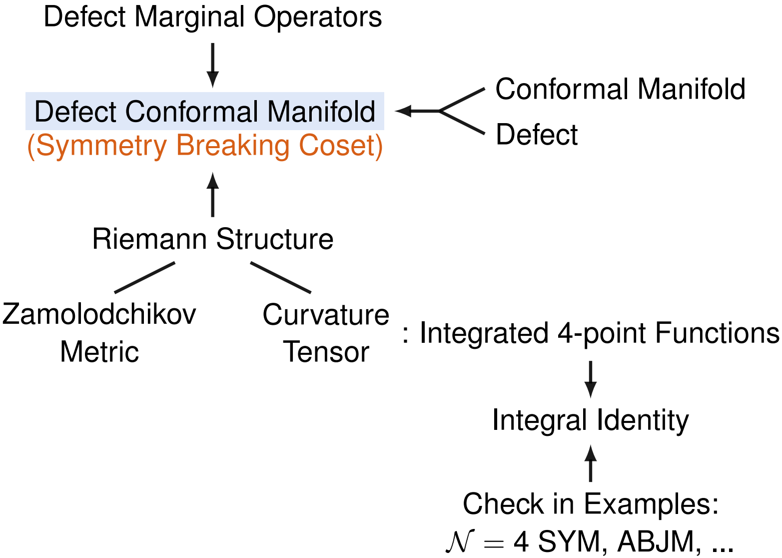
Based on arXiv: 2003.17157 with N. Drukker and G. Sakkas
and work in progress

Content


Defect Marginal Operators
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(Symmetry Breaking Coset)
Conformal Manifold
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: Integrated 4-point Functions
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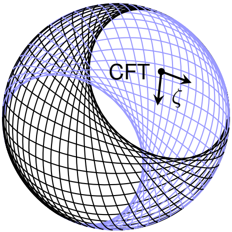
Check in Examples:
$\mathcal{N} = 4$ SYM, ABJM, ...

Conformal Manifold

A conformal manifold $\mathcal{M}_{\text{CFT}}$ is a family of CFT's parametrized by couplings $\{\zeta^i\}$. For each value of the couplings a different point on the manifold is a different CFT.

$$S \to S + \sum_i \zeta^i \int d^D x \mathcal{O}_i$$



where $\mathcal{O}_i$ are exactly marginal operators.

This manifold admits a Riemannian structure:

- Zamolodchikov Metric
- Curvature tensor

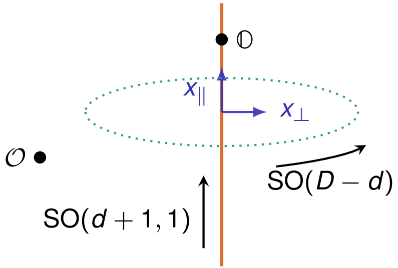
Defect

A conformal defect of dimension d in a D dimensional CFT breaking

- conformal symmetry:

$$SO(D+1,1) \to SO(d+1,1) \times SO(D-d)$$

- **global-symmetry**: $G \to G'$



Defect Conformal Manifold

- Deforming a defect by **exactly marginal** operators $\mathbb{O}_i$ where $\Delta_{\mathbb{O}_i} = d$, correlation functions of any operator ϕ_a

$$\langle\!\langle \phi_{a_1} \cdots \phi_{a_n} \rangle\!\rangle \to \langle\!\langle e^{-\int \zeta^i \mathbb{O}_i d^d x} \phi_{a_1} \cdots \phi_{a_n} \rangle\!\rangle$$

  - $\langle\!\langle \cdots \rangle\!\rangle$: defect correlation function normalized by the expectation value of the defect.
- If the defect breaks $G \to G'$, the conservation equation is

$$\partial_\mu J^{\mu a} = \mathbb{O}_i(x_\parallel)\delta^{ia}\delta^{D-d}(x_\perp)$$

generators of G

generators broken by the defect

When a defect breaks a global symmetry, the resulting defect conformal manifold is the symmetry breaking coset.

