Three-Point Functions of Short Operators

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Introduction

- Due to AdS/CFT, there has been a huge development in the study of strongly coupled conformal field theories. The best understood example is $\mathcal{N}=4$ SYM, which is dual to type IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$.
- Using the TBA, Y-system, or the more recent Quantum Spectral Curve, dimensions of operators can in principle be computed at any value of the coupling, at least numerically.
- The obvious next step is to investigate the structure constants of the theory, for which there is some limited knowledge at both weak and strong coupling.
- In this work, we focus on the strong coupling limit of three-point functions of operators dual to *short strings* (small compared to the curvature scale).

The Operators

- We consider operators with dimensions $\sqrt{\lambda} \gg \Delta \gg 1$, such that the dual string states can be treated as semiclassical point particles. (String length $\sim \Delta\alpha' = \Delta/\sqrt{\lambda}$ by AdS/CFT.)
- We specialize to scalar operators dual to massless and first massive level string states. In the conformal field theory, these correspond to:
  - Chiral primaries with no anomalous dimension and R-charge J which equals their bare dimension:
  $$\mathcal{O}_{\mathrm{CP}} = C_{I_1 \dots I_J} \operatorname{Tr}[\Phi^{I_1} \dots \Phi^{I_J}] .$$
  - Operators with R-charge J and dimension $\Delta \sim \lambda^{1/4}$ at strong coupling, e.g. $\mathcal{O}_J = \operatorname{Tr}[\Phi^I \Phi_I Z^J] + \dots$

  $\mathcal{O}_0$ is the Konishi operator.

Semiclassical Limit

- Given the boundary to bulk propagator
$$K = \frac{\Gamma(\Delta)}{\pi^2 \Gamma(\Delta-2)} \left(\frac{z_0}{z_0^2 + (\vec{z}-\vec{x})^2} \right)^{\Delta} ,$$
the three-point function of scalar operators has been computed, up to the supergravity couplings.
- To obtain these couplings, we consider the path integral for the propagation in AdS of particles dual do scalars with dimensions Δ_i.
- Taking a semiclassical limit (saddle point) we notice two important facts:
  1. All three propagators as well as the intersection point are completely localized, and the propagator's canonical momenta are conserved at the intersection point.
  2. The size of the overlaps of trajectories in AdS is much smaller than the radius of AdS for operators with dimensions $\sqrt{\lambda} \gg \Delta_i \gg 1$.

Flat Space Approximation

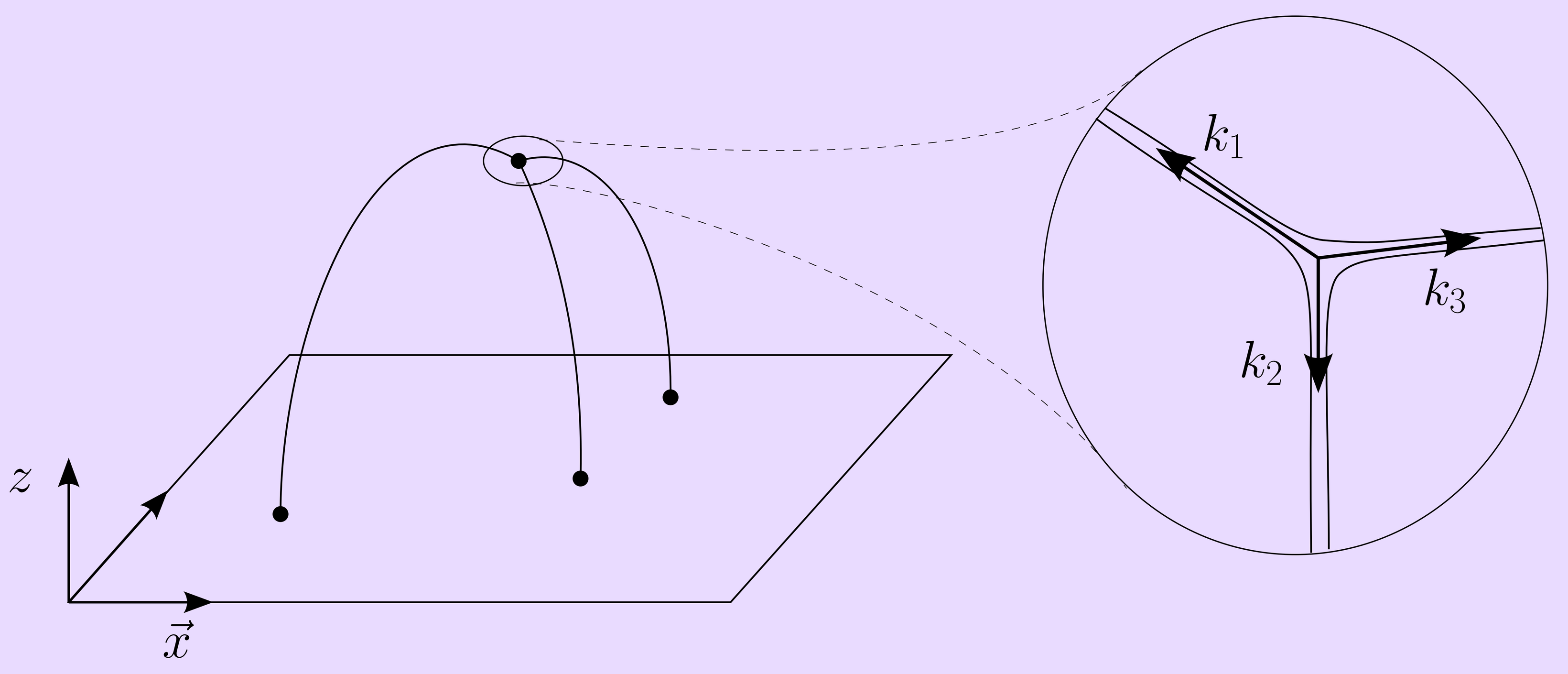

Figure 1: Conservation of momenta at the intersection point.

