MESONIC BOUND STATES IN QCD_2

Based on [F.A., S. Komatsu] to appear soon

PLAN (i) Introduction & Motivation
(ii) Review QCD_2 & mesons
(iii) Analytical solution for mesons' spectral problem
(iv) 3-, 4- point amplitudes & further applications.

STRONG INTERACTION!

~~4d~~ 2d QCD ~~SU(3)~~ SU(N) $N\to\infty$ is confining in the IR?

Spectrum? → MESONS

Why 2d QCD?

FIELD THEORY

- 2d QCD confining!

STRING T. perspective

(i) ST confining
(ii) (AdS_3) Can we do the same?

Overall: SIMPLE (?) INSI...

(ii) $\mathcal{L} = \frac{1}{4} F^{\mu\nu}F_{\mu\nu} + \bar\psi_A (i\not\partial - m_A)\psi_A$ $\quad A=1,\dots,N_f$ $\quad \sigma \in SU(N)$

$x_\pm = x_0 \pm x_1 \qquad A_- = 0 = A^+$

$A_+(y) = \int d^2x\, |x-y|\, \psi^+_A(x)\psi_A(x) = \int d^2x\, j(x)\, |x-y|$

$\mathcal{L} \supset \int d^2x\, d^2y\, |x-y|\, j(x)\, j(y)$

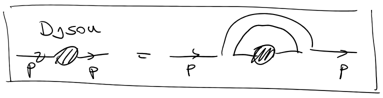
$N\to\infty \qquad g^2_{YM} N = g^2 \qquad \boxed{N=\infty}$

$\rightarrow \; = \frac{ip_-}{2p_+p_- - m^2 + i\epsilon}$

$= -ig$

$= -i/p_-^2$

Dyson



$= \Gamma(p)$

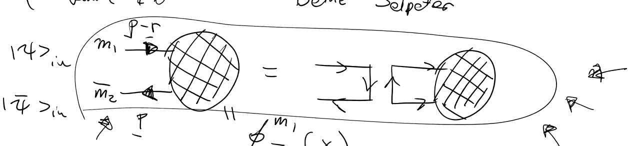
$\Gamma(p) = \frac{g^2 N}{\pi}\left(\frac{\mathrm{sgn}(p_-)}{\epsilon} - \frac{1}{p_-}\right) \qquad \epsilon \to 0 \quad$ IR cutoff

$$\boxed{S(p) = \frac{ip_-}{2p_+p_- - m^2 + i\epsilon + \frac{g^2N}{\pi}\left(\frac{|p_-|}{\epsilon} - \mathrm{sgn}(p_-)\right)}}$$

$m^2 \to M^2 = m^2 - \frac{g^2N}{\pi}\left(\frac{|p_-|}{\epsilon} - \mathrm{sgn}(p_-)\right)$

$\epsilon \to 0$

ϵ finite $\neq 0$ Bethe-Salpeter



$= \phi^{m_1}_{m_2}(x)$

$\frac{p-r}{p} = x \in [0,1] \qquad \boxed{x = x_1}$

$$\boxed{\varphi^{m_1}_{m_2}(x) = \int dx_0\, \phi(x)}$$

$$\boxed{\psi_{\bar{m}_2}(x) = \int^1 dx_0 \; \phi(\underline{x})}$$

$$\boxed{\mu^2 \varphi^{m_1}_{\bar{m}_2}(x) = \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x}\right)\varphi(x) - \fint_0^1 dy \frac{\varphi(y)}{(x-y)^2}}$$