Geometric Structure

Similar to the ambient conformal manifold, one can define

- Zamolodchikov Metric [Zamolodchikov]

$$g_{ij} = \langle\!\langle \mathbb{O}_i(\infty)\mathbb{O}_j(0) \rangle\!\rangle = C_{\mathbb{O}}\delta_{ij}, \quad \mathbb{O}_i(\infty) \equiv \lim_{x_\parallel \to \infty} |x_\parallel|^{2d}\mathbb{O}_i(x_\parallel)$$

- Curvature tensor [Kutasov]

$$R_{ijkl} = \frac{1}{2}(\partial_j\partial_k g_{il} - \partial_i\partial_k g_{jl} - \partial_j\partial_l g_{ik} + \partial_i\partial_l g_{jk})$$

  - Take the first term for instance

$$\partial_j\partial_k g_{il} = \left(\partial_j\partial_k \langle\!\langle e^{-\int \zeta^i \mathbb{O}_i d^d x}\mathbb{O}_i(\infty)\mathbb{O}_l(0) \rangle\!\rangle \right)\Big|_{\zeta^i=0}$$

$$= \iint d^d x_1 d^d x_2 \langle\!\langle \mathbb{O}_j(x_1)\mathbb{O}_k(x_2)\mathbb{O}_i(\infty)\mathbb{O}_l(0) \rangle\!\rangle$$

The curvature tensor a sum of double integrals of 4-pt functions $\Rightarrow$ integrals over cross-ratios. [Friedan, Konechny]

Example I: Maldacena-Wilson loop in $\mathcal{N} = 4$ SYM

The 1/2 BPS Wilson loop

$$W = \mathrm{Tr}\,\mathcal{P} e^{\int (iA_0 + \Phi_6) dt}$$

The breaking of global symmetry (R-symmetry) gives 5 exactly marginal defect operators Φ_i.

The defect conformal manifold is $S^5 = SO(6)/SO(5)$.

- Zamolodchikov Metric: $g_{ij} = \langle\!\langle \Phi_i(0)\Phi_j(1) \rangle\!\rangle = C_\Phi \delta_{ij}$ with C_Φ twice the bremsstrahlung function [Drukker, Forini], [Correa, Henn, Maldacena, Sever], [Fiol, Garolera, Lewkowycz]

$$C_\Phi = \begin{cases} \frac{\lambda}{8\pi^2} - \frac{\lambda^2}{192\pi^2} + \frac{\lambda^3}{3072\pi^2} - \frac{\lambda^4}{46080\pi^2} + O(\lambda^5), & \lambda \ll 1 \\ \frac{\sqrt{\lambda}}{2\pi^2} - \frac{3}{4\pi^2} + \frac{3}{16\pi^2\sqrt{\lambda}} + \frac{3}{16\pi^2\lambda} + O(\lambda^{-3/2}), & \lambda \gg 1 \end{cases}$$

Curvature Tensor

Curvature Tensor: [Friedan, Konechny]

$$R_{ijkl} = -\mathrm{RV}\int_{-\infty}^{+\infty} d\eta \log|\eta| \Big[\langle\!\langle \Phi_i(1)\Phi_j(\eta)\Phi_k(\infty)\Phi_l(0) \rangle\!\rangle_c + \langle\!\langle \Phi_i(0)\Phi_j(1-\eta)\Phi_k(\infty)\Phi_l(1) \rangle\!\rangle_c \Big] \quad (*)$$

- 4-pt function: $\langle\!\langle \Phi_i(0)\Phi_j(\eta)\Phi_k(1)\Phi_l(\infty) \rangle\!\rangle$

$$= \frac{C_\Phi^2}{\eta^2}\left(\delta_{ik}\delta_{jl} h_2(\eta) + \delta_{il}\delta_{jk} h_1(\eta) + \delta_{ij}\delta_{kl} h_0(\eta)\right)$$

Plugging into $(*)$

$$R_{ijkl} = 2(g_{ik}g_{jl} - g_{il}g_{jk}) \int_0^1 \frac{d\eta}{\eta^2} \log(\eta)\, (h_2 + h_1 - 2h_0)$$

Introduce $H = h_2 + h_1 - 2h_0$.

Integral Identity

Generally, for a sphere with radius r we have

$$R_{ijkl} = (g_{ik}g_{jl} - g_{il}g_{jk})/r^2$$

From the Zamolodchikov Metric, we expect $r = \sqrt{C_\Phi}$.