- Due to the small overlap for operators with dimensions $\sqrt{\lambda} \gg \Delta_i \gg 1$, one can ignore curvature and RR-flux effects at the intersection point.
- It is thus possible to use standard flat-space techniques, so that obtaining the coupling reduces to the simpler problem of computing string amplitudes in flat space using RNS vertex operators.

Vertex Operators

- One constructs a physical vertex operator by imposing $[Q_{\mathrm{BRST}}, V_{\mathrm{phys}}] = 0$. The momentum of the vertex operator is given by the canonical momentum at the intersection point. For the n'th mass level, it obeys
$$k^2 \approx -\Delta^2 + J^2 = \frac{4}{\alpha'} n .$$
- At the massless level the NS and R sector left movers are:
$$g_c\, \varepsilon_M\, \psi^M e^{-\phi} e^{ik\cdot X} , \qquad g_c\, t_A\, \Theta^A e^{-\frac{1}{2}\phi} e^{ik\cdot X} .$$
- For the first massive level there are two NS vertices
$$g_c\, \alpha_{MNP}\, \psi^M \psi^N \psi^P e^{-\phi} e^{ik\cdot X} , \quad g_c\, \varepsilon_{MN}\, \psi^M \partial X^N e^{-\phi} e^{ik\cdot X} ,$$
and one in the R sector
$$g_c\, t_{A,M} \left(i\partial X^M \Theta - \tfrac{1}{8} \psi^M (\not{k} \not{\psi} \Theta) \right)^A e^{-\frac{1}{2}\phi} e^{ik\cdot X} .$$

CFT Operators in the String Formalism

- Taking the flat-space limit, the $PSU(2,2|4)$ algebra reduces to a ten dimensional super-Poincaré. The condition for primary operator $S^a_\alpha \mathcal{O}(0) = 0$ then becomes an algebraic condition on the vertex operators:
$$Q^{\mathrm{L}}_{\alpha\tilde{a}} V = i\, Q^{\mathrm{R}}_{\alpha\tilde{a}} V , \qquad Q^{\mathrm{L}\tilde{a}}_{\dot{\alpha}} V = -i\, Q^{\mathrm{R}\tilde{a}}_{\dot{\alpha}} V ,$$
where $(\alpha, \dot{\alpha})$ defines the space orthogonal to the momentum in AdS.
- The equation implies that primary operators are linear combinations of NS-NS and R-R vertices.
- For the massless and first massive level, the solution is unique.
- The dilaton is given by the massless NS-NS scalar vertex operator.

Results

- We test our method by computing the three-point function for three chiral primaries, obtaining total agreement with previous supergravity calculation.
- For two chiral primaries with R-charge J and a Konishi-like operator we get
$$\mathcal{C}_{\mathrm{CCK}} \approx \frac{\pi^{3/2}}{2\sqrt{2}\, N\, \Gamma(J)\Gamma(J-1)} \lambda^{J/2-1/4}\, 2^{-2\lambda^{1/4}} .$$
- We can also compute the structure constant of one chiral primary and two Konishi-like operators with R-charges $J_i \ll \lambda^{1/4}$, thus obtaining
$$\mathcal{C}_{\mathrm{CKK}} \approx \frac{\pi^{1/2}}{N} \lambda^{1/4}\, 2^{-J} .$$
- For three Konishi-like operators:
Horrific combinatorics, but huge simplification:
$$\langle VVV \rangle = g_c^3 \frac{3^8}{2^9} , \qquad \mathcal{C}_{\mathrm{KKK}} \approx \frac{64\pi^{1/2}}{N} \lambda^{1/4} \left(\frac{3}{4} \right)^{3\lambda^{1/4}+5/2} .$$
- In all cases, the structure constant is exponentially suppressed in λ (or J).

Conclusion

- We successfully computed structure constants for scalar operators dual to short string states. Our method works for massless states with R-charge $J \gg 1$, and for any operator at the first massive level.
- The procedure is in principle applicable to any CFT with a string dual, up to the construction of the right vertex operators.
- Big simplifications suggest that integrability plays a role.
- Method perhaps can help lead us to exact vertex operators in $\mathrm{AdS}_5 \times \mathrm{S}^5$.
- For future work it would be interesting to compute three-point functions of operators with spin, and to investigate four-point functions, as well as subleading corrections in the α' expansion.

References

[1] T. Bargheer, J. A. Minahan and R. Pereira, *"Computing Three-Point Functions for Short Operators"*, JHEP 1403, 096 (2014), arxiv:1311.7461.

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