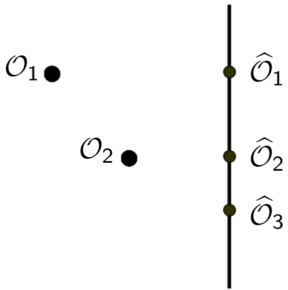
# Recent work on (supersymmetric) defects

Pedro Liendo

April 12 2022

*DESY Journal Club*

# The supersymmetric Wilson line



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Half-BPS defect in $\mathcal{N} = 4$ SYM

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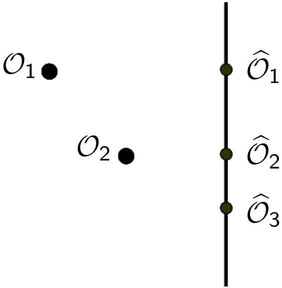

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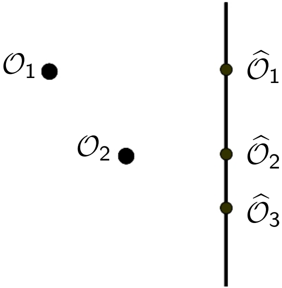


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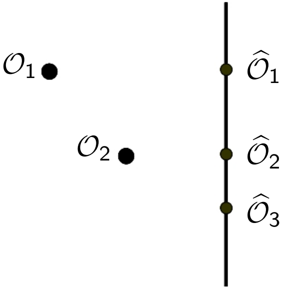


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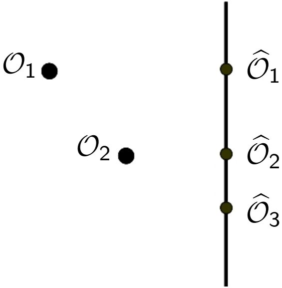


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*Perturbation theory, holography, localization, integrability, bootstrap*

# The main characters

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**Wilson loop**

$$\mathcal{W} = Pe^{\int d\tau (iA_\tau + \phi^6)}$$

$$\langle \mathcal{W} \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

**Chiral fields**

$$\mathcal{O}_p = \mathrm{Tr}\, \phi^{\{i_1} \dots \phi^{i_p\}}$$

$$\langle \mathcal{W} \mathcal{O}_{p_1} \dots \mathcal{O}_{p_n} \rangle$$

**Defect chiral fields**

$$\widehat{\mathcal{O}}_k = \mathrm{Tr}\, \widehat{\phi}^{\{a_1} \dots \widehat{\phi}^{a_k\}}$$

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- Bulk chiral field transform in the $SO(6)$ of R-symmetry.
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- Bulk chiral field transform in the $SO(6)$ of R-symmetry.
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Several combinations are possible

$$\langle \mathcal{W} \mathcal{O}_{p_1} \mathcal{O}_{p_2} \ldots \rangle \quad \langle \mathcal{W} \widehat{\mathcal{O}}_{k_1} \widehat{\mathcal{O}}_{k_2} \ldots \rangle \quad \langle \mathcal{W} \mathcal{O}_{p_1} \widehat{\mathcal{O}}_{k_1} \ldots \rangle$$

# The $1d$ defect CFT

# The displacement operator

The displacement operator (gauge theory)

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Setup: half-BPS correlators in $1d$ $OSP(4|4)$ SCFT.

# Superconformal kinematics

In $1d$ we have one cross-ratio

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = \chi^2 , \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-\chi)^2$$

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We also have R-symmetry indices

$$\begin{aligned} \langle \phi^a \phi^b \phi^c \phi^d \rangle &= \delta^{ab}\delta^{cd} G_S(\chi) \\ &+ (\delta^{ac}\delta^{bd} - \delta^{bc}\delta^{ad}) G_A(\chi) \\ &+ (\delta^{ac}\delta^{bd} + \delta^{bc}\delta^{ad} - \tfrac{5}{2}\delta^{ab}\delta^{cd}) \tfrac{1}{2} G_T(\chi) \end{aligned}$$

# Holographic calculation

The results is

Separating out the singlet ($S$), symmetric traceless ($T$) and antisymmetric ($A$) channels as in (4.3),(4.4)

$$G_{(1)}^{a_1a_2a_3a_4}(\chi) = \frac{1}{\sqrt{\lambda}}\Big[G_S^{(1)}(\chi)\,\delta^{a_1a_2}\delta^{a_3a_4} + G_T^{(1)}(\chi)\left(\delta^{a_1a_3}\delta^{a_2a_4} + \delta^{a_2a_3}\delta^{a_1a_4} - \tfrac{2}{5}\delta^{a_1a_2}\delta^{a_3a_4}\right) + G_A^{(1)}(\chi)\left(\delta^{a_1a_3}\delta^{a_2a_4} - \delta^{a_2a_3}\delta^{a_1a_4}\right)\Big]\,, \tag{4.18}$$

we find

$$G_S^{(1)}(\chi) = -\frac{2\left(\chi^4 - 4\chi^3 + 9\chi^2 - 10\chi + 5\right)}{5(\chi-1)^2} + \frac{\chi^2\left(2\chi^4 - 11\chi^3 + 21\chi^2 - 20\chi + 10\right)}{5(\chi-1)^3}\log|\chi| - \frac{2\chi^4 - 5\chi^3 - 5\chi + 10}{5\chi}\log|1-\chi|\,,$$

$$G_T^{(1)}(\chi) = -\frac{\chi^2\left(2\chi^2 - 3\chi + 3\right)}{2(\chi-1)^2} + \frac{\chi^4\left(\chi^2 - 3\chi + 3\right)}{(\chi-1)^3}\log|\chi| - \chi^3\log|1-\chi|\,, \tag{4.19}$$

$$G_A^{(1)}(\chi) = \frac{\chi\left(-2\chi^3 + 5\chi^2 - 3\chi + 2\right)}{2(\chi-1)^2} + \frac{\chi^3\left(\chi^3 - 4\chi^2 + 6\chi - 4\right)}{(\chi-1)^3}\log|\chi| - (\chi^3 - \chi^2 - 1)\log|1-\chi|$$