So the key point is to check

$$2\int_0^1 \frac{d\eta}{\eta^2} \log(\eta) H(\eta) = \frac{1}{C_\Phi}$$

Strong Coupling Expansion

Expanding in power series: $H(\eta) = \sum_{n=1}^{\infty} \lambda^{-\frac{n}{2}} H^{(n)}(\eta)$

For the first orders: [Giombi, Roiban, Tseytlin], [Liendo, Meneghelli, Mitev], [Ferrero, Meneghelli]

Basis of $H^{(n)}(\eta)$:

$$n = 1 : \{\log(\eta), \log(1-\eta)\}$$

$$n = 2 : \{\log^2(\eta), \log(\eta)\log(1-\eta), \log(1-\eta)^2, \mathrm{Li}_2(\eta)\}$$

$$n = 3 : \{\log^3(\eta), \log^2(\eta)\log(1-\eta), \log(\eta)\log^2(1-\eta), \log(1-\eta)^3, \mathrm{Li}_2(\eta)\log(\eta), \mathrm{Li}_2(\eta)\log(1-\eta), \mathrm{Li}_3(\chi), \mathrm{Li}_3(\frac{\eta}{\eta-1})\}$$

$$n = 4 : \{\log^4(\eta), \mathrm{Li}_2(\eta)\log^2(\eta), \mathrm{Li}_3(\eta)\log(\eta), \mathrm{Li}_4(\eta), \cdots\}$$

The integrand involves terms of the form

$$r(x)\log^a x \log^b(1-x)\,\mathrm{Li}_n(y(x)), \quad y(x) \in \left\{x,\ 1-x,\ \frac{x}{x-1}\right\}$$

where $r(x)$ is a rational function with poles at $x = 0$ and $x = 1$.

At tree level $O(\lambda^{-1/2})$

Decomposing $H^{(1)}$ with its basis [Giombi, Roiban, Tseytlin], [Liendo, Meneghelli, Mitev], [Ferrero, Meneghelli]

$$
\begin{aligned}
H^{(1)} =& \frac{-2\eta^5 + 3\eta^4 + 2\eta^3 - 11\eta^2 + 12\eta - 4}{(\eta - 1)^3} 1 \\
&+ \frac{2\left(\eta^4 - 2\eta^3 + 4\eta - 2\right)\eta^2}{(\eta - 1)^3} \log(\eta) \\
&- \frac{2\left(\eta^4 + \eta^3 + \eta - 2\right)}{\eta} \log(1 - \eta)
\end{aligned}
$$

- The integral

$$2 \int_0^1 \frac{d\eta}{\eta^2} \log(\eta) H^{(1)}(\eta) = 2\pi^2$$

Matching results: The leading term of $1/C_\Phi$ is $2\pi^2/\sqrt{\lambda}$.

At 1-loop order $O(\lambda^{-1})$

[Liendo, Meneghelli, Mitev], [Ferrero, Meneghelli]

$$
\begin{aligned}
H^{(2)} =& \frac{2\left(\eta^4+\eta^3-3\eta^2+4\eta-2\right)}{(\eta-1)^2} 1 \\
&+ \frac{-26\eta^6+33\eta^5+63\eta^4-184\eta^3+72\eta^2+24\eta-8}{4(\eta-1)^3} \log(\eta) \\
&+ \frac{26\eta^6-3\eta^5-72\eta^4+61\eta^3-90\eta^2+126\eta-52}{4(\eta-1)^2\eta} \log(1-\eta) \\
&+ \frac{\eta^2\left(9\eta^6-28\eta^5+12\eta^4+52\eta^3-76\eta^2+60\eta-20\right)}{2(\eta-1)^4} \log^2(\eta) \\
&+ \frac{-9\eta^8+18\eta^7+9\eta^6-45\eta^5+37\eta^4-5\eta^3-9\eta^2+7\eta-2}{(\eta-1)^3\eta} \log(\eta)\log(1-\eta) \\
&+ \frac{9\eta^6+10\eta^5-8\eta^4+\eta^2+16\eta-18}{2\eta^2} \log^2(1-\eta)
\end{aligned}
$$

- The integral

$$2\int_0^1 \frac{d\eta}{\eta^2}\log(\eta)H^{(2)}(\eta) = 3\pi^2$$

Matching results: The subleading term of $1/C_\Phi$ is $3\pi^2/\lambda$.