$$\boxed{\alpha_i = \frac{m_i^2 g^2 N}{\pi} - 1}$$

$\alpha_1 = \alpha_2 = 0$

$\int dy \frac{1}{(x-y)^2} \sim \mathcal{O}(1/\rho_1)$

$\varphi(x) \overset{x \sim 0}{\sim} x^{\pm \beta_1}$

$\varphi(x) \overset{x \sim 1}{\sim} (1-x)^{\pm \beta_2}$

$\boxed{\pi \beta_i \cot \pi \beta_i + \alpha_i = 0}$

1 of the 2

$\varphi(x) \equiv 1$ $\qquad \varphi(x) \in \mathbb{L}^2(0,1)$

$\mu^2 \to \infty$ $\quad$ sch kx

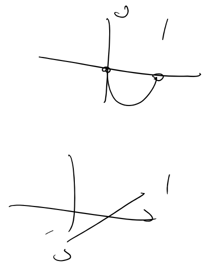
$$\boxed{\Phi(\nu) = \int \frac{dx}{x(1-x)} \left(\frac{x}{1-x}\right)^{i\nu/2} \varphi(x)}$$

$z = \frac{1}{2} \ln\left(\frac{x}{1-x}\right)$

$\boxed{\alpha_1 = \alpha_2 = 0}$ $\quad$ NO $\quad$ $\boxed{\alpha_1 \neq \alpha_2 \neq 0}$

$\varphi(x)$

$$\underbrace{\left(\pi\frac{\alpha_1+\alpha_2}{2} + \nu \coth\frac{\pi\nu}{2}\right)}_{f(\nu)} \Phi_{\pm}(\nu) = \lambda \int_{-\infty}^{\infty} d\nu' \underbrace{\frac{\pi(\nu-\nu') + \frac{\alpha_1-\alpha_2}{2}\pi}{2\sinh\frac{\pi}{2}(\nu-\nu')}}_{\mathcal{K}(\nu,\nu')} \Phi_{\pm}(\nu') + f_{\pm} = \begin{cases} \pi\nu / 2\sinh\frac{\pi}{2}\nu \; \} \to \\ \frac{\pi}{2\cosh\frac{\pi\nu}{2}} \; \} \to \end{cases}$$



$\phi(\nu) = \sqrt{f(\nu)}\, \Phi(\nu)$

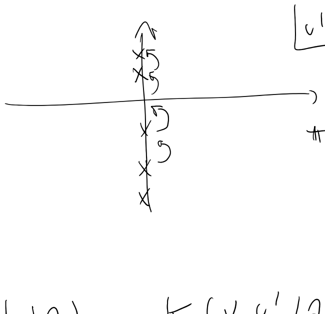
Fr... $\quad$ I kind

$\phi(\nu) = \lambda \int d\nu' \; K(\nu,\nu')\, \phi(\nu')$

$$\left(K(\nu,\nu') = \frac{\mathcal{K}(\nu,\nu')}{\sqrt{f(\nu)}\sqrt{f(\nu')}} \right)$$

$\lambda = 1/\mu^2 \qquad \alpha_1 - \alpha_2 = 0$

$\lfloor \nu'$



$\pi(\nu-\nu') \to \frac{\pi(\nu-\nu') + 2i}{2\sinh\frac{\pi}{2}(\nu-\nu'+2i)}$

Integrable kernels

$$\mathrm{Res}(\nu,\nu'|\lambda) = \lambda \int K(\nu,t|\lambda)\, R(\nu',t|\lambda) - K(\nu,\nu'|\lambda)$$

$$\mathrm{Res}(\nu,\nu'|\lambda) = f(\Phi_{\pm}) = \frac{\sinh(\;)}{\sinh(\;)\sinh(\;)} \left(\Phi_+(\nu)\,\Phi_-(\nu') - \Phi_+(\nu')\,\Phi_-(\nu)\right)$$

$$\boxed{Q(\nu) = \left(\frac{\alpha_1+\alpha_2}{2} + \nu\coth\frac{\pi\nu}{2}\right) 2\sinh\frac{\pi\nu}{2}\, \Phi(\nu)}$$

$$Q(v+2i) + Q(v-2i) - 2Q(v) = T(v)\,Q(v)$$

$$\boxed{T(v) = \frac{-4\pi\lambda}{\frac{\alpha_1+\alpha_2}{2} + v\coth\frac{\alpha v}{2}}}$$