Here and in what follows $\log|\chi| \equiv \frac{1}{2}\log(\chi^2)$ and $\log|1-\chi| \equiv \frac{1}{2}\log\left[(1-\chi)^2\right]$ where $\chi \in (-\infty, \infty)$. Alternatively, we may assume that $\chi \in (0,1)$ (which, in particular, is sufficient for considerations of

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[Giombi, Roiban, Tseytlin (2017)]

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$$G_S(\chi) = -\frac{((\chi-5)\chi+5)\chi f'(\chi) + 5(\chi-2)f(\chi) + c\chi^3}{5\chi}$$

$$G_A(\chi) = -\frac{1}{2}\chi\left((\chi-2)f'(\chi) + c\chi\right) - f(\chi)$$

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Everything is fixed in terms of $c$ and $f(\chi)$, and $f(\chi)$ can be bootstrapped

[PL, Meneghelli, Mitev (2018)]

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Using the Quantum Spectral Curve [Grabner, Gromov, Julius (2020)]

$$\Delta_{\mathcal{K}} = 1 + \frac{\lambda}{4\pi^2} - \frac{\lambda^2}{16\pi^4} + \left(\frac{56}{45}\pi^4 + 128\right)\frac{\lambda^3}{4096\pi^6} + \dots$$

$$\Delta_{\mathcal{K}} = 2 - \frac{5}{\sqrt{\lambda}} + \frac{295}{24}\frac{1}{\lambda} - \frac{305}{16}\frac{1}{\sqrt{\lambda}^3} - 19.6254(10^{-7})\frac{1}{\lambda^2}$$

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$$\frac{351845}{13824} - \frac{75}{2}\zeta(3)$$

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- RG flows [Polchinski, Sully (2011)]

# Bulk excitations

# Ward identities

Recall

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Similar to the $c$ and $f(\chi)$ parameterization.

# Two-point function configuration

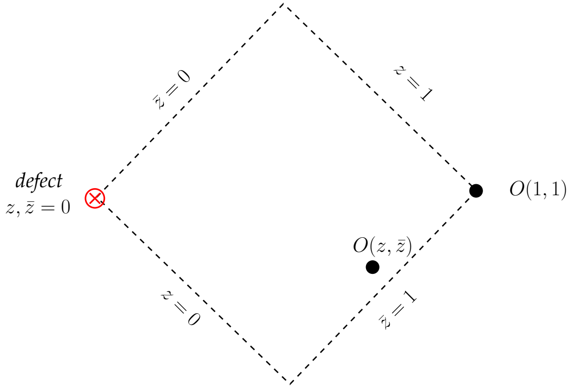
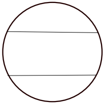
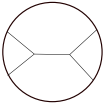



Figure: Configuration of the system in the plane orthogonal to the defect.

# Strong coupling



$\sim N^0$



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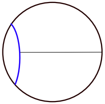

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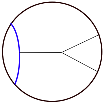

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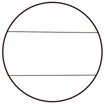

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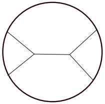

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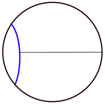

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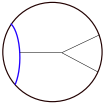

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The natural parameters are $\lambda/N^2$ and $1/\sqrt{\lambda}$:

$$\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle = \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(0)} + \frac{\lambda}{N^2}\left( \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(1)} + \frac{1}{\sqrt{\lambda}} \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(2)} \right) \ldots$$

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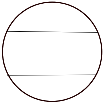

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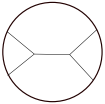

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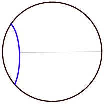

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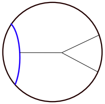

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For the holographic setup see [Giombi, Pestun (2012)].

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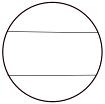

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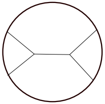

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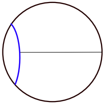

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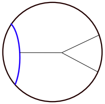

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$$\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle = \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(0)} + \frac{\lambda}{N^2}\left(\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(1)} + \frac{1}{\sqrt{\lambda}}\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(2)}\right)\ldots$$

For the holographic setup see [Giombi, Pestun (2012)].

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At strong coupling only one bulk block contributes to Disc

# The leading order result

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After resummation

$$F_0(z,\bar{z}) = -\frac{\sqrt{\lambda}}{N^2}\frac{2z\bar{z}}{(1-z)(1-\bar{z})}\left[\frac{1+z\bar{z}}{(1-z\bar{z})^2}+\frac{2z\bar{z}\log z\bar{z}}{(1-z\bar{z})^3}\right]$$

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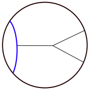
Recall

 $\sim \frac{\lambda^{1/2}}{N^2}$

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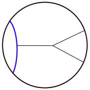

$$\sim \frac{\lambda^{1/2}}{N^2}$$

There is also a protected point

$$\mathcal{F}(z,\bar{z},w)|_{z=\bar{z}=w} = h(\lambda, N)$$

See [Beccaria, Tseytlin (2020)]

# More results, future directions

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Also for 't Hoof lines, boundaries, interfaces, surface defects.

# More general theories

# Defects in $(2,0)$ theories

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- Calculate superconformal blocks
- Obtain Disc from a single block

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Can be compared with Montecarlo and experiment

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In SUSY theories we have

$$D^2\Phi = \Phi^2$$

Thank you!