At 2-loop order $O(\lambda^{-3/2})$

[Liendo, Meneghelli, Mitev], [Ferrero, Meneghelli]

- $H^{(3)} =$

$$
\begin{aligned}
&\frac{(-660x^2+3648x^3-8322x^4+10770x^5-10116x^6+8172x^7-4488x^8+420x^9+906x^{10}-330x^{11})\,\mathrm{Log}[1-x]}{24(-1+x)^5x^5}+\\
&\frac{(-1368x+8124x^2-19872x^3+25488x^4-18756x^5+11652x^6-13584x^7+14760x^8-7032x^9-1146x^{10}+2400x^{11}-666x^{12})\,\mathrm{Log}[1-x]^2}{24(-1+x)^5x^5}+\\
&\frac{(-576+3636x-9620x^2+13670x^3-10990x^4+4796x^5-1900x^6+4184x^7-7664x^8+7020x^9-2304x^{10}-1170x^{11}+1206x^{12}-288x^{13})\,\mathrm{Log}[1-x]^3}{24(-1+x)^5x^5}+\\
&\left(\frac{-36x^3+180x^4-1902x^5+6528x^6-9204x^7+5520x^8-528x^9-888x^{10}+330x^{11}}{24(-1+x)^5x^5}+\right.\\
&\quad\frac{(120x^2-660x^3+1356x^4+888x^5-11640x^6+27456x^7-30480x^8+14076x^9+2376x^{10}-4824x^{11}+1332x^{12})\,\mathrm{Log}[1-x]}{24(-1+x)^5x^5}+\\
&\quad\left.\frac{(108x-636x^2+1554x^3-2010x^4+996x^5+3300x^6-13116x^7+23196x^8-21060x^9+6912x^{10}+3510x^{11}-3618x^{12}+864x^{13})\,\mathrm{Log}[1-x]^2}{24(-1+x)^5x^5}\right)\mathrm{Log}[x]+\\
&\left(\frac{-2040x^5+8160x^6-14520x^7+15000x^8-7104x^9-1272x^{10}+2442x^{11}-666x^{12}}{24(-1+x)^5x^5}+\frac{(600x^5-4344x^6+13908x^7-23892x^8+21360x^9-6720x^{10}-3720x^{11}+3672x^{12}-864x^{13})\,\mathrm{Log}[1-x]}{24(-1+x)^5x^5}\right)\mathrm{Log}[x]^2+\\
&\frac{(-400x^5+1600x^6-4736x^7+8608x^8-7280x^9+2080x^{10}+1388x^{11}-1260x^{12}+288x^{13})\,\mathrm{Log}[x]^3}{24(-1+x)^5x^5}+\\
&\left(\frac{72(-1+x)x-312(-1+x)x^2+492(-1+x)x^3-360(-1+x)x^4+240(-1+x)x^5-384(-1+x)x^6+432(-1+x)x^7-168(-1+x)x^8-48(-1+x)x^9+36(-1+x)x^{10}}{24(-1+x)^5x^5}+\right.\\
&\quad\left.\frac{(18-88x+171x^2-164x^3+70x^4+24x^5-76x^6+56x^7+6x^8-26x^9+9x^{10})\,\mathrm{Log}[1-x]}{2(-1+x)^4x^4}-\frac{(4-18x+32x^2-28x^3+48x^4-92x^5+52x^6+12x^7-28x^8+9x^9)\,\mathrm{Log}[x]}{2(-1+x)^4x^3}\right)\mathrm{PolyLog}[2,x]+\\
&\frac{(-6+20x-21x^2+9x^3-11x^4+21x^5-15x^6-x^7+3x^8)\,\mathrm{PolyLog}\left[2,\frac{x}{-1+x}\right]}{2(-1+x)^3x^4}+\frac{(216x-1320x^2+3372x^3-4620x^4+3528x^5-1464x^6+480x^7-144x^8-120x^9+96x^{10}-24x^{11})\,\mathrm{PolyLog}[3,x]}{24(-1+x)^5x^5}+\\
&\frac{(216x-1272x^2+3108x^3-4020x^4+2808x^5-552x^6-1200x^7+1584x^8-600x^9-384x^{10}+420x^{11}-108x^{12})\,\mathrm{PolyLog}\left[3,\frac{x}{-1+x}\right]}{24(-1+x)^5x^5}+\frac{1}{24(-1+x)^5x^5}\\
&(90x^3-450x^4+810x^5-540x^6-135x^7+405x^8-225x^9+45x^{10}+192x^5\,\mathrm{Zeta}[3]-768x^6\,\mathrm{Zeta}[3]+1392x^7\,\mathrm{Zeta}[3]-1488x^8\,\mathrm{Zeta}[3]+600x^9\,\mathrm{Zeta}[3]+384x^{10}\,\mathrm{Zeta}[3]-\\
&\quad 420x^{11}\,\mathrm{Zeta}[3]+108x^{12}\,\mathrm{Zeta}[3])
\end{aligned}
$$

- The integral $\quad 2\int_0^1 \frac{d\eta}{\eta^2}\log(\eta)H^{(3)}(\eta) = \frac{15\pi^2}{4}$

Matching results: $1/C_\Phi$ at order $O(\lambda^{-3/2})$ is $\frac{15\pi^2}{4\lambda^{3/2}}$.

At 3-loop order $O(\lambda^{-2})$

[Liendo, Meneghelli, Mitev], [Ferrero, Meneghelli]

- $H^{(4)} =$

- The integral $\quad 2\int_0^1 \frac{d\eta}{\eta^2}\log(\eta)H^{(4)}(\eta) = \frac{15\pi^2}{4}$

Matching results: $1/C_\Phi$ at order $O(\lambda^{-2})$ is $\frac{15\pi^2}{4\lambda^2}$.

Overall, we see that

$$2\int_0^1 \frac{d\eta}{\eta^2}\log(\eta)H(\eta) = \frac{2\pi^2}{\sqrt{\lambda}} + \frac{3\pi^2}{\lambda} + \frac{15\pi^2}{4\lambda^{\frac{3}{2}}} + \frac{15\pi^2}{4\lambda^2} + O(\lambda^{-\frac{5}{2}}) = \frac{1}{C_\Phi}$$

Example II: 1/2 BPS loop in ABJM

The 1/2 BPS Wilson loop $W = \mathrm{Tr}\, \mathcal{P} e^{\int i\mathcal{L}dt}$ [Drukker, Trancanelli]

$$\mathcal{L}_1 = \begin{pmatrix} A_\tau^{(1)} + \alpha\bar{\alpha}(M_1)^I_J C_I \bar{C}^J & i\bar{\alpha}\bar{\psi}^1_+ \\ -i\alpha\psi_1^+ & A_\tau^{(2)} + \alpha\bar{\alpha}(M_1)^I_J \bar{C}^J C_I \end{pmatrix}$$

with $M_1 = \mathrm{diag}(-1,1,1,1)$ and $\bar{\alpha}\alpha = -2\pi i/k$. Now the 3 pairs of exactly marginal defect operators $\mathbb{O}_i, \bar{\mathbb{O}}_{\bar{\imath}}$ are chiral.

Defect conformal manifold is $\mathbb{CP}^3 = SU(4)/SU(3)\times U(1)$.

- Zamolodchikov Metric: $g_{i\bar{\jmath}} = \langle\!\langle \mathbb{O}_i(0)\bar{\mathbb{O}}_{\bar{\jmath}}(1)\rangle\!\rangle = 4B_{1/2}\delta_{i\bar{\jmath}}$ with the bremsstrahlung function $B_{1/2} = \sqrt{2\lambda}/4\pi + \ldots$ [Lewkowycz, Maldacena], [Bianchi, Griguolo, Mauri, Penati, Preti, Seminara], [Bianchi, Preti, Vescovi]

Curvature Tensor

- 4-pt functions [Bianchi, Bliard, Forini, Griguolo, Seminara]:

$$\langle\!\langle \mathbb{O}_i(x_1)\bar{\mathbb{O}}_{\bar{\jmath}}(x_2)\mathbb{O}_k(x_3)\bar{\mathbb{O}}_{\bar{l}}(x_4)\rangle\!\rangle = \frac{g_{i\bar{\jmath}}g_{k\bar{l}}K_1 - g_{i\bar{l}}g_{k\bar{\jmath}}K_2}{x_{12}^2 x_{34}^2}$$

$$\langle\!\langle \mathbb{O}_i(x_1)\bar{\mathbb{O}}_{\bar{\jmath}}(x_2)\bar{\mathbb{O}}_{\bar{k}}(x_3)\mathbb{O}_l(x_4)\rangle\!\rangle = \frac{g_{i\bar{\jmath}}g_{l\bar{k}}H_1 - g_{i\bar{k}}g_{l\bar{\jmath}}H_2}{x_{12}^2 x_{34}^2}$$

Plugging into $(*)$

$$R_{i\bar{j}k\bar{l}} = (g_{i\bar{l}}g_{j\bar{k}} - g_{i\bar{k}}g_{j\bar{l}})\mathcal{R}_1\,, \quad R_{i\bar{\jmath}k\bar{l}} = (g_{i\bar{l}}g_{k\bar{\jmath}} + g_{i\bar{\jmath}}g_{k\bar{l}})\mathcal{R}_2$$

where

$$\mathcal{R}_1 = \int_0^1 \frac{d\chi}{\chi^2}\left[\log\frac{\chi}{1-\chi}(K_1+K_2) + 2\log\chi(H_1+H_2)\right]$$

$$\mathcal{R}_2 = \int_0^1 \frac{d\chi}{\chi^2}\left[\log(1-\chi)(2H_1 - 2H_2 - K_1) + \log\chi(2H_2+K_2)\right]$$

Strong coupling expansion at tree level

- At tree level
  - Explicit expressions

$$
\begin{aligned}
H_1 =& \epsilon \left(-\chi^2 \log\chi + \left(\chi^2 - \frac{4}{\chi} + 3\right)\log(1-\chi) + \chi - 4 \right) \\
H_2 =& \epsilon \left(\chi^2(3-4\chi)\log\chi + (\chi-1)(4\chi^2+\chi+1)\log(1-\chi) + 4\chi^2 - \chi \right) \\
K_1 =& \epsilon \left(-\frac{\chi^2}{(1-\chi)^2}\log\chi + \frac{\chi-4}{\chi}\log(1-\chi) - \frac{3\chi^2-7\chi+4}{(1-\chi)^2} \right) \\
K_2 =& \epsilon \left(\log(1-\chi) + \frac{\chi^2(\chi+3)}{(1-\chi)^3}\log\chi - \frac{\chi\left(3\chi^2-2\chi-1\right)}{(1-\chi)^3} \right)
\end{aligned}
$$

  - $\epsilon = 1/(2\pi\sqrt{2\lambda})$.
  - The integrals $\mathcal{R}_1 = 0$, $\mathcal{R}_2 = \pi/\sqrt{2\lambda} \approx 1/4B_{1/2}$.
- Higher order?

Example III: 1/3 BPS loop in ABJM

- Another 1/2 BPS Wilson loop where

$$\mathcal{L}_4 = \begin{pmatrix} A_\tau^{(1)} + \alpha\bar{\alpha}(M_4)^I_{\ J} C_I \bar{C}^J & i\bar{\alpha}\bar{\psi}^4_- \\ -i\alpha\psi_4^- & A_\tau^{(2)} + \alpha\bar{\alpha}(M_4)^I_{\ J} \bar{C}^J C_I \end{pmatrix}$$

with $M_4 = \mathrm{diag}(-1, -1, -1, 1)$.
• Other 3 pairs of exactly marginal defect operators $\mathbb{O}_a, \bar{\mathbb{O}}^a$.

- The simplest 1/3 BPS Wilson loop

$$\mathcal{L}' = \begin{pmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_4 \end{pmatrix}$$

- Permutation operator $\sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \sigma\mathcal{L}'\sigma^{-1} = \begin{pmatrix} \mathcal{L}_4 & 0 \\ 0 & \mathcal{L}_1 \end{pmatrix}$

Exactly marginal operators of 1/3 BPS loops

The 10 broken R-symmetry generators

$$J^2_1 \quad J^1_2 \quad J^3_1 \quad J^1_3 \quad J^4_1 \quad J^1_4 \quad J^4_2 \quad J^2_4 \quad J^4_3 \quad J^3_4$$

give 5 pairs of exactly marginal operators

$$\mathbb{O}^2 = \begin{pmatrix} \mathbb{O}^2 & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbb{O}^3 = \begin{pmatrix} \mathbb{O}^3 & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbb{O} = \begin{pmatrix} \mathbb{O}^4 & 0 \\ 0 & \bar{\mathbb{O}}_1 \end{pmatrix}$$

$$\mathbb{O}_2 = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{O}_2 \end{pmatrix} \quad \mathbb{O}_3 = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{O}_3 \end{pmatrix}$$

and their conjugates.

Defect conformal manifold is $SU(4)/SU(2)\times U(1)\times U(1)$?

Geometric Properties

- Metric

$$g_{i\bar{j}} = 2B_{1/2}\delta_{i\bar{j}}, \quad g_{3\bar{3}} = 4B_{1/2}, \quad g_{a\bar{b}} = 2B_{1/2}\delta_{a\bar{b}}$$

- A naive prediction of non-vanishing components of curvature tensor

$$R_{i\bar{j}k\bar{l}} = \frac{1}{2B_{1/2}}(g_{i\bar{l}}g_{k\bar{j}} + g_{i\bar{j}}g_{k\bar{l}})$$

$$R_{3\bar{3}3\bar{3}} = \frac{1}{2B_{1/2}}g_{3\bar{3}}g_{3\bar{3}}$$

$$R_{a\bar{b}c\bar{d}} = \frac{1}{2B_{1/2}}(g_{a\bar{d}}g_{c\bar{b}} + g_{a\bar{b}}g_{c\bar{d}})$$

$$R_{i\bar{3}3\bar{l}} = \frac{1}{4B_{1/2}}g_{i\bar{l}}g_{3\bar{3}}, \quad R_{a\bar{3}3\bar{d}} = \frac{1}{4B_{1/2}}g_{a\bar{d}}g_{3\bar{3}}$$

and their permutations.

Coset manifold $SU(4)/SU(2)\times U(1)\times U(1)$

- Components in directions within single $\mathbb{CP}^3$ match the prediction.
- Four unusual components and their permutations

$$R_{1\bar{1}4\bar{4}},\qquad R_{2\bar{2}5\bar{5}},\qquad R_{1\bar{2}4\bar{5}},\qquad R_{2\bar{1}5\bar{4}}$$

- How to address them?
  - Integrated four-point functions?

$$\iint dx_1 dx_2 \left\langle\!\left\langle \mathbb{O}^2(x_1)\bar{\mathbb{O}}_2(x_2)\mathbb{O}_2(0)\bar{\mathbb{O}}^2(\infty)\right\rangle\!\right\rangle$$

  - Two-point functions?

$$\left\langle\!\left\langle \mathbb{O}^2(x_1)\bar{\mathbb{O}}_2(x_2)\right\rangle\!\right\rangle + \left\langle\!\left\langle \mathbb{O}_2(x_1)\bar{\mathbb{O}}^2(x_2)\right\rangle\!\right\rangle$$

Other examples

- At weak coupling in SYM up to 2-loop order
  [Cavaglia, Gromov, Julius, Preti]
- 1/2 BPS surface operator in 6d $\mathcal{N} = (2,0)$ theory
  [Drukker, Giombi, Tseytlin, Zhou]

$$S^4 = SO(5)/SO(4)$$

Defect conformal manifold is the symmetry breaking coset

$$\mathcal{M}_{\mathrm{dCFT}} = G/G'$$

Outlook

1. $\mathcal{M}_{\text{dCFT}} = G/G' \Rightarrow \mathcal{M}_{\text{dCFT}} \supset G/G'$?
2. Generalization:
   i) More examples
      - in 2d CFT

   ii) A manifold comprised of
      - Defect marginal & non-marginal operators
      - Bulk & defect marginal operators
3. Similar constraints for higher point functions
4. $\cdots$

Thank you!